

First Principle Model Simulation Study

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0.1 Quadratic Model

0.1.1 Intake Equation

$$E(t) = \beta_\theta + \beta_1 t + \beta_2 t^2$$

0.1.2 Time equation

$$t = \min\left(\frac{-\beta_1 \pm (\beta_1^2 - 4(\beta_\theta - E(t))\beta_2)^{1/2}}{2\beta_2}\right)$$

0.2 Thompson et al., 2017 First-Principles Dynamic Model

0.2.1 Oringinal Equation in paper

$$E(t) = \frac{E_{max}\theta \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} - 1 \right)}{\theta \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} + E_{max}r \right)}$$

0.2.2 Correct intake equation

- took the integral of the derivative/eating rate equation

$$\begin{aligned} \frac{dE}{dt} &= (rE(t) = \theta) \left(1 - \frac{E(t)}{E_{max}} \right) \\ \int \frac{dE}{dt} &= \int (rE(t) = \theta) \left(1 - \frac{E(t)}{E_{max}} \right) \\ &\vdots \\ E(t) &= \frac{E_{max}\theta \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} - 1 \right)}{\theta \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} + \frac{E_{max}r}{\theta} \right)} \\ &= \frac{E_{max} \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} - 1 \right)}{e^{\frac{t(E_{max}r+\theta)}{E_{max}}} + \frac{E_{max}r}{\theta}} \end{aligned} \tag{1}$$

$\theta \neq 0$; integral constant $c \neq 1$; $\theta \neq +E_{max}r$

0.2.3 Correct time equation

$$\begin{aligned}
E(t) &= \frac{E_{max} \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} - 1 \right)}{e^{\frac{t(E_{max}r+\theta)}{E_{max}}} + \frac{E_{max}r}{\theta}} \\
E(t) \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} + \frac{E_{max}r}{\theta} \right) &= E_{max} \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} - 1 \right) \\
E(t) \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} \right) + E(t) \left(\frac{E_{max}r}{\theta} \right) &= E_{max} \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} \right) - E_{max} \\
E(t) \left(\frac{E_{max}r}{\theta} \right) + E_{max} &= E_{max} \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} \right) - E(t) \left(e^{\frac{t(E_{max}r+\theta)}{E_{max}}} \right) \\
E_{max} \left(\frac{E(t)r}{\theta} + 1 \right) &= e^{\frac{t(E_{max}r+\theta)}{E_{max}}} (E_{max} - E(t)) \\
\frac{E_{max} \left(\frac{E(t)r}{\theta} + 1 \right)}{E_{max} - E(t)} &= e^{\frac{t(E_{max}r+\theta)}{E_{max}}} \\
\ln \left(\frac{E_{max} \left(\frac{E(t)r}{\theta} + 1 \right)}{E_{max} - E(t)} \right) &= \frac{t(E_{max}r + \theta)}{E_{max}} \\
\frac{E_{max}}{E_{max}r + \theta} \ln \left(\frac{E_{max} \left(\frac{E(t)r}{\theta} + 1 \right)}{E_{max} - E(t)} \right) &= t
\end{aligned}$$

0.3 Log likelihood

$$\begin{aligned}
L(\hat{E}(t) + \epsilon, \sigma^2 | t, \theta, r, E_{max}) &= \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(E_i - E(t_i))^2}{2\sigma^2}} \\
&= \prod_{i=1}^n (2\pi\sigma^2)^{-\frac{1}{2}} e^{-\frac{1}{2\sigma^2} (E_i - E(t_i))^2} \\
&= (2\pi\sigma^2)^{-\frac{n}{2}} \prod_{i=1}^n e^{-\frac{1}{2\sigma^2} (E_i - E(t_i))^2}
\end{aligned}$$

$$\begin{aligned}
\ln \left(L(\hat{E}(t) + \epsilon, \sigma^2 | t, \theta, r, E_{max}) \right) &= \ln \left((2\pi\sigma^2)^{-\frac{n}{2}} \prod_{i=1}^n e^{-\frac{1}{2\sigma^2} (E_i - E(t_i))^2} \right) \\
&= \ln \left((2\pi\sigma^2)^{-\frac{n}{2}} \right) + \ln \left(\prod_{i=1}^n e^{-\frac{1}{2\sigma^2} (E_i - E(t_i))^2} \right) \\
&= \ln \left((2\pi\sigma^2)^{-\frac{n}{2}} \right) + \ln \left(e^{-\frac{1}{2\sigma^2} \sum_i^n (E_i - E(t_i))^2} \right) \\
&= \frac{-n}{2} \ln (2\pi\sigma^2) + -\frac{1}{2\sigma^2} \sum_i^n (E_i - E(t_i))^2
\end{aligned} \tag{2}$$

$$\sigma^2 = \frac{\sum_i^n (E_i - E(t_i))^2}{n}$$

$$\ln \left(L(\hat{E}(t) + \epsilon, \sigma^2 | t, \theta, r, E_{max}) \right) = \frac{-n}{2} \ln \left(2\pi \left(\frac{\sum_i^n (E_i - E(t_i))^2}{n} \right)^2 \right) + -\frac{1}{2 \left(\frac{\sum_i^n (E_i - E(t_i))^2}{n} \right)^2} \sum_i^n (E_i - E(t_i))^2$$

1 Replication of Figure 1 from Thompson et al 2017

1.1 Compare by Sampling Rate

1.2 Parameter Recovery by Sampling Rate

1.2.1 Parameter Recovery with Infrequent Bites (500 sims)

2 Simulations based on Fogel et al., 2017

2.1 Simulated Microbehavior

Data were simulated from a multivariate normal distribution for the following microbehaviors:

- : Number of Bites - rounded to whole number
- : Bite Size (g) - restricted to be > 0
- : Percent of Time Active (ie. eating) - restricted to be $\leq 100\%$
- : Oral Exposure per Bite (sec)

2.1.1 Computed Microbehavior

The following microbehavior characteristics were calculated based on simulated data:

- : Total Oral Exposure (min); (Oral Exposure per Bite * Number of Bites)/60 - restricted to be > 2 min to fit range reported in Fogel et. al., 2017
- : Total Intake (g); Bite Size (g) * Number of Bites
- : Eating Rate (g/min); Total Intake (g)/Total Oral Exposure (min) - restricted to be > 0 and < 25 to approximate range depicted in Figure 1 in Fogel et al., 2017
- : Meal Duration (min); Total Oral Exposure/(Percent of Time Active/100) - restricted to be ≤ 30 min as that was the longest duration the meal could go in the Fogel et al., 2017 paper

2.2 Correlations between Microstructure Behaviors

2.2.1 Correlations Reported in Fogel et al., 2017 (Table 1)

Table 1: Correlations between Microstructure Behaviors - Simulated

	nBites	EatRate	TotalOE_min	ActiveMeal_pcent	BiteOE_sec
nBites	NA	NA	NA	NA	NA
EatRate	0.15**	NA	NA	NA	NA
TotalOE_min	0.54*	-0.05	NA	NA	NA
ActiveMeal_pcent	0.11*	-0.02	0.33***	NA	NA
BiteOE_sec	-0.58****	-0.25***	0.02	0.16**	NA
TotalIntake_g	NA	NA	NA	NA	NA
BiteSize_g	-0.42***	0.55*	-0.01	0.17**	0.54***

2.2.2 Correlations Simulated

Table 2: Correlations between Microstructure Behaviors - Simulated

	nBites	EatRate	TotalOE_min	ActiveMeal_pcent	BiteOE_sec	TotalIntake_g
nBites						
EatRate	0.28*					
TotalOE_min	0.4*	-0.2*				
ActiveMeal_pcent	0.17*	0	0.38*			
BiteOE_sec	-0.68*	-0.4*	0.23*	0.14*		
TotalIntake_g	0.47*	0.67*	0.5*	0.22*	-0.14*	
BiteSize_g	-0.37*	0.44*	0.09*	0.14*	0.53*	0.53*

