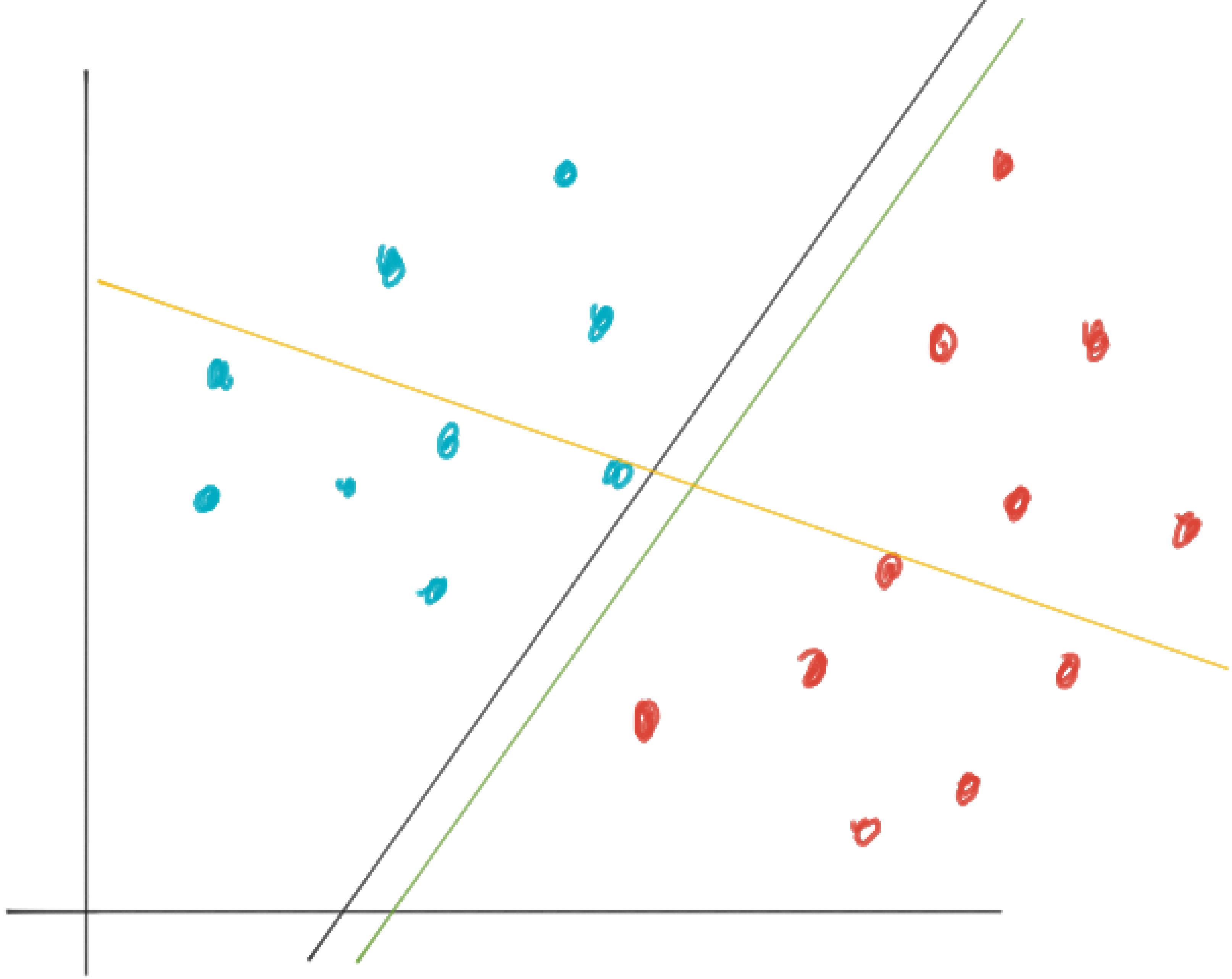
☆ Calculus

→ Why calculus

→ Functions

→ Limit

→ Some imp. functions



① Future
② Distance
} → Gain Function

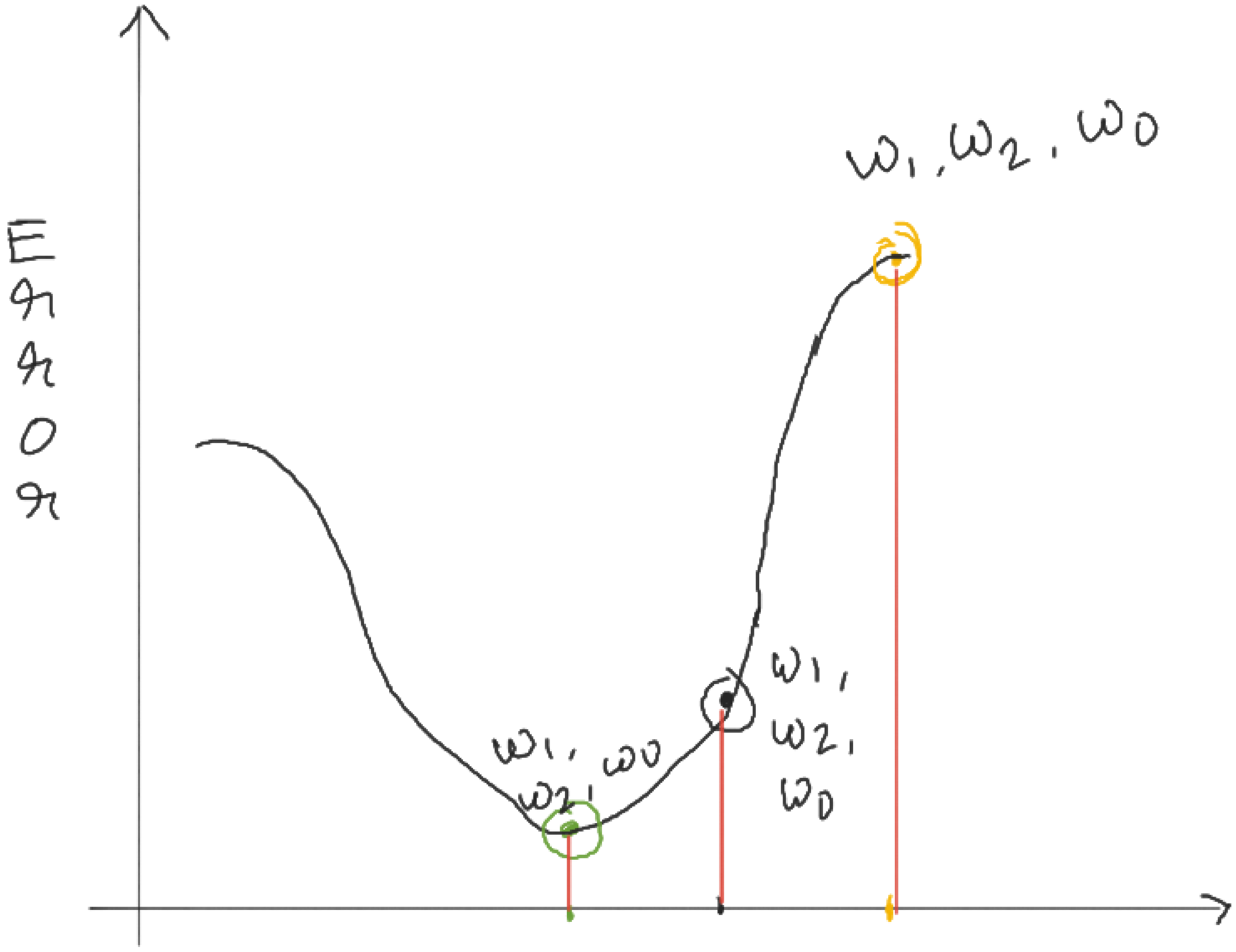
Loss Function → Error

black = less
green = very less
yellow = very high

Accuracy:

black = 100

green = 100



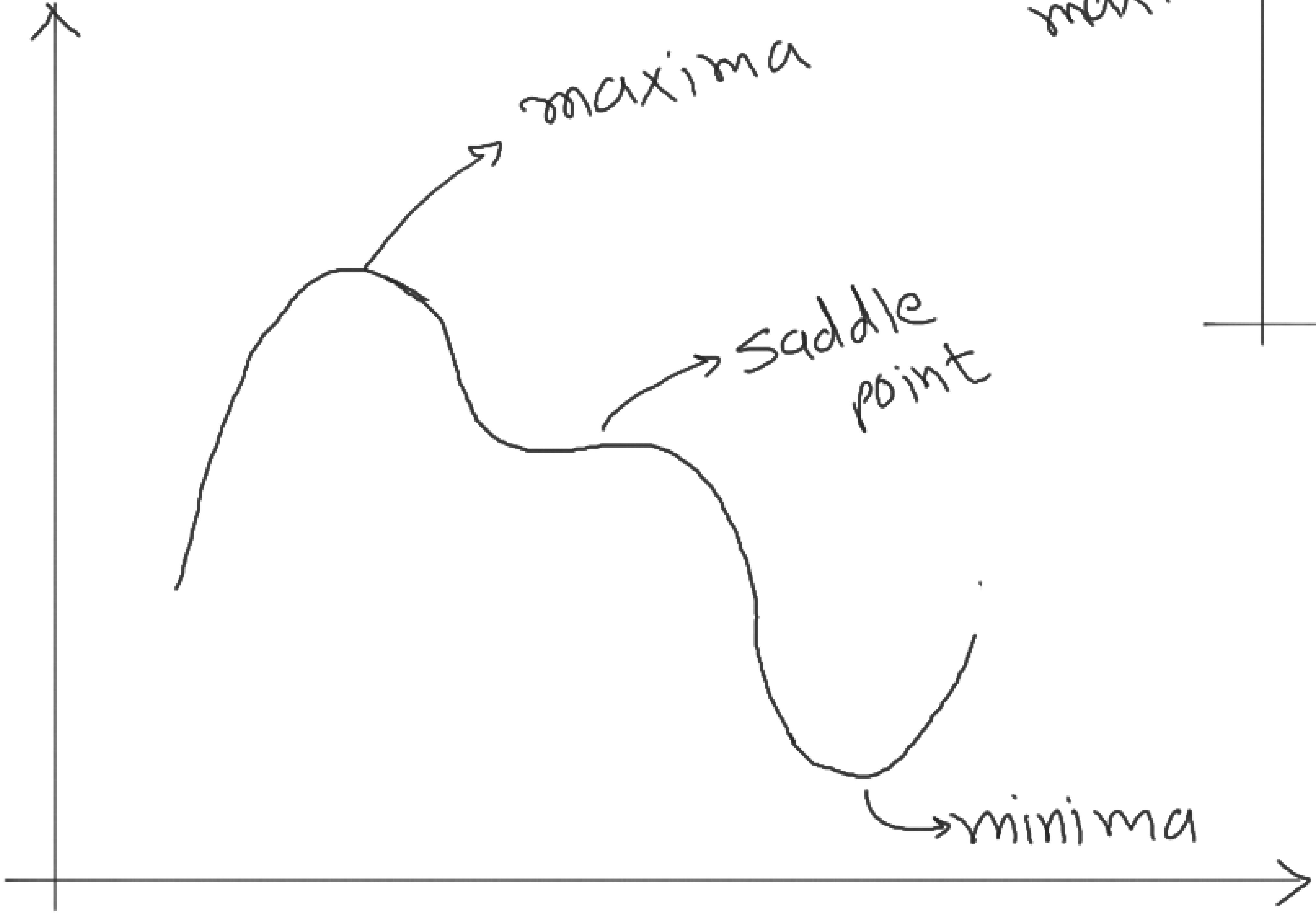
Let's assume possible values of w_1, w_2 & w_0 is in [0, 20] at the interval of 0.1

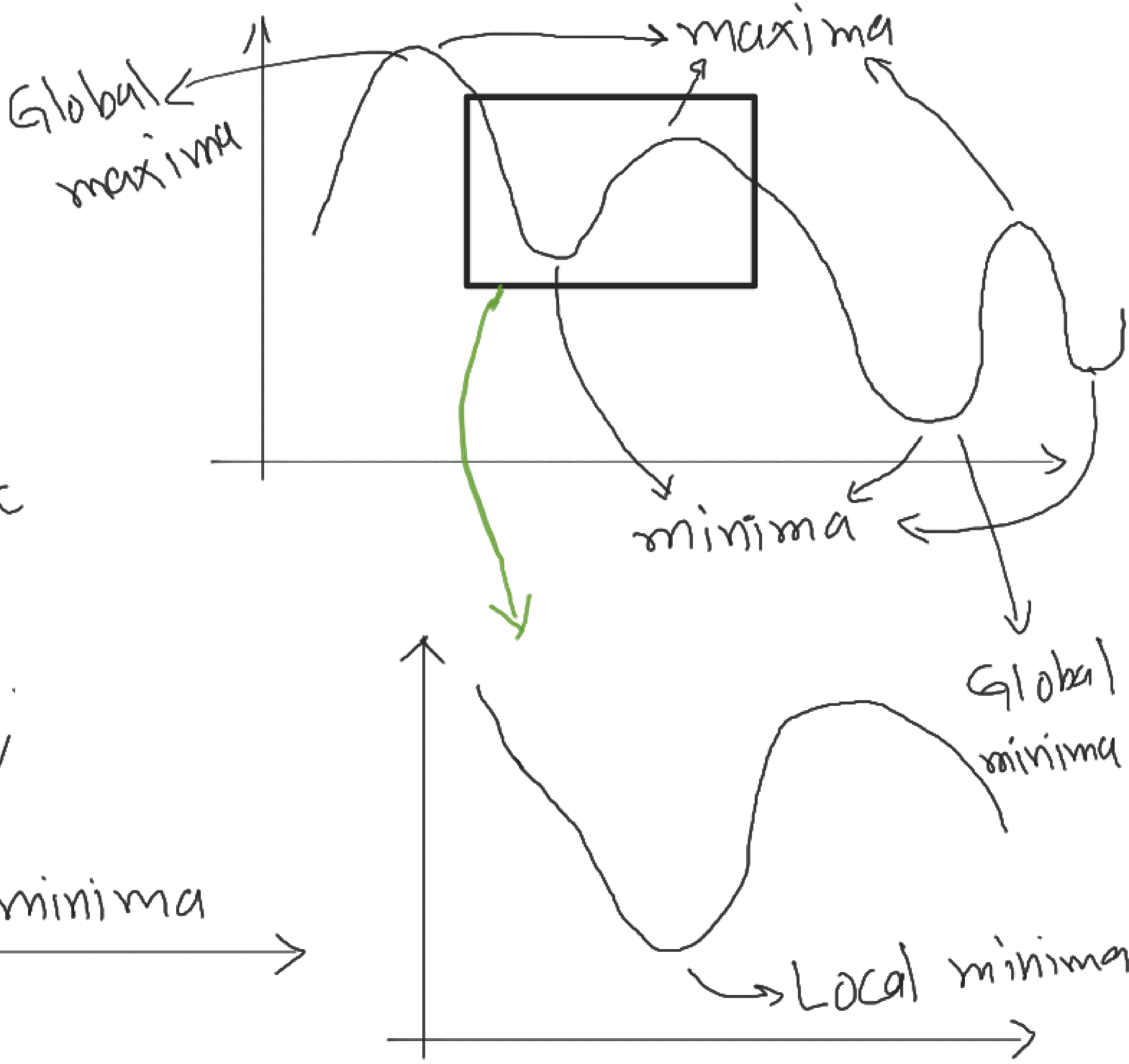
variable	Possible value
w_1	201
w_2	201
w_0	201

