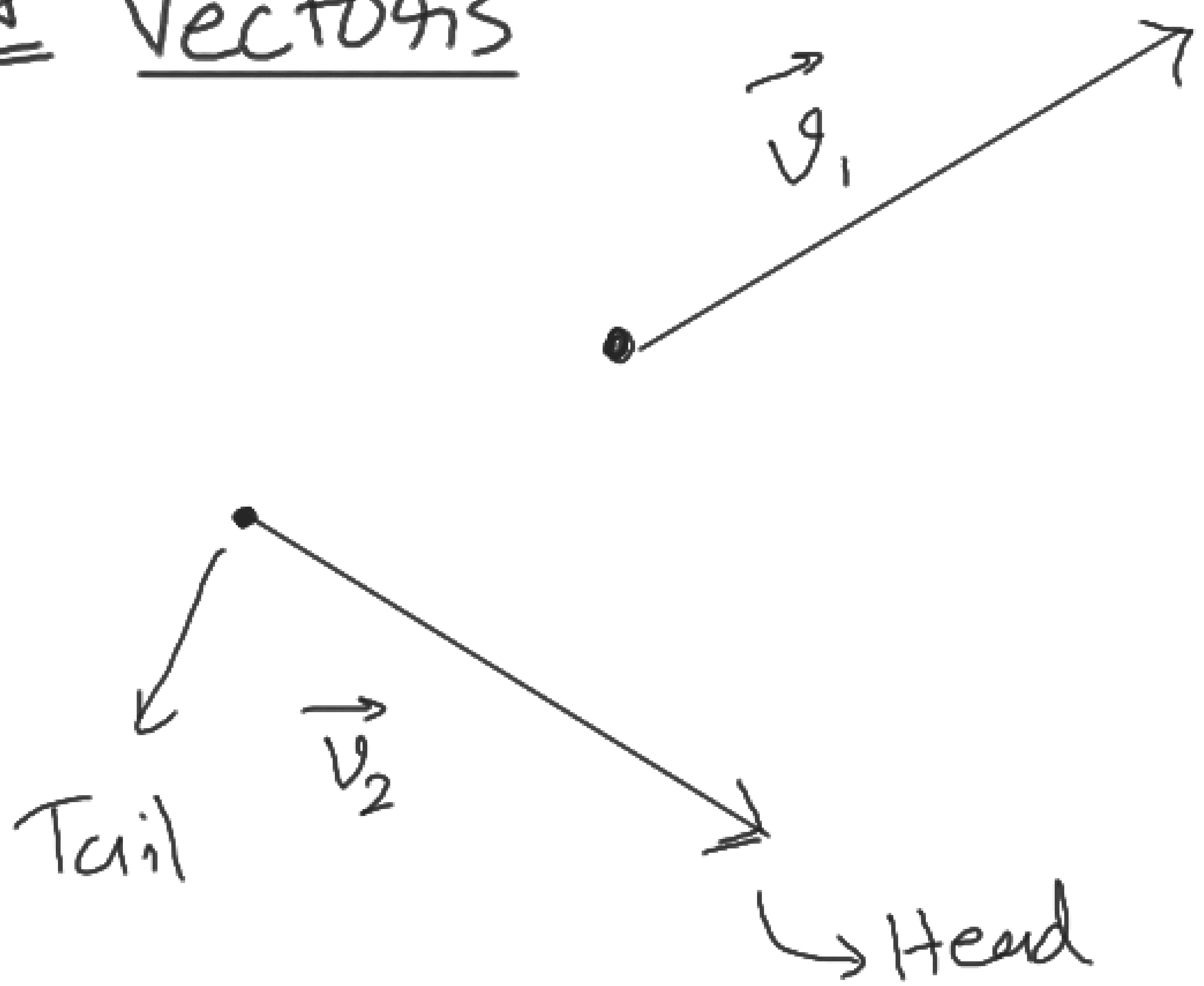
☆ **Vectors** : A quantity that need magnitude (value) and direction both to full describe it.

