First Principle Model Simulation Study

Contents

0.1 Quadratic Model

0.1.1 Intake Equation

$E(t) = \beta_\theta + \beta_1 t + \beta_2 t^2$

0.1.2 Time equation

$t = min(\frac{-\beta_1 \pm \left(\beta_1^2 - 4(\beta_\theta - E(t))\beta_2\right)^{1/2}}{2\beta 2}$

0.2 Thompson et al., 2017 First-Principles Dynamic Model

0.2.1 Oringinal Equation in paper

$$E(t) = \frac{E_{max}\theta\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} - 1\right)}{\theta\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} + E_{max}r\right)}$$

0.2.2 Correct intake equation - took the integral of the derivative/eating rate equation

$$\begin{aligned}
\frac{dE}{dt} &= (rE(t) = \theta)\left(1 - \frac{E(t)}{E_{max}}\right) \\
\int \frac{dE}{dt} &= \int (rE(t) = \theta)\left(1 - \frac{E(t)}{E_{max}}\right) \\
&\vdots \\
E(t) &= \frac{E_{max}\theta\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} - 1\right)}{\theta\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} + \frac{E_{max}r}{\theta}\right)} \\
&= \frac{E_{max}\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} - 1\right)}{e^{\frac{t(E_{max}r+\theta)}{E_{max}}} + \frac{E_{max}r}{\theta}}
\end{aligned} \tag{1}$$

$\theta \neq 0;$ *integral constant* $c \neq 1;$ $\theta \neq +E_{max}r$

0.2.3 Correct time equation

$$
\begin{aligned}
E(t) &= \frac{E_{max}\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}}-1\right)}{e^{\frac{t(E_{max}r+\theta)}{E_{max}}}+\frac{E_{max}r}{\theta}} \\
E(t)\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}}+\frac{E_{max}r}{\theta}\right) &= E_{max}\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}}-1\right) \\
E(t)\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}}\right)+E(t)\left(\frac{E_{max}r}{\theta}\right) &= E_{max}\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}}\right)-E_{max} \\
E(t)\left(\frac{E_{max}r}{\theta}\right)+E_{max} &= E_{max}\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}}\right)-E(t)\left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}}\right) \\
E_{max}\left(\frac{E(t)r}{\theta}+1\right) &= e^{\frac{t(E_{max}r+\theta)}{E_{max}}}\left(E_{max}-E(t)\right) \\
\frac{E_{max}\left(\frac{E(t)r}{\theta}+1\right)}{E_{max}-E(t)} &= e^{\frac{t(E_{max}r+\theta)}{E_{max}}} \\
ln\left(\frac{E_{max}\left(\frac{E(t)r}{\theta}+1\right)}{E_{max}-E(t)}\right) &= \frac{t(E_{max}r+\theta)}{E_{max}} \\
\frac{E_{max}}{E_{max}r+\theta}ln\left(\frac{E_{max}\left(\frac{E(t)r}{\theta}+1\right)}{E_{max}-E(t)}\right) &= t
\end{aligned}
$$

0.3 Log likelihood

$$
\begin{aligned}
L(\hat{E(t)} + \epsilon, \sigma^2 | t, \theta, r, E_{max}) &= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(E_i - E(t_i))^2}{2\sigma^2}} \\
&= \prod_{i=1}^{n} (2\pi\sigma^2)^{\frac{-1}{2}} \; e^{-\frac{1}{2\sigma^2}(E_i - E(t_i))^2} \\
&= (2\pi\sigma^2)^{\frac{-n}{2}} \; \prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}(E_i - E(t_i))^2}
\end{aligned}
$$

