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Highlights

- We develop an approach to analyze the disruption of community structures in networks
- We explore a measure of the resilience of these structures based on community similarity before and after disruption
- We apply the approach to study community structures in an electric power network

QUANTIFYING THE RESILIENCE OF COMMUNITY STRUCTURES IN NETWORKS

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Abstract

Many networks contain community structures, or collections of densely connected nodes with sparse connections to other dense groups in the network. Communities may coalesce for a number of reasons, including friendships in a social network, physical connections in an infrastructure network, or spatial distribution in a neighborhood. Several approaches have been proposed to identify communities and compare the partition of networks into communities. This work explores community structures from the perspective of their resilience, or their ability to withstand degradation in network performance and recover to a desired level of network performance. In this context, network performance is defined as the similarity of a network partition (or the characterization of the network into community structures) formed after the disconnection of one or more links to the initial partition. This work provides an approach to measure how the initial set of community structures survive after a disruption and how these structures return after restoration commences. The approach is illustrated with an electric power network case study.

Keywords

Resilience, vulnerability, networks, partitions

1. Introduction and Motivation

In recent years, the concept of resilience, or the ability to withstand, adapt to, and recover from a disruption, has been discussed, measured, and modeled in a variety of perspectives including social science[1]-[3] and engineering[4]-[7], as well as the relationship of the two perspectives[8],[9]. Resilience-related work published in archival journals has increased significantly in the last decade[10], and guidance and policy dealing with resilience has grown in the government sector as well[11],[12].

Among the planning documents by government agencies on resilience is a particular recent emphasis on the resilience of communities, or networks of socially connected individuals[13],[14], after a disruptive event. The National Academies of Science[15] suggest “One way to reduce the impacts of disasters on the nation and its communities is to invest in enhancing resilience […].” The National Institute for Standards and Technology[16] defines community resilience as “the ability of a community to prepare for anticipated hazards, adapt to changing conditions, and withstand and recover rapidly from disruptions.” Measuring and quantifying this ability to prepare, adapt, withstand, and recover is vital to planning for and implementing community resilience. And doing so requires the identification of how communities are defined and how they emerge.

Quantifying the resilience of community structures in networks is a first step toward quantifying community resilience. Communities are often thought of as entities who group together due to commonalities, such as interests or geography[17]. A more general, network-centric definition is offered by Porter et al.[18] as a network structure consisting of a group of nodes that are “relatively densely connected to each other but sparsely connected to other dense groups in the network.” These two concepts of community, one from a social science perspective and one from a network science perspective, are congruent: connectivity is a matter of how relationships are defined. Communities of scholars[19] and actors[20] emerge from thousands of entities based on collaborative relationships. Neighborhood communities are based on geographical relationships[21]. In general, these systems are modeled using the concept of a network, where scholars, actors, are represented by nodes and their relationships as links. At the end, no matter the cohesiveness concept used, a set of initial communities are derived. We distinguish the resilience of the set of initial community structures as being quantified from a network-centric perspective (i.e., the ability of the set to withstand and recover from perturbations), whereas community resilience is a social science realization of that network structure. We focus on the former in this paper, and the latter could be addressed by applying our proposed approach to social networks and studying what measures could be implemented to reduce social vulnerability to disrupted (network-centric) community structures.

As such, this work analyzes communities from a network perspective and studies relevant community resilience phenomena. Resilience is modeled as a function of the similarity of the initial partition of the network (i.e., the community structure) before and after a disruption, suggesting that community structures that stay intact and recover quickly after a disruption are more resilient. As such, resilience depends on the structure and characteristic of the network and its partition in communities (i.e., our main analysis refers to the partition of the network as a whole). Note that, in some cases, communities could adjust their internal structure to cope with a disruptive event. Such analysis at the community level is also possible but requires the definition of a different performance function. For example, the authors of [22] consider the effects of a disruption on the performance of the communities in a network, where the assessment is based on a performance function simulating the electricity load in each community as well as the interaction among communities.

While the example illustrated here deals with the topology of an electric power network, the work deals with the general definition of community structures, no matter how they are derived, and the general approach to modeling their resilience, thus, a variety of communities could be described.

The remaining of the paper is as follows: Section 2 offers background to a resilience modeling technique developed by the authors as well as background on community structures in networks. The proposed approach to model community resilience is discussed in Section 3, with a case study of network behavior under disruption is provided in Section 4. Concluding remarks are given in Section 5.

2. METHODOLOGICAL BACKGROUND

This section offers background on the approaches to quantify resilience and identify community structures in networks.

2.1. Modeling Resilience

Fig. 1 is a graphical depiction[23]-[25] of the performance of a system before, during, and after a disruptive event, e^l. The performance of the system is measured over time with function $\varphi(t)$, which reduces after the disruptive event suggesting that a larger value of $\varphi(t)$ is desired (e.g., flow along a network, utilization of an asset, well-being of entities). Fig. 2 depicts a system whose performance measure increases after a disruption (e.g., count of entities without service, delays in flow, unsatisfied customers).

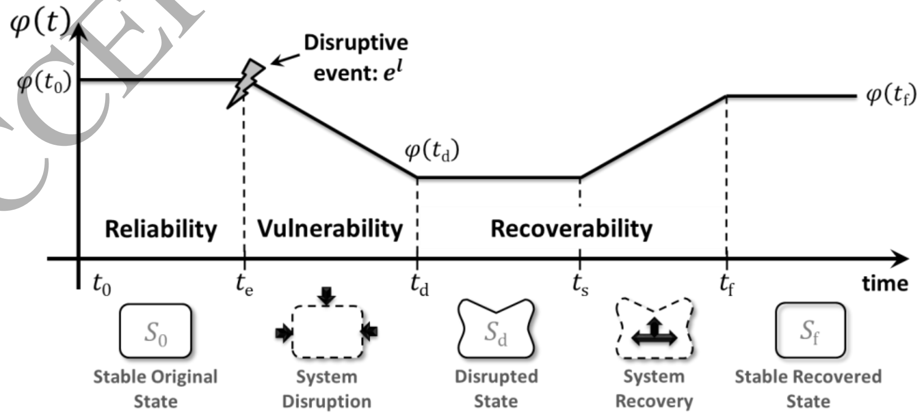


Fig. 1. Graphical depiction of decreasing system performance, $\varphi(t)$, across several state transitions over time[23].