2.3 Fast vs Slow Eaters Microstructure Characteristics

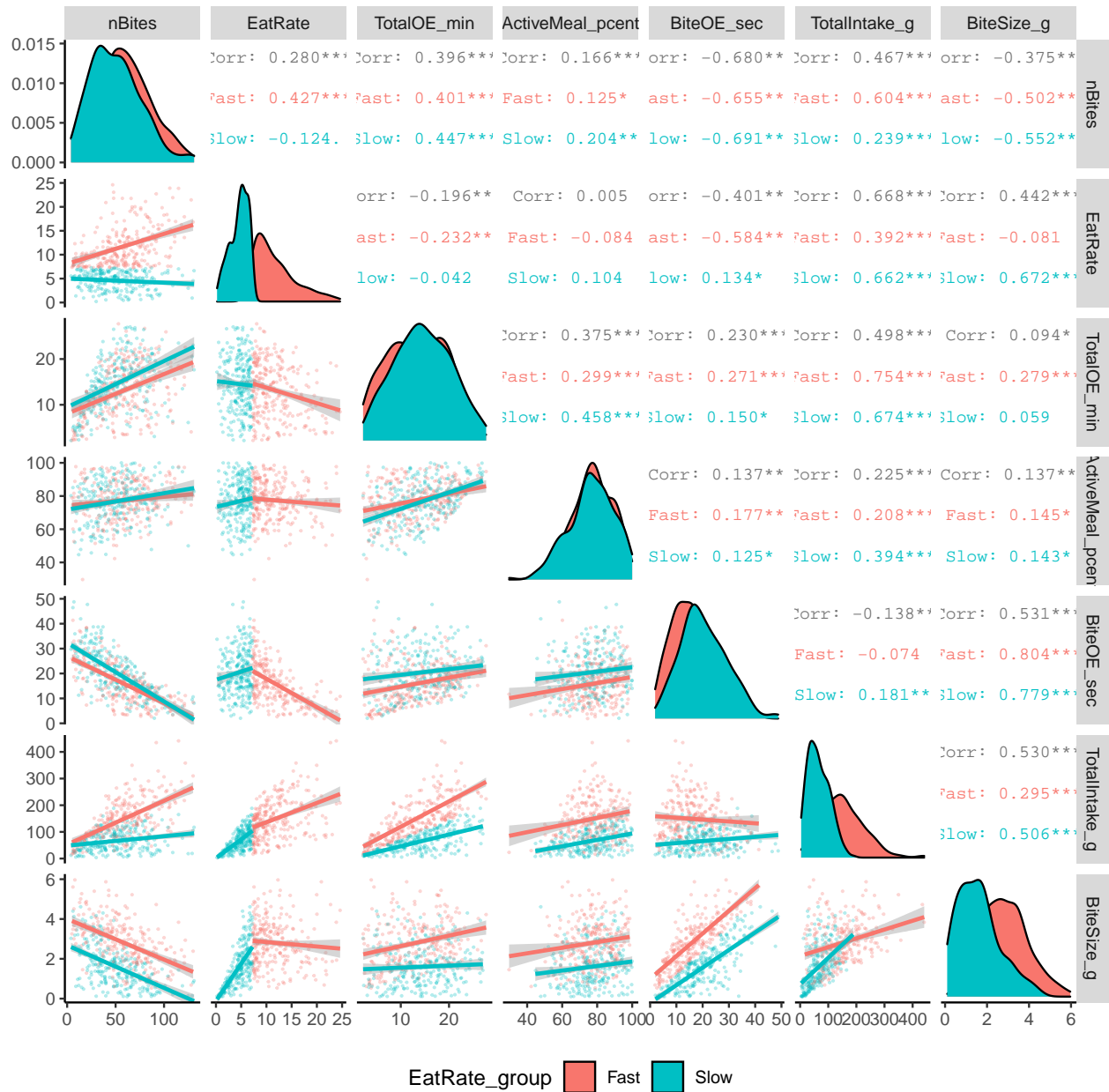
2.3.1 Means (SEM): Reported in Fogel et al., 2017 (Table 2)

	Slow	Fast	t	p
Bites(#)	57.7 (2.5)	68.4 (2.5)	3.04	0.003
Bite Size (grams/bite)	1.4 (0.1)	2.4 (0.1)	9.17	<0.001
Oral Exposure per Bite (sec)	20.1 (0.9)	15.6 (0.5)	4.11	<0.001
Active Mealtime (%)	75.0 (1.0)	76.0 (1.0)	0.56	0.570
Total Oral Exposure (min)	15.1 (0.4)	15.2 (0.4)	0.08	0.930
Total Intake (kCal)	175.3 (6.09)	306.7 (9.9)	11.28	<0.001

2.3.2 Means (SEM): From Simulated and Calculated Data

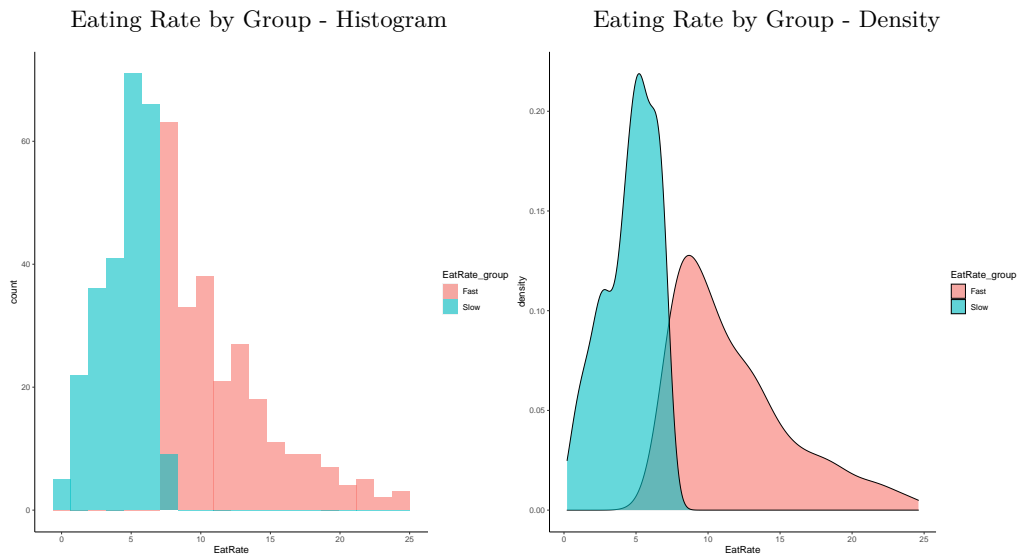
	Slow		Fast		t	p
	Mean (SE)	Range	Mean (SE)	Range		
Bites(#)	50.28 (1.6)	4 - 131	58.74 (1.7)	5 - 130	15.53	<0.001
Bite Size (grams/bite)	1.6 (0.1)	0.1 - 4.6	2.8 (0.1)	0.5 - 6.0	12.56	<0.001
Oral Exposure per Bite (sec)	20.5 (0.6)	2.0 - 48.8	15.7 (0.5)	2.0 - 41.4	-6.14	<0.001
Active Mealtime (%)	76.8 (0.8)	44.7 - 100	73.6 (0.7)	29.7 - 98.6	0.553	0.581
Total Oral Exposure (min)	14.5 (0.4)	2.2 - 27.0	13.1 (0.34)	2.2 - 27.7	-2.66	0.008
Total Intake (grams)	66.2 (2.4)	3.7 - 188.5	149.0 (4.8)	16.3 - 441.3	15.53	<0.001

