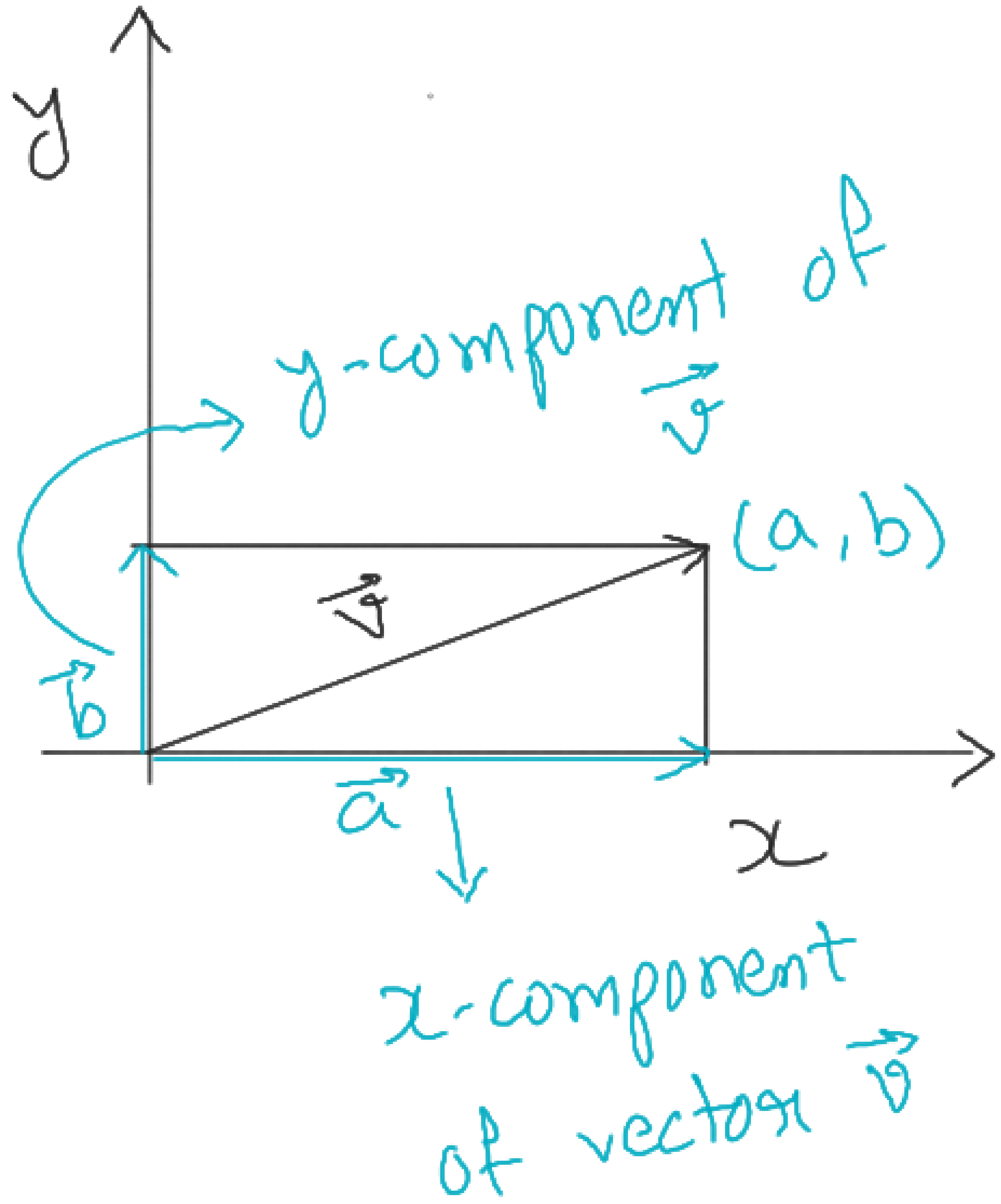
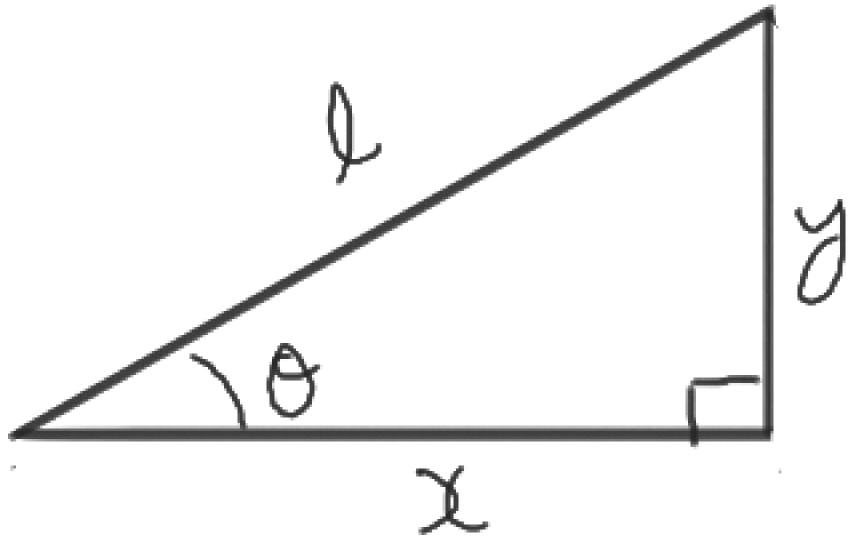
☆ Components of a vector



$\therefore \vec{v} = \vec{a} + \vec{b}$ And as $\vec{a} = a \cdot \hat{i}$ & $\vec{b} = b \cdot \hat{j}$