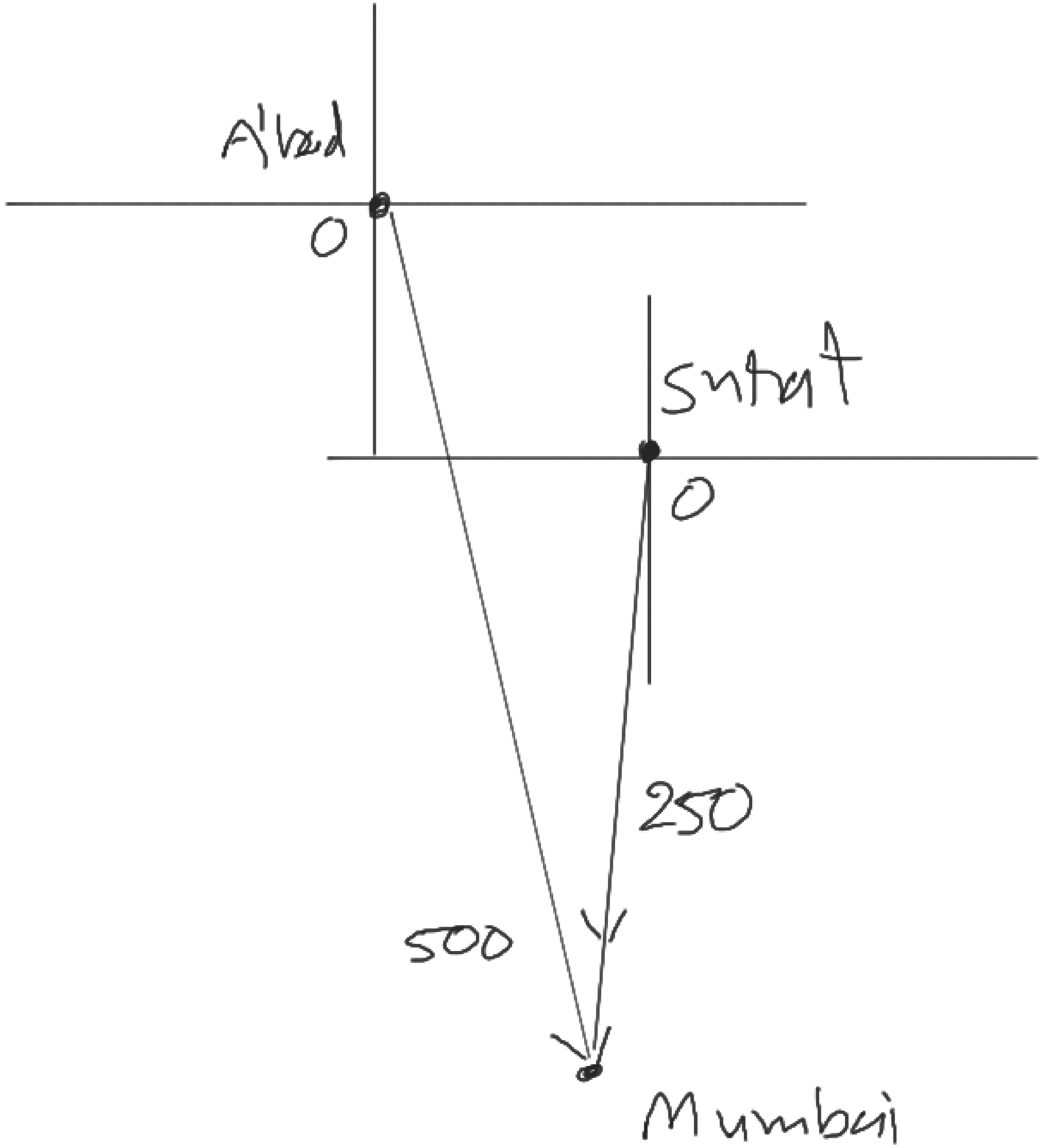
examples - Force, velocity, displacement