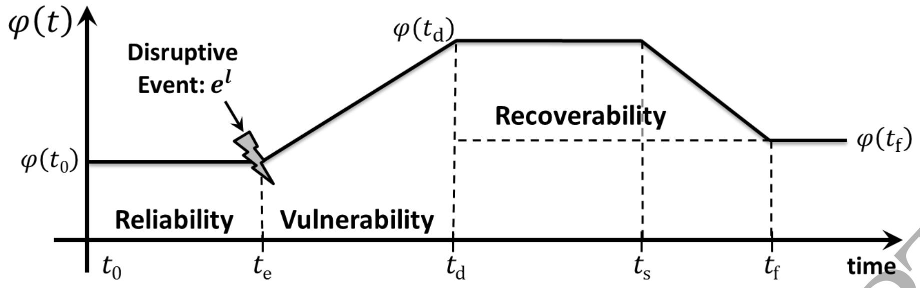


Fig. 2. Graphical depiction of increasing system performance, $\varphi(t)$, over time[25].

Three dimensions of resilience are exhibited in Figs. 1 and 2: reliability, vulnerability, and recoverability. In the *reliability* dimension, the steady-state behavior of the system is exhibited in the time interval $t_\mathrm{e} - t_0$. The behavior of the system prior to the disruptive event is typically described with reliability theory[26],[27], which provides models and techniques and measure the probability that under normal operating conditions, the common-cause failure time is greater than some value t, $R(t) = P(T > t), t \in (t_0, t_\mathrm{e})$. In the *vulnerability* dimension, the reduction in system performance in the interval $t_\mathrm{d} - t_\mathrm{e}$ to its disrupted state after the disruptive event[28]. Methods in this area are used to: (i) understand how disruptive events adversely affect the service function (e.g., analyzing probability that a disruptive event e^k does not affect the service function below some threshold b, $P(\varphi(t) > b|e^l)$, and (ii) identifying the components that are critical to the system (i.e., those components that, when degraded, have the most adverse effect on system performance)[26]-[33]. Finally, in the *recoverability* dimension, actions are taken during interval $t_\mathrm{f} - t_\mathrm{d}$ to restore system performance to a desired level (perhaps similar, better, or worse than $\varphi(t_0)$) in a timely fashion. Recent work in this area has offered optimization models to restore system performance[34]-[36].

A number of studies have described related resilience metrics. Cimellaro et al.[37] quantify resilience as the "normalized shaded area underneath" the function $\varphi(t)$. Similarly, Zobel[38],[39] analyzes the difference between steady-state performance and disrupted performance to measure system resilience to different events. Francis and Bekera[40] offer a similar measure assuming exponential recovery. Sterbenz et al.[41] provide a temporal description of resilience but no mathematical formulation, Nair et al.[42] provide a demand-based perspective, and Rose[43] analyzes at the economic impact of resilience.

Note that Figs. 1 and 2 depict situations where a disruptive event causes an undesirable decrease and an undesirable increase in system performance, respectively. However, situations could arise in non-coherent systems where, for example, disruptions actually improve performance (e.g., the removal of caustic links in a social network), but such work is not addressed here.

2.2. Modeling Resilience

Discussed previously, when nodes with similar characteristics have close connections, they form a *community*, and the connections between communities are relatively sparse. A simple illustration of three communities, from Rocco and Ramirez-Marquez[44], is provided in Fig. 3: Community A consists of {1,2}, Community B of {3,4,5}, and Community C of {6,7,8,9}.

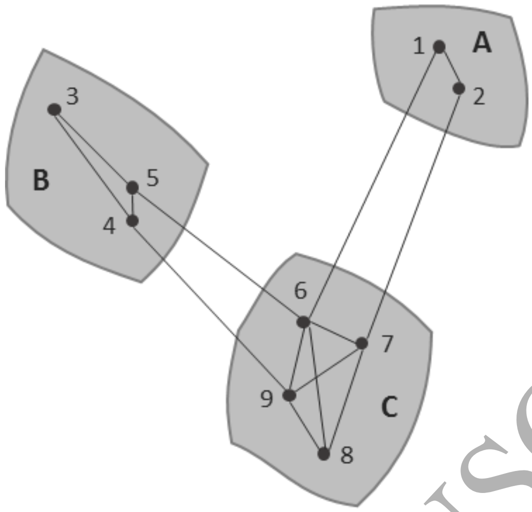


Fig. 3. An example partition of communities in a network[44].

Community detection techniques have been used by Rocco and Ramirez-Marquez[44] as a mechanism to identify the vulnerability of the different communities in a network, finding the most important nodes and links that when disconnected produce a disconnection of the network from a community. For example, in Fig. 3, links between communities A and C or between B and C, the inter-community links (ICL), are considered important for the connectivity of the network.

In general, studies on community structures consider only the topology of the network, also known as un-weighted networks. However, most real-world networks contain weighted edges[45],[46]. In general, such weights represent a physical characteristic of the link, such as distance, capacity, and reliability, among others. However, no matter its physical meaning, they are normalized to [0,1]. In an un-weighted network, a zero weight means that nodes are not connected, while a weight equal to one means that nodes are connected. It is important that the number of sets of communities can vary depend on whether weights are present and how they are defined[46]. In this paper, such weights correspond to the probability of failure of the link between two nodes. Such probabilities will allow for the simulation of disruptions in the network. From a community point of view, a link with low probability of failure suggests that its end nodes are likely more highly connected.

Several approaches have been proposed to identify communities in a network, and Fortunato[48] offers a survey of these approaches. In this work, the Fast Modularity algorithm[49] is used as it has the capability to handle weighted graphs and is available in the *igraph* library in the R platform. As mentioned in [44], the algorithm is based on the concepts of modularity "which attempts to measure how well a given partition of a network compartmentalizes its communities"[18]. This means that the partition with the highest modularity is selected as the best.

The disconnection of elements (nodes or links) due to failure, intentional attacks, or a change in the relationship among entities could generate a new partition that is different from the initial one. Several methods for measuring similarities or differences between community structures or partitions of a network have been proposed in the literature: the Rand index (RI)[50], the Adjusted Rand index (ARI)[51], the Normalized Mutual Information (NMI)[52], and the distance-based Variation Information (VI)[53], as well as the distance measures recently presented by Labatut[54]. Such measures could compare community structures before and after some disconnection in the network.