Total possible lines

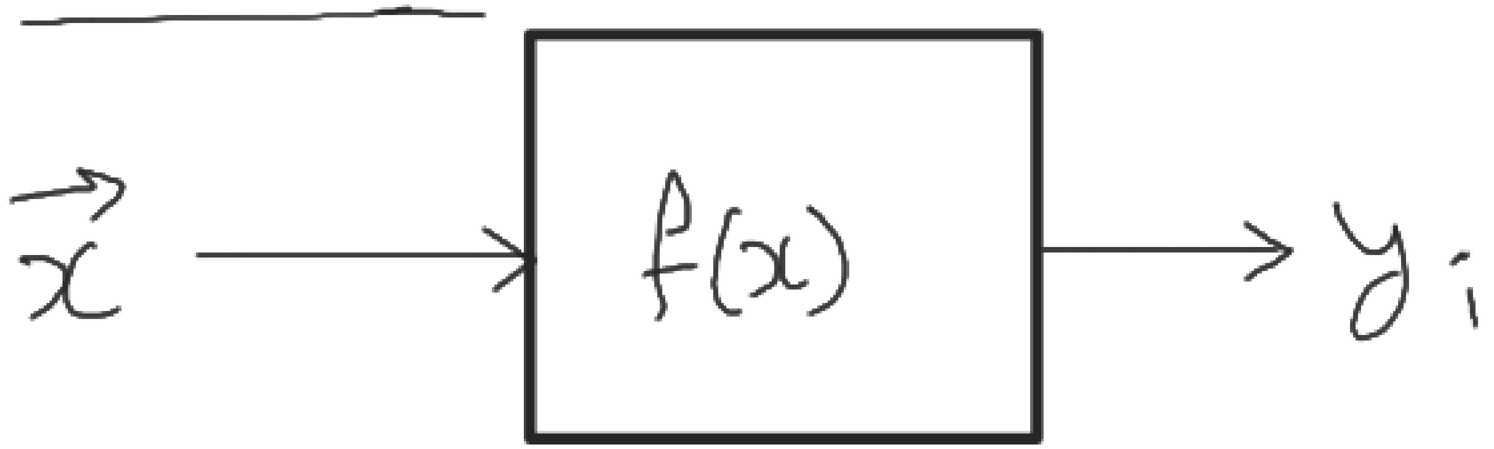
$= 201^3$

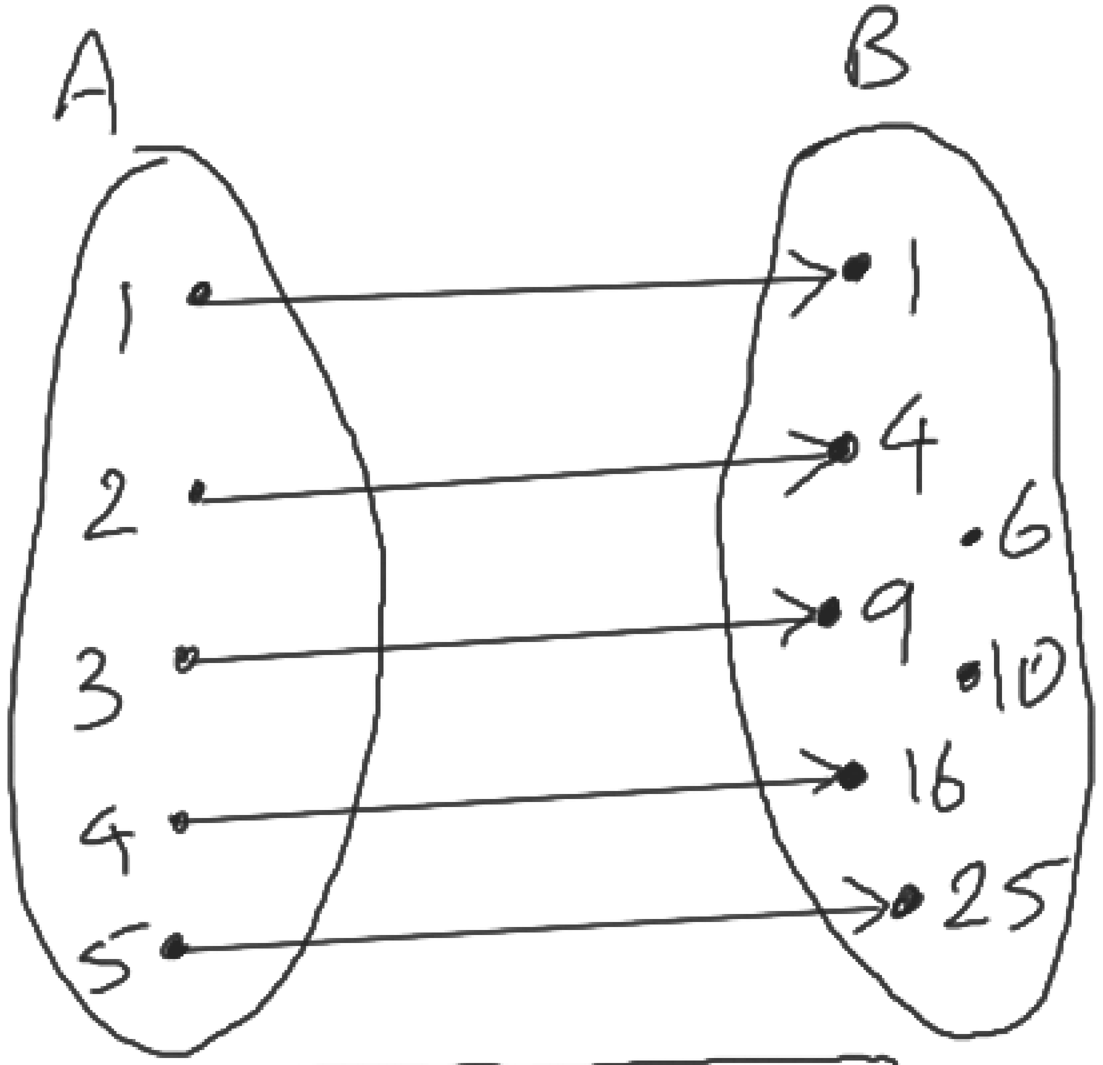
✩ Maxima & minima





☆ Functions

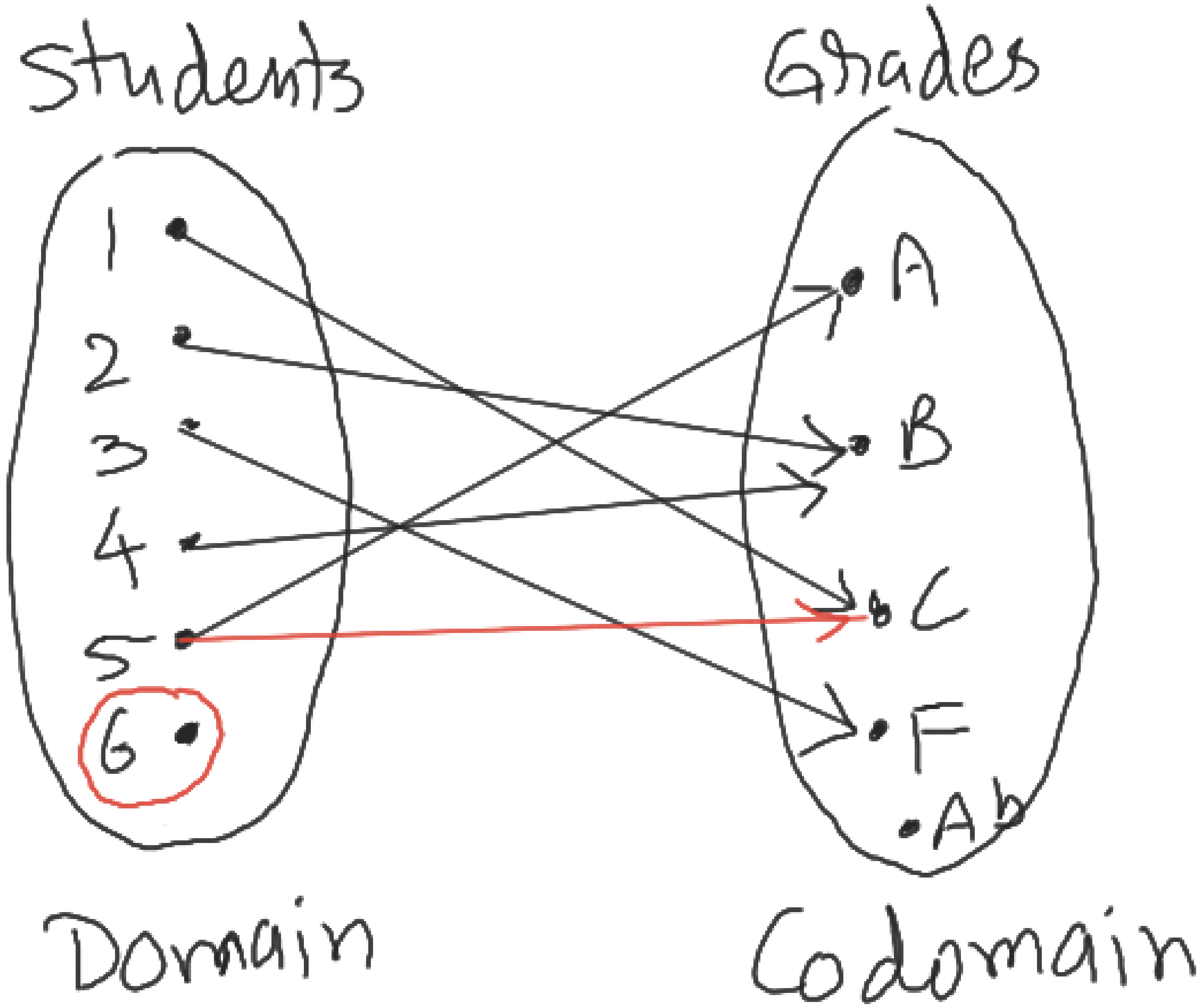


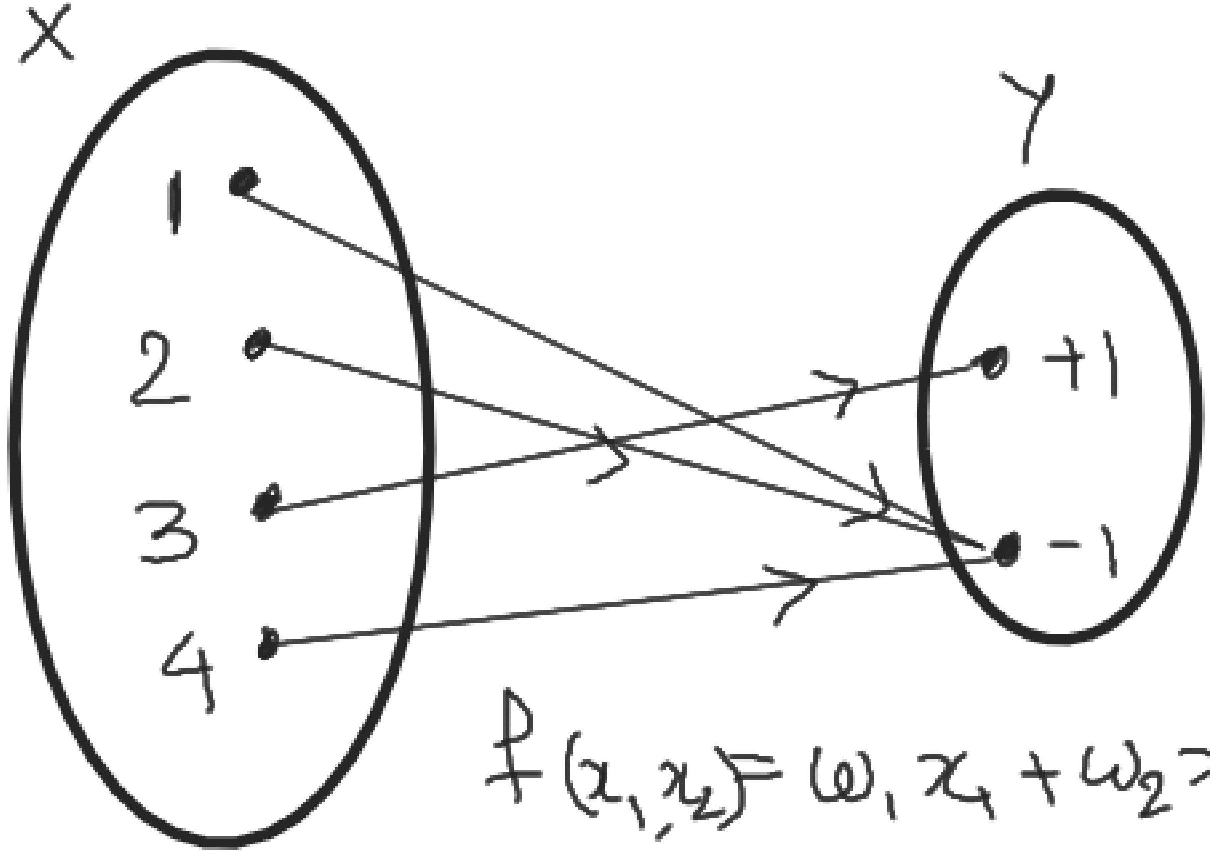


$$\boxed{f(x) = x^2}$$

$f : A \to B$