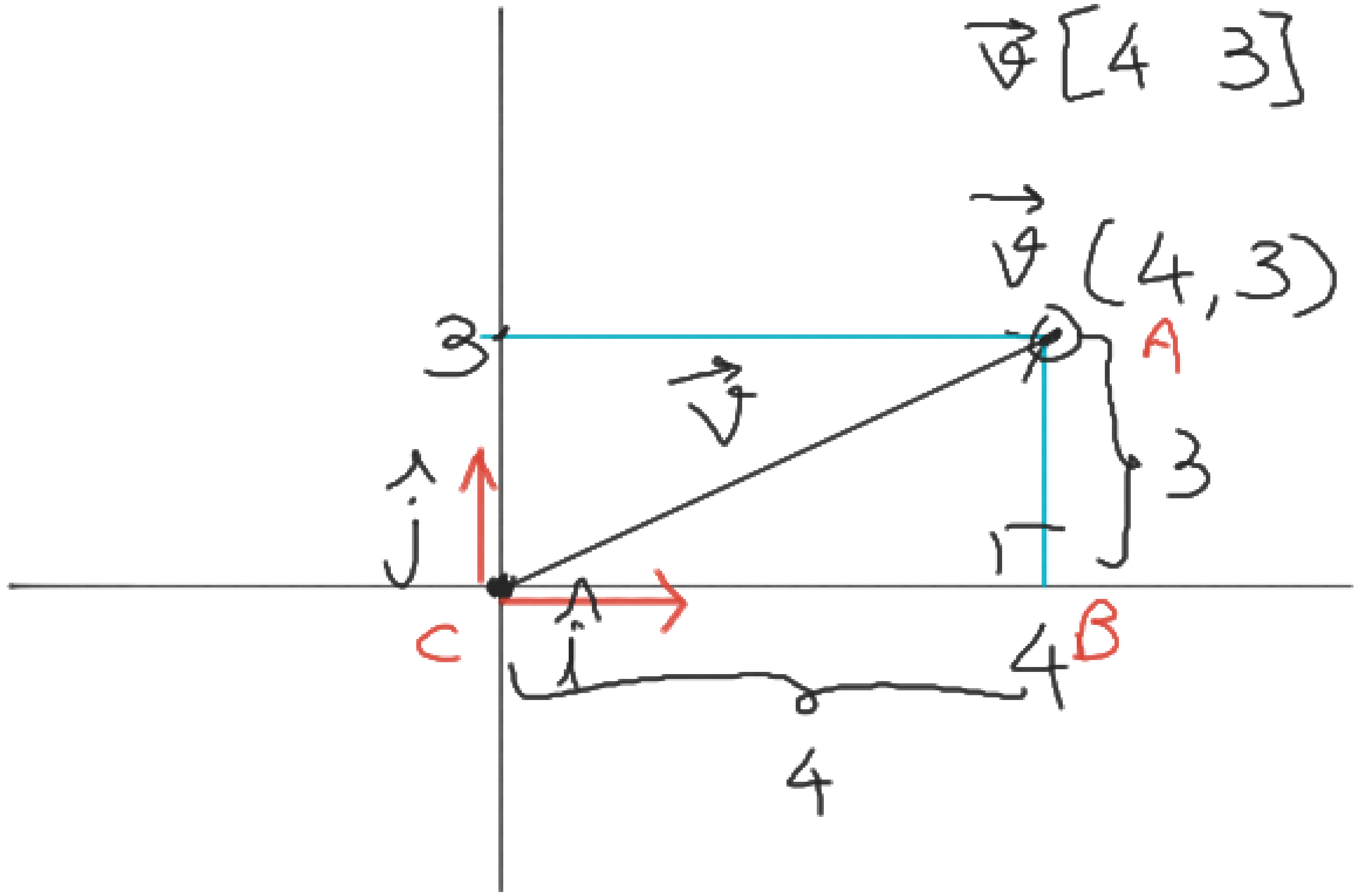
☆ **Scalar** : A quantity that needs just magnitude (value) to describe it fully.

examples - Temperature

Vectors







$\vec{v}\,[4 \quad 3]$

$\vec{v}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$\vec{v}\,(4, 3)$

$\triangle ABC \quad m\angle B = 90^\circ$

$\therefore AB^2 + BC^2 = AC^2$

$\therefore AC = |\vec{v}| = \sqrt{AB^2 + BC^2}$

$= \sqrt{9 + 16}$

$|\vec{v}| = 5$

→ In general, for a vector $\vec{v}\,[x \quad y]$, $|\vec{v}| = \sqrt{x^2 + y^2}$