$\vec{v} = a\hat{i} + b\hat{j}$

☆ Trigonometry



$\cos\theta = \frac{x}{l}$ $\qquad$ $\sin\theta = \frac{y}{l}$

$\therefore x = l\cos\theta$ $\qquad$ $\therefore y = l\sin\theta$

$\tan\theta = \frac{y}{x}$

$\therefore y = x\tan\theta$

According to Pythagoras Theorem,

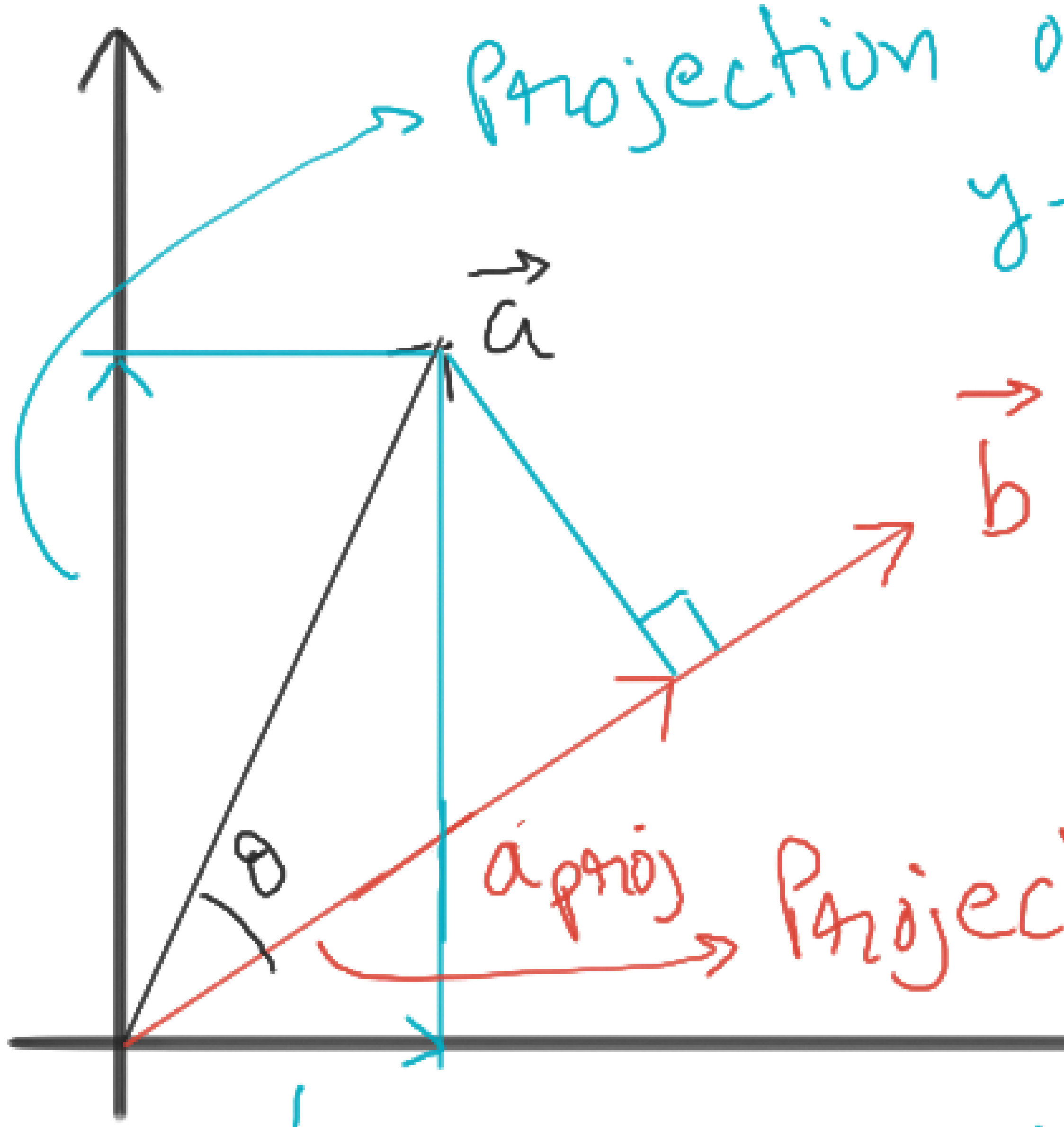
$$l = \sqrt{x^2 + y^2}$$

$$l = \sqrt{x^2 + x^2\tan^2\theta} = \sqrt{x^2(1 + \tan^2\theta)}$$

$$l = x\sqrt{(1 + \tan^2\theta)}$$

☆ Projction of a vector

Projection of $\vec{a}$ on y-axis = y-component of $\vec{a}$



Projection of $\vec{a}$ on $\vec{b}$

Projection of $\vec{a}$ on x-axis = x-component of $\vec{a}$

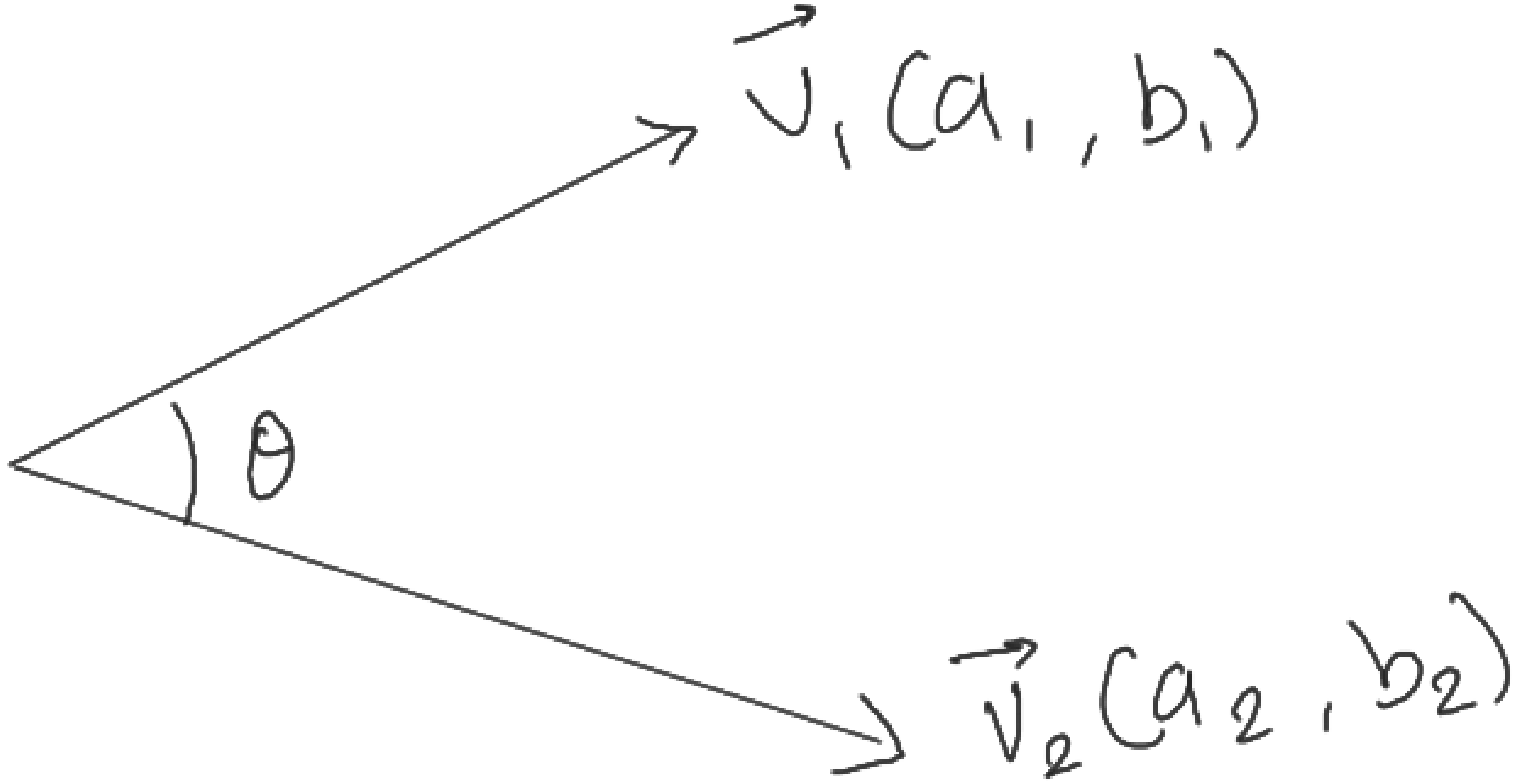
$$|a_{proj}| = a \cdot \cos\theta$$

$\vec{a}_{proj}$ = magnitude of a_{proj} * unit vector in that direction

$$\vec{a}_{proj} = a \cdot \cos\theta \cdot \frac{\vec{b}}{|\vec{b}|}$$

☆ An interesting fact about dot product:

$$\vec{v}_1 \cdot \vec{v}_2 = (a_1 \cdot a_2) + (b_1 \cdot b_2)$$



☆ Normal Equation Formula of a straight line:

General form of a line: $Ax + By + C = 0$

Let $\vec{n}\,[A, B]$ and hence, $\vec{n} = A\hat{i} + B\hat{j}$ &

let $\vec{p}\,[x, y]$ therefore, $\vec{p} = x\hat{i} + y\hat{j}$

$\therefore\ \vec{n} \cdot \vec{p} = Ax + By$

Substituting in gen formula,

$\vec{n} \cdot \vec{p} + C = 0$ OR $\boxed{\vec{n} \cdot \vec{p} = -C}$

Normal equation formula

☆ Why is it called 'Normal Eq^n Formula'?