2.4 Correlations Between Microstructure Behaviors



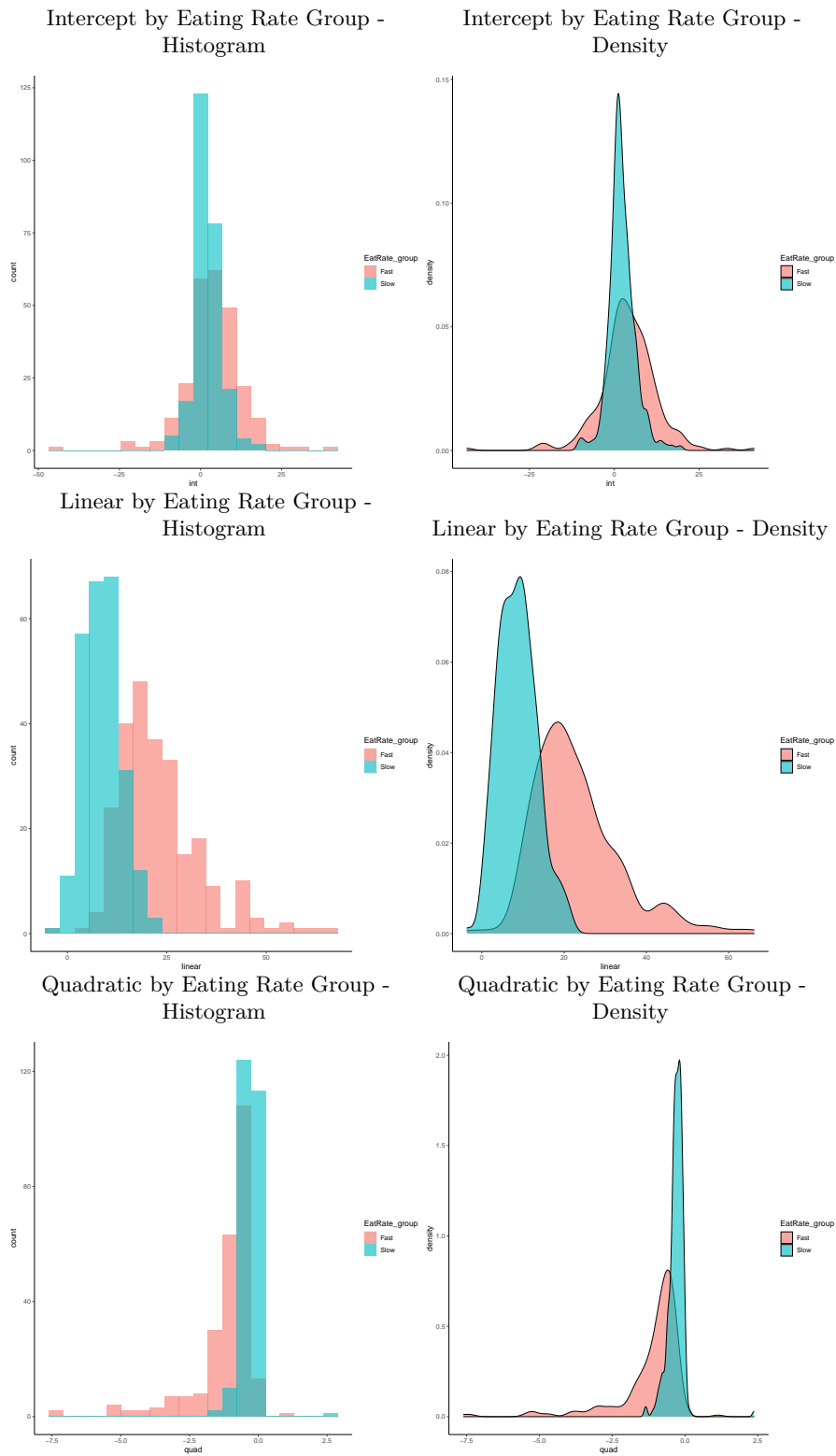
3 Parameter Distributions

3.1 Eating Rate Distribution



3.2 Kissileff's Quadratic Model

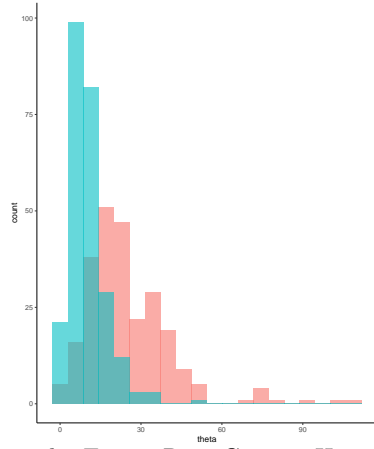
Kissileff Model



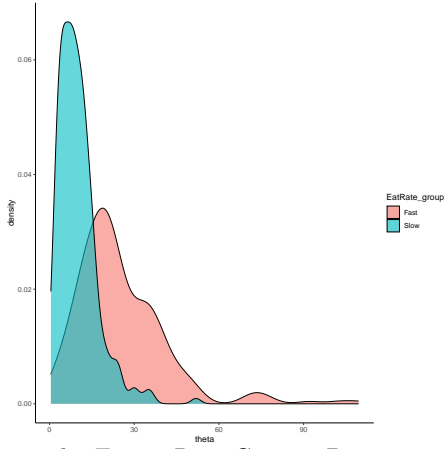
3.3 First Principles Model

First Principles Model

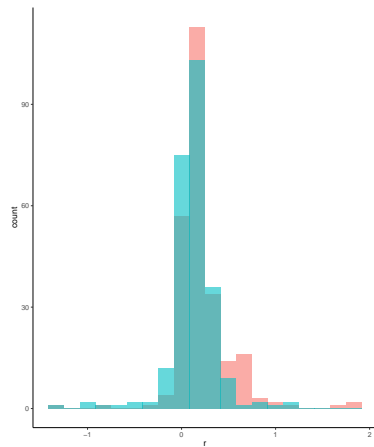
θ by Eating Rate Group - Histogram



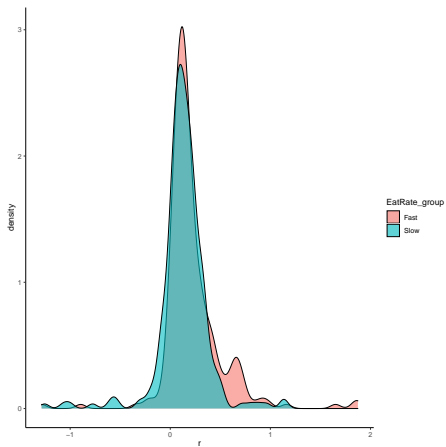
θ by Eating Rate Group - Density



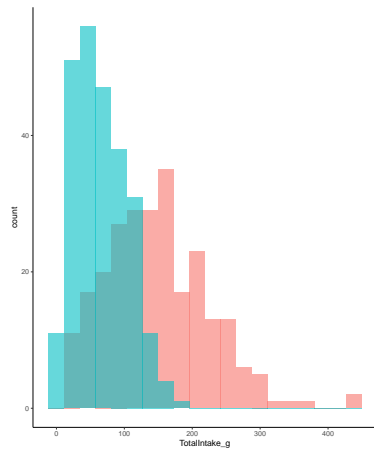
r by Eating Rate Group - Histogram



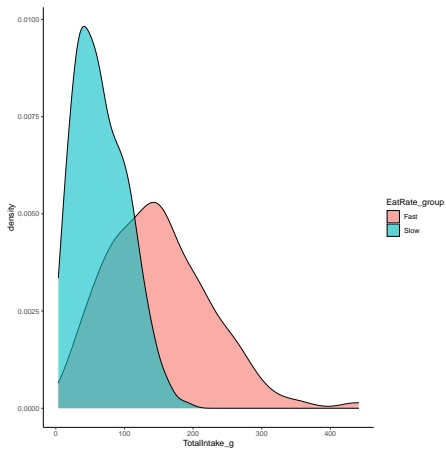
r by Eating Rate Group - Density



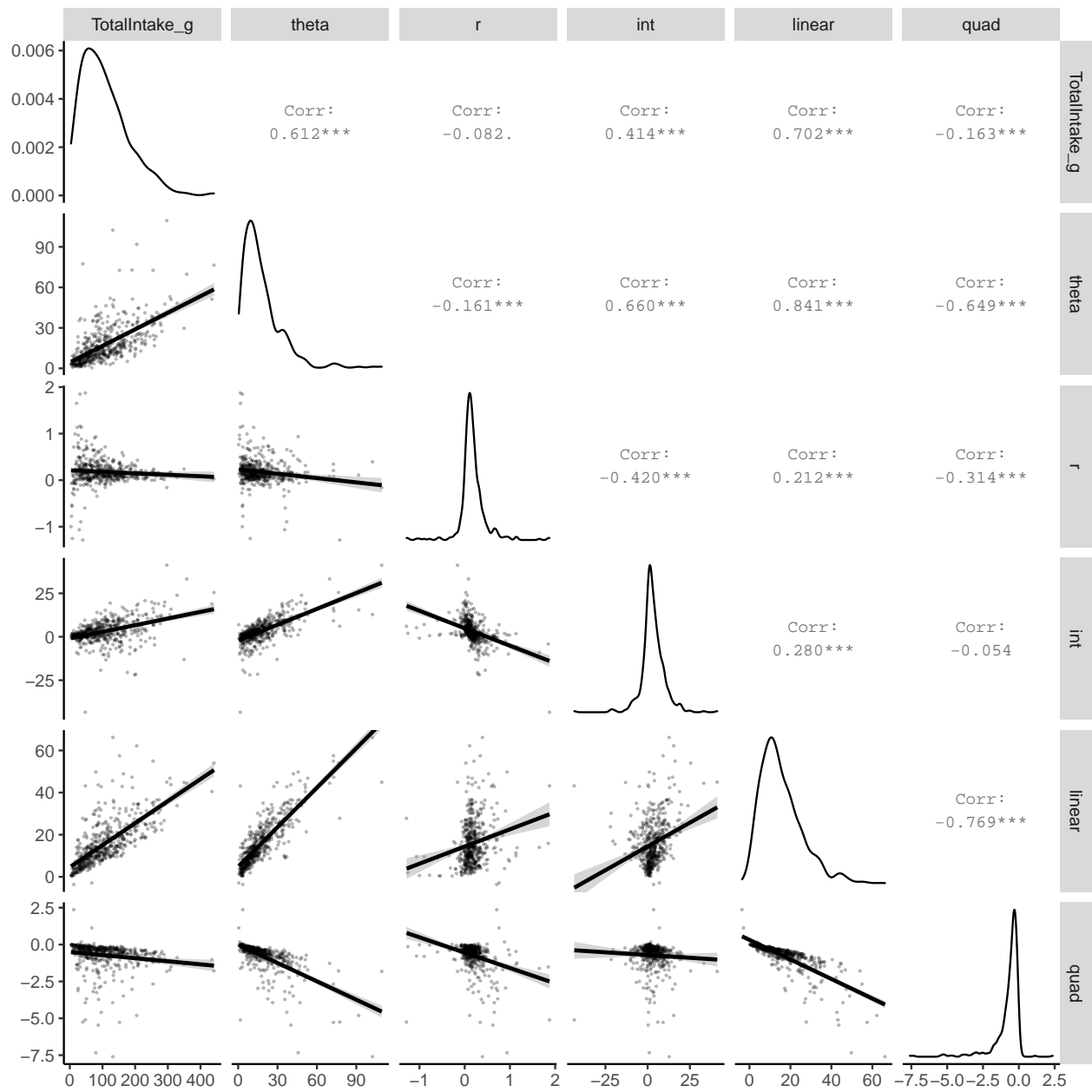
E_{max} by Eating Rate Group - Histogram



E_{max} by Eating Rate Group - Density



3.4 Correlations Between Parameters



4 Cumulative Intake Curves Accross Parameter Distributions

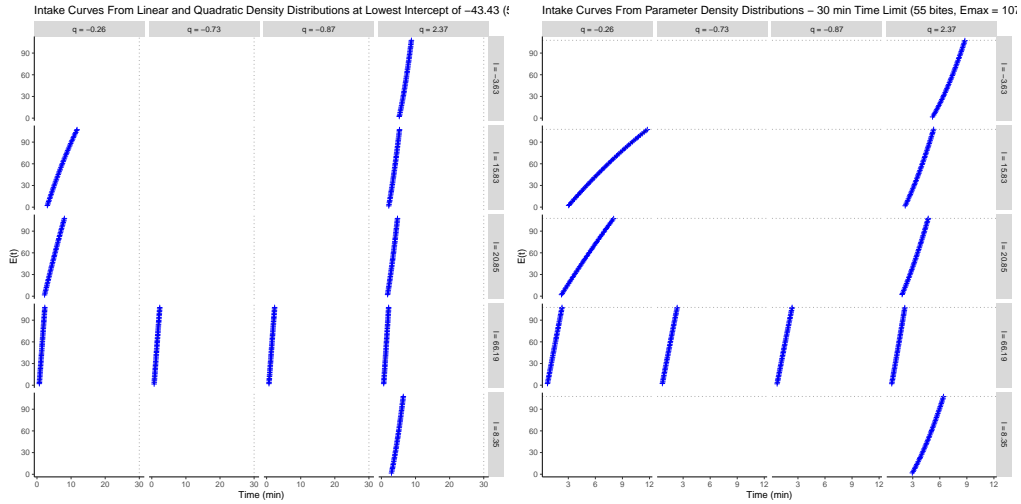
Set Emax and number of bites to the average of the distributions

4.1 Kissileff Quadratic Model

Kissileff Cumulative Intake Curves: Lowest Intercept (-43.43)