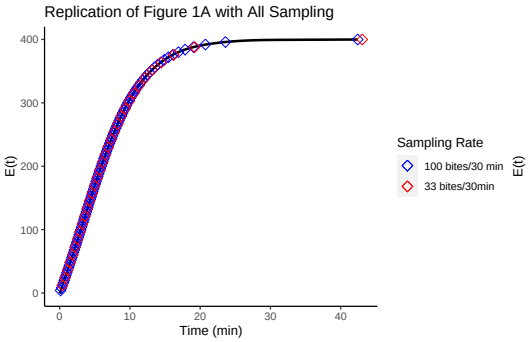
$$
\begin{aligned}
\ln\left(L(\hat{E(t)} + \epsilon, \sigma^2 | t, \theta, r, E_{max})\right) &= \ln\left((2\pi\sigma^2)^{\frac{-n}{2}} \; \prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}(E_i - E(t_i))^2}\right) \\
&= \ln\left((2\pi\sigma^2)^{\frac{-n}{2}}\right) \; + ln\left(\prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}(E_i - E(t_i))^2}\right) \\
&= \ln\left((2\pi\sigma^2)^{\frac{-n}{2}}\right) \; + ln\left(e^{-\frac{1}{2\sigma^2} \; \sum_i^n (E_i - E(t_i))^2}\right) \\
&= \frac{-n}{2} \ln\left(2\pi\sigma^2\right) \; + -\frac{1}{2\sigma^2} \; \sum_{i}^{n} \left(E_i - E(t_i)\right)^2
\end{aligned}
\tag{2}
$$

$$
\sigma^2 = \frac{\sum_i^n \left(E_i - E(t_i)\right)^2}{n}
$$

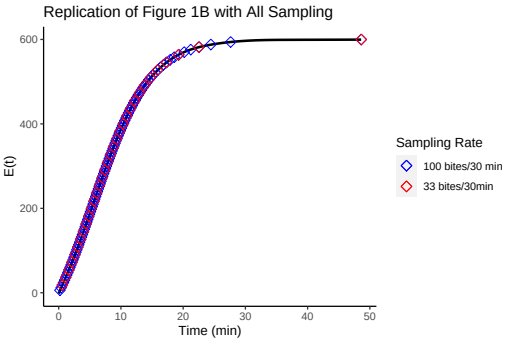
$$
\ln\left(L(\hat{E(t)} + \epsilon, \sigma^2 | t, \theta, r, E_{max})\right) = \frac{-n}{2} \ln\left(2\pi \left(\frac{\sum_i^n \left(E_i - E(t_i)\right)^2}{n}\right)^2\right) \; + -\frac{1}{2\left(\frac{\sum_i^n (E_i - E(t_i))^2}{n}\right)^2} \; \sum_{i}^{n} \left(E_i - E(t_i)\right)^2
$$

already created - only need to redo if something changes

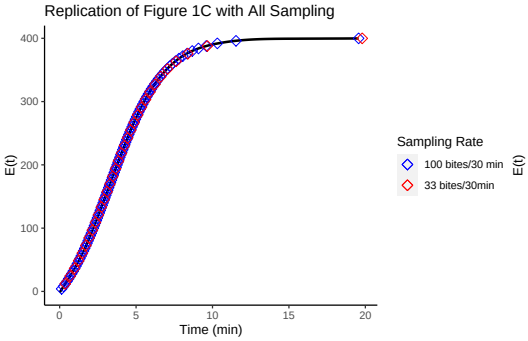
1A - theta: 30 g/min, r: 0.17 1/min, Emax = 400g



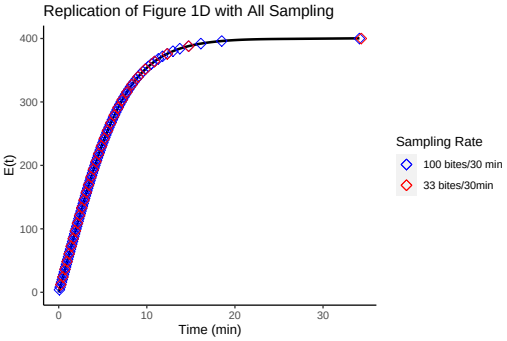
1B - theta: 30 g/min, r: 0.17 1/min, Emax = 600g



