33-777

Jan 21, 2020

Today

- Telescopes
- Surface Brightness (a short note)

In this lecture, we will focus on the primary tool of observational astronomers, the telescope.

For this discussion, I have relied on two sources

- "An Introduction to Modern Astrophysics" by B.W. Carroll and D.A. Ostlie
- "Astrophysical Techniques" by C.R. Kitchin

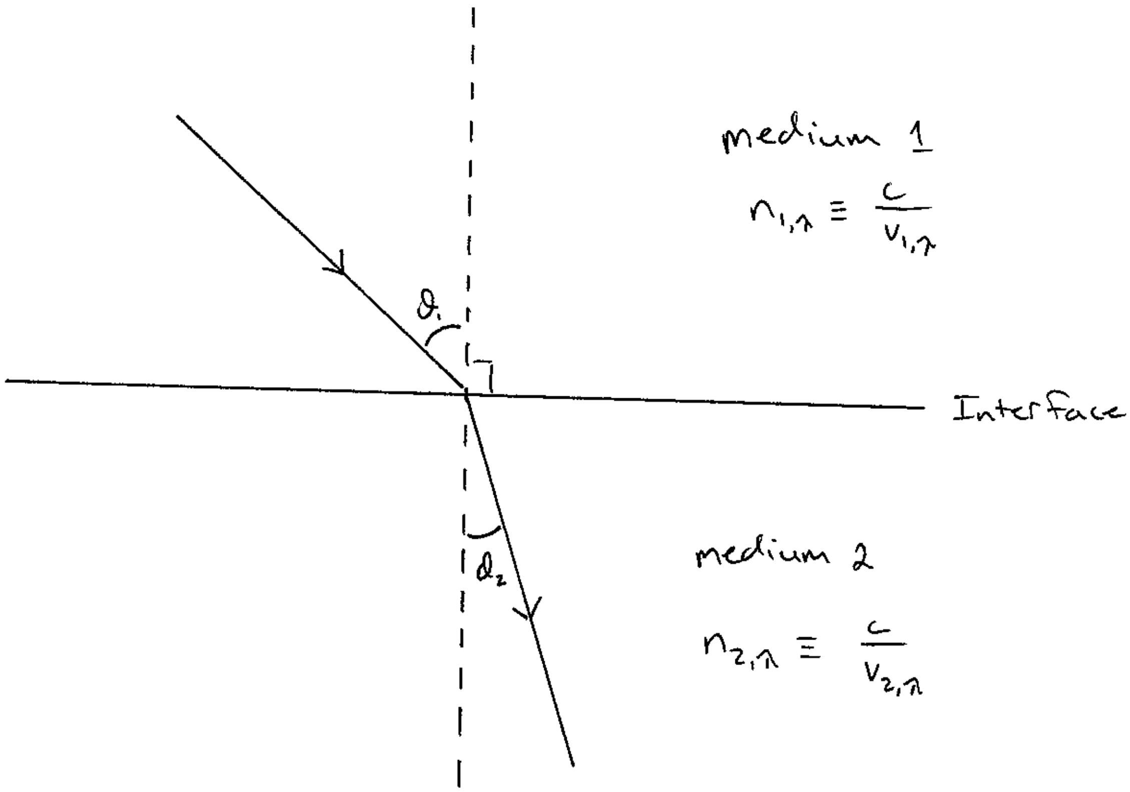
- Telescopes

In this course, we will limit our discussion of telescope to optical telescopes. This is not meant to diminish the importance of other types of telescopes. In fact throughout this course we will utilize observations from radio, microwave, and high energy observatories.

There are two catagories of optical telescopes

- Reflecting
- Refracting

Refracting telescopes are primarily of historical importance; nevertheless, it is important to understand their design and limitations. Refractors use optical lenses to bring parallel rays of light to a common focus. Given this, all refractor designs are based on the manipulation of lens shapes to exploit Snell's Law.