No Time Constraint

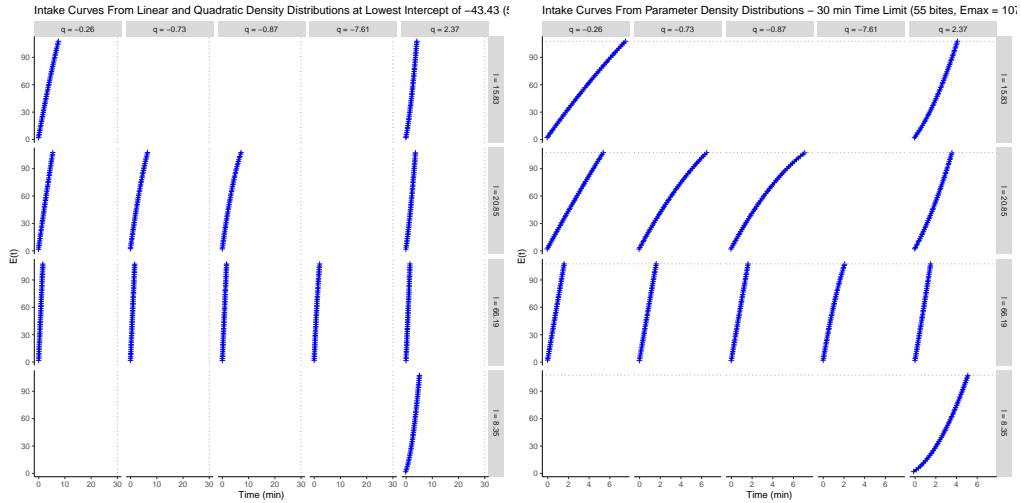
All Bites Occured in 30 min



Kissileff Cumulative Intake Curves: Mean Intercept (3.13)

No Time Constraint

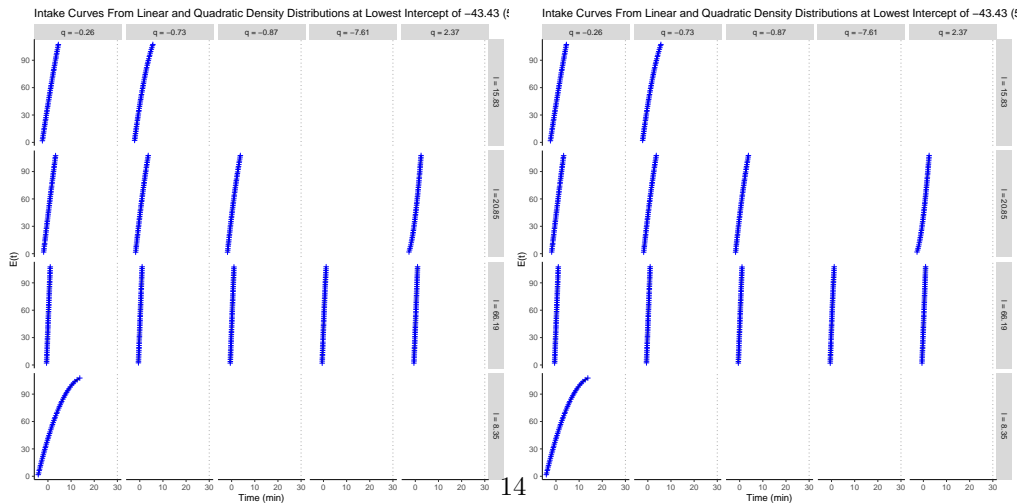
All Bites Occured in 30 min



Kissileff Cumulative Intake Curves: Highest Intercept (41.2)

No Time Constraint

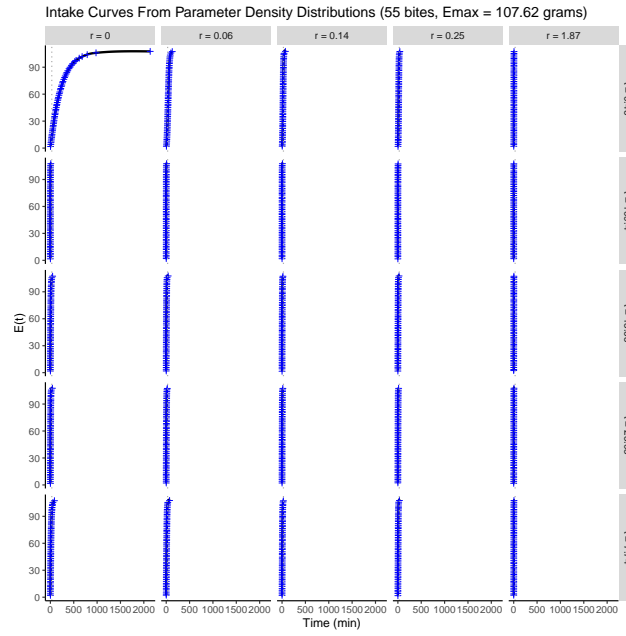
All Bites Occured in 30 min



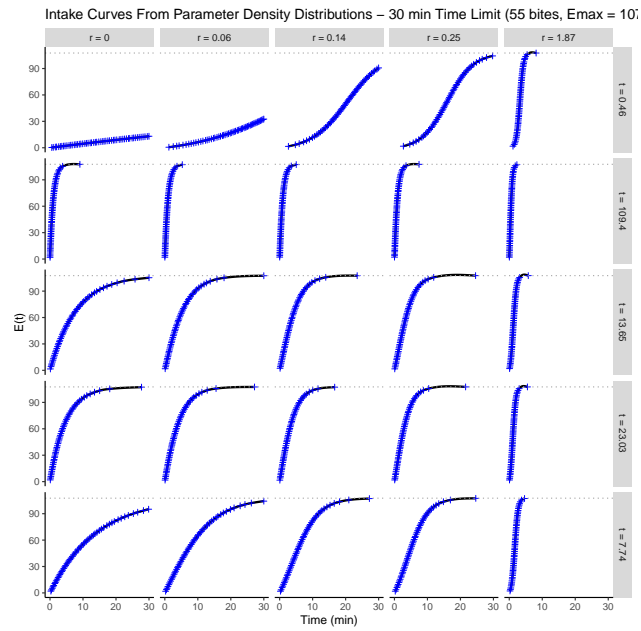
4.2 First Principles Model

First Principles Model Cumulative Intake Curves

No Time Constraints



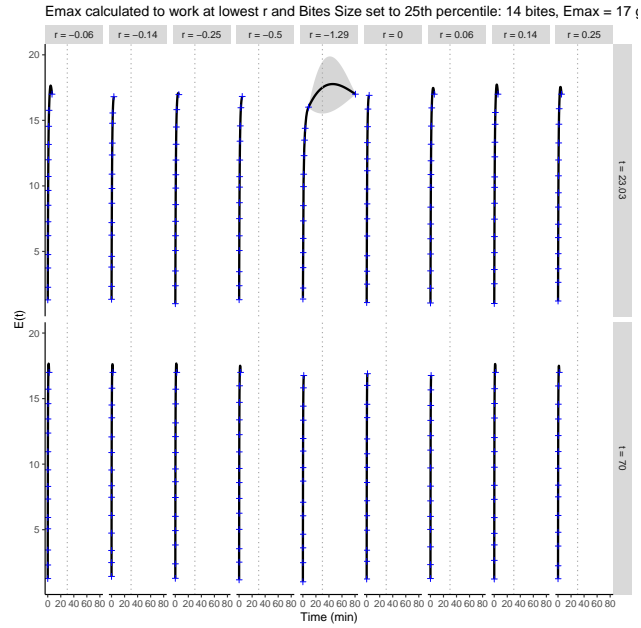
All Bites Occured in 30 min



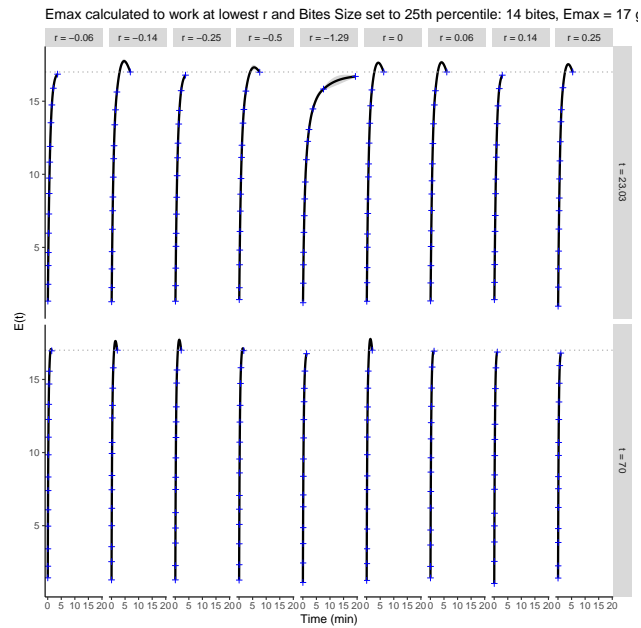
4.2.1 Zoom in on r - negative and small values

First Principles Model Cumulative Intake Curves

No Time Constraints



All Bites Occured in 30 min



5 Parameter Recovery

5.1 Recovery Tables from 100 Random Samples

5.1.1 First Principles Model

Table 3: Recovery of r within 95 percent confidence bounds

	Measurement noise	No noise	Process noise
FALSE	32	27	30
TRUE	68	73	70

Table 4: Recovery of θ within 95 percent confidence bounds

	Measurement noise	No noise	Process noise
FALSE	3	0	1
TRUE	97	100	99

5.1.2 Kissileff's Quadratic Model

Table 5: Recovery of intercept within 95 percent confidence bounds

	Measurement noise	No noise	Process noise
FALSE	10	8	8
TRUE	90	92	92

Table 6: Recovery of linear coefficient within 95 percent confidence bounds

	Measurement noise	No noise	Process noise
FALSE	15	15	14
TRUE	85	85	86

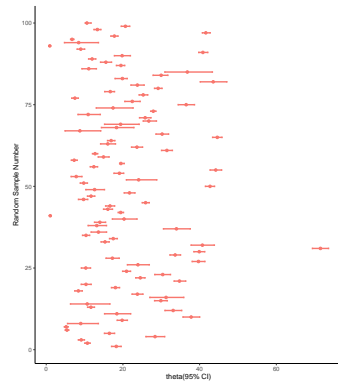
Table 7: Recovery of quadratic coefficient within 95 percent confidence bounds

