Thesis, Chap 1: pair approximation: the 4 state model

- We have 4 states (n, p, 0, $-$).
- We have ten different pairs (ρ_{nn}, ρ_{n0}, ρ_{np}, ρ_{n-}, ρ_{pp}, ρ_{p0}, ρ_{p-}, ρ_{00}, ρ_{0-}, ρ_{--}) since $\rho_{\sigma\sigma'} = \rho_{\sigma'\sigma}$.
- We have five conservation equations:

1. $\rho_n + \rho_p + \rho_0 + \rho_- = 1$
2. $\rho_n = \rho_{nn} + \rho_{n0} + \rho_{np} + \rho_{n-}$
3. $\rho_0 = \rho_{00} + \rho_{n0} + \rho_{p0} + \rho_{0-}$
4. $\rho_p = \rho_{pp} + \rho_{p0} + \rho_{np} + \rho_{p-}$
5. $\rho_- = \rho_{--} + \rho_{0-} + \rho_{n-} + \rho_{p-}$

- There are 4 singleton variables: ρ_n, ρ_p, ρ_0, ρ_-

So we need $10 + 4 - 5 = 9$ equations to solve this system:

- $\frac{d\rho_{nn}}{dt}$
- $\frac{d\rho_{n-}}{dt}$
- $\frac{d\rho_{np}}{dt}$
- $\frac{d\rho_{pp}}{dt}$
- $\frac{d\rho_{p-}}{dt}$
- $\frac{d\rho_{--}}{dt}$
- $\frac{d\rho_{n}}{dt}$
- $\frac{d\rho_{p}}{dt}$
- $\frac{d\rho_{-}}{dt}$

For clarity, here is the list of replacement rules:

- $\rho_0 = 1 - \rho_n - \rho_p - \rho_-$
- $\rho_{n0} = \rho_n - \rho_{nn} - \rho_{np} - \rho_{n-}$
- $\rho_{p0} = \rho_p - \rho_{pp} - \rho_{np} - \rho_{p-}$
- $\rho_{0-} = \rho_- - \rho_{--} - \rho_{n-} - \rho_{p-}$

Equation system:

$$\begin{aligned}\frac{d\rho_{nn}}{dt} =& \rho_{n0}(\delta\rho_n + \frac{(1-\delta)}{z} + (1-\delta)\frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0})(b - \frac{c_n}{z} - c_n\frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0} - c_{pn}\frac{(z-1)}{z}\frac{\rho_{p0}}{\rho_0}) \\ & - 2\rho_{nn}m\end{aligned}$$

$$\begin{aligned}\frac{d\rho_{n-}}{dt} =& \rho_{n0}d + \rho_{0-}(\delta\rho_n + (1-\delta)\frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0})(b - c_n\frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0} - c_{pn}\frac{(z-1)}{z}\frac{\rho_{p0}}{\rho_0}) \\ & - \rho_{n-}m - \rho_{n-}(r + f(\frac{\rho_{n0}}{\rho_0} + \frac{\rho_{p0}}{\rho_0})\frac{(z-1)}{z})\end{aligned}$$

$$\begin{aligned}\frac{d\rho_{np}}{dt} =& \rho_{n0}(\delta\rho_p + (1-\delta)\frac{(z-1)}{z}\frac{\rho_{p0}}{\rho_0})(b - \frac{(z-1)}{z}c_p\frac{\rho_{p0}}{\rho_0} - \frac{c_{np}}{z} - \frac{(z-1)}{z}c_{np}\frac{\rho_{n0}}{\rho_0} - g(1 - \frac{n}{z} - \frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0}n))\\ &+ \rho_{p0}(\delta\rho_n + (1-\delta)\frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0})(b - c_n\frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0} - \frac{c_{pn}}{z} - c_{pn}\frac{(z-1)}{z}\frac{\rho_{p0}}{\rho_0})\\ &- 2\rho_{np}m\end{aligned}$$

$$\begin{aligned}\frac{d\rho_{pp}}{dt} =& \rho_{p0}(\delta\rho_p + \frac{(1-\delta)}{z} + (1-\delta)\frac{(z-1)}{z}\frac{\rho_{p0}}{\rho_0})(b - \frac{c_p}{z} - \frac{(z-1)}{z}c_p\frac{\rho_{p0}}{\rho_0} - \frac{(z-1)}{z}c_{np}\frac{\rho_{n0}}{\rho_0} - g(1 - \frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0}n))\\ &- 2\rho_{pp}m\end{aligned}$$

$$\begin{aligned}\frac{d\rho_{p-}}{dt} =& \rho_{p0}d + \rho_{0-}(\delta\rho_p + (1-\delta)\frac{(z-1)}{z}\frac{\rho_{p0}}{\rho_0})(b - \frac{(z-1)}{z}c_p\frac{\rho_{p0}}{\rho_{p0}} - \frac{(z-1)}{z}c_{np}\frac{\rho_{n0}}{\rho_0} - g(1 - \frac{(z-1)}{z}\frac{\rho_{n0}}{\rho_0}n))\\ &- \rho_{p-}m - \rho_{p-}(r + f(\frac{\rho_{n0}}{\rho_0} + \frac{\rho_{p0}}{\rho_0})\frac{(z-1)}{z})\end{aligned}$$

$$\frac{d\rho_{--}}{dt} = \rho_{0-}d - 2\rho_{--}(r + f(\frac{\rho_{n0}}{\rho_0} + \frac{\rho_{n0}}{\rho_0})\frac{(z-1)}{z})$$

$$\begin{aligned}\frac{d\rho_p}{dt} =& \rho_0(\delta\rho_p + (1-\delta)\frac{\rho_{p0}}{\rho_0})(b - c_p\frac{\rho_{p0}}{\rho_0} - c_{np}\frac{\rho_{n0}}{\rho_0} - g(1 - \frac{\rho_{n0}}{\rho_0}n))\\ &- \rho_p m\end{aligned}$$

$$\begin{aligned}\frac{d\rho_n}{dt} =& \rho_0(\delta\rho_n + (1-\delta)\frac{\rho_{n0}}{\rho_0})(b - c_n\frac{\rho_{n0}}{\rho_0} - c_{pn}\frac{\rho_{p0}}{\rho_0})\\ &- \rho_n m\end{aligned}$$

$$\frac{d\rho_-}{dt} = \rho_0 d - \rho_-(r + f(\frac{\rho_{n0}}{\rho_0} + \frac{\rho_{p0}}{\rho_0}))$$