

BELIEFS: STATE UNCERTAINTY

AA228/CS238 DECISION MAKING UNDER UNCERTAINTY¹

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¹Mykel J. Kochenderfer, Tim A. Wheeler, and Kyle H. Wray. *Algorithms for Decision Making*. MIT Press, 2020.

INTRODUCTION: BELIEFS

- A POMDP² is an MDP with *state uncertainty*
- The agent receives an *observation* of the current state rather than the true state (potentially imperfect observations)
- Using past observations, the agent builds a *belief* of their underlying state
 - Which can be represented by a probability distribution over true states

²Partially observable Markov decision process. “Partially observable” is key in understanding beliefs.

BELIEF REPRESENTATION

Beliefs can be represented in different ways:

- **Parametric:** The belief distribution is represented by a set of parameters for a fixed distribution family
 - E.g., Categorical distribution³ or multivariate normal (Gaussian) distribution
- **Non-parametric:** The belief distribution is represented by particles (or points sampled from the state space)

Depending on the representation, different algorithms can be used to update beliefs.

³A probability mass is assigned to each discrete category.

ALGORITHMS FOR UPDATING BELIEFS

Various algorithms can update the current belief:

- If the state space is *discrete* (or certain linear Gaussian assumptions are met), then we can perform *exact belief updates*:⁴
 - Recursive Bayesian estimation
 - Kalman filter
- Otherwise, we can use approximations based on linearization or sampling:
 - Extended Kalman filter
 - Unscented Kalman filter
 - Particle filter
 - Particle filter with rejection
 - Injection particle filter
 - Adaptive injection particle filter

⁴Meaning we arrive at an analytical solution without approximations.

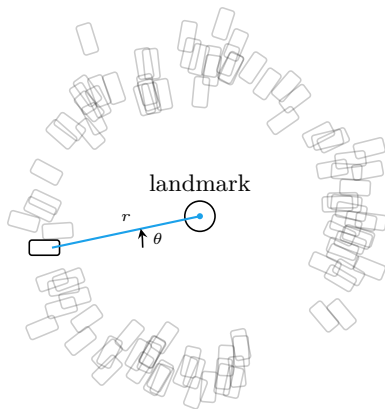
BELIEF INITIALIZATION

Before any actions or observations, we start with an initial belief distribution

- We can encode prior knowledge in the initial distribution
- Generally want to use diffuse (i.e. spread out) initial distributions to avoid over confidence in the absence of information
 - In non-parametric representations, a diffuse initial prior may cause difficulties
 - Thus, we may wait until an informative observation is made to initialize our beliefs

EXAMPLE: LANDMARK BELIEF INITIALIZATION

Figure: Localization of an autonomous car using a landmark (Example 19.1).



Making a range r and bearing θ observation, we initialize our belief around the landmark.

OBSERVATION SPACE

- The agent receives an observation o , which belongs to some *observation space* \mathcal{O}
- The probability of observing o given action a and next state s' is: $O(o \mid a, s')$
 - If \mathcal{O} is continuous, then $O(o \mid a, s')$ is a probability density

DYNAMIC DECISION NETWORK FOR POMDPs

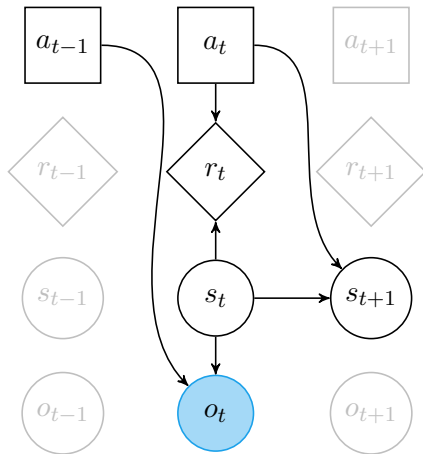


Figure: A dynamic decision network for the POMDP problem formulation.

BELIEF INFERENCE

- To infer the unknown belief distribution, we use *recursive Bayesian estimation*
 - Updates belief estimate recursively over time
 - Markov assumption: Only requires the current state, action, and observation
- Let $b(s)$ represent the probability⁵ assigned to state s
 - A particular belief b belongs to a *belief space* \mathcal{B} (which contains all possible beliefs)
- For finite state and observation spaces, we can use a *discrete state filter* to perform exact inference

⁵or probability density for continuous state spaces

BELIEF VECTOR

- In the finite state case, we can represent beliefs using a categorical distribution⁶
 - Represented as a *belief vector* \mathbf{b} of length $|\mathcal{S}|$, therefore $\mathcal{B} \subset \mathbb{R}^{|\mathcal{S}|}$
 - Sometimes \mathcal{B} is referred to as a *probability simplex* or *belief simplex*⁷
- The belief vector \mathbf{b} must be strictly non-negative and sum to one:

$$b(s) \geq 0 \text{ for all } s \in \mathcal{S} \quad \sum_s b(s) = 1$$

- In vector notation:

$$\mathbf{b} \geq \mathbf{0} \quad \mathbf{1}^\top \mathbf{b} = 1$$

- In Julia syntax:

`all(b . \geq 0) && sum(b) \approx 1`

⁶A probability mass is assigned to each discrete state.

⁷Simplex being the generalization of a triangle to arbitrary dimensions.

DISCRETE STATE FILTER

A *filter* is a process that remove noise from data.⁸

Due to the independence assumptions, if an agent with belief b takes an action a and receives an observation o , then the new belief b' becomes:

$$\begin{aligned} b'(s') &= P(s' \mid b, a, o) \\ &\propto P(o \mid b, a, s')P(s' \mid b, a) && \text{(Bayes' rule)} \\ &\propto O(o \mid a, s')P(s' \mid b, a) && \text{(observation definition)} \\ &\propto O(o \mid a, s') \sum_s P(s' \mid b, a, s)P(s \mid b, a) && \text{(law of total probability)} \\ &\propto O(o \mid a, s') \sum_s T(s' \mid s, a)b(s) && \text{(state transition model)} \end{aligned}$$

⁸Often used in signal processing, effectively “filtering” out the noise.

EXACT BELIEF STATE PLANNING

REFERENCES

Kochenderfer, Mykel J., Tim A. Wheeler, and Kyle H. Wray. *Algorithms for Decision Making*. MIT Press, 2020.