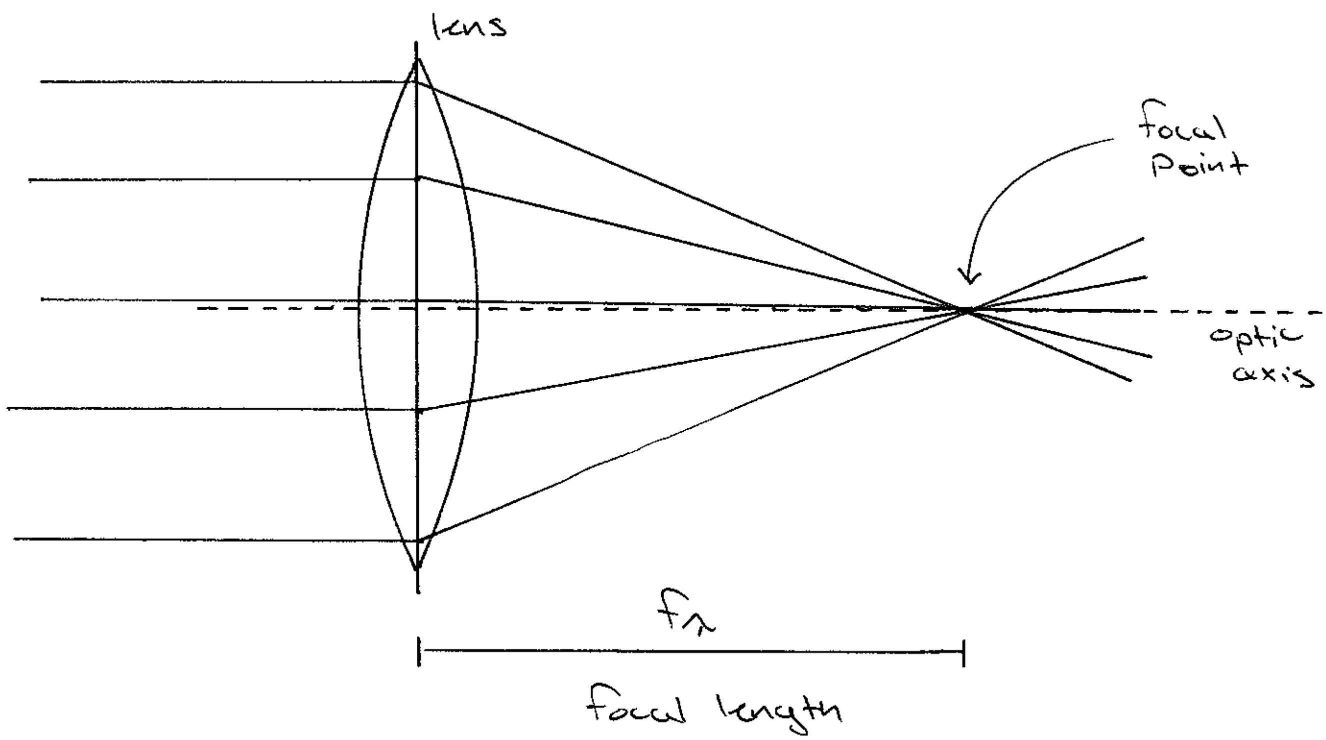
Here θ_1, θ_2 is the angle a ray makes with respect to a line perpendicular to the interface between two different materials.

The index of refraction of a medium is the ratio of the speed of light in a vacuum to the speed of light in the medium, v_λ. The subscript λ makes it explicit that this speed depends on the wavelength / frequency, $n_\lambda \equiv \frac{c}{v_\lambda}$.

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

In the most simple refracting telescope, the <u>objective</u> lens is made with a curvature necessary to produce the following geometry of refractions as light rays pass between air, the glass of the lens, and back.



- note that the focal length depends on λ given that the index of refraction of the lens is a function of λ

- the focal plane is defined to be the plane perpendicular to the optic axis that contains the focal point

- note that we will often make use of the thin lens approximation wich holds when the radii of curvature of the lens surfaces is << than the thickness of the lens.

