1.1(a) Row operation $\Rightarrow$ multiply with matrix from the left

Column operation $\Rightarrow$ multiply with matrix from the right.

**

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}}_{\text{Subtract row 2 from other rows}} \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Add row 3 to row 1}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Half row 3}} B \underbrace{\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Double col 1}} \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\text{Interchange cols. 1 and 4}} \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Delete col. 1}}$$

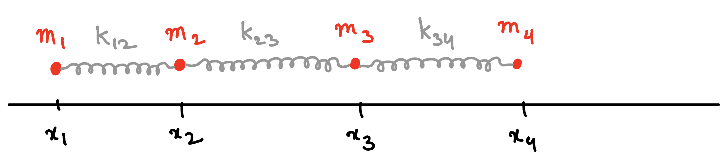
$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{Replace col. 4 by col. 3}}$$

(b) A = product of matrices on the left

C = product of matrices on the right

** The key idea in obtaining the transformation matrix is to take an identity matrix and perform desired operation on it and then multiply either from the left (for row operation) or from the right (for column operation).

1.2



$F \propto$ extension length on spring / compression length

$$f_1 = k_{12}(x_2 - x_1 - l_{12})$$

$$f_2 = k_{12}(x_2 - x_1 - l_{12}) - k_{23}(x_3 - x_2 - l_{23})$$

$$f_3 = k_{23}(x_3 - x_2 - l_{23}) - k_{34}(x_4 - x_3 - l_{34})$$

$$f_4 = k_{34}(x_4 - x_3 - l_{34})$$

(a)
$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} -k_{12} & k_{12} & 0 & 0 \\ -k_{12} & k_{12}+k_{23} & -k_{23} & 0 \\ 0 & -k_{23} & k_{23}+k_{34} & -k_{34} \\ 0 & 0 & -k_{34} & k_{34} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} -k_{12}l_{12} \\ -k_{12}l_{12}+k_{23}l_{23} \\ -k_{23}l_{23}+k_{34}l_{34} \\ -k_{34}l_{34} \end{bmatrix}$$

(b) Spring constant N/m or kg/sec^2

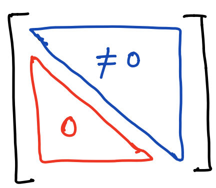
(c) $[kg/sec^2]^4$

(d) $1\ kg = 10^3\ gm$

$\therefore\ K' = 10^3\ K$

$\det(K') = (10^3)^4 \det(K)$

1.3 R is upper-triangular if $r_{ij} = 0$ for $i > j$



Given, R is an $m \times m$ nonsingular, upper Δr matrix. Let Z be such that $I = ZR$, then

$$\begin{bmatrix} e_1 & e_2 & \cdots & e_m \end{bmatrix} = \underbrace{\begin{bmatrix} z_1 & z_2 & \cdots & z_m \end{bmatrix}}_{\text{need to show that this is upper } \Delta r} \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ 0 & r_{22} & \cdots & r_{2m} \\ 0 & & \ddots & \vdots \\ \vdots & & & \\ 0 & 0 & \cdots & r_{mm} \end{bmatrix}$$

☆☆ For $B = AC$, each column of B is a linear combination of the columns of A.

$$\begin{bmatrix} b_j \end{bmatrix} = c_{1j}\begin{bmatrix} a_1 \end{bmatrix} + c_{2j}\begin{bmatrix} a_2 \end{bmatrix} + \cdots + c_{mj}\begin{bmatrix} a_m \end{bmatrix}$$

we use this to write by induction:

Base case: $e_1 = r_{11} z_1$

$$\Rightarrow \quad z_1 = r_{11}^{-1} e_1$$

$\therefore$ z_1 has only the 1^{st} entry non-zero.

Inductive case: Let $C^m(k)$ denote the column space of vectors of dimensionality m which have at most 1^{st} k non-zero elements.

$C^m(m) = C^m$ and each $C^m(k)$ is a linear subspace of C^m.

$z_1 \in C^m(1)$, from base case.

$$e_{i+1} = \sum_{k=1}^{m} z_k r_{k(i+1)} = \sum_{k=1}^{i} z_k r_{k(i+1)} + z_{i+1} r_{(i+1)(i+1)}$$

for $k > i+1$, $r_{k(i+1)} = 0$ by definition.

$$\Rightarrow \quad z_{i+1} = \underbrace{\underset{\text{scalar}}{r_{(i+1)(i+1)}^{-1}} \Big(e_{i+1} - \underbrace{\sum_{k=1}^{i} z_k r_{k(i+1)}}_{\in C^m(i)} \Big)}_{\in C^m(i+1)}$$

1.4 $\displaystyle\sum_{j=1}^{8} c_j f_j(i) = d_i \quad , \quad i = 1, \dots, 8$

(a)
$$\begin{bmatrix} f_1(1) & f_2(1) & \cdots & f_8(1) \\ f_1(2) & \cdots & & \vdots \\ \vdots & & \ddots & \vdots \\ f_1(8) & \cdots & & f_8(8) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_8 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_8 \end{bmatrix}$$

$$F \qquad\qquad c \qquad d$$

The condition says that $Fc = d$ is true for any $d \in \mathbb{C}^8$

$\therefore$ $\text{Range}(F) = \mathbb{C}^8$, i.e., F is a full-rank matrix and hence $c \leftrightarrow d$ is a one-one mapping.

(b) $Ad = c \Rightarrow A^{-1} = F$

$\therefore$ $A^{-1}(i,j) = f_j(i)$