Range = {1, 4, 9, 16, 25, . . .}





$$f(x_1, x_2) = w_1 x_1 + w_2 x_2 + w_0$$

☆ Misc

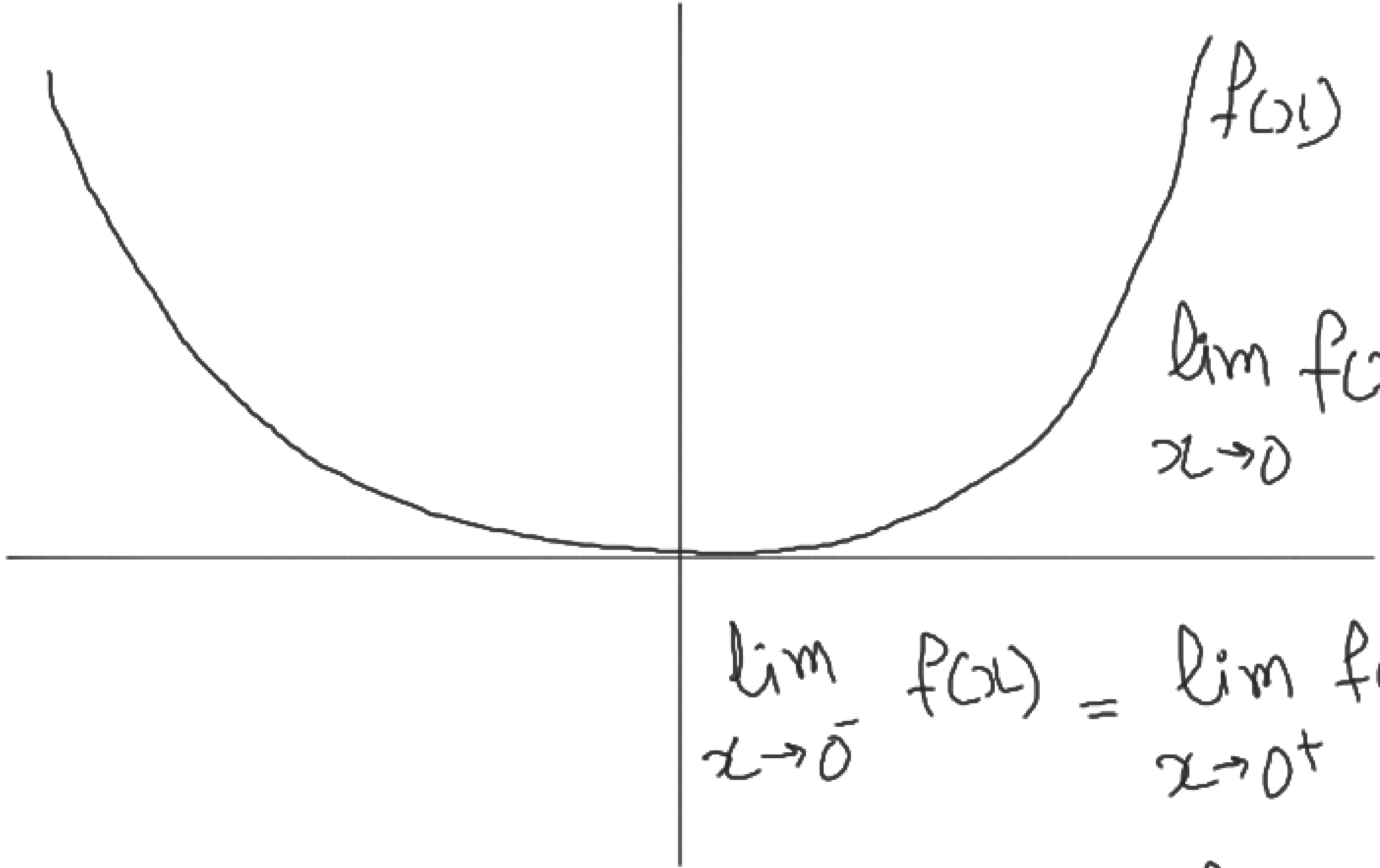
$$\underset{\vec{\omega}, \omega_0}{\arg\max}\ \sigma(\vec{\omega}, \omega_0, x, y) = \sum_{i=0}^{n} \frac{\vec{\omega}^T \cdot \vec{x} + \omega_0}{\|\vec{\omega}\|} \cdot y_i$$