$\vec{v} = 4\hat{i} + 3\hat{j}$

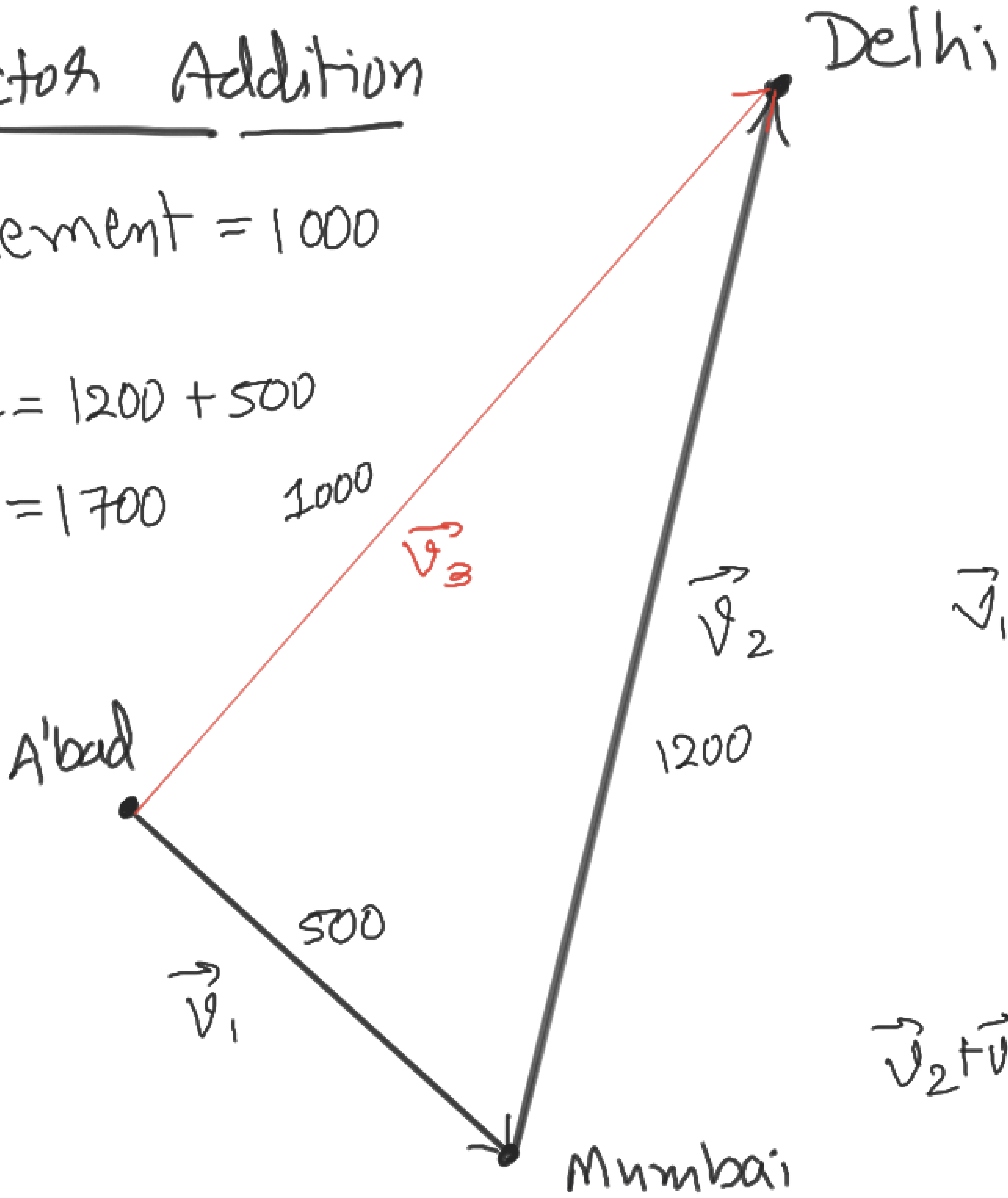
$\hat{i}$ & $\hat{j}$ are <u>unit</u> <u>vectors</u> in direction of x-axis & y-axis respectively.