In general, measure SI(A,B) quantifies the similarity between two partitions A and B, or the similarity between the community structures A and B. Each SI(A,B) is normalized in [0,1], and its limits are interpreted depending on the index used. For example, for RI, ARI, and NMI discussed above, the value of 1 suggests perfect matching, while 0 indicates "no matching"[55]. The interpretation for VI is different, since VI is a distance measure. In this case SI(A,B) = VI(A,B) = 0 suggests that the two community structures are equal. In this paper the index SI(A,B) = 1 – VI(A,B) will be used, as the concept of perfect matching between communities is easy to understand. However, any of the above mentioned measures could be used for the similarity index, SI(A,B).

Note that the candidates for SI(A,B) listed above quantify a global measure of similarity between two partitions A and B, but they do not evaluate the communities that are common in both partitions. To address this, community similarity indices (CSI) have been defined, including the purity index[56] and the recall score[57]. The comparison of a specific community C_a with the partition $P_b = \{C_{1b}, C_{2b}, \ldots, C_{rb}\}$ found after the disconnection of one or more components basically requires the count of the proportion of nodes shared by C_a and each of the community in P_b. The purity index, which is the community similarity index used in this paper, represents the maximum fraction of nodes of C_a shared with any community in P_b. The values of the index are in [0,1]. If the index is equal to 1, it means that there is at least one community in P_b that is identical to C_a.

3. Proposed Approach

The proposed approach for measuring the resilience of community structures is discussed in this section.

3.1. Notation for a Network, its Communities, and its Disruption

Let $G(N, A, W)$ define a network where N represents the set of nodes, $i \in N$, $i = 1, \ldots, n$, and A represents the set of links, $A = \{(i,j) | a_{ij} = 1\}$, with the cardinality of $|A| = m$ and $a_{ij} = w_{ij}$, where $w_{ij} \in W$ represents the probability that two nodes maintain its relationship after a disruption.

Let W_0 represent the set of initial weights defined for each link in A, and let P_0 be the initial partition, or the initial group of communities, of the network, derived using a selected approach. Let C_k, $k = 1, \ldots, K$ represent the kth community in the network with $C_k \cap C_q = \emptyset$ for all $k \neq q$ and $\cup\, C_k = N$. Let P_l be the partition corresponding to the set of weights W_l generated when a set of links D_l is disconnected (the partition P_l is derived using the same selected approach). In this case, the weights corresponding to disrupted links are set of 0. It is assumed that all of the disconnections occur simultaneously.

Assume that the performance of the network $\varphi(t) = \mathrm{SIM}_l = \mathrm{SI}(P_0, P_l)$ is defined as the similarity between the initial partition P_0 and the partition P_l generated after the disruption. That is, network performance is defined in a sense as the "normalcy" of the community structures in the network after the disruption, and recoverability is measured as the return to this normalcy. The similarity index chosen here is the Variation Information index, or more specifically $1 - VI$, though any similarity index could be chosen.

Let $\mathbf{SR}_l$ be the $|D_l| \times 1$ vector listing sequence of the links to be restored Let $\mathbf{T}_l$ be the $|D_l| \times 1$ vector of restoration time for each of the links in D_l. Let $\mathbf{TR}_l$ be the $|D_l| \times 1$ vector containing the points in time at which each link restoration has completed, which depends on the restoration strategy selected (e.g., in series, all at once, among others).. Note that the last element (the $|D_l|$th element) of $\mathbf{TR}_l$ represents the measure *time to complete restoration of the network*, or TTCR. After the jth disrupted link is restored, the partition of the network is regenerated and referred to as P_{lj}. Finally, define $\mathrm{SIMR}_{lj} = \mathrm{SI}\left(P_0, P_{lj}\right)$ as the restored similarity measure between initial partition P_0 and P_{lj}, which measures how the return of the network to a partition that is approaching P_0.

Several measures of vulnerability and recoverability could be derived to describe the resilience of the community structure of a network after a disruption incapacitates a set of links D_l based on: (i) the effects of the disconnection of D_l and the process of restoration, and (ii) the robustness of the initial partition and its communities.

3.2. Measuring the Disconnection and Restoration of Links

The following procedure, titled *Link Assessment*, evaluates the effects of disconnecting set of links D_l by evaluating the similarity between the initial partition and the partition derived after the disconnection and during the restoration process.

Procedure Link Assessment

Input: $G = (N, A, W_0)$, P_0, W_l, T_l, TR_l, SR_l, D_l, and a selected SI index
Output: SIM_l = Similarity between P_0 and P_l, SIMR_{lj} = Similarity between P_0 and each P_j generated due to the restoration process ($j = 1, \dots, |D_l|$)

Derive P_l *# initial partition using* W_0
Derive P_l *#partition corresponding to* W_l, *whose* k*th element* $w_{ik} = 0$ *indicates that link* k *is disconnected*
$\mathrm{SIM}_l = \mathrm{SI}(P_0, P_l)$ *#similarity between* P_0 *and* P_l *using the chosen SI index*
Determine which community in P_0 has been affected by the disruption with W_l

#restoration process
For $j = 1, \dots, |D_l|$
{
$r = SR_{lj}$ *#select the* j*th link to be restored (the* j*th element) from* $\mathbf{SR}_l$
$w_{lr} = w_{0r}$ *#assign the original weights for link* r
Derive P_{lj} *#partition corresponding to the updated* W_l
$\mathrm{SIMR}_{lj} = \mathrm{SI}(P_0, P_{lj})$ *#similarity between* P_0 *and* P_{lj}
}
#end restoration process

end Procedure Link Assessment

The time for restoration of each disconnected link could also be considered as a random variable. In this case, the elements of $\mathbf{TR}_l$ and $\mathbf{SR}_l$ are also random variables and can be used to derive the approximate distribution of TTCR. To this aim, the *#restoration process* in the Link Assessment procedure could be repeated *NT* times.

3.3. Measuring the Robustness of the Initial Partition

The procedure *Network Assessment* evaluates the effects of disconnection of random sets of links by evaluating the robustness of the initial partition and its communities. This procedure uses a simple Monte Carlo approach (of *NSIMUL* samples) based on the recent work of Rocco et al.[46].

