

1) How many ways to choose 2 students from AI Department, that has a population of 250 students?

$$C(250, 2) = 250! / (2!(250-2)!) = 31125$$

2) How many distinct bit strings can be formed from three 0's and two 1's?

$$P(3+2, 2) = 5! / (2!(5-2)!) = 10$$

3) How many bit strings of length 5, start and end with 1's?

$$2 \times 2 \times 2 = 8$$

4) If three awards are given each year to football team members. If there are 30 players this year in this team, and each one of them can receive at most only one award. How many possible ways are there?

$$C(30, 3) = 30! / (3!(30-3)!) = 4060$$

5) How many ways to select 3 books from 6 Solution If each of the books are distinct?

$$C(6, 3) = 6! / (3!(6-3)!) = 20$$

6) When you are rolling a pair of (fair) dice three times. What is the probability that, least one of the three tries, you roll a 7?

$$P(\text{at least one } 7) = 1 - P(\text{no } 7\text{'s in three rolls})$$

$$1 - (30/36)^3 =$$

$$1 - 0.5787 = 0.4213$$

$$= 42.13\%$$

7) How many ways to select 3 books from 6 Solution If there are 2 books that should not both be chosen together?

The number of ways to select 3 books from 6 is given by the combination formula:

$$|A| \quad C(6, 3) = 20$$

The number of ways to select 3 books from the 4 that do not include the two specific books is:

$$|B| \quad C(4, 3) = 4$$

we need to subtract this number from our count: $|A \cap B| = 4$

$$|A \setminus B| = |A| - |B| + |A \cap B| = 20 - 4 + 4 = 20$$

8) In a Discrete Mathematics class, that contains 25 students, among whom are two students with name "Ahmed", two students with name "Sally", two students with name "Marla", two students with name "Yousef", and two students with name "Ibrahim" (along with the reminder twenty two students whose names are all distinct from those five names and from one another's names).

- (a) How many different 14-student study-groups can be formed such that within the study group, there is exactly one student with name "Ahmed", exactly one student with name "Sally", exactly one student with name "Marla", exactly one student with name "Yousef", and exactly one student with name "Ibrahim"?

First, we need to choose one student with each of the five given names. We have 2 students with each of the five names, so the number of ways to do this is:

$$C(2, 1) \times C(2, 1) \times C(2, 1) \times C(2, 1) \times C(2, 1) = 2^5 = 32$$

Next, we need to choose the remaining 9 students from the 20 students whose names are all distinct from those five names. We have 20 choices for the first remaining student, 19 choices for the second remaining student, and so on, down to 12 choices for the ninth remaining student. So the number of ways to choose the remaining 9 students is:

$$20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12 = 27,907,200$$

the total number of different 14-student study-groups that can be formed such that within the study group there is exactly one student with each of the names "Ahmed", "Sally", "Marla", "Yousef", and "Ibrahim" is:

$$32 \times 27,907,200 = 892,497,664$$

- (b) If the Instructor is randomly assigning 14 of these students to a project-team, in how many ways could this team be formed such that this team will contain two students both with name "Ahmed", two students both with name "Sally", two students both with name "Marla", two students both with name "Yousef", and two students both with name "Ibrahim"? We can approach this problem using the multiplication principle of counting.

First, we need to choose two students with each of the five given names. We have 2 students with each of the five names, so the number of ways to do this is:

$$C(2, 2) \times C(2, 2) \times C(2, 2) \times C(2, 2) \times C(2, 2) = 1$$

Next, we need to choose the remaining 4 students for the project-team from the remaining 20 students whose names are all distinct from those five names. We need to choose 4 students from 20, without any restrictions on their names. We can do this by simply selecting 4 students from the remaining 20 students, which can be done in:

$$C(20, 4) = 4,845$$

the total number of ways to form a 14-student project-team such that the team will contain two students with each of the names "Ahmed", "Sally", "Mariam", "Yousef", and "Ibrahim" is:

$$1 \times 4,845 = 4,845$$