↳ Because $\vec{n}$ is called Normal Vector. Why?

↳ Because $\vec{n} = A\hat{i} + B\hat{j}$ is always perpandicular to line $Ax + By + C = 0$

☆ One more interesting result:

If we have projection of $\vec{p}$ on this $\vec{n}$ then,

$$P_{proj} = |\vec{p}| \cos\theta \cdot \frac{\vec{n}}{|\vec{n}|} \quad \text{———} \quad \text{I}$$

$\vec{a} = \underline{|\vec{a}|} \cdot \text{unit}$

Now, $\vec{p} \cdot \vec{n} = |\vec{p}| \, |\vec{n}| \cos\theta \Rightarrow |\vec{p}| \cos\theta = \dfrac{\vec{p} \cdot \vec{n}}{|\vec{n}|}$ ——— II

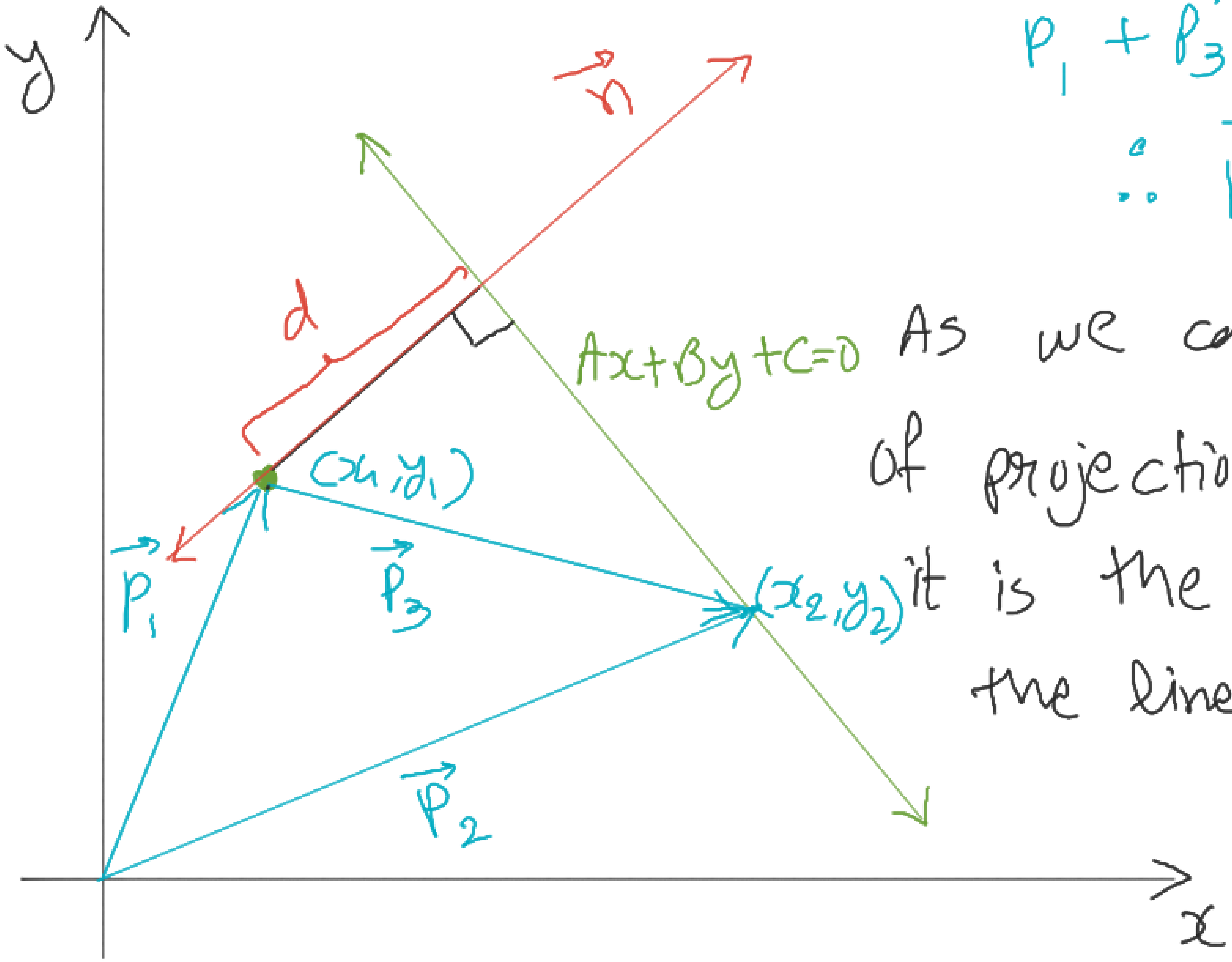
Substituting II into I

$$P_{proj} = \frac{\vec{p} \cdot \vec{n}}{|\vec{n}|} \cdot \frac{\vec{n}}{|\vec{n}|}$$

unit vector in direction of $\vec{n}$

magnitude of P_{proj}

☆ Distance of a point from a line



$$\vec{P_1} + \vec{P_3} = \vec{P_2}$$

$$\therefore \vec{P_3} = \vec{P_2} - \vec{P_1}$$

As we can see, if we find magnitude of projection vector of $\vec{P_3}$ on $\vec{n}$ then it is the distance of point P_1 from the line.

$$\therefore d = \frac{\vec{P_3} \cdot \vec{n}}{|\vec{n}|}$$

Let $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$

$\therefore \vec{P_1} = x_1 \cdot \hat{i} + y_1 \cdot \hat{j}$ & $\vec{P_2} = x_2 \cdot \hat{i} + y_2 \cdot \hat{j}$

$\therefore \vec{P_3} = \vec{P_2} - \vec{P_1} = x_2 \cdot \hat{i} - x_1 \cdot \hat{i} + y_2 \cdot \hat{j} - y_1 \cdot \hat{j}$

$$\vec{P_3} = (x_2 - x_1) \cdot \hat{i} + (y_2 - y_1) \cdot \hat{j}$$

And the normal vector $\vec{n} = A\hat{i} + B\hat{j}$

$$|\vec{n}| = \sqrt{A^2 + B^2}$$

$$\therefore d = \frac{\left[(x_2 - x_1) \cdot \hat{i} + (y_2 - y_1) \cdot \hat{j}\right] \cdot \left[A\hat{i} + B\hat{j}\right]}{\sqrt{A^2 + B^2}}$$