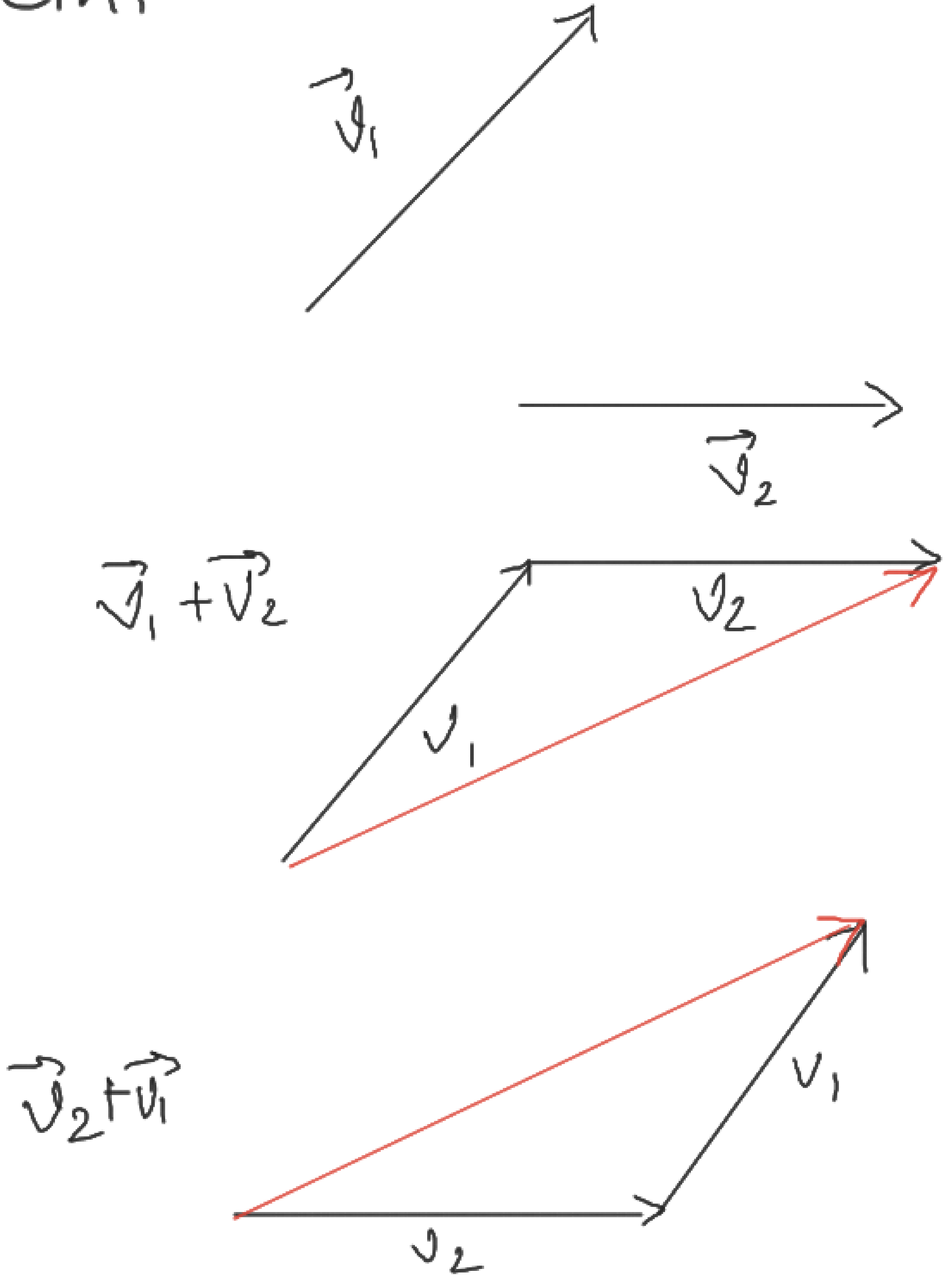
☆ Vector Addition

displacement = 1000

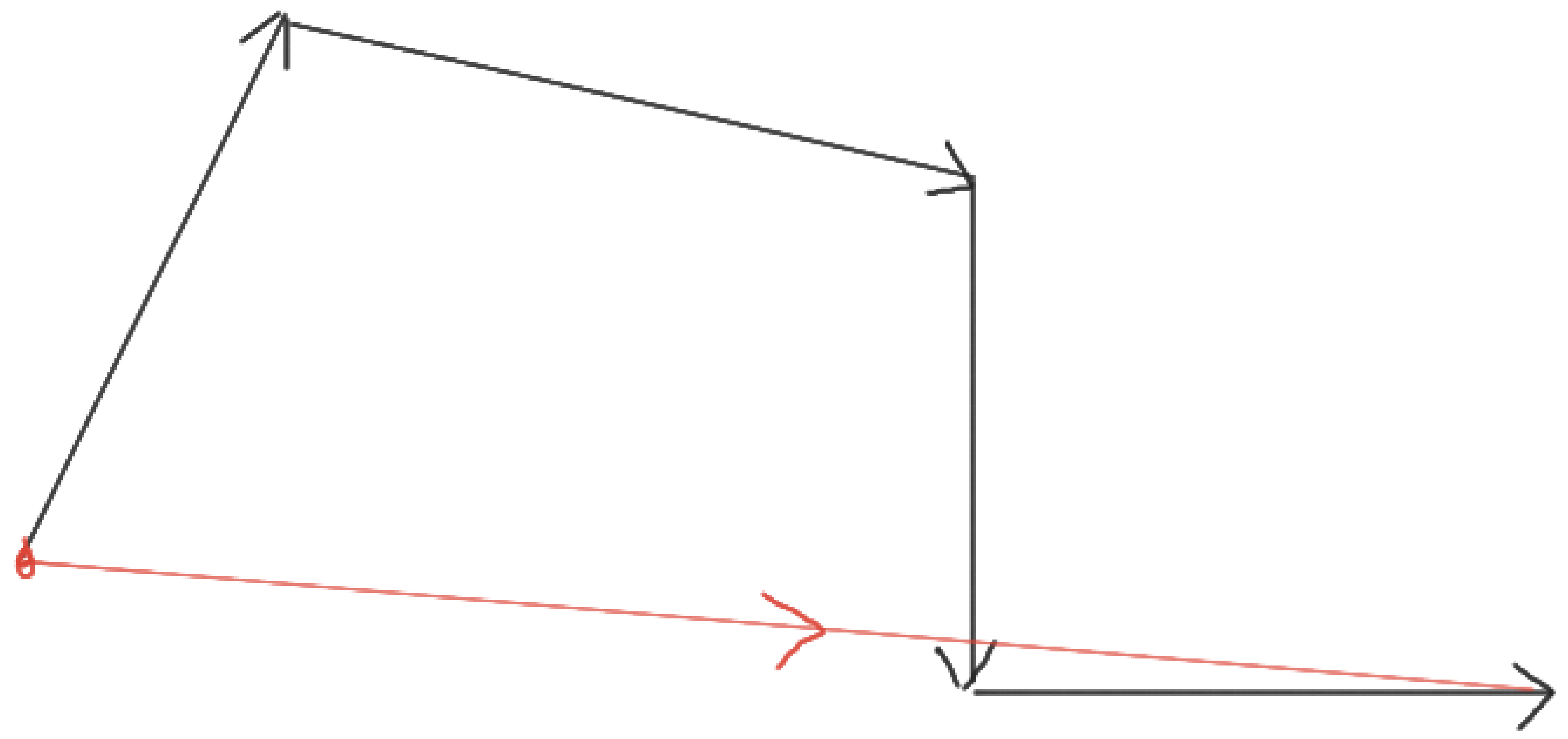