Procedure Network Assessment

Input: $G = (N, A, W_0)$, $NSIMUL$, and a selected CSI index
Output: **Rob** = Vector of community robustness associated with P_0

Derive P_0 and its base communities $C_0 = \{C_{0,1}, C_{0,2}, \dots, C_{0,|C_0|}\}$ *#initial partition using W_0*

For $l = 1, \dots, NSIMUL$
{
For $k = 1, \dots, m$
{
 # generating the set of weights W_l whose kth element $w_{lk} = 0$ indicates that link k is disconnected
$w_{lk} = w_{0k}$
unif = UNI[0,1]
if unif > w_{0k} then $w_{lk} = 0$
}

Derive P_l *#partition corresponding to W_l*

For $j = 1, \dots, |C_0|$
{
Determine CSI(C_0, j, P_l)
}
}

end Procedure Network Assessment

From CSI(C_0, j, P_l), Rob$_j$, ($j = 1, \dots, |C_0|$), the jth average community robustness (and jth element of the **Rob** vector) can be calculated across the $NSIMUL$ simulated disruptions. Each Rob$_j$ represents the percentage of maximum nodes that each base community $C_{0,j}$ had shared during the simulation. For example, if Rob$_j = 1$, then the base community $C_{0,j}$ has been always generated during the simulation, suggesting that this community is not sensitive to random link disconnections. A critical level, say CL, could be defined to determine which communities would be considered robust. For example, if $CL = 0.95$, then all of the communities with Rob$_j \geq CL$ would be considered robust. Also calculated is the cumulative distribution of time to complete restoration, TTCR.

Since all of the outputs reported correspond to estimated proportions (e.g., proportion of nodes shared by C_a and each of the community in P_b), the maximum error, for a 95% confidence level, is approximately $1/\sqrt{NSIMUL}$.

4. Illustrative Example: Community Structure in Electric Power Networks

The topology and community structure of the Italian 380 kV power transmission grid is analyzed. The network, illustrated in Fig. 4, consists of 127 nodes and 171 links[58].

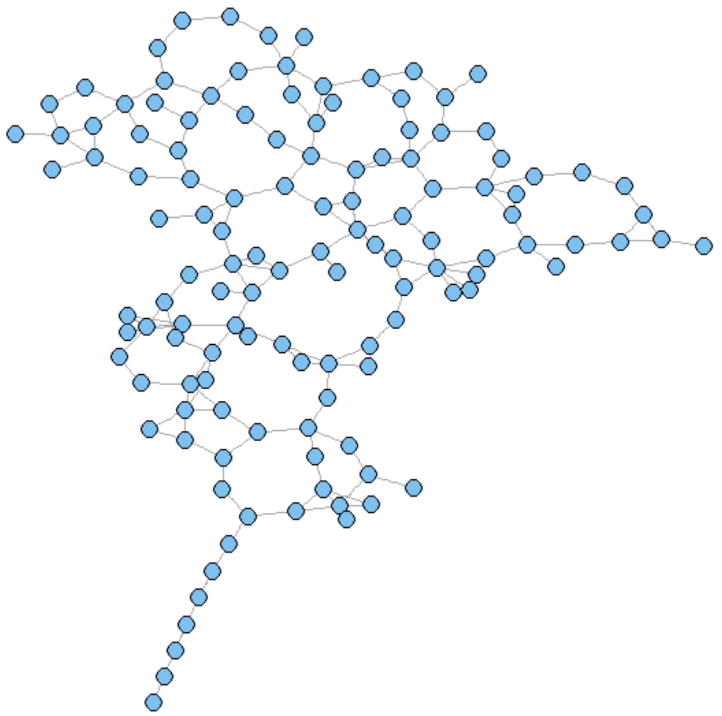


Fig. 4. Topology of the Italian high-voltage (380 kV) power transmission network[58].

The weight vector, W, represents link reliability. The assumptions that were made to illustrate the proposed approach are as follows:

- The partitions of the network only consider the topology of the electric power network.
- No electric phenomena is considered.
- The network topology is free of error.
- The initial probability of disconnection of each link is $w_{0,ij} = 0.01$.
- The SI used is the $1 - VI$ index.
- The CSI used is the Purity index.
- *NSIMUL* is fixed at 10000 (so the maximum error on estimated proportion is 0.01 (at 95% confidence level))
- *NT* is fixed at 1000.
- The time for the link restoration is modeled as a discrete uniform distribution UNI[5,20].
- Links are restored in order of increasing length of restoration time.
- Restoration is performed in series. However, if the times to restore two or more links are equal, then the restoration actions are performed simultaneously.
- Community detection is performed using the algorithm Fast Modularity[49], available in *igraph* 0.7.2 library in the R platform.

Fig. 5 depicts the set of 11 communities detected. There are 25 inter-community links (ICL) connecting the communities, displayed with a thicker line in Fig. 5.

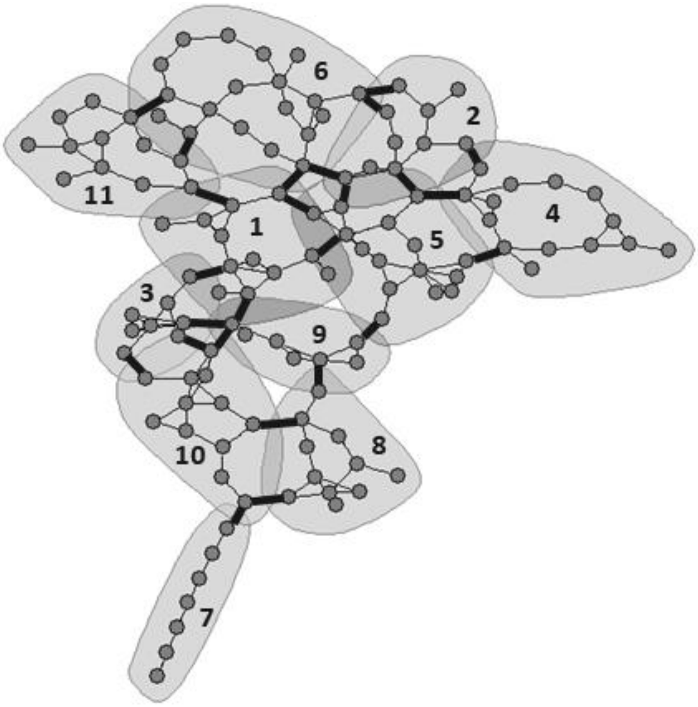


Fig. 5. The 11 communities detected in the 380 kV Italian network.

