Taller Grupal

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Example 1

There is strong evidence that the first level of processing what we see is done in the retina. It involves detecting something called edges or positions of transitions from dark to bright or bright to dark points in images. These points usually coincide with boundaries of objects. To model the edges, derivatives of functions such as

$$f(x)=\begin{cases}1-e^{-ax}, & x\geq 0\\ e^{ax}-1, & x\leq 0\end{cases}$$

need to be found.

a) Use the forward divided difference approximation of the first derivative of $f(x)$ to calculate the functions derivative at $x=0.1$ for $a=0.24$. Use a step size of $\Delta x=0.05$. Also, calculate the absolute relative true error.

b) Repeat the procedure from part (a) with the same data except choose $a=0.12$. Does the estimate of the derivative increase or decrease? Also, calculate the absolute relative true error.

a)

Solution

a) $$f'(x_i)\approx\frac{f(x_{i+1})-f(x_i)}{\Delta x}$$

$$a=0.24$$
$$x_i=0.1$$
$$\Delta x=0.05$$
$$x_{i+1}=x+\Delta x$$
$$=0.1+0.05$$
$$=0.15$$
$$f(0.1)=1-e^{(-0.24\times 0.1)}$$
$$=0.023714$$
$$f(0.15)=1-e^{(-0.24\times 0.15)}$$
$$=0.035360$$

$$f'(0.1)\approx\frac{f(0.15)-f(0.1)}{0.05}$$
$$=\frac{0.035360-0.023714}{0.05}$$
$$=0.23291$$

Area_del_triangulo.m | Areatriangulo.m | PuntoFijo.m | SisEcuaciones.m

```
%Programa para aplicar la derivacion numerica
% del ejemplo de la retina

t = [0.1, 0.15];
v = [0.023714, 0.035360];
dvdt = diff(v) ./ diff(t);
tl = t(1:end-1);
vl = v(1:end-1);
al = dvdt

```

```
gnu.octave.9.2.0
>> Example1a
al = 0.2329
>>
```

b)

b) $a = 0.12$

$$f(0.1) = 1 - e^{-(0.12\times0.1)} = 0.011928$$

$$f(0.15) = 1 - e^{-(0.12\times0.15)} = 0.017839$$

$$f'(0.1) \approx \frac{f(0.15) - f(0.1)}{0.05} = \frac{0.017839 - 0.011928}{0.05} = 0.11821$$

```
1  %Programa para aplicar la derivacion numerica
2  % del ejemplo de la retina
3
4  t = [0.1, 0.15];
5  v = [0.011928, 0.017839];
6  dvdt = diff(v) ./ diff(t);
7  tl = t(1:end-1);
8  vl = v(1:end-1);
9  al = dvdt
10
```

```
gnu.octave.9.2.0
>> Example1b
al = 0.1182
>>
```

Example 2

There is strong evidence that the first level of processing what we see is done in the retina. It involves detecting something called edges or positions of transitions from dark to bright or bright to dark points in images. These points usually coincide with boundaries of objects. To model the edges, derivatives of functions such as

$$f(x)=\begin{cases}1-e^{-ax}, & x\geq 0\\ e^{ax}-1, & x\leq 0\end{cases}$$

need to be found.

a) Use the backward divided difference approximation of the first derivative of $f(x)$ to calculate the functions derivative at $x=0.1$ for $a=0.24$. Use a step size of $\Delta x=0.05$. Also, calculate the absolute relative true error.

b) Repeat the procedure from part (a) with the same data except choose $a=0.12$. Does the estimate of the derivative increase or decrease? Also, calculate the absolute relative true error.

Solution

a) $$f'(x_i)\approx\frac{f(x_i)-f(x_{i-1})}{\Delta x}$$

$$
\begin{aligned}
a &= 0.24 \\
x_i &= 0.1 \\
\Delta x &= 0.05 \\
x_{i-1} &= x_i - \Delta x \\
&= 0.1 - 0.05 \\
&= 0.05 \\
f(0.1) &= 1 - e^{-(0.24\times 0.1)} \\
&= 0.023714 \\
f(0.05) &= 1 - e^{-(0.24\times 0.05)} \\
&= 0.011928 \\
f'(0.1) &\approx \frac{f(0.1) - f(0.05)}{0.05} \\
&= \frac{0.023714 - 0.011928}{0.05} \\
&= 0.23572
\end{aligned}
$$

```
1  %Programa para aplicar la derivacion numerica
2  % del ejemplo2
3
4  t = [0.05, 0.1];
5  v = [0.011928, 0.023714];
6  dvdt = diff(v) ./ diff(t);
7  tl = t(1:end-1);
8  vl = v(1:end-1);
9  al = dvdt
10
```

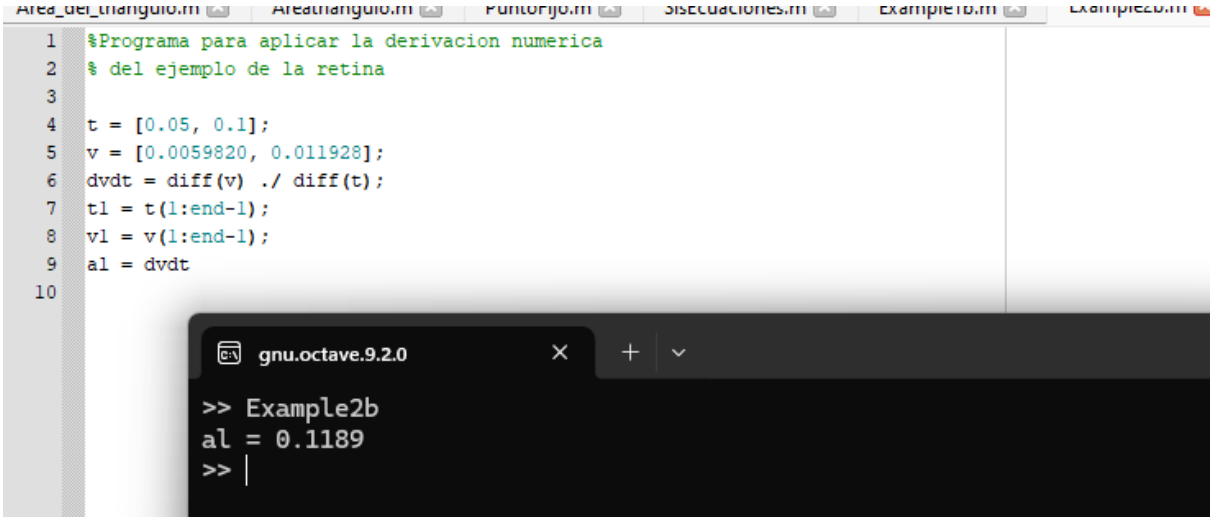
```
gnu.octave.9.2.0
>> Example1a

al = 0.1182
>> Example1a

al = 0.2357
>>
```

b)

$$
\begin{aligned}
a &= 0.12 \\
f(0.1) &= 1 - e^{-(0.12\times 0.1)} \\
&= 0.011928 \\
f(0.05) &= 1 - e^{-(0.12\times 0.05)} \\
&= 0.0059820 \\
f'(0.1) &\approx \frac{f(0.1) - f(0.05)}{0.05} \\
&= \frac{0.011928 - 0.0059820}{0.05} \\
&= 0.11893
\end{aligned}
$$



Conclusiones:

- Se demostró la eficacia de los programas para calcular derivadas de funciones usando diferencias finitas
- Se observo que al cambia el parametro `a` , el resultado de la derivada cambia
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