distance = 1200 + 500

= 1700

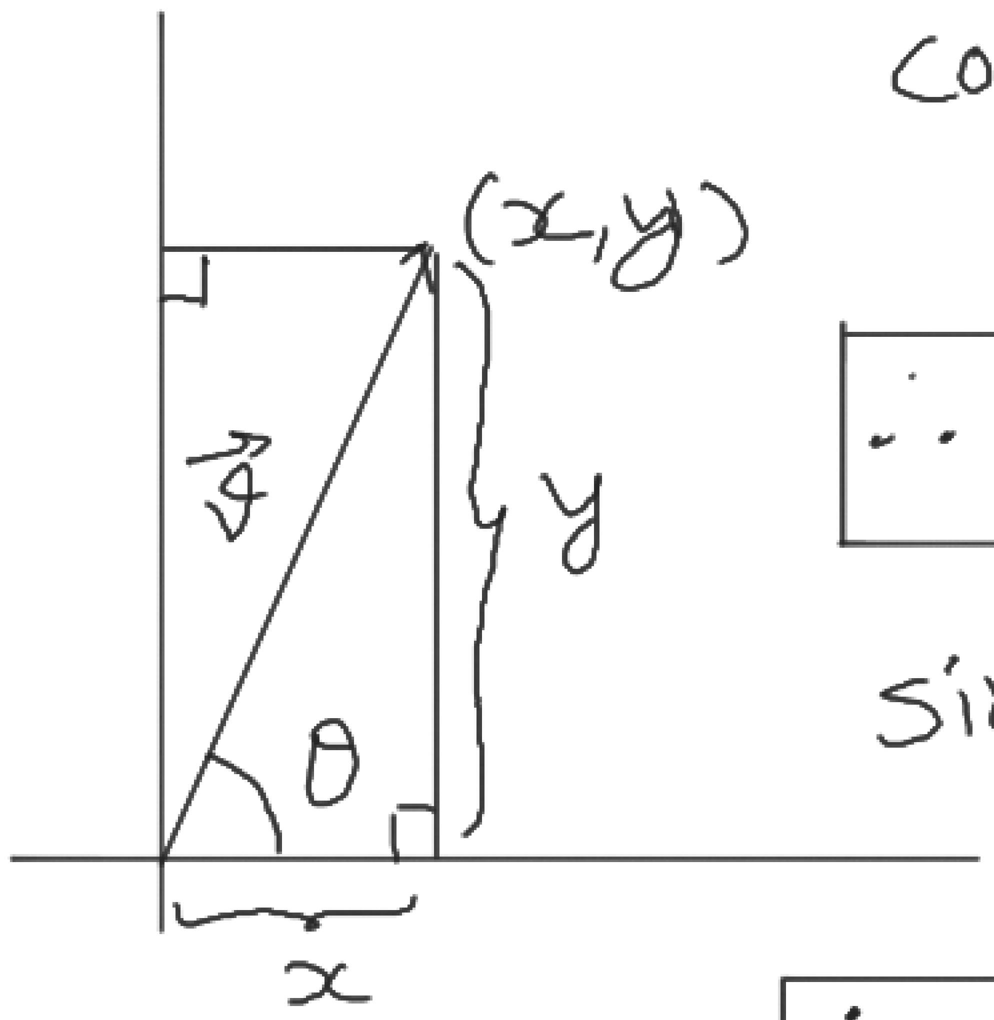




☆ Addition of multiple vectors:



☆ Some results from trigonometry



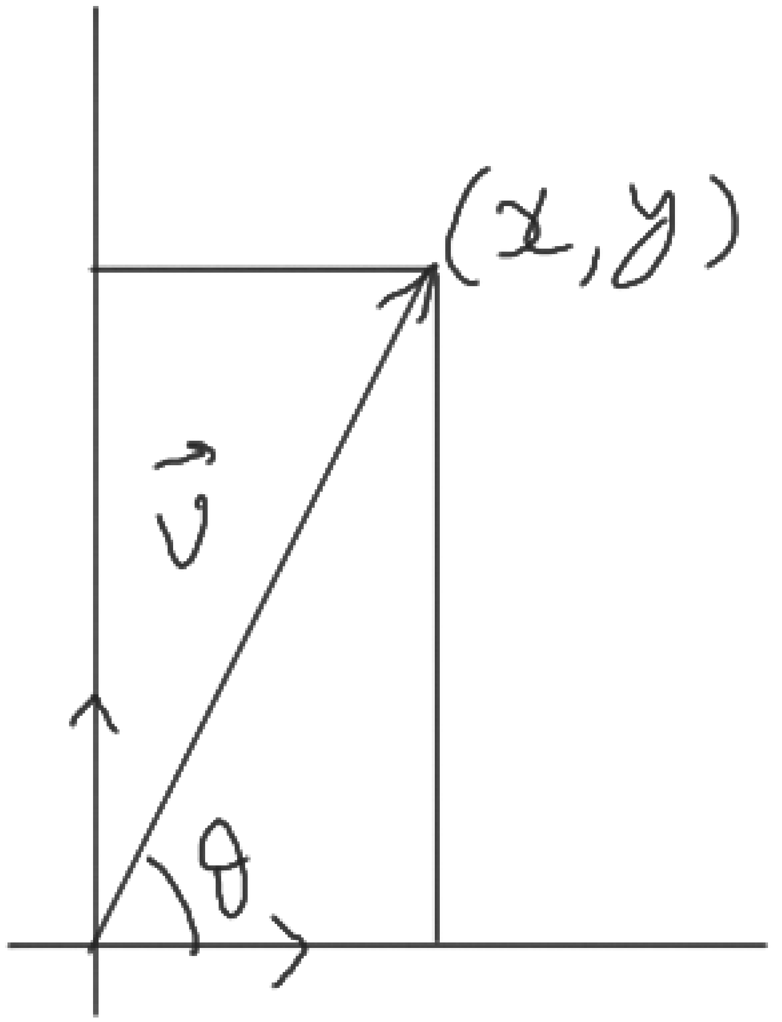
$$\cos\theta = \frac{x}{|\vec{v}|}$$

$$\boxed{\therefore x = |\vec{v}|\cos\theta}$$

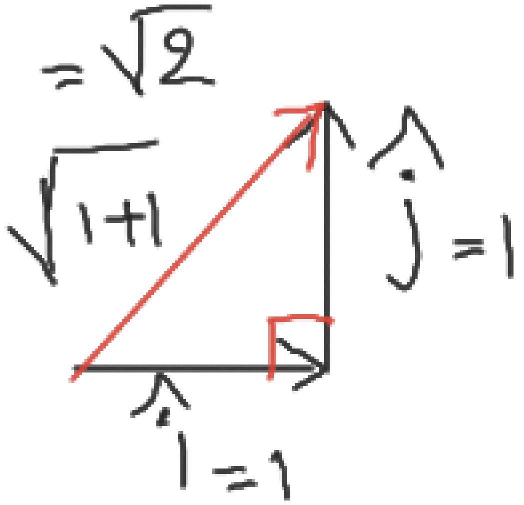
$$\sin\theta = \frac{y}{|\vec{v}|}$$

$$\boxed{\therefore y = |\vec{v}|\sin\theta}$$