From the result of dot product of two vectors $\vec{v_1}(a_1, b_1)$ & $\vec{v_2}(a_2, b_2)$: $\vec{v_1} \cdot \vec{v_2} = a_1 \cdot a_2 + b_1 \cdot b_2$:-

$$d = \frac{A(x_2 - x_1) + B(y_2 - y_1)}{\sqrt{A^2 + B^2}}$$

Recap: (1) Fish Sorting Problem - Features (Independent variables), Target variable (dependent variable), Records (data points), seperator | classifier | boundary line,

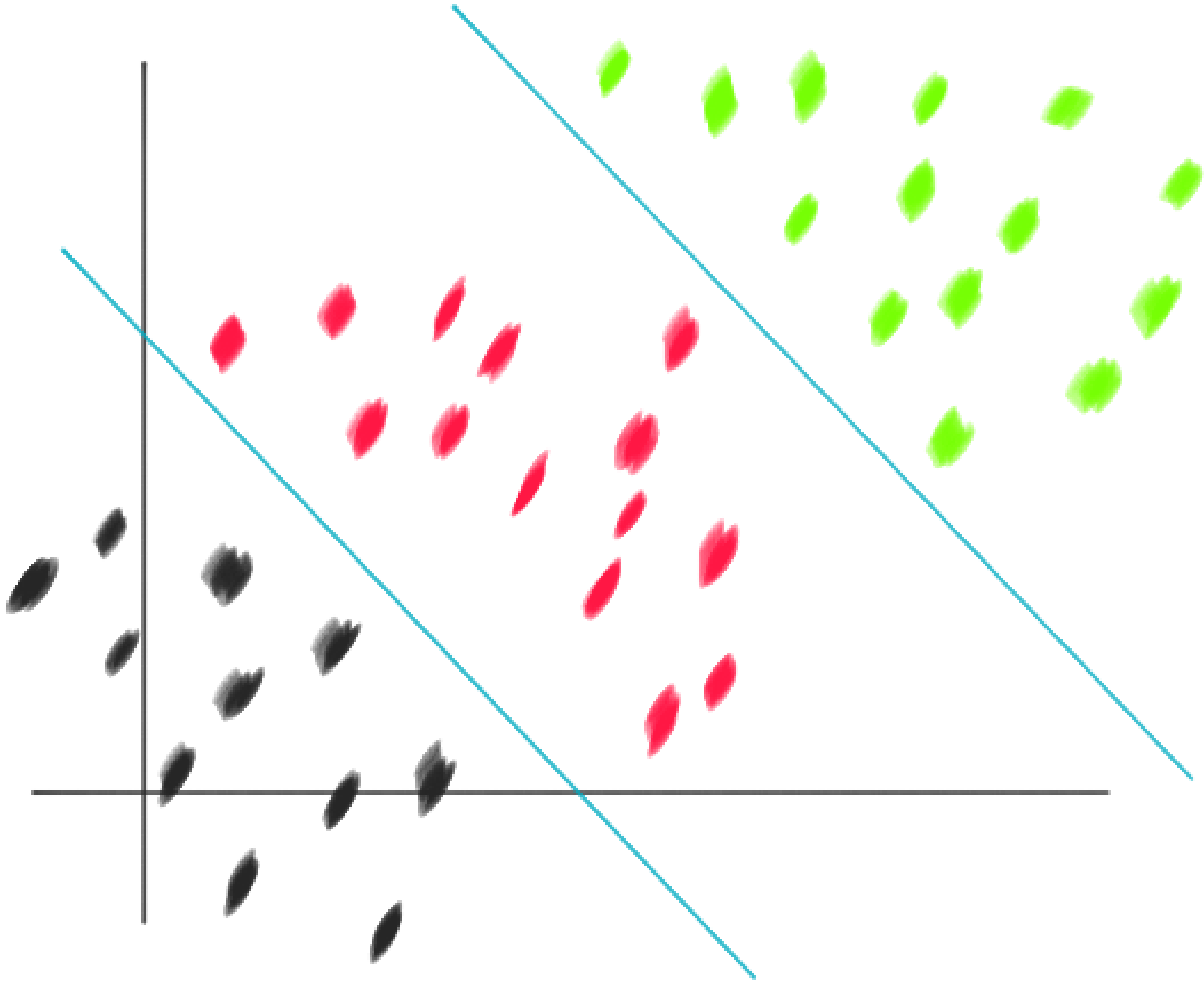
(2) Equations of line - slope-intercept form, Two-point formula, intercept formula, slope-point form, General form

(3) No. of features = no. of dimensions. If we have an n-dimensional space then our boundary will have $(n-1)$ dimensions.

(4) Hyperplane: $w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_n x_n + w_0 = 0$

(5) Binary classification - eg. small fish | big fish

multi-class classification: dog/cat/cow - We need more than on boundaries for a multi-class classification.



☆ Straight Line - $w_1x_1 + w_2x_2 + w_0 = 0$

slope $= \dfrac{-w_1}{w_2}$, x-intercept $= \dfrac{-w_0}{w_1}$

y-intercept $= \dfrac{-w_0}{w_2}$

→ Lines are parallel if their slopes are equal.

→ For lines to be perpendicular, $m_1 = -\dfrac{1}{m_2}$

⑥ Introduction to vectors - magnitude (also known as 'norm')

Euclidian & Manhattan distances (2-norm & 1-norm respectively)