Recall that the focal length of a lens in air is given by the lens makers equation

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

n : index of refraction

R_1 : radius of curvature of outer surface

R_2 : radius of curvature of inner surface

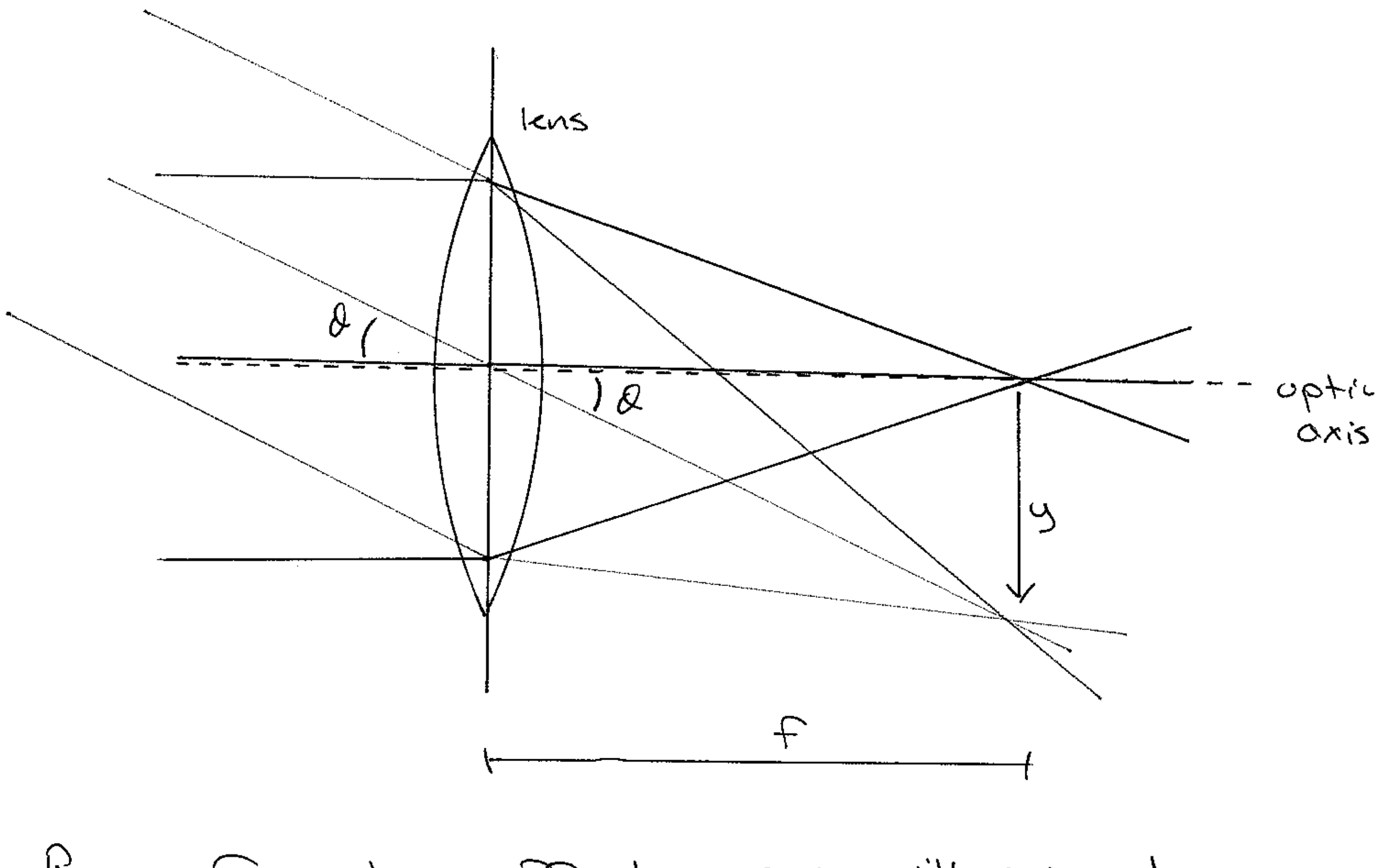
d : thickness of lens at its thickest point

In the thin lens limit

$$\frac{1}{f} \simeq (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Back to our simple refracting telescope set-up. Here, all light rays that enter parallel to the optic axis will converge at the focal point.

Now, consider observing two different sources seperated by an angle θ on the sky.



Rays from the offset source will come to an approximate focus a physical distance, y, from the focus of the central source, as measured on the focal plane.

By applying some basic geometry,

$$\tan(\theta) = \frac{y}{f} \simeq \theta$$

This lets us define the <u>plate scale</u>,

$$\boxed{\frac{d\theta}{dy} = \frac{1}{f}}$$

The plate scale lets an observer translate angular seperation on the sky, $d\theta$, to physical seperation on the focal plane, dy.