4.1. Analysis of Link Disconnection

Fig. 6 shows the distribution of the links disconnected during the disruption simulation. Given the link failure probability of 0.01 in the simulation, in more than 30% of the cases, no links were removed and the maximum number of simultaneously failed links is seven.

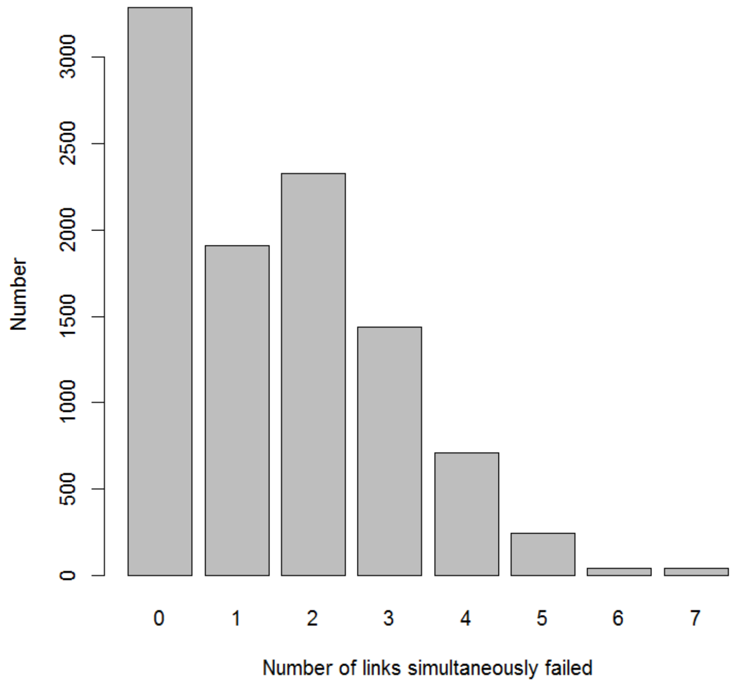


Fig. 6. Count of number of links simultaneously disconnected.

Fig. 7 illustrates how often a link impacts at least one community. For example, link 99 appears 220 times in a disruptive event that led to at least one community being altered, more than any other link. Link 56 occupies the second position with 172 appearances. Fig. 8 shows the location of links 56 and 99 (with more thickly shaded links), which belong to communities 6 and 1, respectively.

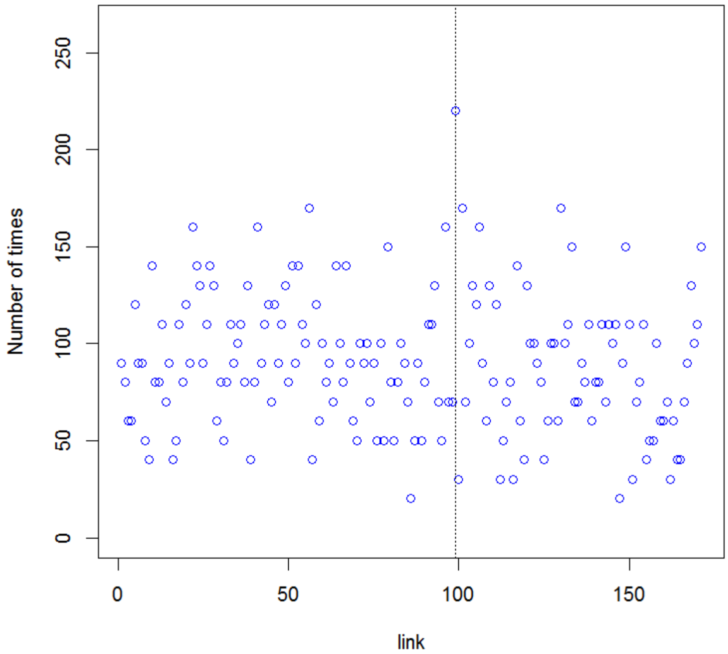


Fig. 7. Frequency of a link impacting at least one community

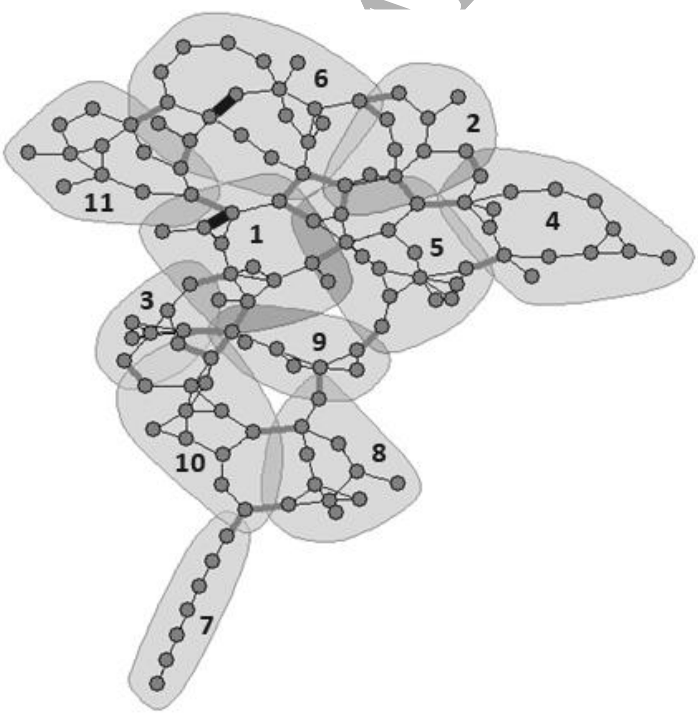


Fig. 8. Location of link 56 (Community 6) and link 99 (Community 1) with thickly shaded lines.

Table 1 provides the percentage of times the initial communities changes due to link disconnections (i.e., at least one node of the initial partition is not included in any simulated partition). Community 8 is the least affected, while Communities 1 and 6 are the most affected, due to the influence of links 56 and 99.