$$= \prod_{i=0}^{n} \frac{\vec{\omega}^T \cdot \vec{x} + \omega_0}{\|\vec{\omega}\|} \cdot y_i$$

$$\prod_{i=5}^{9} x_i = \underline{x_5} * \underline{x_6} * \underline{x_7} * \underline{x_8} * \underset{}{\overset{-ve}{\underline{x_9}}} = 0$$

$$\sum_{i=5}^{9} x_i = x_5 + x_6 + x_7 + x_8 + x_9$$

☆ Limits & Continuity



$$\lim_{x \to 0} f(x) = 0$$

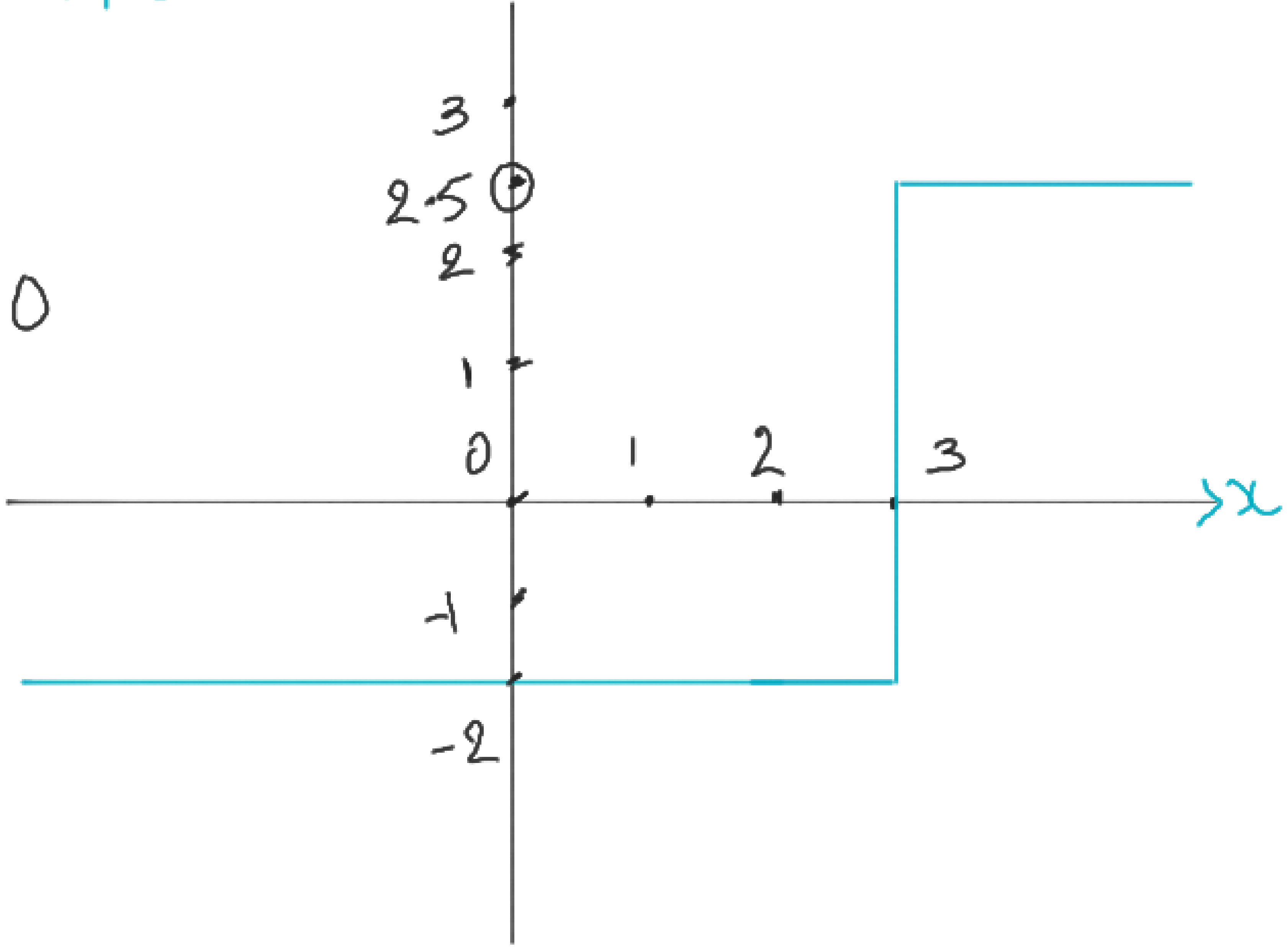
$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$= f(0)$$

$\therefore$ continuous function

$$f(x) = -2 \quad x < 3$$

$$f(x) = +2.5 \quad x > 3$$



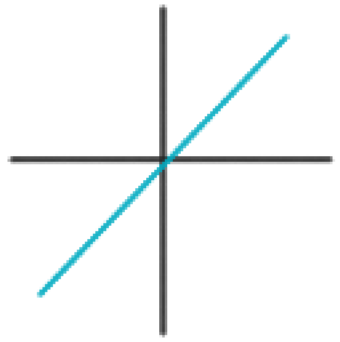
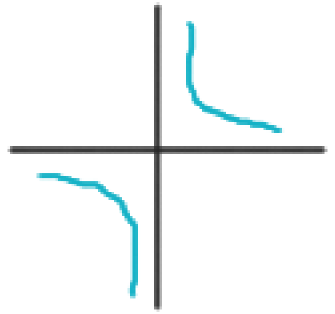
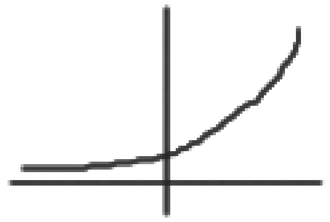
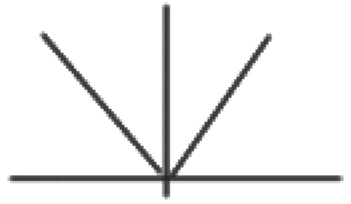
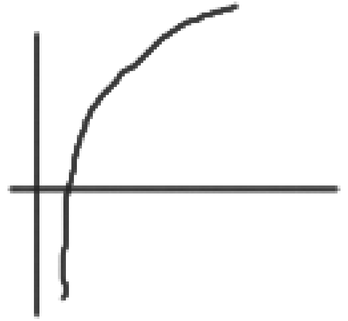
$$\lim_{x \to 3^-} f(x) = -2$$

$$\lim_{x \to 3^+} f(x) = +2.5$$

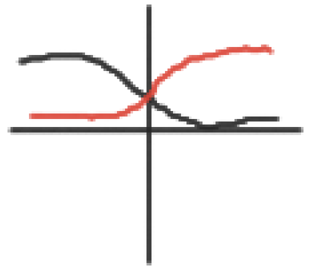
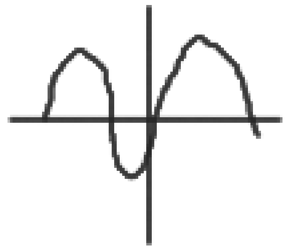
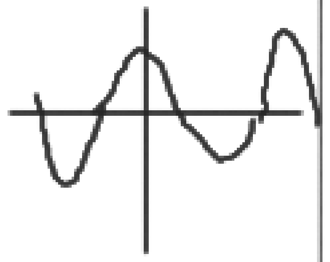
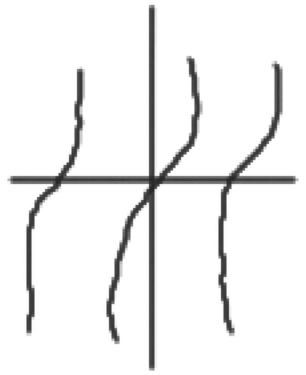
$$f(x) = ? \quad ; \quad x = 3$$

Discontinuous

☆ Some important functions:

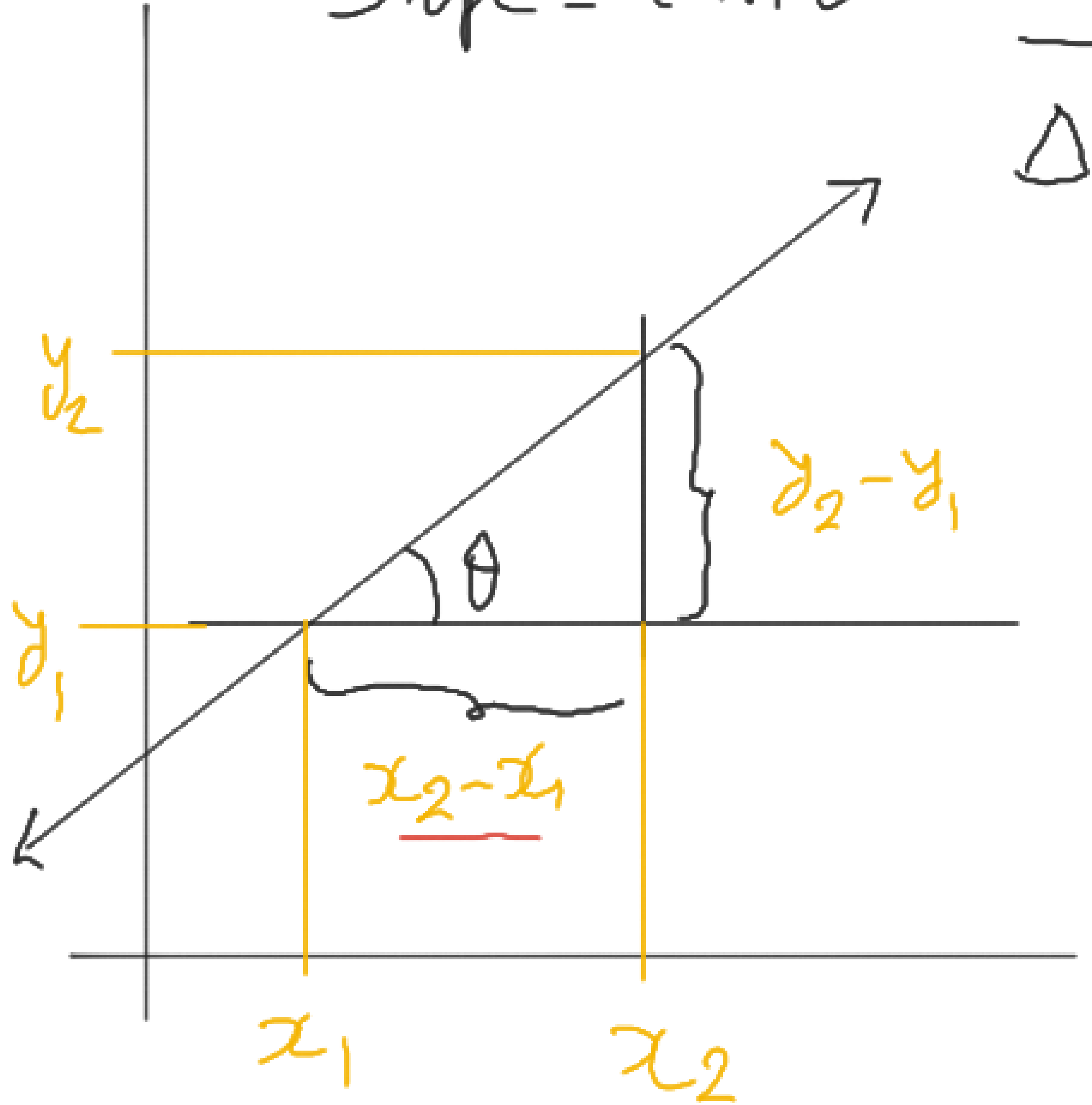
function	domain	range	continuous?	plot
① $y = x$	$[-\infty, +\infty]$	$[-\infty, +\infty]$	Y	
② $y = \frac{1}{x}$	$[-\infty, +\infty]$	$[-\infty, +\infty] - \{0\}$	N	
③ $y = e^x$	$[-\infty, +\infty]$	R^+	Y	
④ $y = \lvert x \rvert$	$[-\infty, +\infty]$	$R^+ + \{0\}$	Y	
⑤ $y = \ln(x)$	R^+	$[-\infty, +\infty]$	Y	

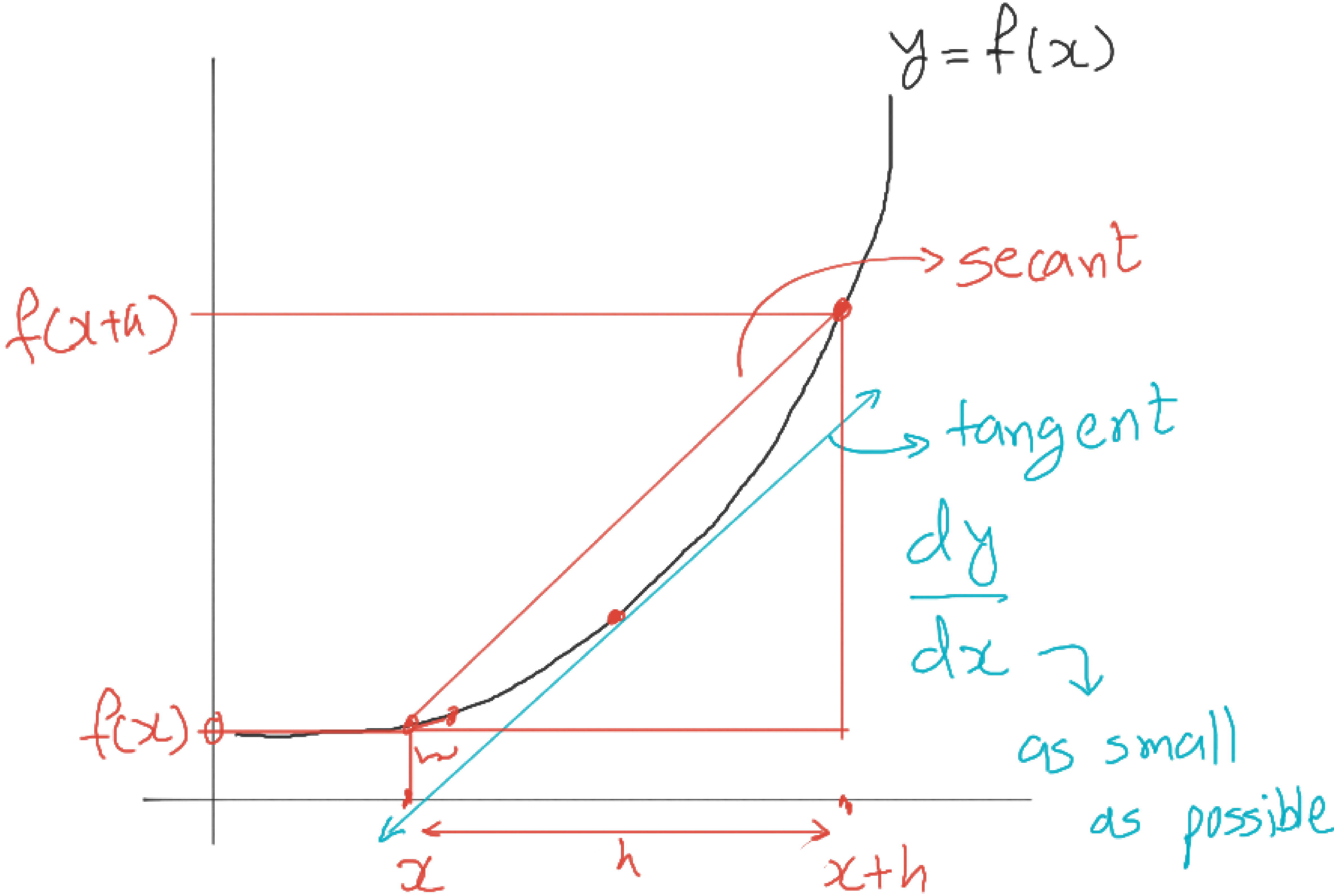
☆ Some Important Functions

function	domain	range	cont?	plot
$y = \frac{1}{1+e^{-x}}$	$[-\infty, +\infty]$	$[0, 1]$	Y	
$y = \sin\theta$	$[-\infty, +\infty]$	$[-1, +1]$	Y	
$y = \cos\theta$	$[-\infty, +\infty]$	$[-1, +1]$	Y	
$y = \tan\theta$	$[-\infty, +\infty]$	$[-\infty, +\infty]$	N	