1C - theta: 30 g/min, r: 0.5 1/min, Emax = 400g



1D - theta: 50 g/min, r: 0.17 1/min, Emax = 400g



0.3.1 Compare Parameter Fits by Sampling Rate

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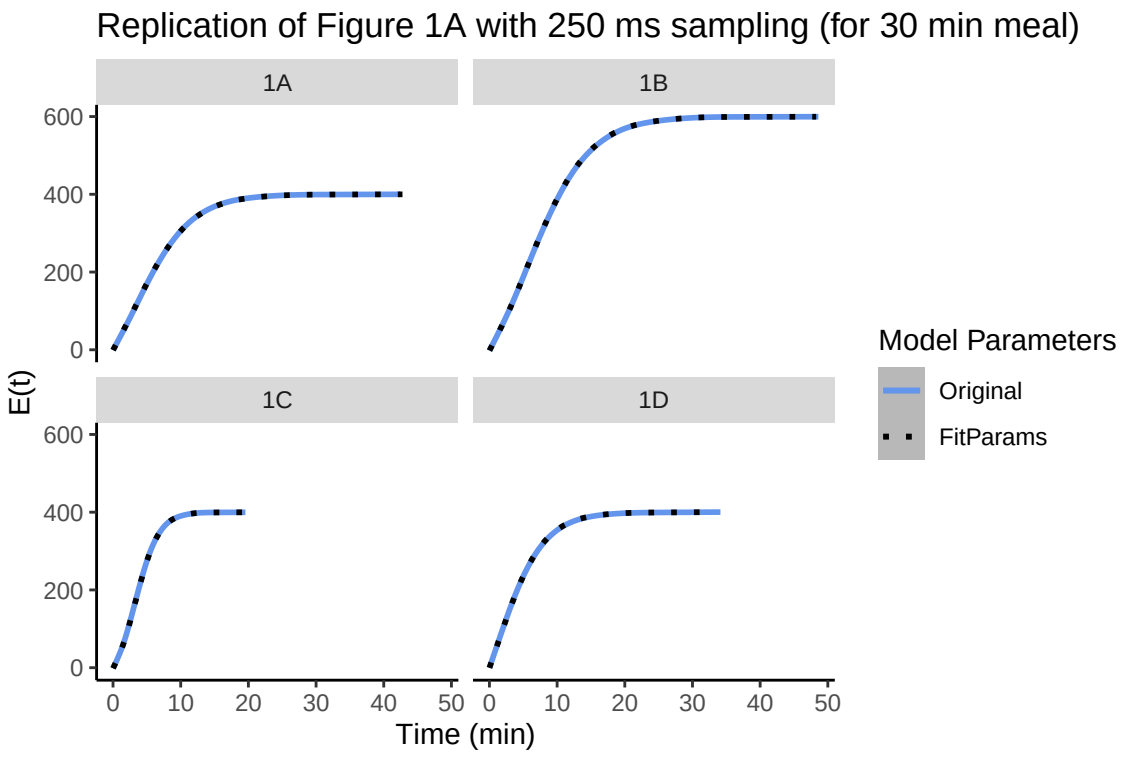
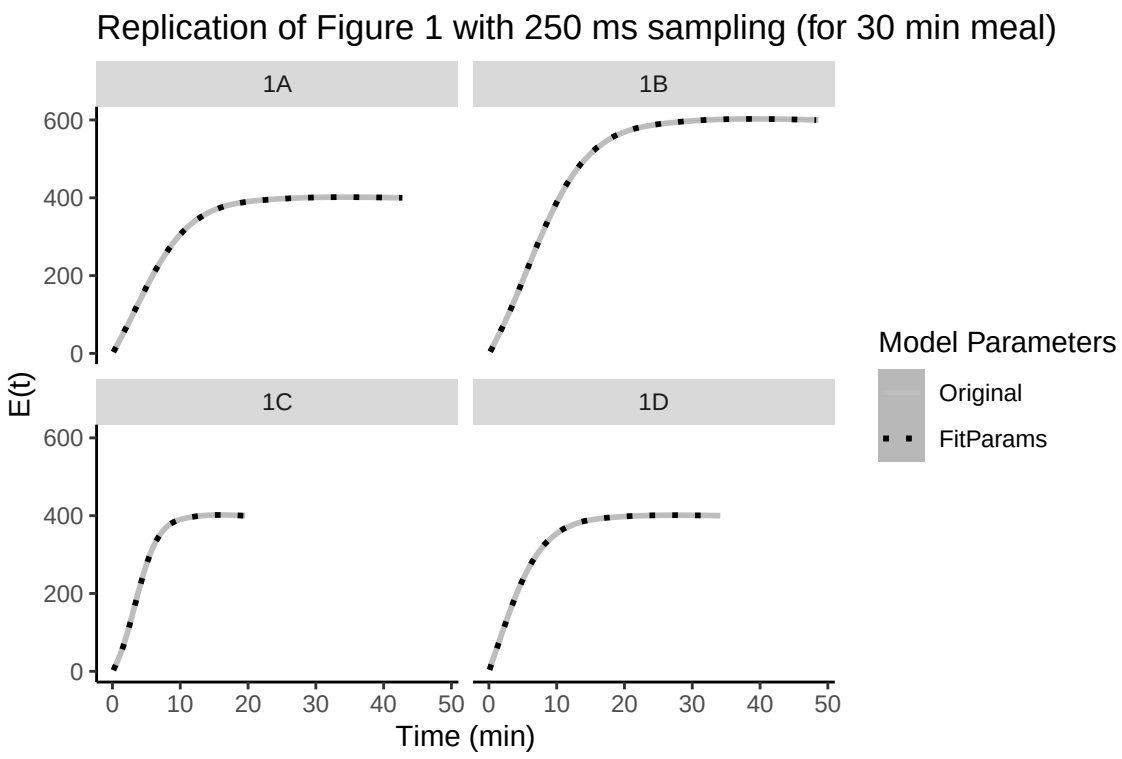
–> –>