Fig. 9 depicts generally how often the initial communities are simultaneously changed by link disconnections. More than 30% of simulated disruptions did not affect any communities since no links were disconnected, as shown in Fig. 6. In only 10 simulated disruptions were seven communities affected simultaneously, and no more than seven communities were ever affected simultaneously. This suggests that the network is relatively robust to disconnections that alter several community structures at once (given the manner in which disruptions were simulated), implying that the network can keep most of its initial community structures intact.

Table 1. Effect of link disconnection on each community.

Community	Percentage frequency affected
1	23.9
2	19.8
3	11.8
4	8.5
5	16.6
6	23.9
7	6.4
8	3.5
9	4.0
10	10.9
11	6.1

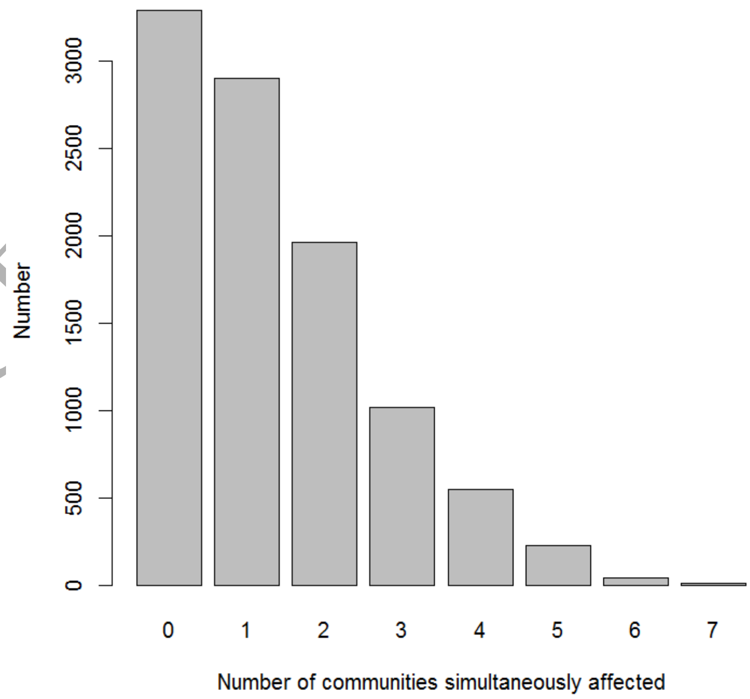


Fig. 9. Number of communities simultaneously affected by link disconnections.

Table 2 provides the robustness values for each of the initial 11 communities in P_0. The robustness value represents the estimated frequency for a community not being affected by disruptive events. Note that only Community 8 has 100% robustness, suggesting that the nodes of this community have always belonged to the same community during all of the simulated disruptive events.

Mentioned previously, a critical level (CL) could be defined to determine which communities achieve a certain level of robustness. For example, if CL $\geq$ 0.95, then eight communities are considered as robust, as depicted in Fig. 10.

The concept of robustness is different from the concept related to the vulnerability of a community presented in [44], where the authors evaluated how a community is vulnerable to become disconnected. In this case, communities that are in the periphery of the network (more isolated) tend to be the most vulnerable, since they have fewer interconnections with other communities.

Table 2. Robustness values for the initial 11 communities.

Community	Robustness
1	0.93
2	0.92
3	0.96
4	0.98
5	0.96
6	0.94
7	0.98
8	1.00
9	0.99
10	0.97
11	0.99

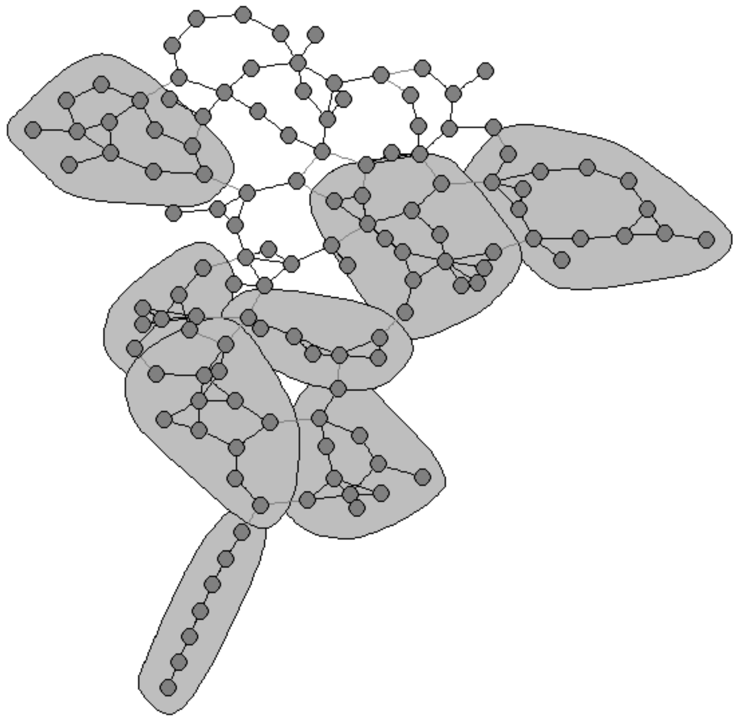
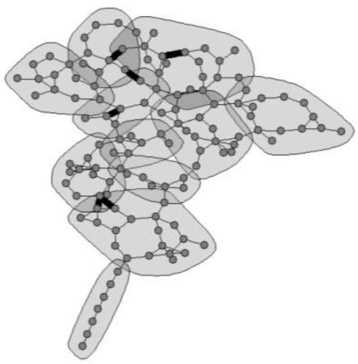


Fig. 10. Communities considered as robust for CL ≥ 0.95.