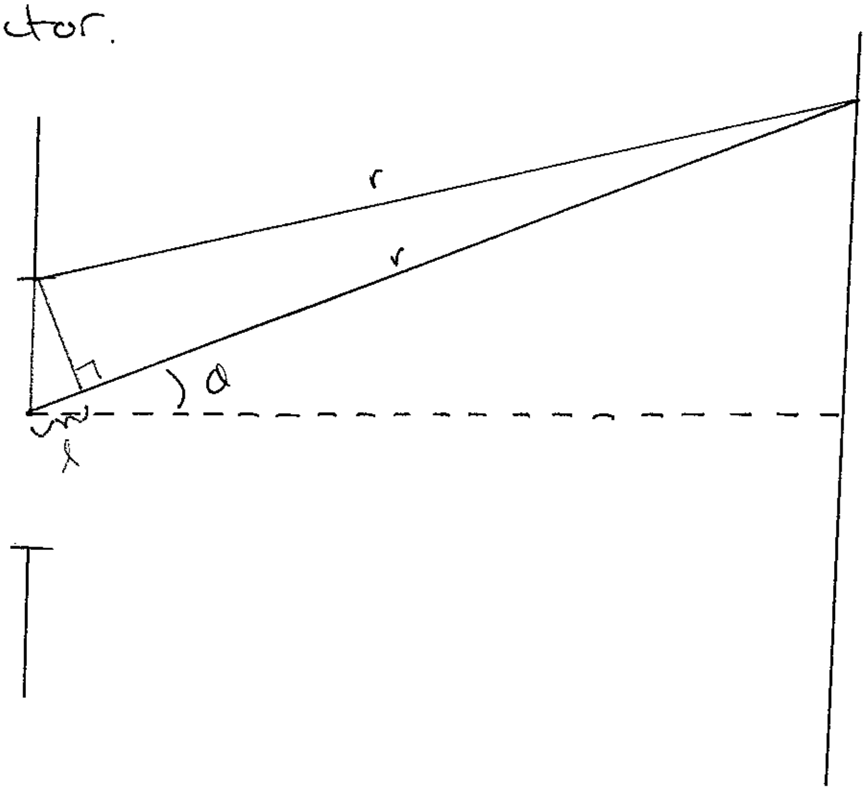
This brings us to the topic of <u>resolution</u>. For many observations in astronomy, it is critical to distinguish two (or more) point sources with small angular seperation or small (in angular size) extended features.

Modern imaging is done using <u>CCDs</u>. A CCD in the most basic description is a photon detector composed of light sensitive areas (pixels) which convert incident photons into one or more electrons. A digital image is constructed when a CCD is read out, and the number of electrons in each pixel is counted.

For applications in astronomical imaging CCDs are generally composed of square pixels that touch eachother (no gaps). Typical pixel sizes are $\sim 10\,\mu m$. A single CCD may be 1024×1024 pixels.