	Measurement noise	No noise	Process noise
FALSE	47	45	37
TRUE	53	55	63

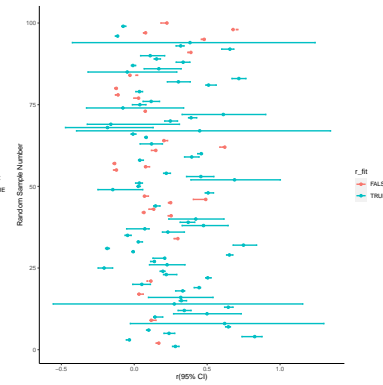
5.2 100 Random Multivariate Samples: No Noise

First Principles Model: No Noise

Theta: Parameter Recovery 95%
CI

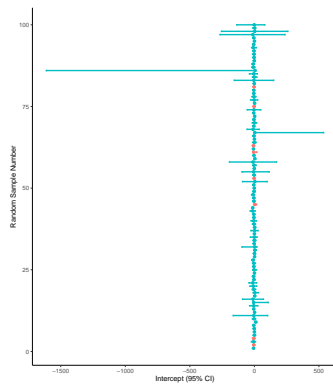


r Parameter Recovery 95% CI

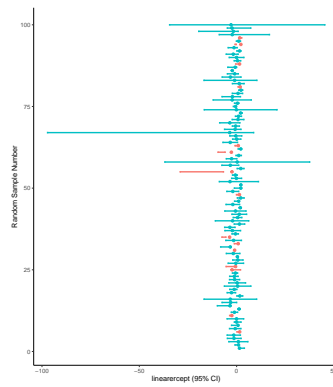


Kissileff Quadratic Model: No Noise

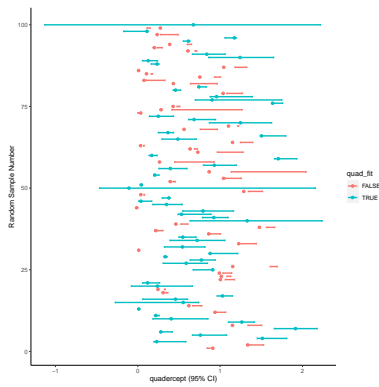
Intercept Parameter Recovery 95%
CI



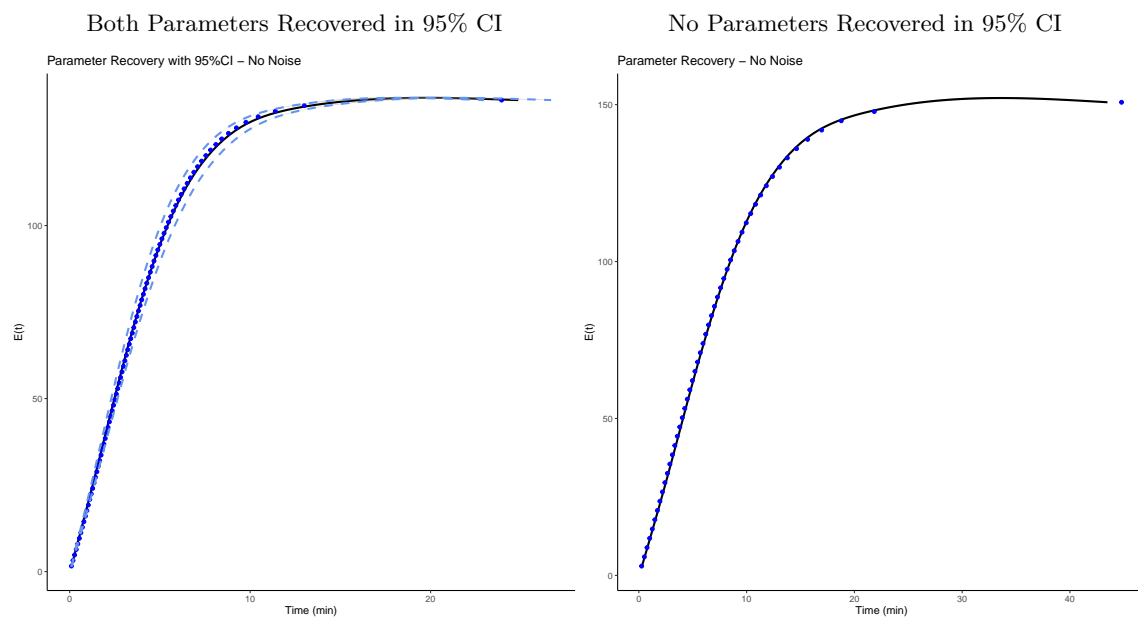
Linear Parameter Recovery 95% CI



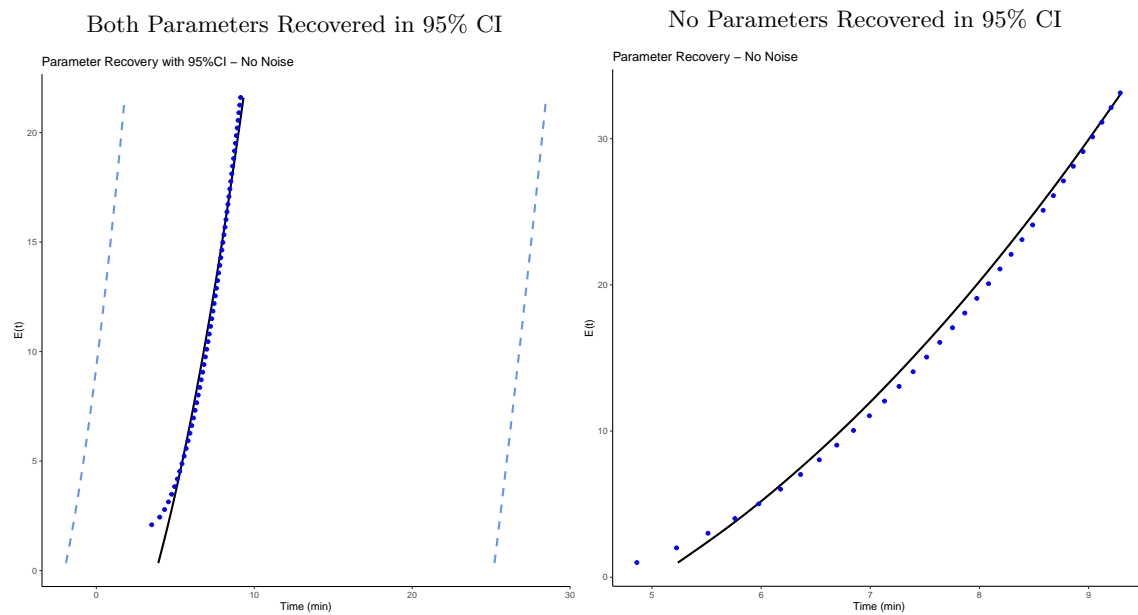
Quadratic Parameter Recovery
95% CI



First Principles Model - No Noise



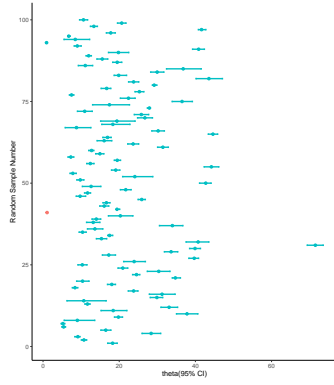
Kissileff Quadratic Model - No Noise



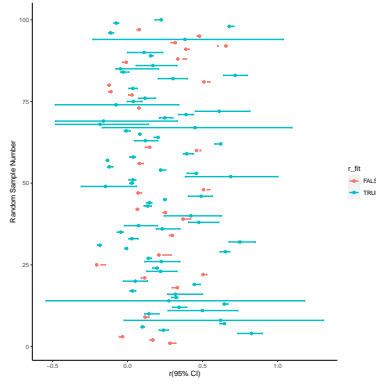
##100 Random Multivariate Samples - Process Noise only