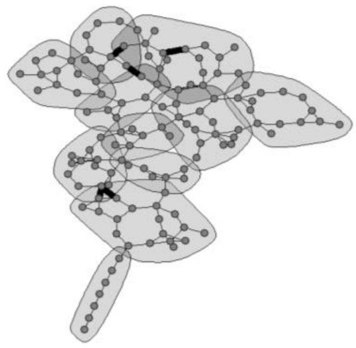
4.2. Analysis of Link Restoration

To illustrate the restoration process, consider disruption l = 566 which disconnected seven links: $D_{566} = \{17, 21, 29, 32, 41,136,137\}$. This particular disruption was among the 10 disruptions that altered the structure of seven communities simultaneously, shown in Fig. 9. Seven restoration times, one for each of the disconnected links, was generated using UNI[5,20] to produce $\mathbf{T}_{566} = \{17,8,12,13,5,9,12\}$. It is assumed that the restoration activities commence at $t = t_s$, as suggested in Fig. 1. Given the restoration heuristic of "shortest repair time first," the sequence of the links to be restored is $\mathbf{SR}_{566} = \{41, 21, 136, 137, 29, 32, 17\}$, and the time at which each link restoration is completed is $\mathbf{TR}_{566} = \{5, 13, 22, 34, 34, 47, 64\}$. This suggests that link 41 is restored first at time $t_s + 5$, the second restored link is link 21 at time $t_s + 13$, and so on. Note that at time $t_s + 34$, links 137 and 29 are restored, and the network is restored at $t_s + 64$.

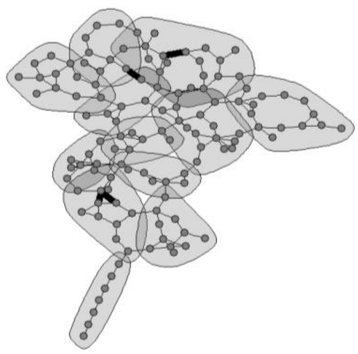
Fig. 11 illustrates the change in community structures as restoration sequence $\mathbf{SR}_{566} = \{41, 21, 136, 137, 29, 32, 17\}$ occurs (thicker links highlight the links yet to be restored). A comparison of Fig. 5 with Fig. 11a, when link 41 is restored, shows that some communities remain the same while others change (e.g., the two communities at the top of Fig. 11a have grown in size). This causes the similarity with the original partition to decrease. Note that there is no variation in the communities when comparing Fig. 11a (when link 41 is restored) to Fig. 11b (when links 41 and 21 have both been restored), suggesting that the restoration of link 21 did not improve the restoration of the community structure back to P_0, as link 21 is an "internal" link in one of the communities. This behavior is also demonstrated in Fig. 12, where the trajectory of $1 - \text{VI}$ does not improve with the restoration of link 21. This has implications for recovery sequence strategies: perhaps a "shortest link restoration time first" heuristic strategy could be improved.



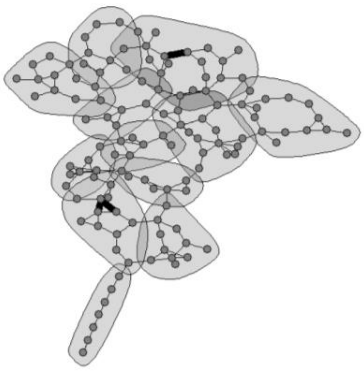
(a) repair of link 41



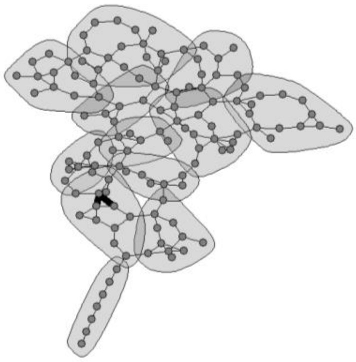
(b) repair of link 21



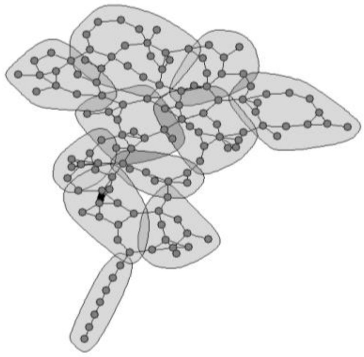
(c) repair of link 136



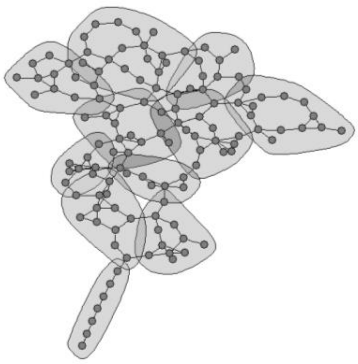
(d) repair of link 137



(e) repair of link 29



(f) repair of link 32



(g) repair of link 17

Fig. 11. Evolution of community structures after the restoration of each of the disconnected links, with the return to original partition P_0 with the restoration of link 17.

Fig. 12 illustrates the behavior of the performance function, $\varphi(t) = \mathrm{SIM}_l = 1 - \mathrm{VI}$, for disruption D_{566} over time assuming restoration governed by $\mathbf{T}_{566}$, $\mathbf{SR}_{566}$, and $\mathbf{TR}_{566}$. In this case, the network is completely restored at $t = \mathrm{TTCR} = t_s + 64$. Again, this is a depiction of how the network returns to its initial community structure of 11 communities as restoration proceeds. Note that following the restoration of link 41, the performance degrades, That is, the network under the performance function defined can be considered as non-coherent.

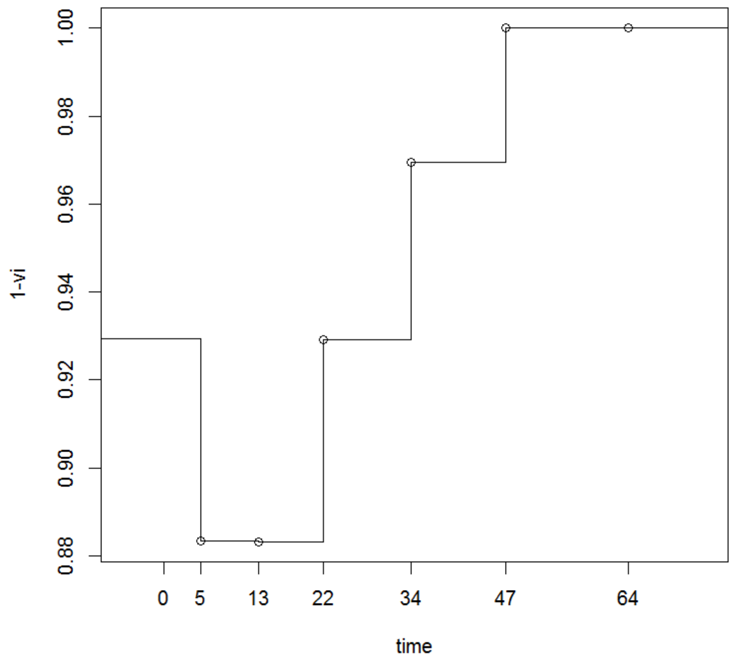


Fig. 12. Restoration trajectory for the similarity index for D_{566}.