☆ Differentiation

$$\text{slope} = \tan\theta = \frac{\Delta y}{\Delta x}$$





$$\tan\theta = \frac{\Delta y}{\Delta x} = \text{slope} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

$$\boxed{\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

☆ $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$ (Derive from previous formula)

$dx = h =$ difference | variation in x

x	y	dx	dy	dy/dx
2	4	0.00001	0.00004	4
3	9	0.00001	0.00006	6
4	16	0.00001	0.00008	8

Let $h = 0.00001$

$\therefore x + h = 2.00001$

$y_{new} = 4.00004$

$dy = y_{new} - y_{old} = 0.00004$

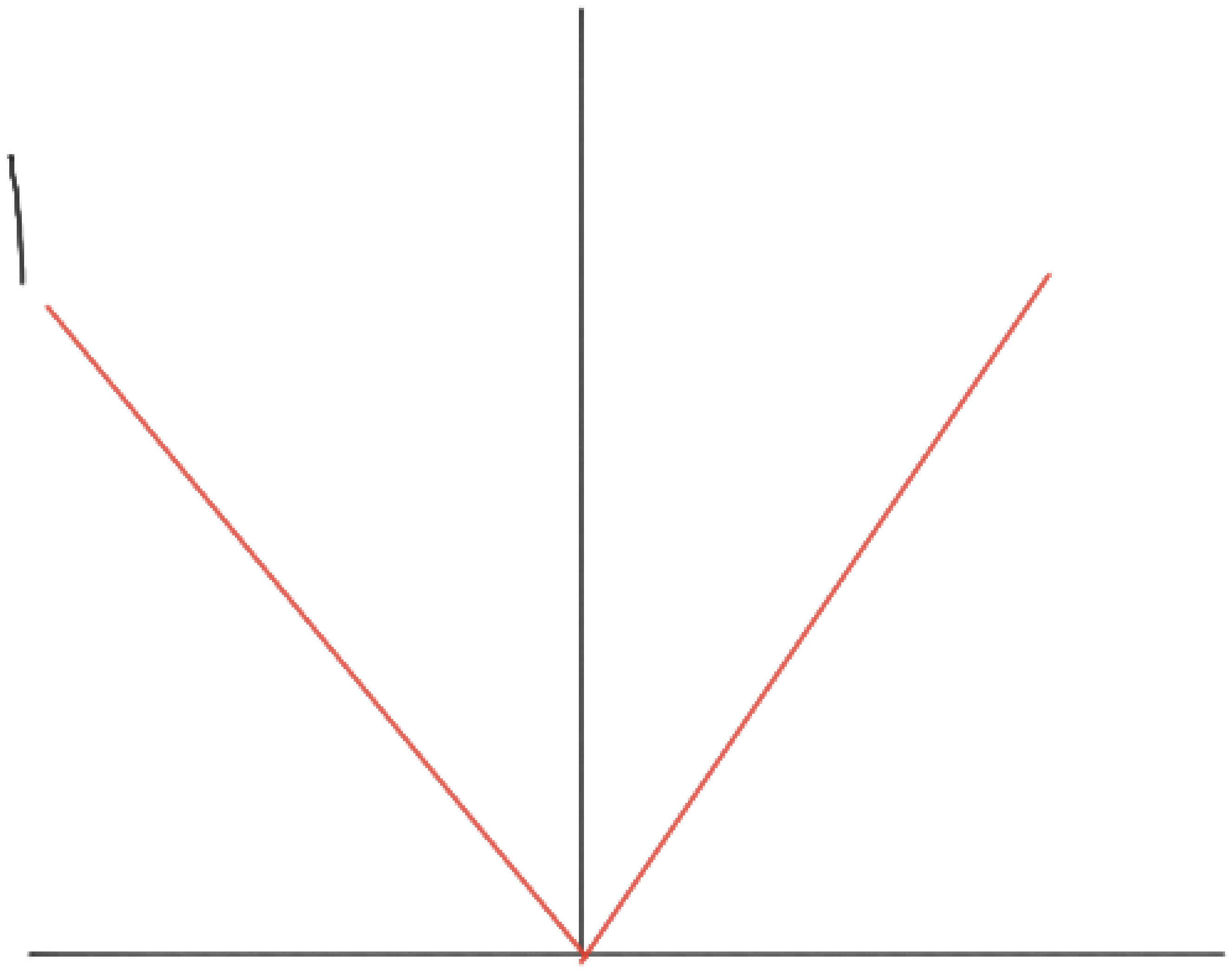
☆ $y=f(x)$

$\frac{dy}{dx}$ at $x=a$

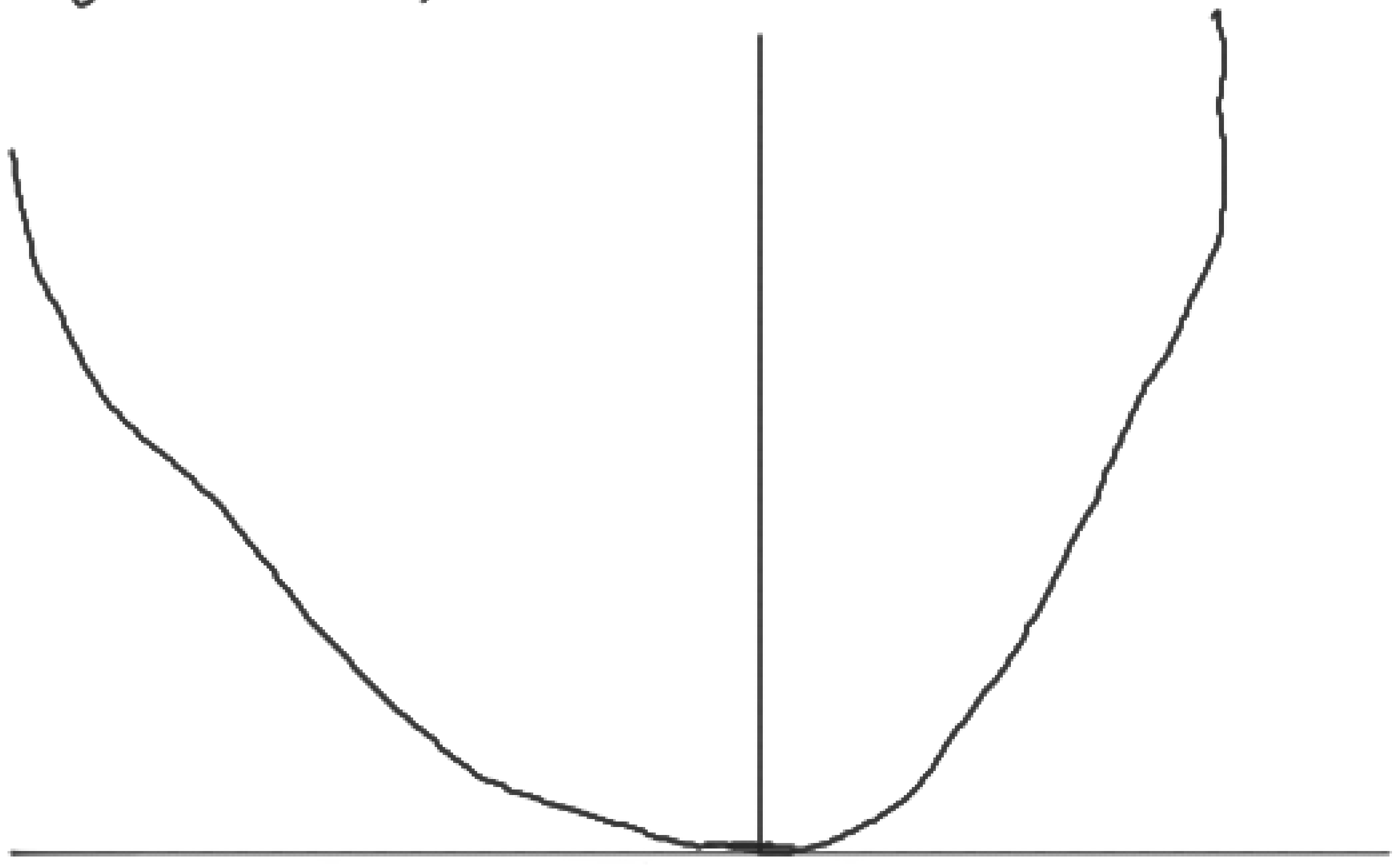
$$\lim_{h\to a^+}\frac{f(x+h)-f(x)}{h}=\lim_{h\to a^-}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to a}\frac{f(x+h)-f(x)}{h}$$