→ x & y components of vectors

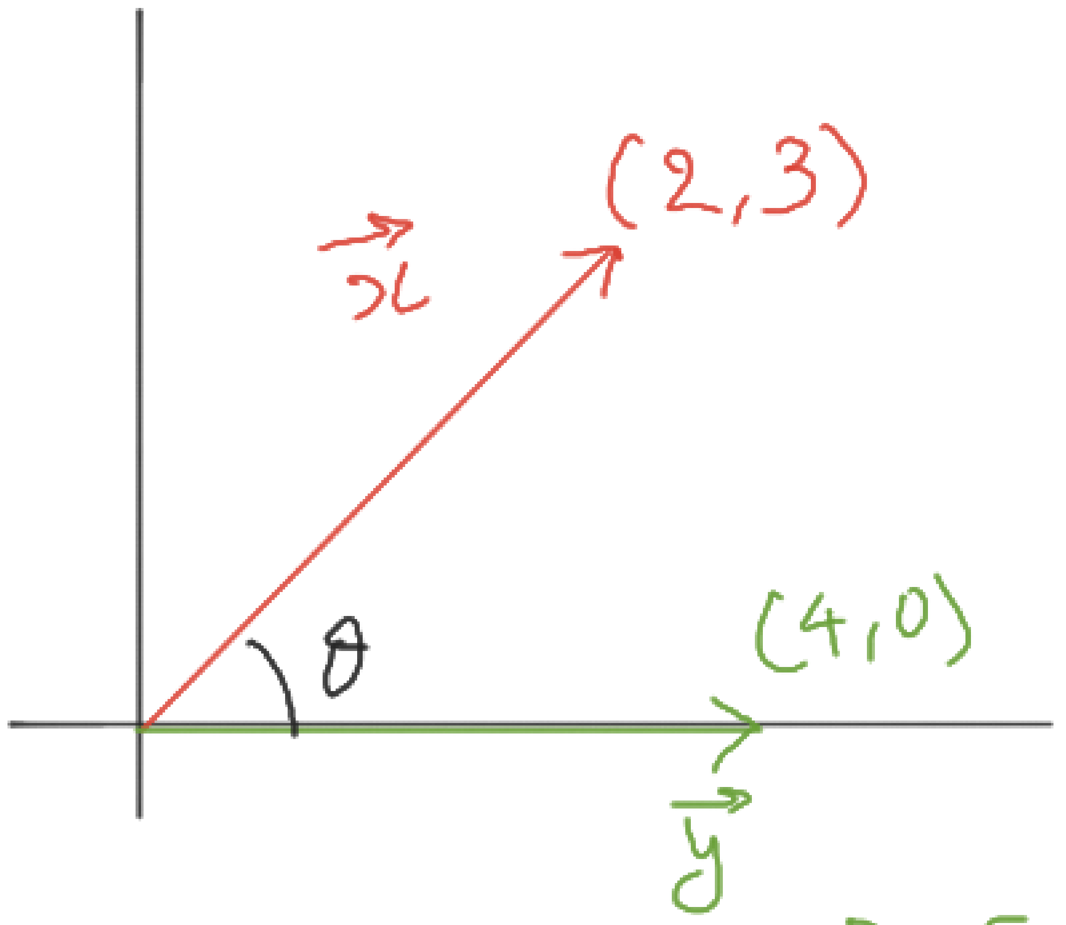
→ Vector Addition

→ Dot product of two vectors

→ Angle between two vectors

$$\vec{x}^T \cdot \vec{y} = \|\vec{x}\| \cdot \|\vec{y}\| \cdot \cos\theta$$

$$\therefore \cos\theta = \frac{\vec{x}^T \cdot \vec{y}}{\|\vec{x}\| \cdot \|\vec{y}\|}$$



$$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

(8) Unit vector of $\vec{w}$: $\hat{w} = \dfrac{\vec{w}}{\|\vec{w}\|} = \dfrac{1}{\|\vec{w}\|} \cdot \vec{w} = \dfrac{1}{\|\vec{w}\|} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

$\therefore \hat{w} = \begin{bmatrix} w_1 / \|\vec{w}\| \\ w_2 / \|\vec{w}\| \end{bmatrix}$ But, $\|\vec{w}\| = \sqrt{w_1^2 + w_2^2}$

$\therefore \hat{w} = \begin{bmatrix} w_1 / \sqrt{w_1^2 + w_2^2} \\ w_2 / \sqrt{w_1^2 + w_2^2} \end{bmatrix}$

(9) Projection of a vector

(7) $\underbrace{w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots + w_n x_n}_{\vec{w}^T \cdot \vec{x}} + w_0 = 0$ — boundary

$+ w_0 = 0$

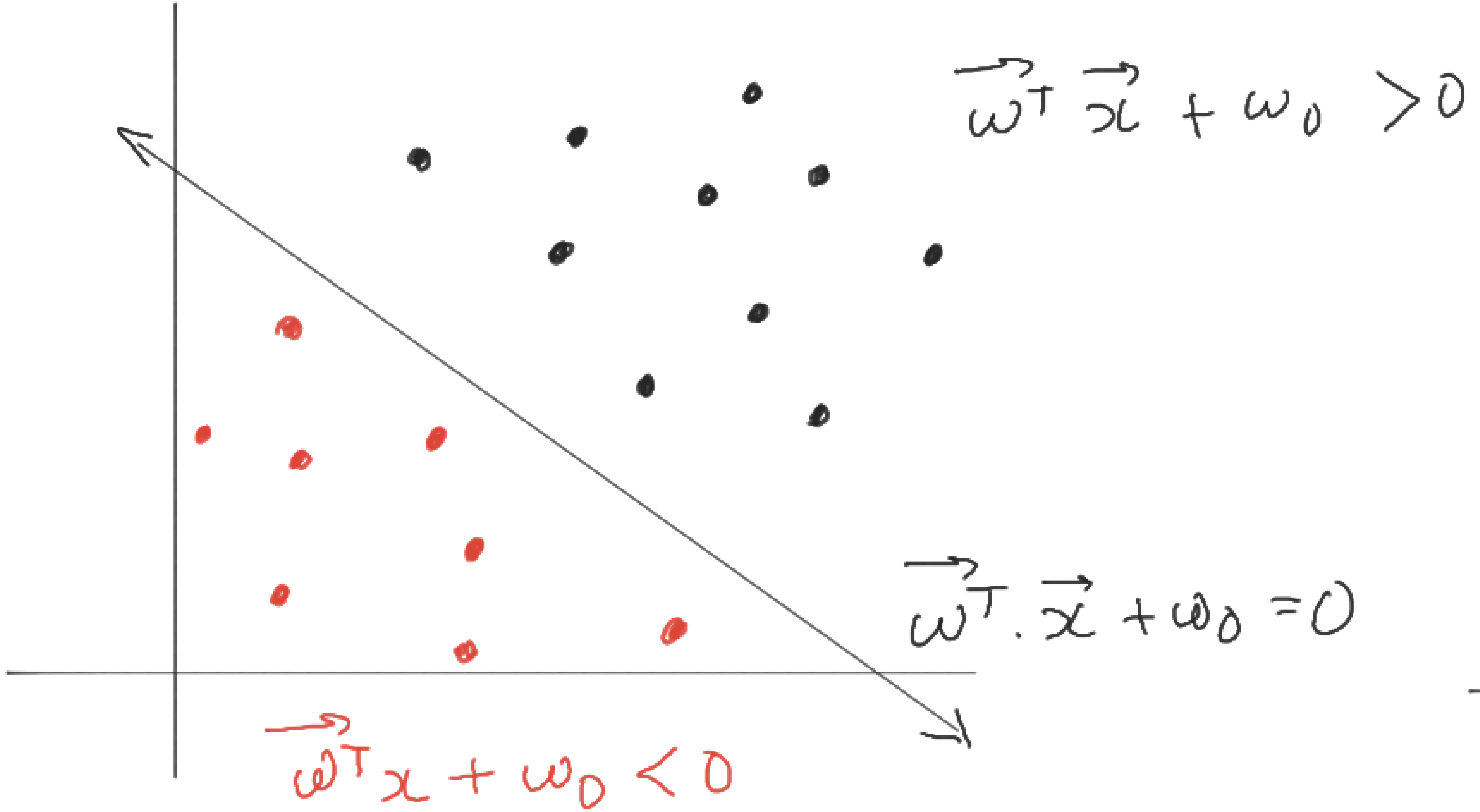
$\Rightarrow \boxed{\vec{w}^T \vec{x} = -w_0}$; $\vec{w}$ = weight vector & $\vec{x}$ = feature vector,

w_0 = bias term

* $\vec{w}$ is always normal to the boundary.