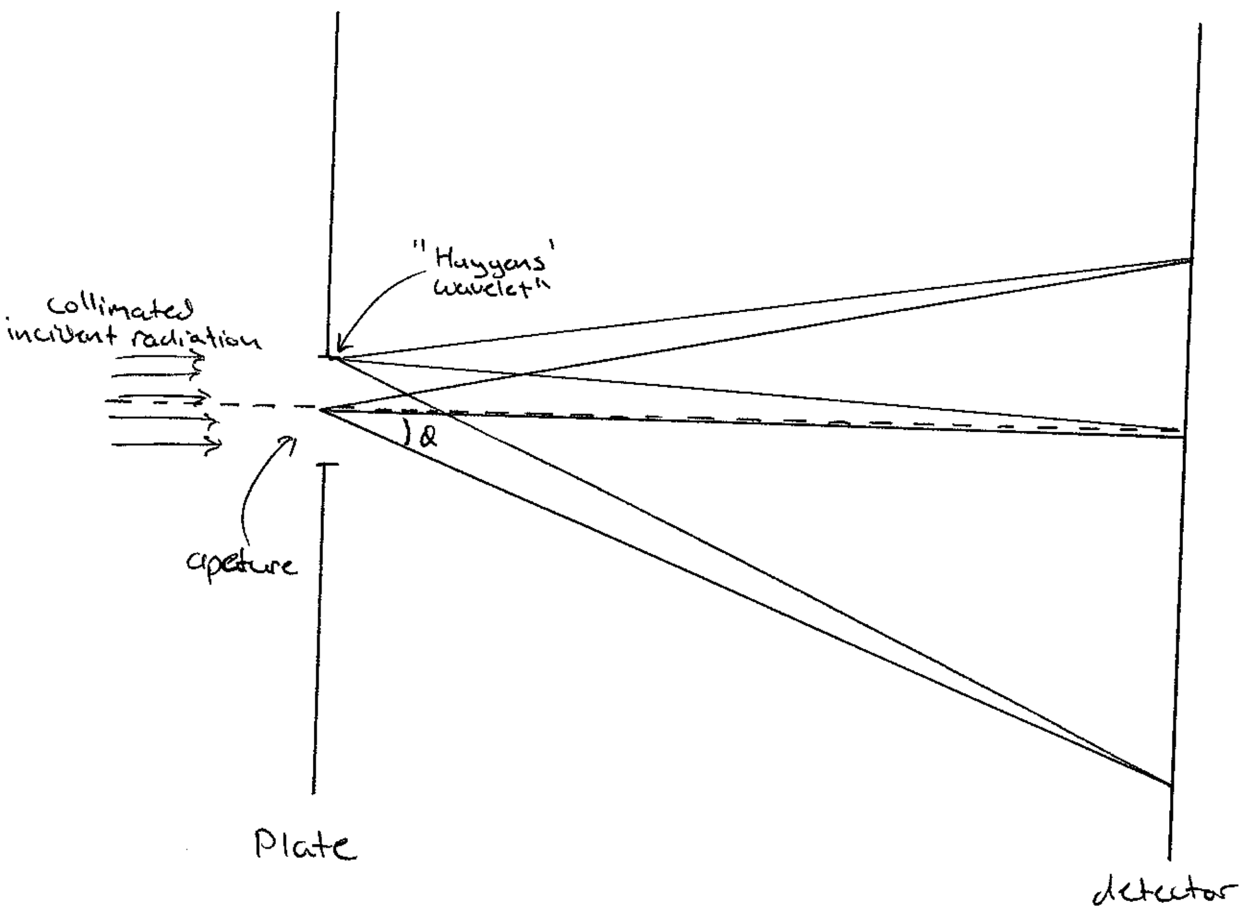
As a result, a minimum requirement to resolve two point sources is that their seperation on the focal plane must be $\gtrsim$ the pixel size. In principle, this could be accomplished for arbitrarily small $d\alpha$. However, in practice resolution is fundamentally limited by diffraction.

The locations of the bright and dark patches can be easily understood. Consider a ray of light originating from the top of the slit, and one originating from the midpoint. The ray originating from the midpoint must travel a distance, l, further before hitting the detector.

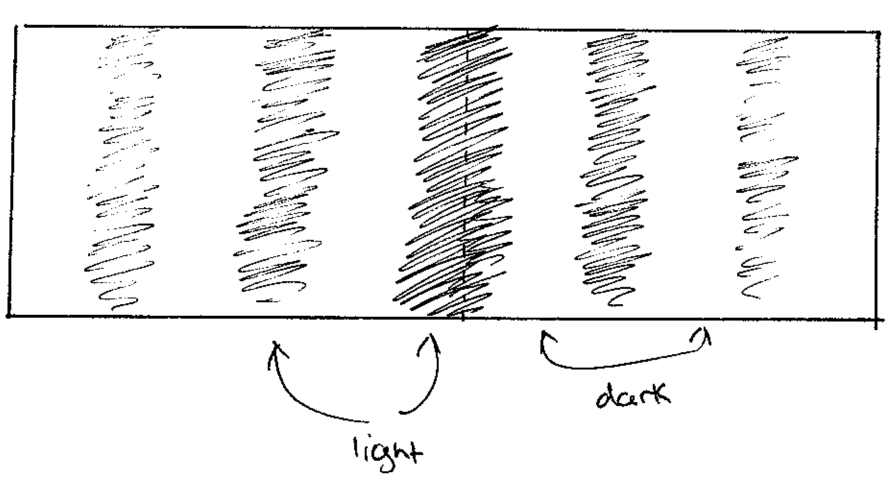


If l is equal to $\frac{1}{2}\lambda$ of the incident light, then the two rays will destructively interfere. The interference pattern seen on the detector is the result of constructive and destructive interference from rays all along the slit.