250 ms Sampling

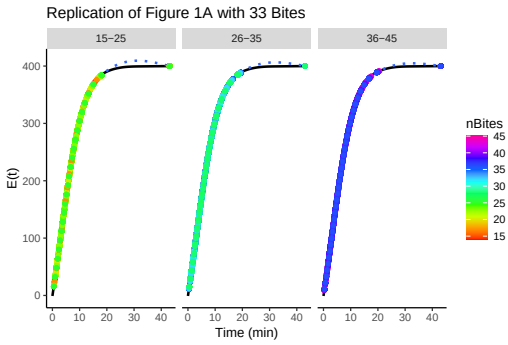


100 bite Sampling

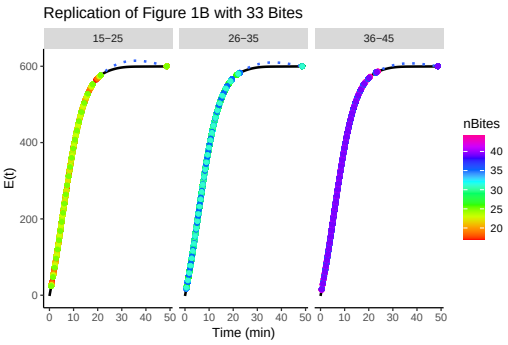


randomly sampled number of bites: mean = 33, sd = 5

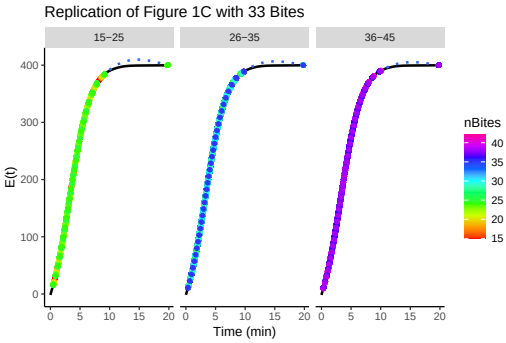
1A - theta: 30 g/min, r: 0.17 1/min, Emax = 400g