If θ that is the angle between $\vec{w}$ & boundary & if $\theta = 90°$

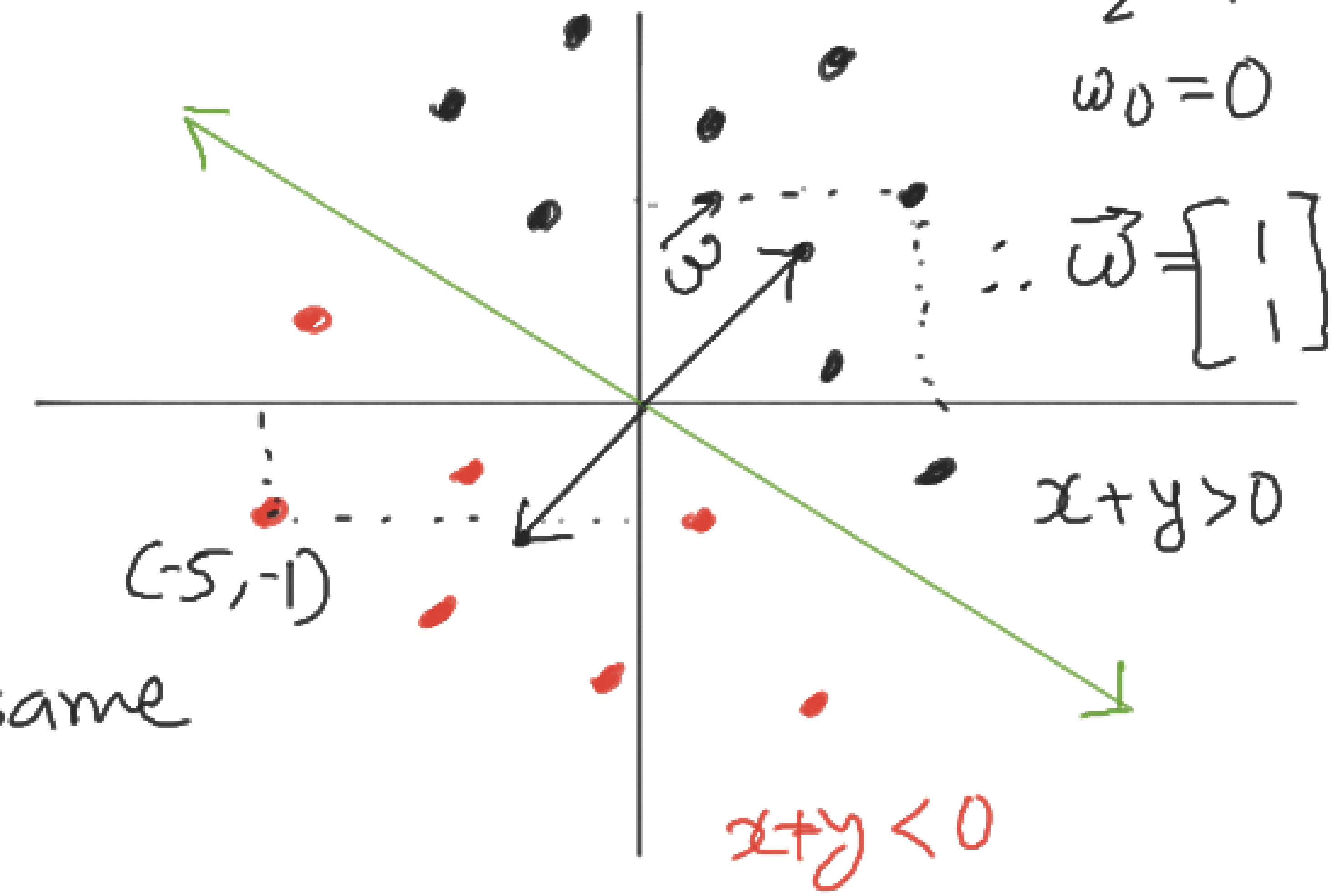
then $\cos\theta = 0$ & vice versa.