Recall that when light passes through a simple 1-Dimensional apeture (i.e. slit), the wave nature of light becomes important to consider.



Detected Photon Pattern



In general the diffraction pattern that results from a given apeture is the power spectrum of the fourier transform of its shape, For a simple slit this is:

$$I_\alpha = I_0 \frac{\sin^2(\pi d \sin\alpha/\lambda)}{(\pi d \sin\alpha/\lambda)^2}$$

α: angle to the normal from the slit
d: width of slit
I_0: central intensity
I_α: intensity at angle, α.

The dark patches then occur at

$$\sin\alpha = m\frac{\lambda}{d}, \quad m = 1, 2, 3, \ldots$$

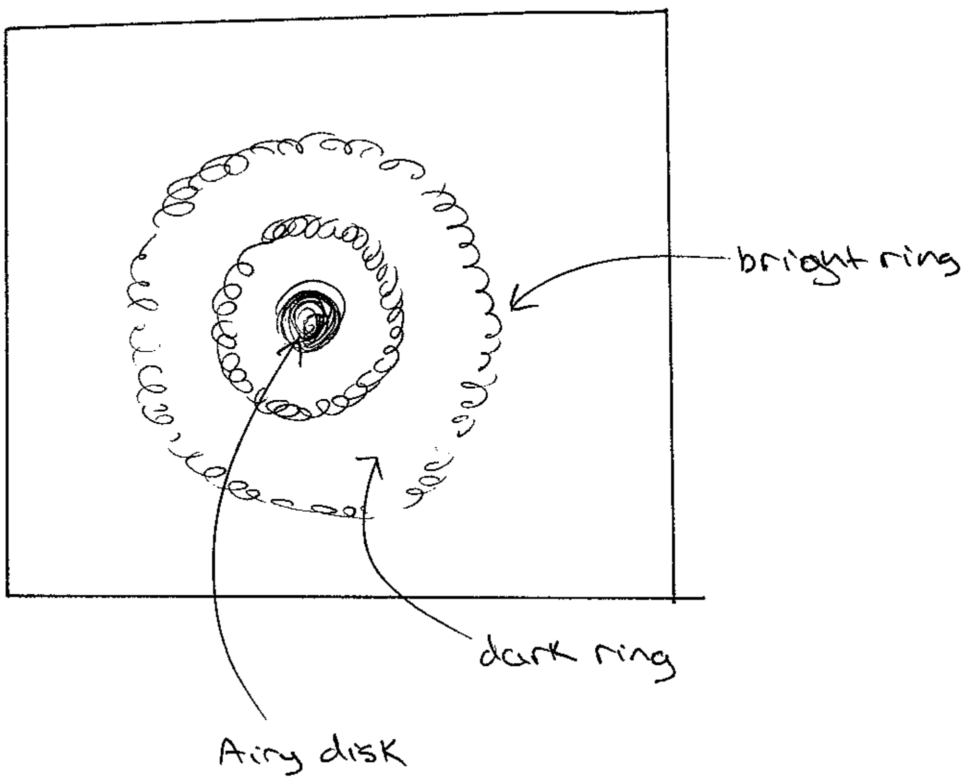
The most relavent apeture for Astronomy is the circular apeture. The calculation for a circular apeture is a litte more complex (but still analytical).

The result for a circular apeture is

$$I(\alpha) = I_0 \left(\frac{2 J_1(m)}{m} \right)^2$$

$$m \equiv \frac{\pi r \sin\alpha}{\lambda}$$

r : radius of the apeture



The dark fringes occur at,

$$\sin(\alpha) = \frac{1.220\lambda}{D}, \frac{2.233\lambda}{D}, \frac{3.238\lambda}{D}, \ldots$$

where D is the diameter of the apeture.

A simple "rule of thumb" for a resolution criteria is the Rayleigh Criteria. Two sources can be considered resolved if their central maxima are outside eachothers first minimum.

$$\Rightarrow \boxed{\Delta\theta_{min} \simeq 1.22 \frac{\lambda}{D}}$$

$D \equiv$ diameter of telescope
$\lambda \equiv$ wavelength of light observed.

