Combinatorics

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India and Pakistan play a 3-match series. How many results are possible? Note that we consider (Ind, Ind, Pak) different from (Ind, Pak, Ind) etc.

Click on an option to submit your answer

A	6
B	9
C	8
D	4

I|P I|P I|P

↓ ↓ ↓

(2) (2) (2)

① Order is important

Is it necessary to play all the 3 matches to complete the series, or playing only 1 match can complete series?

Yes, it is necessary to play all the 3 matches

⇒ Use **PRODUCT Rule**

Product Rule: If a task 'T' can be divided into smaller subtasks t1, t2, t3, … ,tn and to perform task T, it is necessary to perform **all the subtasks** then

n(T) = n(t1) * n(t2) * n(t3) * … * n(tn)

where
n(T) is number of ways to perform T
n(t1) is number of ways to perform t1,
n(t2) is number of ways to perform t2,
….
n(tn) is number of ways to perform tn,

In a bowl-out, for a specific ball you have to choose a bowler and a wicket keeper. Suppose you have 5 bowlers and 3 wicket keepers. How many ways can you select for a ball?

Click on an option to submit your answer

A	8
B	125
C	243
D ✓	15
E	2

There are 3 ways to move from Chennai to Bangalore.
There are 4 ways to move from Bangalore to Delhi.
What are the total ways of moving from Chennai to Delhi?

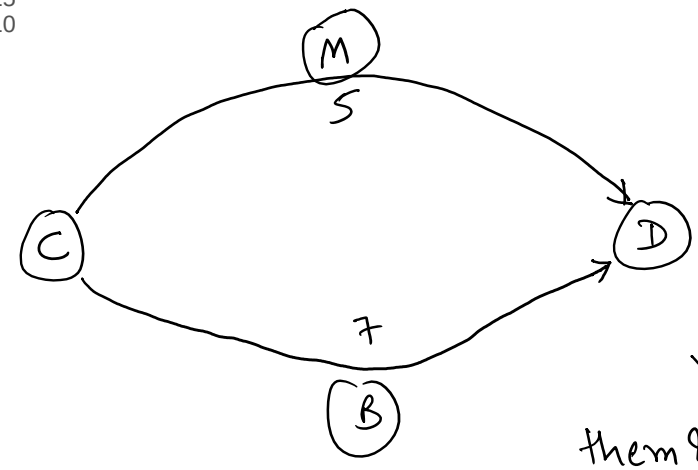
Click on an option to submit your answer

A	7
B ✓	12
C	81
D	64

Quiz-4. Chennai to Delhi II

To reach Delhi from Chennai one can go either via Bangalore or via Mumbai. There are 5 flights that go via Bangalore and 7 flights via Mumbai. Total how many combinations of flights a person can have to go to Delhi from Chennai?

A. 35
B. 12
C. 25
D. 10



To complete the task of flying from Chennai to Delhi, is it necessary to take both flights (via Mumbai & via Bangaluru) or only one of them? → Only one of them

Then use **Sum Rule**

Sum Rule: If a task 'T' can be divided into smaller subtasks t1, t2, t3, … ,tn and to perform task T, it is necessary to perform **only one of the subtasks** then
n(T) = n(t1) + n(t2) + n(t3) + … + n(tn)

where
n(T) is number of ways to perform T
n(t1) is number of ways to perform t1,
n(t2) is number of ways to perform t2,
….
n(tn) is number of ways to perform tn,

A Maruti Showroom has 3 colours in their "Baleno" model and 3 colours in the "Swift" model. In how many ways can they place it such that Baleno and Swift are kept in alternate slots?

Click on an option to submit your answer

A	6
B	36
C	72
D	216
E	720

Option-1 : Starting with Baleno

B S B S B S

$3 \times 3 \times 2 \times 2 \times 1 \times 1$

R R G G B B

$= 36$

Option-2 : Starting with Swift

S B S B S B

$3 \times 3 \times 2 \times 2 \times 1 \times 1$

$= 36$

→ To arrange 6 cars either we can start with Baleno or we can start with Swift.

∴ Total no. of possible ways = $36 + 36 = 72$

✱ Combinations & Permutation

(1) There is a sports competition & 4 students are participating. In how many ways can they win Gold, Silver & Bronze medals?

G S B

$4 \times 3 \times 2 = 24$

Listing all the ways:

A B C ~~BAC~~ ~~CBA~~ ~~DBC~~

A B D ~~BAD~~ ~~CBD~~ ~~DBA~~

(2) We want to select 3 student to play in inter-school sports competition from among 4 students, in how many ways is this possible?

ABC = BAC = CBA = ACB = BCA = CAB

ABD = ADB = BAD = BDA = DAB = DBA

ACD = ADC = CAD = CDA = DCA = DAC

BCD = BDC = CBD = CDB = DBC = DCB

ORDER DOES NOT MATTER

ABD ~~BAD~~ ~~CBD~~ ~~DBA~~

~~ACB~~ ~~BCA~~ ~~CAB~~ ~~DCB~~

ACD BCD ~~CAD~~ ~~DCA~~

~~ADB~~ ~~BDA~~ ~~CDB~~ ~~DAB~~

~~ADC~~ ~~BDC~~ ~~CDA~~ ~~DAC~~

ORDER MATTERS

- PERMUTATIONS

ORDER DOES NOT MATTER

- COMBINATIONS

→ Both the above examples can be thought as:

Organising n - items (students) into r - boxes.

n = no. of items = 4

r = no. of boxes = 3

Permutation: $\_\_ \quad \_\_ \quad \_\_$

$\uparrow \quad \uparrow \quad \uparrow$

$n \quad (n-1) \quad (n-2)$

$= n \cdot (n-1) \cdot (n-2)$

$$ {}^{n}P_r = {}^{n}P^r = \frac{n!}{(n-r)!} = {}^{4}P_3 = \frac{4!}{(4-3)!} = \frac{4 \times 3 \times 2 \times 1}{1} = 4 \times 3 \times 2 $$

$$ {}^{n}C_r = {}^{n}C^r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!} \Rightarrow {}^{4}C_3 = \frac{4!}{3!\,1!} = \frac{4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}} = 4 $$

Quiz-11. In how many ways can we choose two coders from 5 students to represent our college in a national hackathon?