First Principles Model - Process Noise

Theta - Parameter Recovery 95%
CI

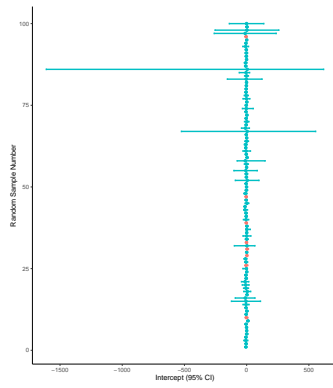


r - Parameter Recovery 95% CI

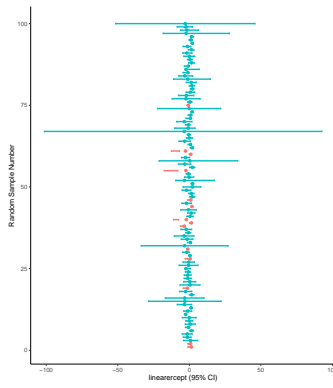


Kissileff Quadratic Model - Process Noise

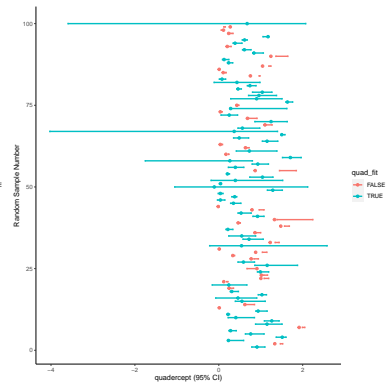
Intercept - Parameter Recovery
95% CI



Linear - Parameter Recovery 95%
CI

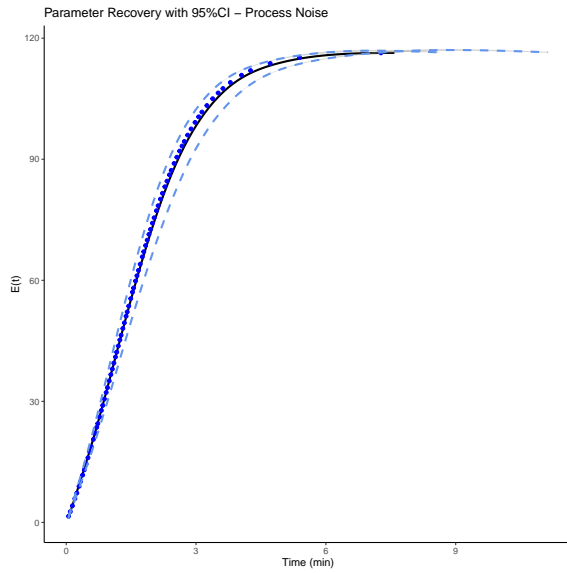


Quadratic - Parameter Recovery
95% CI

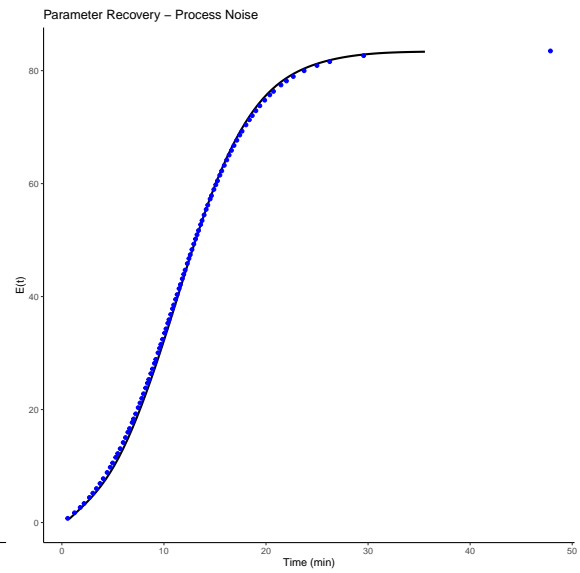


First Principles Model - Process Noise

Both Parameters Recovered in 95% CI

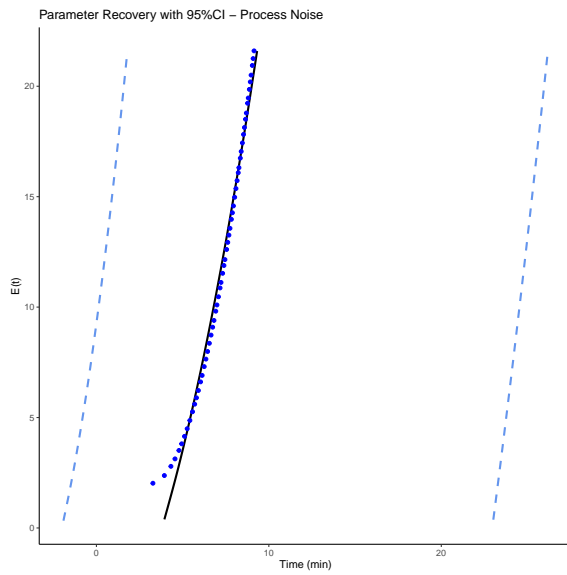


No Parameters Recovered in 95% CI

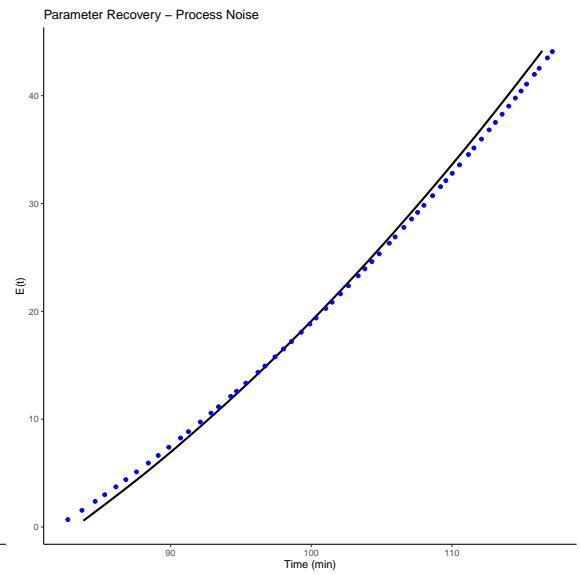


Kissileff Quadratic Model - No Noise

Both Parameters Recovered in 95% CI



No Parameters Recovered in 95% CI



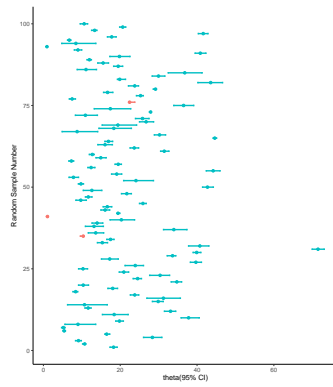
5.3 100 Random Multivariate Samples - Process Noise and Measurement Error

Measurement error: uses average bite size and jittered bite timing

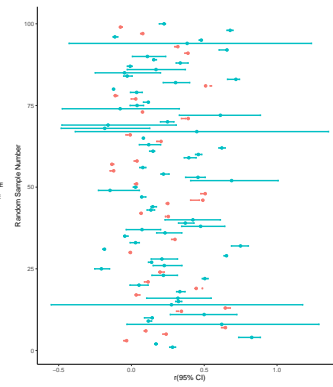
First Principles Model - Process Noise and Measurement Noise