Let's take an example of a line-

$x + y = 0$

$\omega_1 = 1$
$\omega_2 = 1$
$\omega_0 = 0$

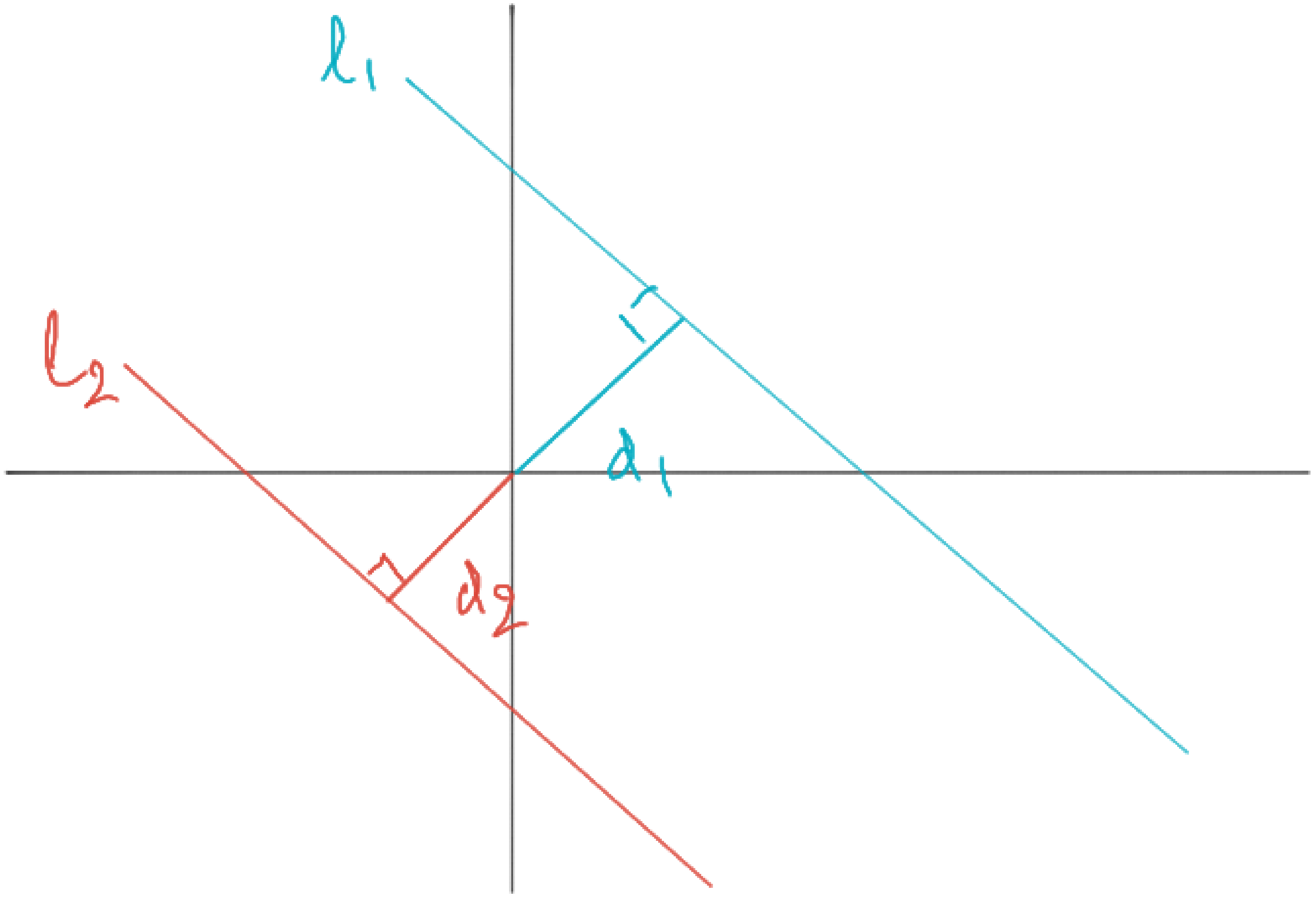


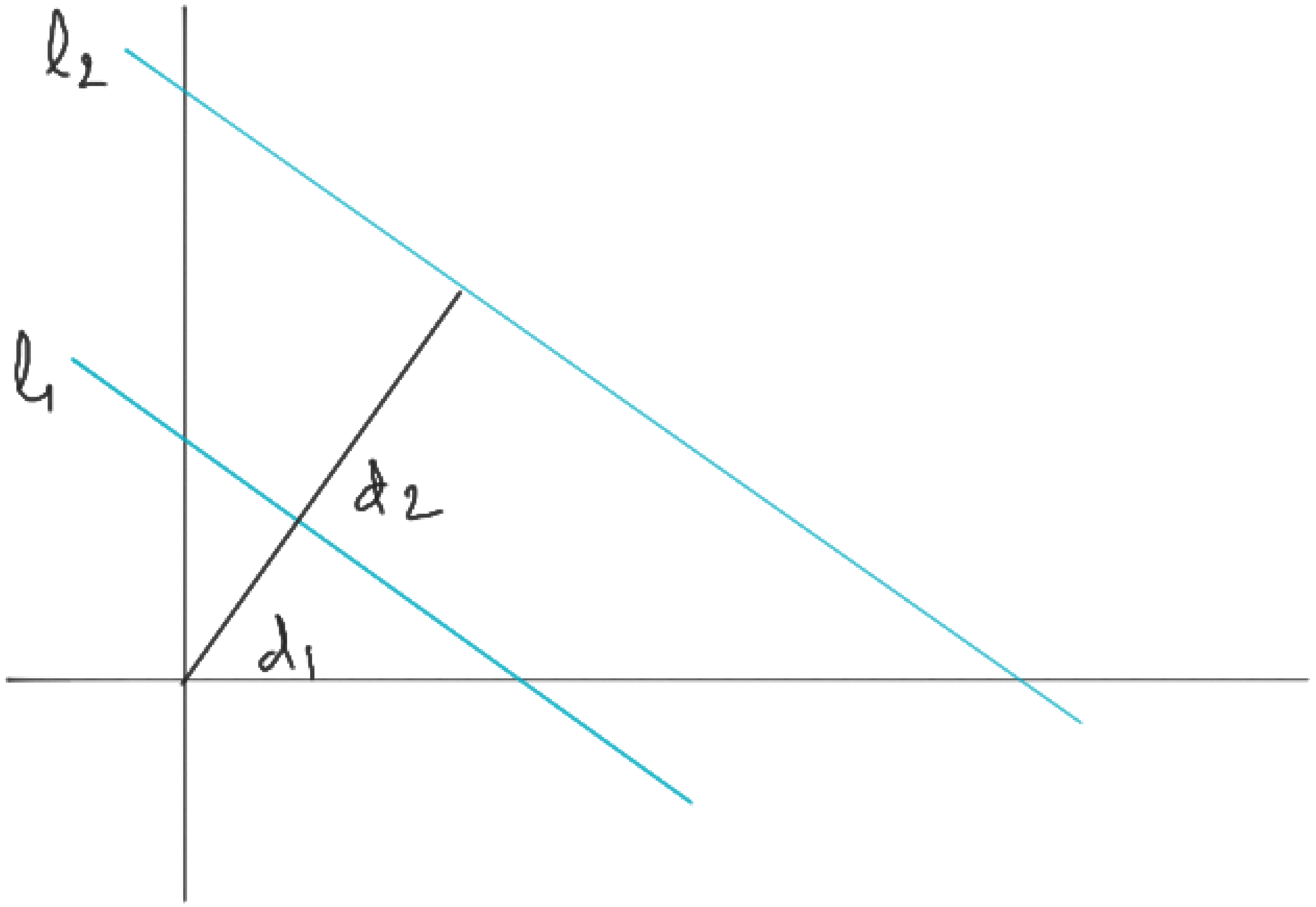
If we draw line $-x - y = 0$, it is the same line but with $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$ as $\vec{\omega}$.

$-x - y$ for point $(-5, -1) > 0$

$-x - y$ for point $(3, 3) < 0$

$\therefore \vec{\omega}$ changes our perception!