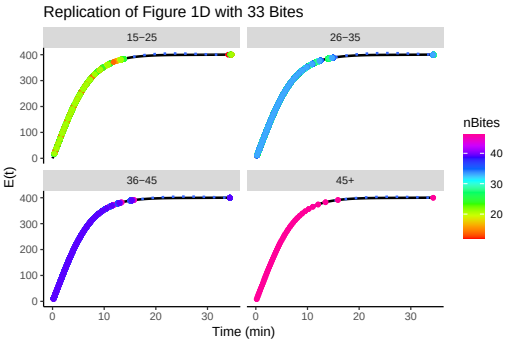
1B - theta: 30 g/min, r: 0.17 1/min, Emax = 600g



1C - theta: 30 g/min, r: 0.5 1/min, Emax = 400g



1D - theta: 50 g/min, r: 0.17 1/min, Emax = 400g



1 Simulations based on Fogel et al., 2017

1.1 Correlations between Microstructure Behaviors

1.1.1 Corrleations Reported in Fogel et al., 2017 (Table 1)

Table 1: Correlations between Microstructure Behaviors - Simulated

nBites	BiteSize_g	BiteOralExposure_sec	ActiveMeal_pcent	TotalOralExposure_min	EatRate_g.min
NA	NA	NA	NA	NA	NA
-0.42*	NA	NA	NA	NA	NA
-0.58*	0.54*	NA	NA	NA	NA
0.11*	0.17*	0.16*	NA	NA	NA
0.54*	-0.01	0.02	0.33	NA	NA
0.15*	0.55*	-0.25*	-0.02	-0.05	NA

1.1.2 Corrleations Simulated

Table 2: Correlations between Microstructure Behaviors - Simulated

nBites	BiteSize_g	BiteOralExposure_sec	ActiveMeal_pcent	TotalOralExposure_min	EatRate_g.min	TotalIn
-0.42*						
-0.58*	0.54*					
0.11*	0.17*	0.16*				
0.42*	0.15*	0.49*	0.3*			
0.04	0.66*	-0.27*	0.05	-0.26*		
0.34*	0.71*	0.11*	0.27*	0.49*	0.71*	
0.02	0.4*	-0.14*	0.04	-0.14*	0.58*	0.43*
0.4*	0.09*	0.45*	-0.06	0.93*	-0.29*	0.41*

Table 3: Correlations between Microstructure Behaviors - Simulated, Rounded

nBites	BiteSize_g	BiteOralExposure_sec	ActiveMeal_pcent	TotalOralExposure_min	EatRate_g.min	TotalIn
-0.42*						
-0.58*	0.54*					
0.09*	0.17*	0.16*				
0.43*	0.15*	0.48*	0.28*			
0.04	0.66*	-0.27*	0.05	-0.26*		
0.35*	0.7*	0.1*	0.25*	0.49*	0.71*	
0.01	0.37*	-0.19*	0.04	-0.2*	0.6*	0.39*
0.42*	0.09*	0.44*	-0.08	0.94*	-0.29*	0.42*

1.1.3 Corrleations Simulated - After Rounding Simulated Number of Bites

1.2 Fast vs Slow Eaters Microstructure Characteristics

1.2.1 Means (SEM) Reported in Fogel et al., 2017 (Table 2)

	Slow	Fast	t	t
Bites(#)	57.7 (2.5)	68.4 (2.5)	3.04	0.003
Bite Size (grams/bite)	1.4 (0.1)	2.4 (0.1)	9.17	<0.001
Oral Exposure per Bite (sec)	20.1 (0.9)	15.6 (0.5)	4.11	<0.001
Active Mealtime (%)	75.0 (1.0)	76.0 (1.0)	0.56	0.570
Total Oral Exposure (min)	15.1 (0.4)	15.2 (0.4)	0.08	0.930
Total Intake (kCal)	175.3 (6.09)	306.7 (9.9)	11.28	<0.001