Theta - Parameter Recovery 95% CI

CI

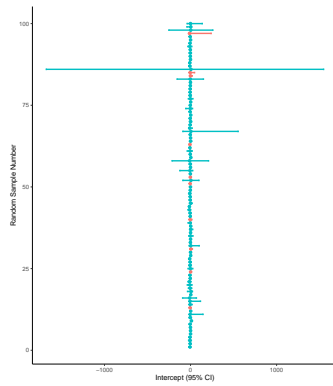


r - Parameter Recovery 95% CI

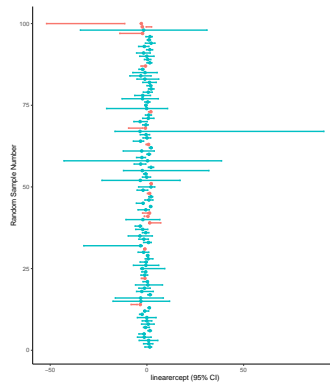


Kissileff Quadratic Model - Process Noise

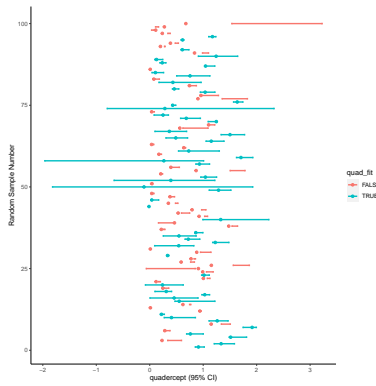
Intercept - Parameter Recovery 95% CI



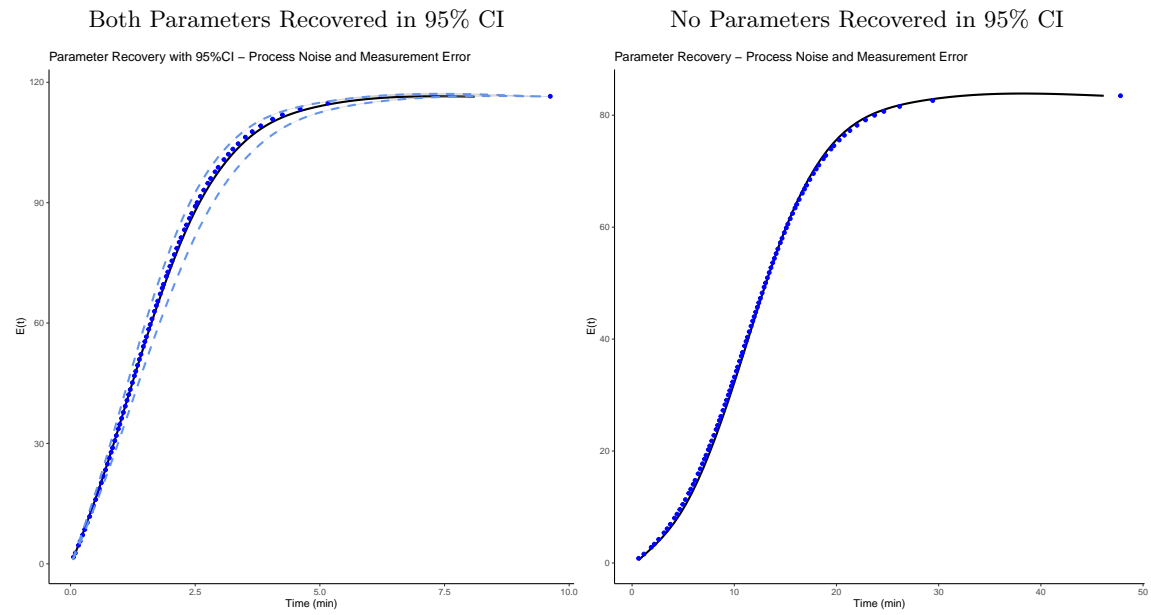
Linear - Parameter Recovery 95% CI



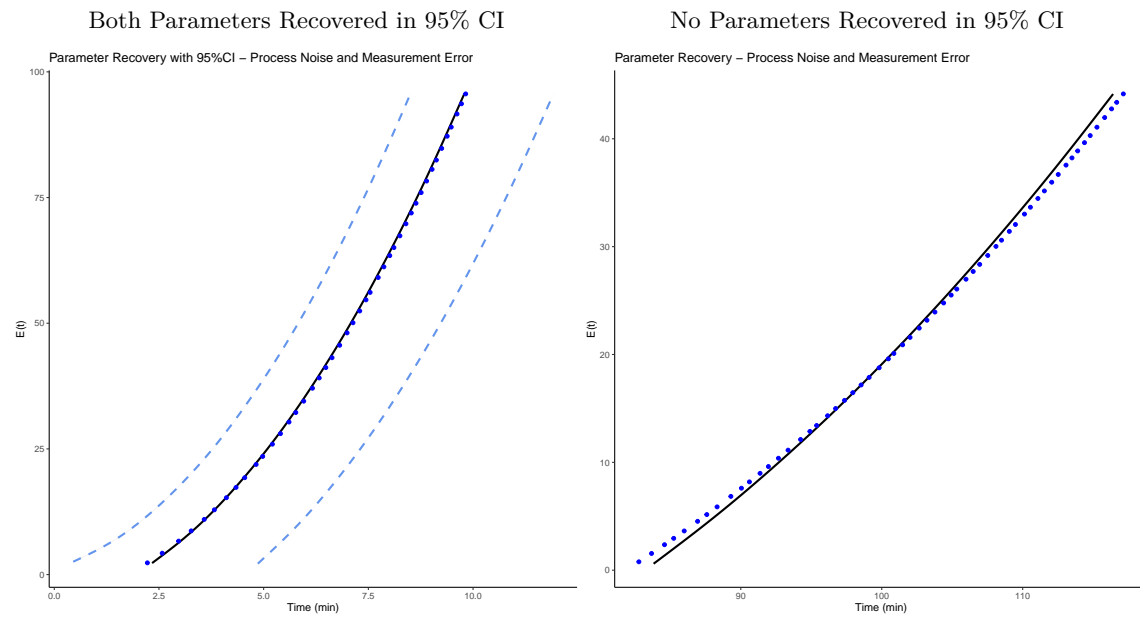
Quadratic - Parameter Recovery 95% CI



First Principles Model - Process Noise



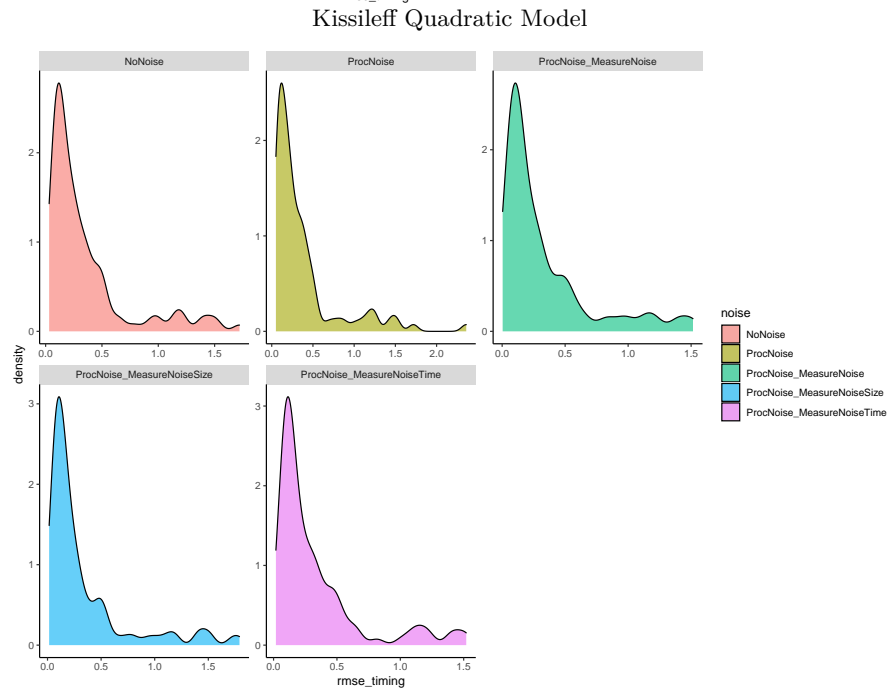
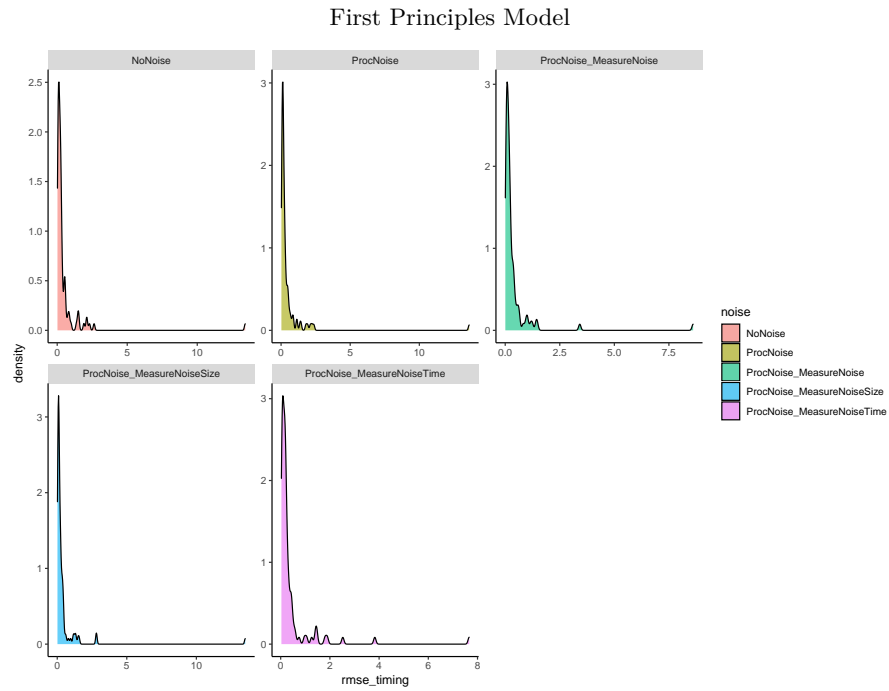
Kissileff Quadratic Model - No Noise



6 Root Mean Square Error

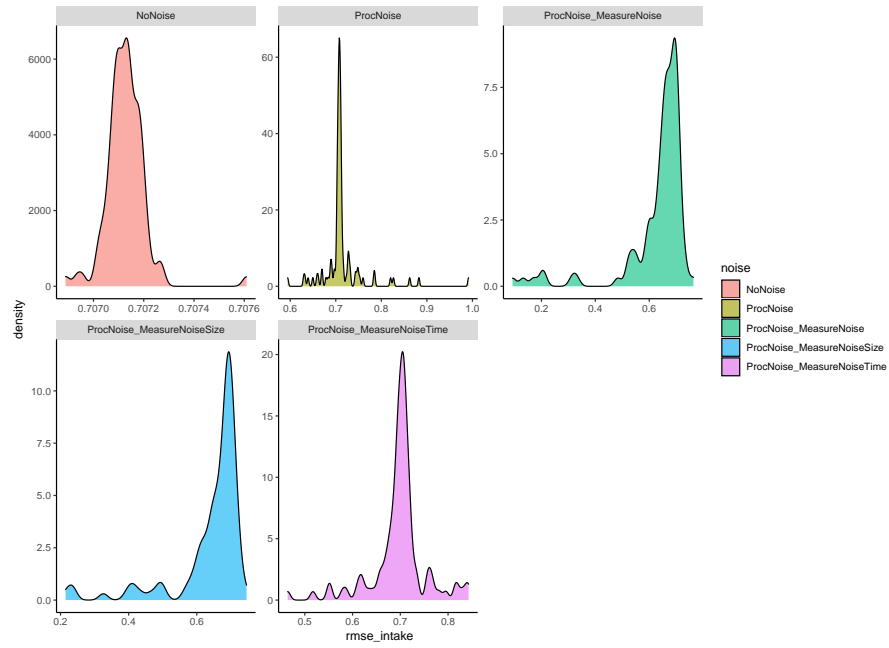
6.1 Distributions

RMSE Distribution for Bite Timing

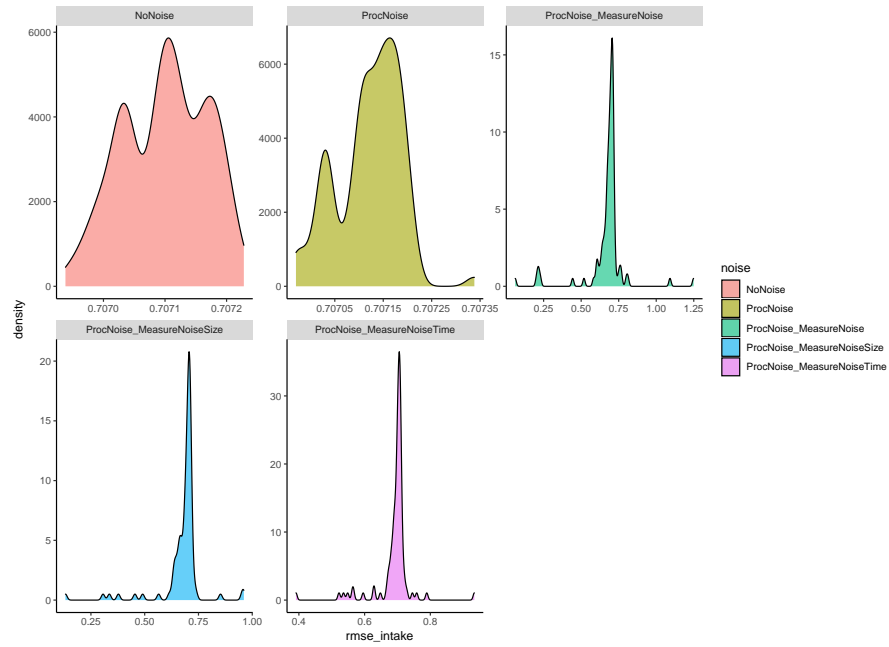


RMSE Distribution for Intake

First Principles Model



Kissileff Quadratic Model



6.2 number of NA's (estimated from recovered parameters)

Table 8: Number of random draws with an NAs in estimation of bite timing

	NoNoise	ProcNoise	ProcNoise_MeasureNoise	ProcNoise_MeasureNoiseSize	ProcNoise_MeasureNoiseTime
N	325	330	345	365	335
Y	175	170	155	135	165