This indicates that larger telescopes are capable of resolving finer angular resolutions.

Example

Consider a 0.5 m telescope ($D = 0.5$ m), observing at $\lambda = 500$ nm.

$$\Delta\theta_{min} = \frac{1.22\,(500 \times 10^{-9}\ m)}{0.5\ m} = 1.22 \times 10^{-6} \text{ radians}$$

$$= 0.79''$$

However, for ground based observatories (as opposed to space) in the optical are generally limited to resolutions of $\sim 1''$. This is a result of atmospheric turbulence. This bluring effect due to the atmosphere is called "<u>seeing</u>". On a good night, at a good location, $0.5''$ is considered spectacular seeing.

This limit is only overcome by moving your telescope to space, or through the use of techniques meant to correct for atmospheric turbulence. These techniques generally fall under the term <u>adaptive optics</u>.

In the optical, the primary reason to build larger telescopes is to increase the <u>illumination</u>. The illumination quantifies the amount of energy per unit time a telescope brings to focus per unit area of the detector. The illumination,

$$J \propto \frac{D^2}{f^2} = \frac{1}{F^2}$$

where this is often quantified as the focal ratio,

$$F = \frac{f}{D}$$

f: effective focal length of system
D: diameter of primary apeture.

Large optical telescopes are primarily <u>light buckets</u>.

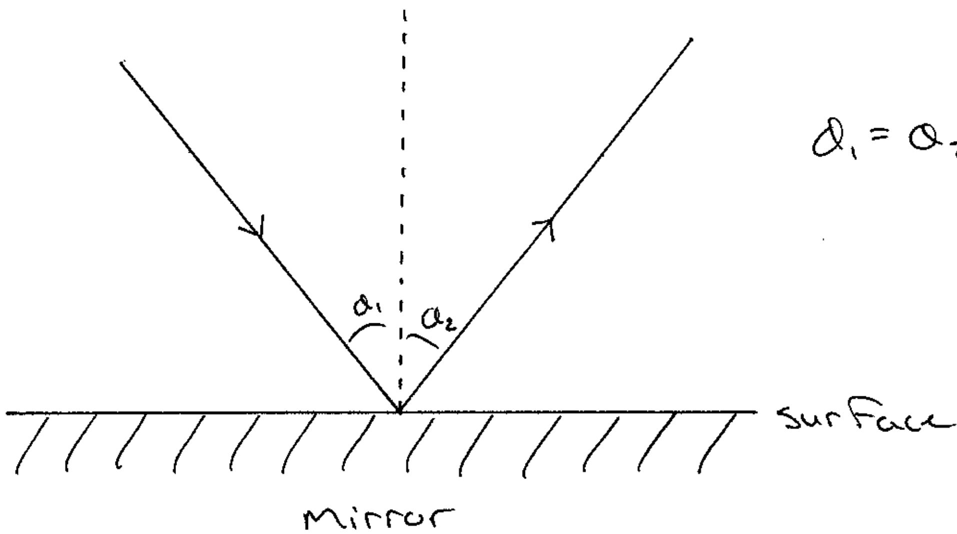
Refracting telescopes are now primarily of historic interest. Refractors suffer from two serious limitations:

1. chromatic aberation
2. heavy + hard to support primary lens.