Alluded to previously, perhaps a different link restoration strategy could restore the initial community structure more quickly. Perhaps a planner is interested in the trajectory of $1 - \mathrm{VI}$ to be more steep earlier in the restoration process, which may not be the result of the "shortest link restoration time first" heuristic. If the time to restore is used as additional criterion, then the best (more quickly restored sequence $\{136,29,32,41,137,17,21\}$) and worst (sequence $\{41,32,137,21,17,136,29\}$) case restoration sequences to recover from D_{566} are shown in Fig. 13. For this small example the 7! possible restoration sequences could be compared fairly easily, though a heuristic would need to be developed for larger cases.

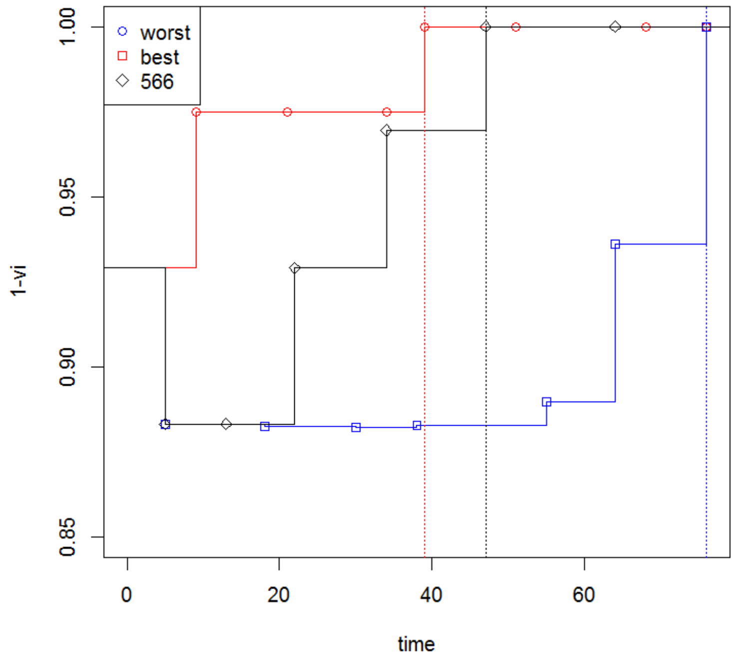


Fig. 13. Best (red line) and worst (blue line) case restoration trajectories for the similarity index for D_{566}, as well as the trajectory illustrated previously.

Finally, Fig. 14 illustrates the cumulative distribution of TTCR for disruption D_{566} when $\mathbf{T}_{566}$ (and, thus, $\mathbf{SR}_{566}$ and $\mathbf{TR}_{566}$) are considered random variables. The *NT* simulations result in a TTCR ranging from 25 to 103 time units, illustrating how variability in link restoration time and restoration sequence can impact network restoration time.

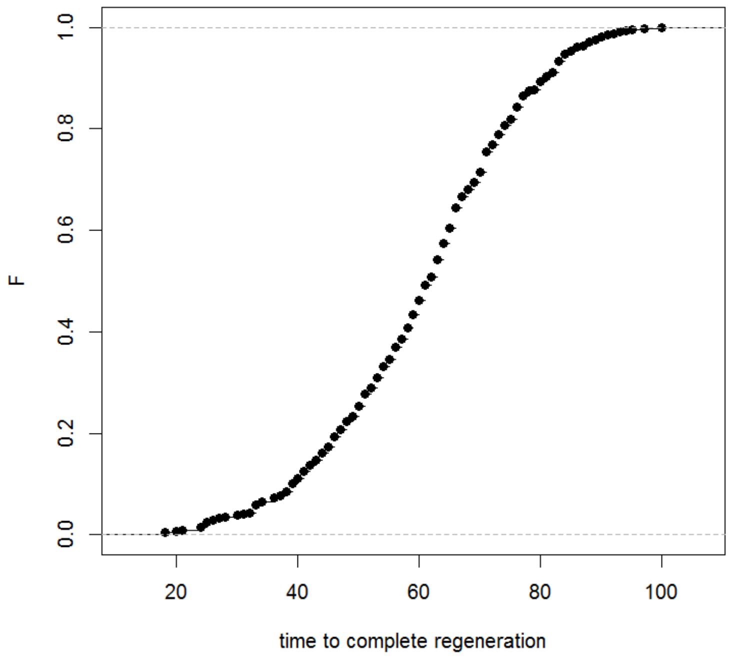


Fig. 14. Cumulative distribution of TTCR for D_{566}.

5. Concluding Remarks

Many networks contain community structures, or collections of densely connected nodes with fewer connections between communities. Communities may coalesce for a number of reasons, including friendships in a social network, physical connections in an infrastructure network, or spatial distribution in a neighborhood. Several approaches have been proposed to identify communities[48],[49], compare the partition of networks into communities[50],[53], and measure the vulnerability of communities to disruption[44],[46].

This work explores community structures from the perspective of their resilience, or their ability to withstand and recover from a disruption, defined as the disconnection of one or more links. The resilience of a system is graphically depicted as the degradation and subsequent recovery of the performance of the system, and in this work, system performance is defined as the similarity of a partition (or set of communities) formed after a disruption to the original partition. This provides a measure of how community structures survive after a disruption, and how these structures return after restoration commences.

This work offers insights into choosing a restoration sequence, comparing the trajectories of recovery for different strategies and providing an understanding of how uncertainty in restoration time impacts network restoration time.

Future work will explore more rigorous algorithms for restoration optimization, and develop tradeoffs for investments in vulnerability and recoverability for spatially located disruptions. Further, while community structures exist in many different kinds of networks, including the electric power network discussed here, such structures have larger implications in social interactions and communities of people. This initial exploration enables a quantitative extension of the methods described here applied to study the resilience of community structures in society.

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