A. 35
B. 12
C. 25
D. 10

Ans - combination : $n = 5, \; r = 2$

$$ {}^{5}C_2 = \frac{5!}{} = \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{} = 10 $$

$$^5C_2 = \frac{5!}{2!(5-2)!} = \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{2} \times \cancel{3!}} = 10$$

Quiz-12: Get me the number of options to select batting order of first 4 batsmen from team of 11 players.

A. 135
B. 1200
C. 2350
D. 7920

Quiz-13: In how many ways can we pick three fruits from a basket of fruits containing apples, oranges & mangoes?

Combination with Repeatition:

If there are n different types of items & we need to choose r items from it then no. of ways

$$= {}^{n+r-1}C_r = \binom{n+r-1}{r}$$

$$n = 3,\ r = 3 \Rightarrow \binom{3+3-1}{3} = \binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!} \times \cancel{2}} = 10$$

List: OOO OOA AAO MMO OAM
AAA OOM AAM MMA
MMM

Quiz-14: How many 4 letter words can be formed using alphabets A, B, C & D? Note: the words might not have any meaning in English.

Here the order is important therefore we will use '—' method (permutation).

— — — —

$4 \times 4 \times 4 \times 4 = 256$

(∵ Repetition is allowed)

Q1. Keep Revising

A Machine Learning course contains **5** 'Supervised Learning', **6** 'Unsupervised Learning', and **2** 'Reinforcement Learning' modules. One of the students wants to revise these topics and picks **two** modules at random. Find the probability that one module is of 'Unsupervised Learning' and one is of 'Supervised Learning'.

A. 11/13
B. 5/13
C. 5/26
D. 11/26

Sample space = selecting 2 modules out of 13.

$\therefore |S| = {}_{13}C_2$

Event = 1 module of Supervised Learning AND 1 module of unsupervised learning.

$|E| = {}_5C_1 \times {}_6C_1 = 5 \times 6 = 30$

$P(E) = \frac{|E|}{|S|} = \frac{30}{{}_{13}C_2} = \frac{30}{\frac{13!}{2!(13-2)!}} = \frac{\cancel{30}^{5} \times \cancel{2} \times \cancel{11!}}{13 \times \cancel{12} \times \cancel{11!}_{6}} = \frac{5}{13}$

Q2. Student Committee

In a school, one **President**, one **treasurer**, and one **secretary** are to be chosen, from a students committee of 10 persons.
No person can hold two posts at a time.
In how many ways can this happen?

A. 880
B. 1000
C. 120
D. 720

Q3. Give a Medal

A student participates in **four** different sports on Sport's Day at her school. Each sport has three medals assigned, **gold**, **silver**, and **bronze**.

It is possible that the student wins no medals. How many possible ways can she get **at least one** medal?

A. 80
B. 255
C. 81
D. 256

Q4. Creating Passwords

How many **five-character-long** passwords can be created such that the first two characters can use any digit from **0 to 9**, and the last three characters can use letters from **A to Z**?

A. 1,581,840
B. 1,560,000
C. 1,757,600
D. 1,263,600

Q5. Elite team

A company wants to form an elite team of **5 members** to assign a particular project. Suppose the company has **5** Data Scientists, **4** Data Engineers, and **6** Data Analysts. Find the probability that the company will select a team that contains **2** Data Scientists, **1** Data Engineer, and **2** Data Analysts.

A. 0.158
B. 0.199
C. 0.302
D. 0.245

Q6. Cars

Twenty distinct cars park in the same parking lot every day. **Ten** of these cars are **India-made**. while the other **ten** are **foreign-made**.

The parking lot has exactly **twenty spaces** all in a row, so the cars park side by side.

What is the probability that on a given day, the cars will park in such a way that they **alternate** (e.g., India-made, foreign-made, India-made, foreign-made, etc)?

A) 1/ (30C10)
B) 2/ (10C2)
C) 4/ (20C20)
D) 2/ (20C10)

$|S| = \underline{20} \quad \underline{19} \quad \underline{18} \quad \cdots \quad \underline{3} \quad \underline{2} \quad \underline{1} = 20!$

Opt.-1 :- start with an India-made car :

Opt.-1 :- start with an India-made car:

$$\frac{I}{10}\ \frac{F}{10}\ \frac{I}{9}\ \frac{F}{9}\ \cdots\ \frac{I}{2}\ \frac{F}{2}\ \frac{I}{1}\ \frac{F}{1} = 10! \times 10!$$

Opt-2 : Starting with a foriegn-made car:

same way = $10! * 10!$

$$|E| = 10! \times 10! + 10! + 10! = 2(10! \times 10!)$$

$$P(E) = \frac{2 * 10! * 10!}{20!} = 2 * \frac{1}{\frac{20!}{10! \cdot 10!}}$$

$$= 2 * \frac{1}{{}^{20}C_{10}} = \frac{2}{{}^{20}C_{10}}$$

Q7. Not to include 5s

Suppose you're making a list of **three digit** numbers.
Answer the following questions:
i) How many three digit numbers are there that **do not contain 5**?
ii) Which contain 5 **at least** once?
iii) Which contains 5 **at most** once?

A. 648, 252, 873
B. 729, 873, 252
C. 648, 252, 874
D. 729, 873, 253