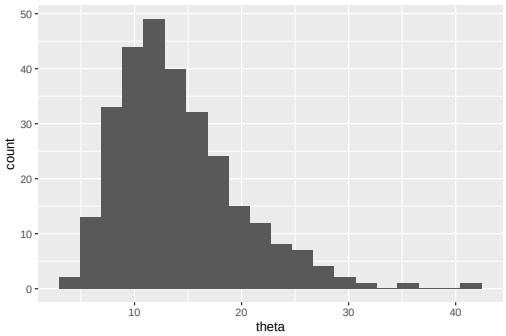
1.2.2 Means (SEM) After Rounding Simulated Number of Bites

	Slow	Fast	t	t
Bites(#)	57.7 (2.5)	68.4 (2.5)	3.04	0.003
Bite Size (grams/bite)	1.4 (0.1)	2.4 (0.1)	9.17	<0.001
Oral Exposure per Bite (sec)	20.1 (0.9)	15.6 (0.5)	4.11	<0.001
Active Mealtime (%)	75.0 (1.0)	76.0 (1.0)	0.56	0.570
Total Oral Exposure (min)	15.1 (0.4)	15.2 (0.4)	0.08	0.930
Total Intake (kCal)	175.3 (6.09)	306.7 (9.9)	11.28	<0.001

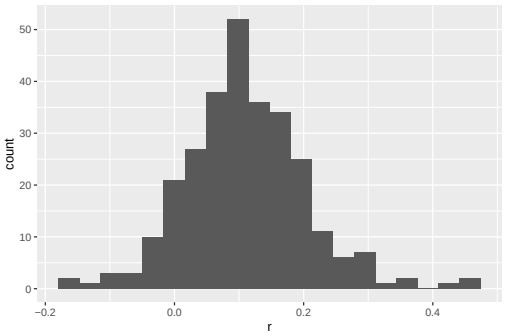
1.2.3 Parameter Distributions - Quadratic