Table 9: Average number of bites with NA timepoints (limited to the those with NAs)

	x
NoNoise	1.971429
ProcNoise	2.147059
ProcNoise_MeasureNoise	1.967742
ProcNoise_MeasureNoiseSize	2.296296
ProcNoise_MeasureNoiseTime	1.969697

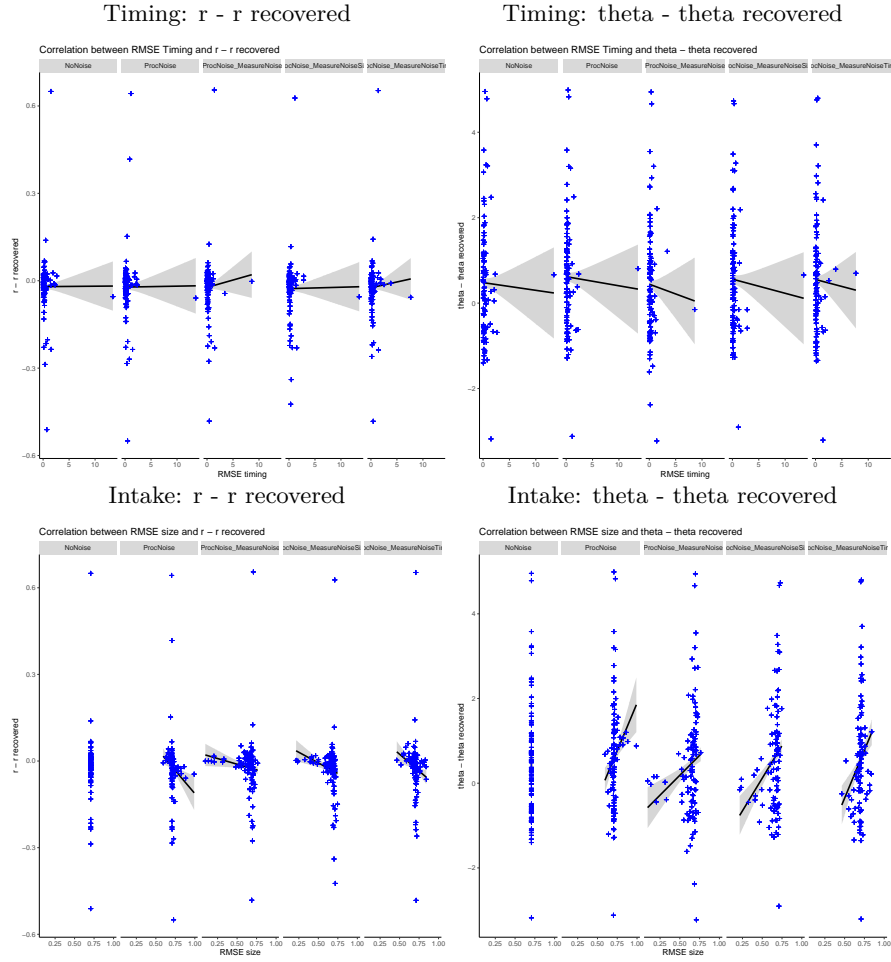
Table 10: Range in the number of bites with NA timepoints (limited to the those with NAs)

	x
NoNoise	1, 5
ProcNoise	1, 6
ProcNoise_MeasureNoise	1, 5
ProcNoise_MeasureNoiseSize	1, 6
ProcNoise_MeasureNoiseTime	1, 6

6.3 Correlation between Parameter Recovery Difference and RMSE

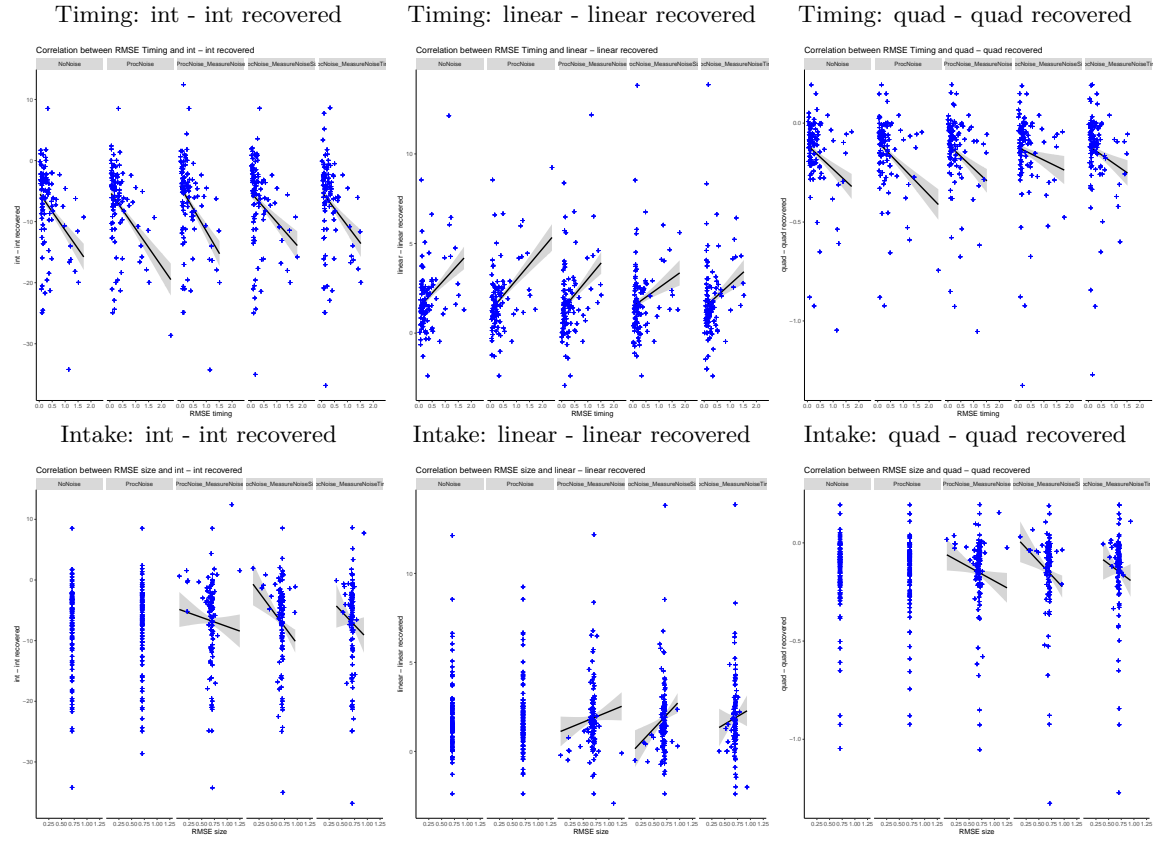
6.3.1 First Principles Model

RMSE correlation with parameter - recovered



6.3.2 Kissileff Quadratic Model

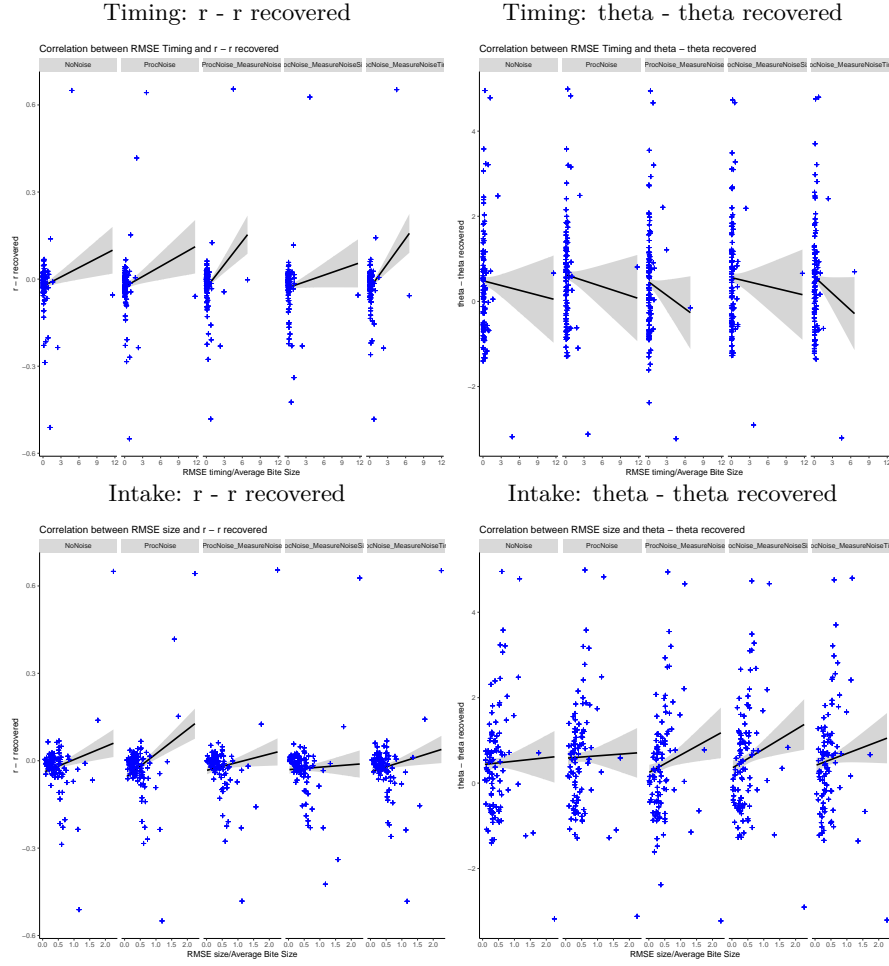
RMSE correlation with parameter - recovered



6.4 Correlation between Parameter Recovery Difference and RMSE/Avg Bite Size

6.4.1 First Principles Model

RMSE correlation with parameter - recovered



6.4.2 Kissileff Quadratic Model

RMSE correlation with parameter - recovered