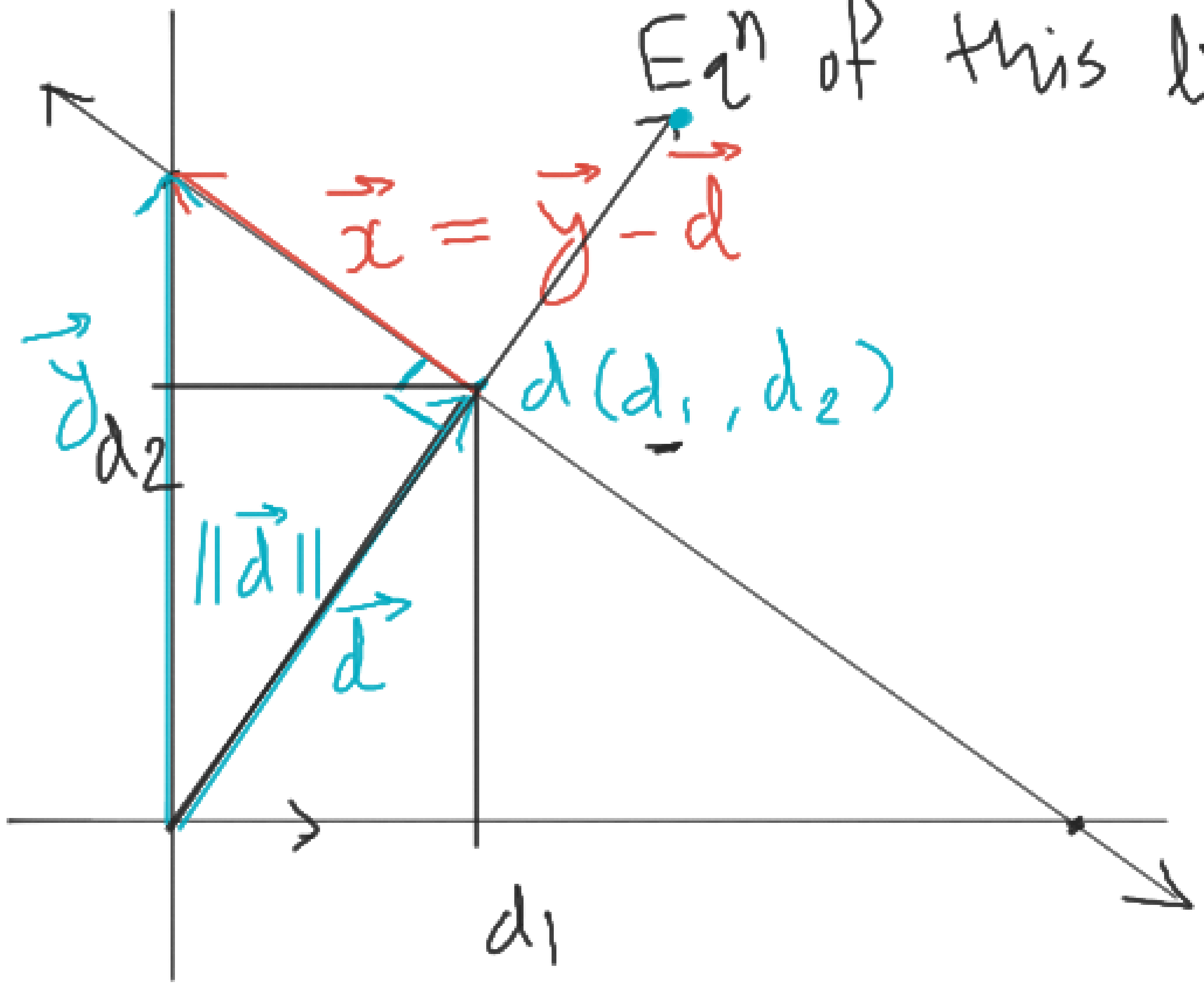


distance $= d_2 - d_1$

distance between

l_1 & $l_2 = d_1 + d_2$

If sign of w_0 of both the line is same then $d = |d_1 - d_2|$ but if sign of w_0 of both lines is different then $d = d_1 + d_2$



Eq^n of this line : $w_1 x_1 + w_2 x_2 + w_0 = 0$

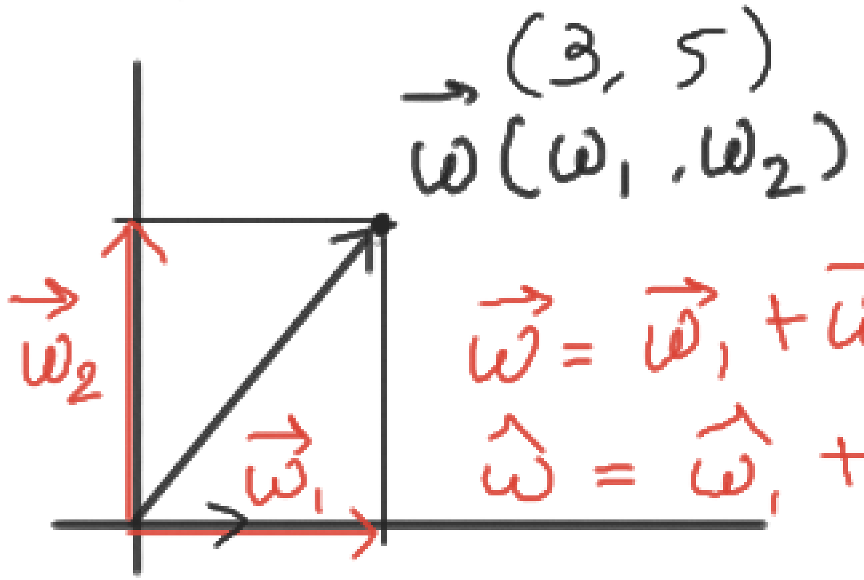
$$\vec{w} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \Rightarrow \|\vec{w}\| = \sqrt{w_1^2 + w_2^2}$$

$$\|\vec{d}\| = \sqrt{d_1^2 + d_2^2}$$

As $\vec{d}$ is in the same direction of $\vec{w}$,

$\vec{d} = k\hat{w}$ where k is some constant

H.W.- If $\vec{d} = k\hat{w}$ then prove that $d_1 = k\hat{w}_1$ & $d_2 = k\hat{w}_2$



$\vec{w} = \vec{w}_1 + \vec{w}_2$

$\hat{w} = \hat{w}_1 + \hat{w}_2$

$$\therefore d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} k\hat{w}_1 \\ k\hat{w}_2 \end{bmatrix} = \begin{bmatrix} \frac{k\, w_1}{\|\vec{w}\|} \\ \frac{k\, w_2}{\|\vec{w}\|} \end{bmatrix}$$

→ Also try to figure this out

$d_1 = \frac{k\omega_1}{\|\vec{\omega}\|}$ & $d_2 = \frac{k\omega_2}{\|\vec{\omega}\|}$; Our eq^n of line is : $\omega_1 x + \omega_2 y + \omega_0 = 0$

As, point $d(d_1, d_2)$ is on the line, $\omega_1 d_1 + \omega_2 d_2 + \omega_0 = 0$

$$\omega_1 \frac{k\omega_1}{\|\vec{\omega}\|} + \omega_2 \frac{k\omega_2}{\|\vec{\omega}\|} + \omega_0 = 0 \Rightarrow k\left(\frac{\omega_1^2}{\|\vec{\omega}\|} + \frac{\omega_2^2}{\|\vec{\omega}\|}\right) = -\omega_0$$

$$k = \frac{-\omega_0}{\left(\frac{\omega_1^2 + \omega_2^2}{\|\vec{\omega}\|}\right)} \Rightarrow k = \frac{-\omega_0 \cdot \|\vec{\omega}\|}{\omega_1^2 + \omega_2^2}$$ Substituting in d_1 & d_2 :-

$$d = \begin{bmatrix} \frac{\omega_1}{\cancel{\|\vec{\omega}\|}} \cdot \frac{-\omega_0 \cancel{\|\vec{\omega}\|}}{\omega_1^2 + \omega_2^2} \\ \frac{\omega_2}{\cancel{\|\vec{\omega}\|}} \cdot \frac{-\omega_0 \cancel{\|\vec{\omega}\|}}{\omega_1^2 + \omega_2^2} \end{bmatrix} = \begin{bmatrix} \frac{-\omega_0 \omega_1}{\omega_1^2 + \omega_2^2} \\ \frac{-\omega_0 \omega_2}{\omega_1^2 + \omega_2^2} \end{bmatrix}$$

Now $x = \vec{y} - \vec{d}$ where $y = \begin{bmatrix} 0 \\ -\frac{\omega_0}{\omega_2} \end{bmatrix}$ $\therefore x = \begin{bmatrix} 0 + \frac{\omega_0 \omega_1}{\omega_1^2 + \omega_2^2} \\ \frac{-\omega_0}{\omega_2} + \frac{\omega_0 \omega_2}{\omega_1^2 + \omega_2^2} \end{bmatrix}$

$$\Rightarrow \vec{x} = \begin{bmatrix} \frac{\omega_0 \omega_1}{\omega_1^2 + \omega_2^2} \\ \frac{\omega_0 \omega_2^2 - \omega_0(\omega_1^2 + \omega_2^2)}{\omega_2(\omega_1^2 + \omega_2^2)} \end{bmatrix}$$

$$\vec{\omega}^T \cdot \vec{x} = [\omega_1 \quad \omega_2] \cdot \vec{x}$$

$$= \frac{\cancel{\omega_0 \omega_1^2}}{\omega_1^2 + \omega_2^2} + \frac{\cancel{\omega_0 \omega_2^2} - \cancel{\omega_0 \omega_1^2} - \cancel{\omega_0 \omega_2^2}}{\omega_1^2 + \omega_2^2}$$

$$= 0$$

As $\vec{\omega}^T \cdot \vec{x} = 0$,

$\cos\theta = 0 \Rightarrow \theta = 90^\circ$

Hence $\vec{\omega} \perp \vec{x}$

$\therefore \vec{\omega}$ is also perpendi-
-cular to the line.