$y=f(x)=|x|$



$y=f(x)=x^2$



☆ Important derivatives:

① $\frac{d}{dx} x^n = nx^{n-1}$

② $\frac{d}{dx} c = 0$

③ $\frac{d}{dx} \sin x = \cos x$

④ $\frac{d}{dx} \cos x = -\sin x$

⑤ $\frac{d}{dx} \log x = \frac{1}{x}$

⑥ Sum Rule:

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

⑦ Product Rule:

$$\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx} g(x) + g(x) \cdot \frac{d}{dx} f(x)$$

⑧ Division Rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)}$$

☆ Try these out:

Ⓐ $x \log x = x \cdot \frac{1}{x} + \log x = 1 + \log x$

Ⓑ $\frac{\log x}{x} = \frac{x\left(\frac{1}{x}\right) - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}$

⑨ Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

☆ Try this out: Ⓒ $y = e^{5x^2+2} \Rightarrow \frac{d}{dx} y = e^{5x^2+2} \cdot \frac{d}{dx}(5x^2+2)$

$= e^{5x^2+2} \cdot 10x$

(D) $\boxed{y = \dfrac{1}{1+e^{-x}}}$ $\Rightarrow \dfrac{(1+e^{-x})\frac{d}{dx}\cdot 1 - 1\,\frac{d}{dx}(1+e^{-x})}{(1+e^{-x})^2} = \dfrac{0 - e^{-x}\cdot\frac{d}{dx}(-x)}{(1+e^{-x})^2}$

Het: $\dfrac{e^{x}}{(e^{x}+1)^2}$

$$= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{(1+e^{-x})}\cdot\frac{e^{-x}}{(1+e^{-x})}$$

$$= \frac{1}{(1+e^{-x})}\cdot\left(1 - \frac{1}{(1+e^{-x})}\right)$$

Harsh & krisha: $\dfrac{e^{-x}}{(e^{-x}+1)^2}$

Vittal: $\dfrac{-e^{-x}}{(1+e^{-x})}$

$\dfrac{d}{dx} f(x) = f(x)\cdot(1-f(x))$

only for $f(x) = \dfrac{1}{1+e^{-x}}$

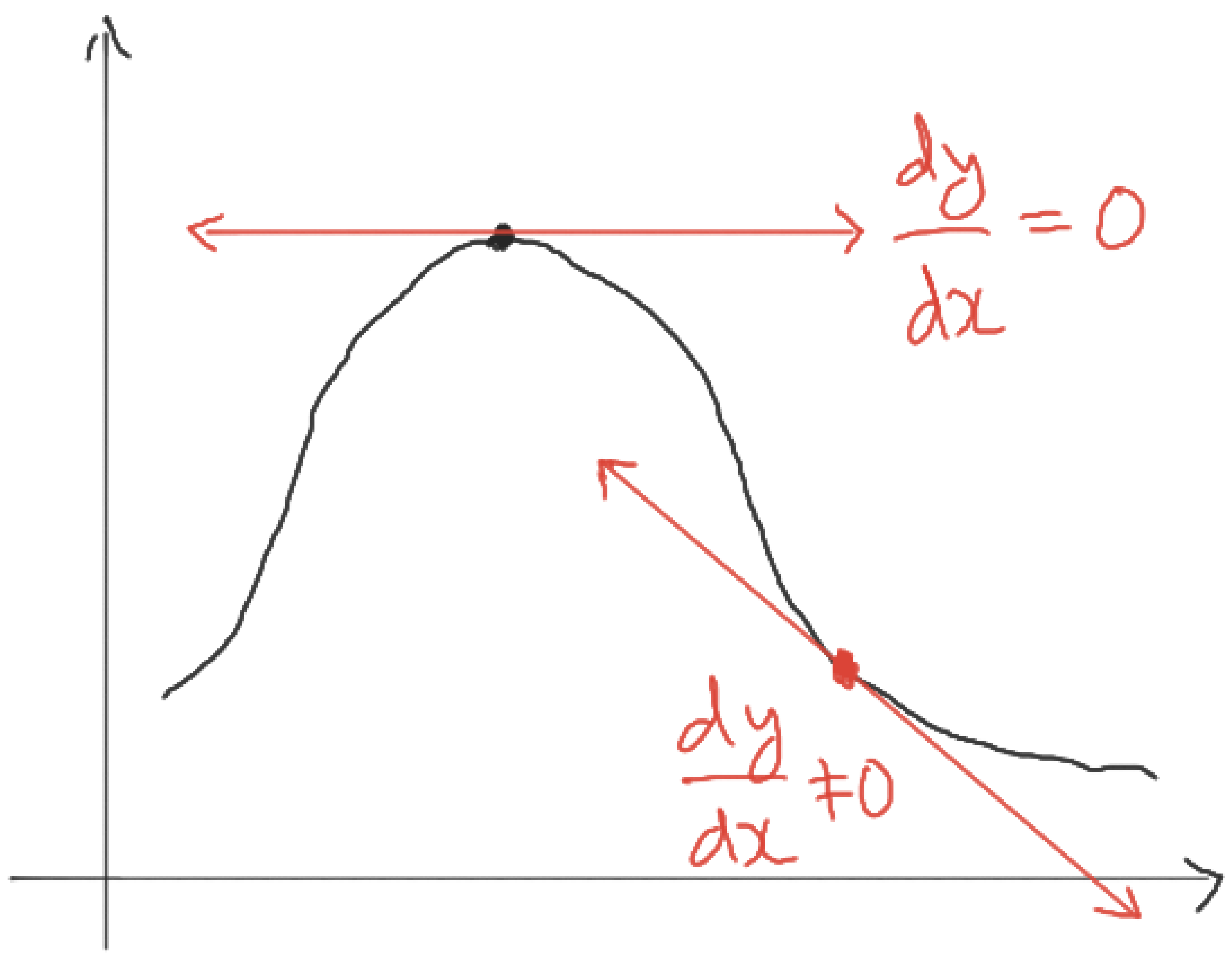
☆ Towards Gradient Descent - What is argmax or max?

$f(x) = -(x-2)^2 \Rightarrow \underset{x}{\operatorname{argmax}} f(x)$ means the value of x for which $f(x)$ is maximum.

$\max(f(x)) = 0$ but $\underset{x}{\operatorname{argmax}} f(x) = 2$

$\therefore \underset{\vec{w}, w_0}{\operatorname{argmax}} \sigma(\vec{w}, w_0, x, y)$ means values of $\vec{w}$ & w_0 when gain function σ is maximum!

☆ Let's say, graph of our gain function is as below:



Suppose $f(x) = 41 - 72x - 18x^2$ is the function of profit

$\therefore f'(x) = -72 - 36x$

$\therefore f''(x) = -36$

☆ If $f''(x) < 0 \Rightarrow$ function is concave downward ∩

→ If $f''(x) > 0$ then function is concave upward ∪

☆ Steps to reach optima

1. Calculate $f'(x)$
2. Equate $f'(x) = 0$ to find value of x at $x = c$
3. Calculate value of $f''(x)$ at each value of x.
4. If $f''(x) > 0 \Rightarrow x = c$ is minima

   If $f''(x) < 0 \Rightarrow x = c$ is maxima

   If $f''(x) = 0 \Rightarrow x = c$ can be neither maxima nor minima but a saddle point

☆ Example: $f(x) = 41 - 72x - 18x^2$

(1) $f'(x) = -72 - 36x$

(2) $-36x - 72 = 0 \Rightarrow -36x = 72 \Rightarrow x = -2$

But if $f(x)$ was $41 - 32x - 72x^2 - 18x^3$ then

$f'(x) = -32 - 144x - 54x^2 \Rightarrow 54x^2 + 144x + 32 = 0 \Rightarrow$ Either $x = \underline{-0.245}$ OR $x = \underline{-2.422}$

In this case we will get more than one value of x.

(3) Calculate value of $f''(x)$ for each such value of x

$f''(x) = -36$

$f''(x) = 108x + 144 \Rightarrow f''(-0.245) = 117.54$

$f''(-2.422) = -117.57$

(4) $f''(x) < 0 \Rightarrow x = -2$ is maxima

$\therefore$ Minima is at $x = -0.245$ & maxima is at $x = -2.422$