1.2.4 Parameter Distributions - First Principles Model

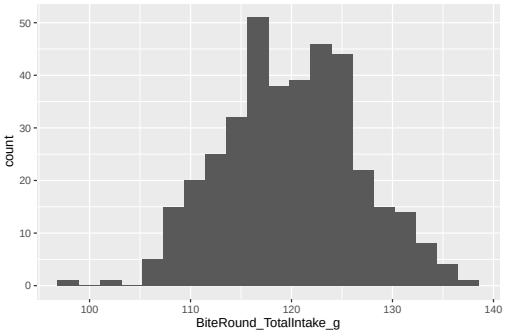
θ



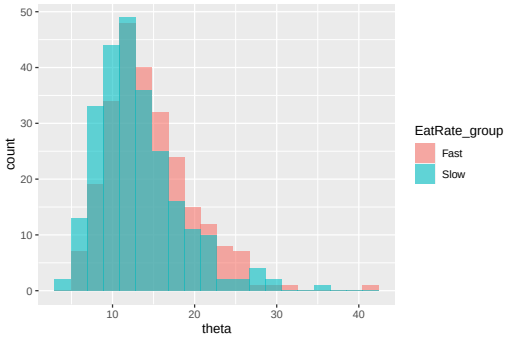
r



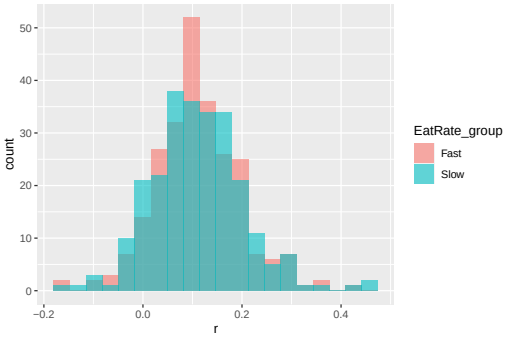
Emax



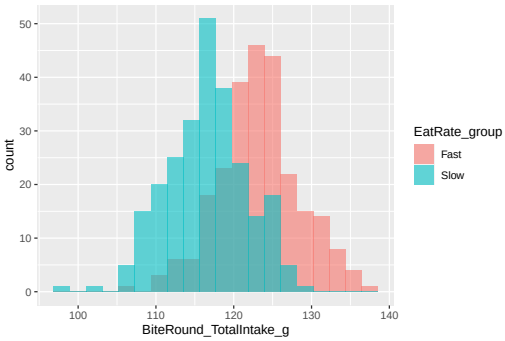
θ by Eating Rate Group

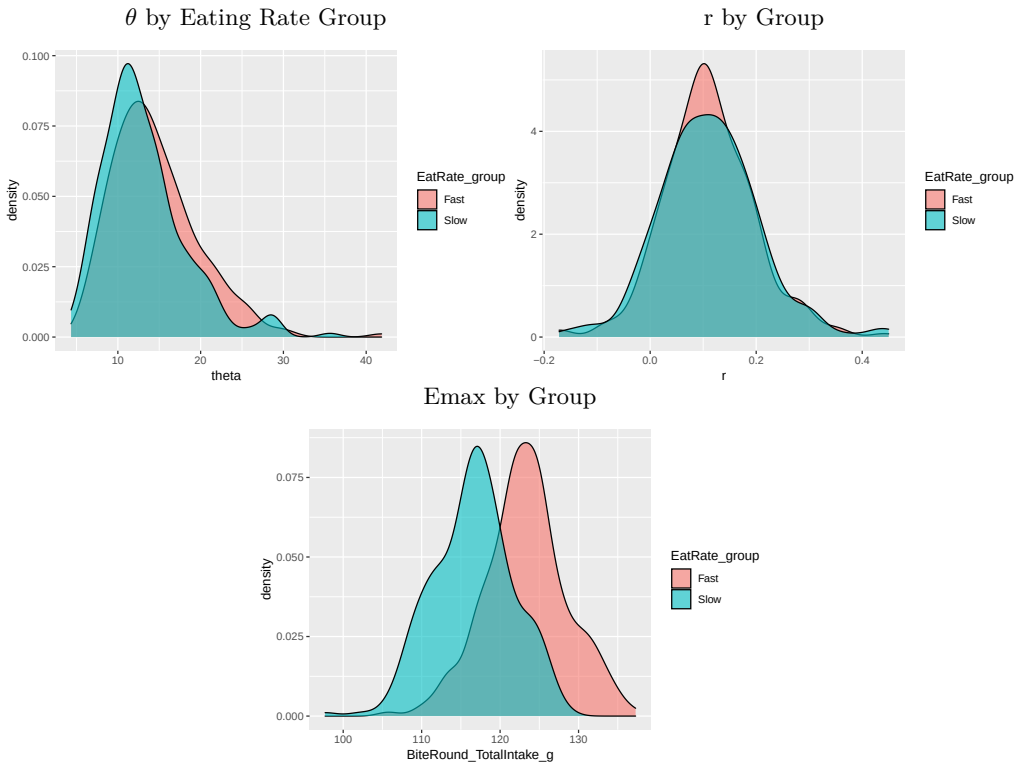


r by Group



Emax by Group




θ by Eating Rate Group
density
theta
EatRate_group
Fast
Slow
r by Group
density
r
EatRate_group
Fast
Slow
Emax by Group
density
BiteRound_TotalIntake_g
EatRate_group
Fast
Slow