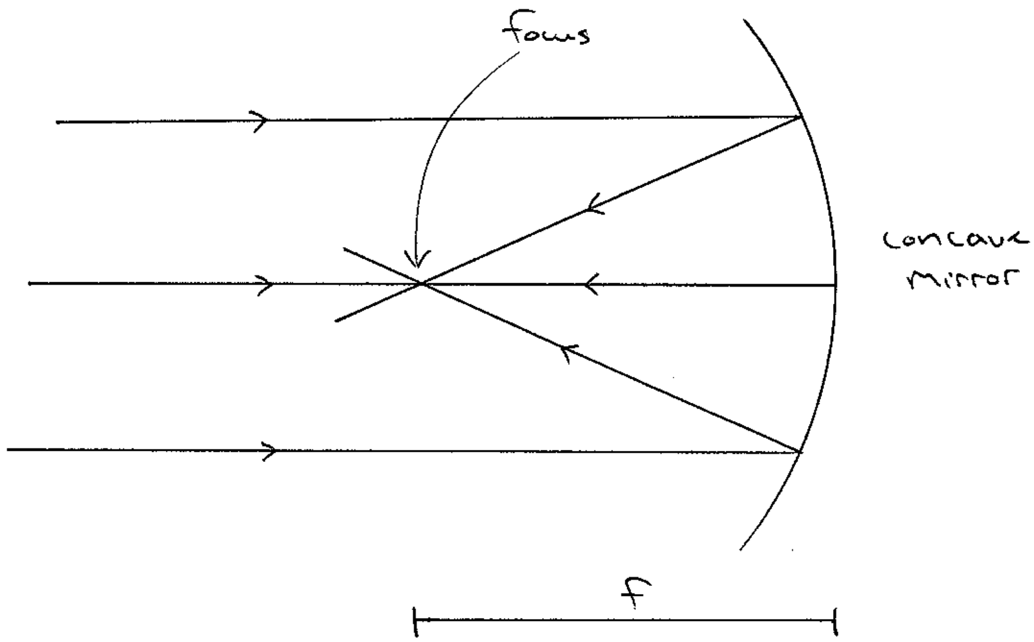
- chromatic aberation results from light of different wavelengths being brought to different focii. This is because the index of refraction of the lens material is wavelength dependent. So called "achromatic" refractors can be built by combing lenses of different materials. Furthermore, it is still only possible to bring (at maxium) light of a few different λ to a common focus.

- Because light passes through a lens, lenses can only be supported by their edges. This eventually places a mechanical limit on the upper limit for lens size.

The most common type of research grade telescope is the reflecting telescope.
This type of telescope uses mirrors instead of lenses. Instead of manipulating Snell's Law, reflecting telescopes make use of the law of reflection.



A very nice feature of mirrors is that their focal length is wavelength idependent.
Similar to lenses, mirrors can also be made to be concave or convex.



In addition to avoid chromatic aberation, mirrors can be supported from behind. of further practical importance is that only the surface of the mirror needs to be of optical quality.

However, because mirrors bring light to focus back in the direction from which it came, careful designs must be used to minimize the amount of light is blocked while bringing light rays to a usable focus.

Some of the most important designs used for research grade telescopes include:

1. prime focus
2. Cassegrain
3. Coudé

There are many variations on these designs.

While reflecting telescopes do not suffer from chromatic aberation, they do suffer from a host of other aberations (along with refractors). These include,

- spherical aberation
- Coma
- astigmatism
- distortion
- vignetting
- field curvature

- Surface Brightness

The surface brightness, I, is the energy in photons received by an observer on a detector per unit area, per unit time, per unit solid angle in a given direction.

It is common to give I in units of $erg \cdot s^{-1} \cdot cm^{-2} \cdot sr^{-1}$.
steradians

It is also common to use more astrophysical units, $L_\odot / pc^2$.

A more "observer oriented" units for surface brightness is $mag/arcsec^2$. In this case surface brightness is denoted by μ. The two are related by

$$\mu = -2.5 \log(I) + c$$

The constant C depends on the units of I. We can work it out for $L_\odot / pc^2$. First, calculate the total luminosity of a patch, a

$$\delta\Omega = \delta\theta^2$$

The physical area is given by

$$dA = d^2\,\delta\Omega$$

The luminosity of that patch is given by

$$dL = I\,dA$$

The luminosity of a square arcsecond patch is

$$L = I d^2 \delta\theta^2 \quad \text{where} \quad \delta\theta = 1'' = \frac{1}{206265} \text{ radians}$$

The distance modulus is given by

$$m - M = 5\log\left(\frac{d}{pc}\right) - 5$$

$$\Rightarrow m = M + 5\log\left(\frac{d}{pc}\right) - 5$$

The absolute magnitude of the patch is given by

$$M = -2.5\log\left(\frac{L}{L_\odot}\right) + M_\odot$$

If we plug in the luminosity of the small patch

$$\mu = m = -2.5\log(I d^2 \delta\Omega^2) + M_\odot + 5\log(d) - 5$$

$$\Rightarrow \mu = -2.5\log(I) - 5\log(\delta\Omega) - 5 + M_\odot$$

$$= -2.5\log(I) + 21.572 + M_\odot$$

note: I must be in units of $L_\odot/pc^2$.