☆ **Unit Vector:** Any vector whose magnitude is 1 is called unit vector.



Unit vector in direction of $\vec{v} = \hat{i} + \hat{j}$

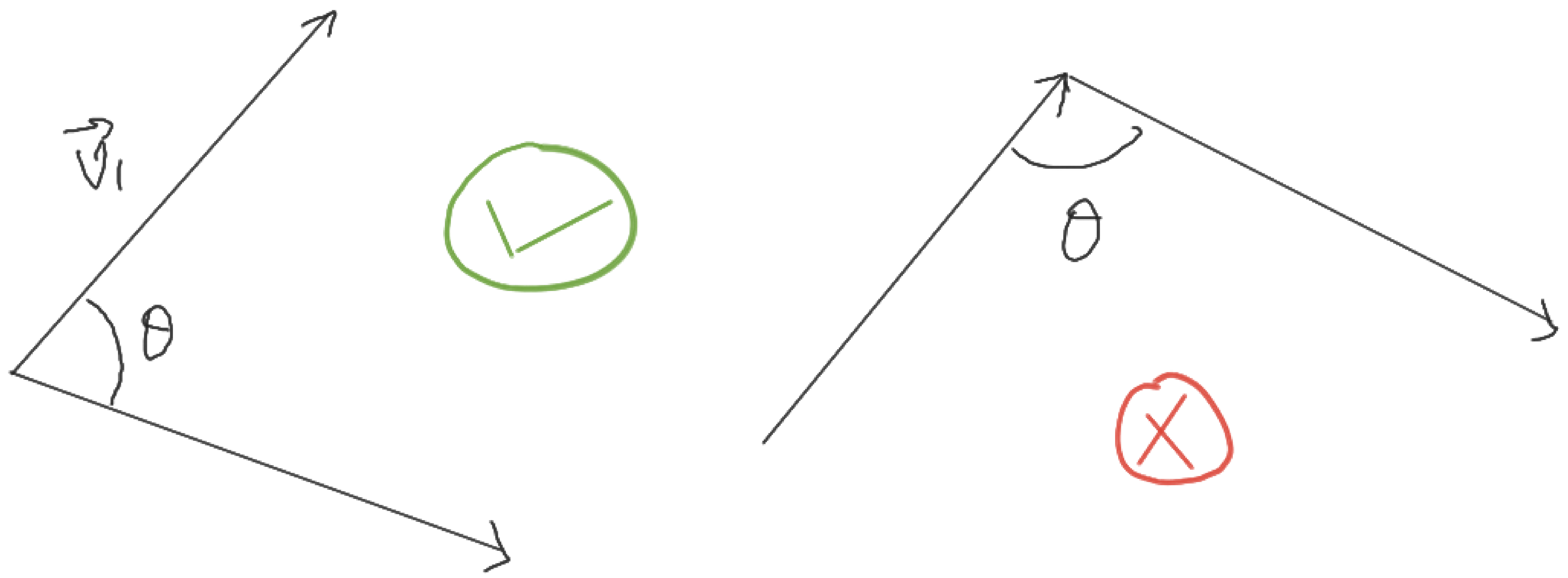


Unit vector in direction of

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$$

$$\boxed{= \frac{x}{\sqrt{x^2+y^2}} \cdot \hat{i} + \frac{y}{\sqrt{x^2+y^2}} \cdot \hat{j}}$$

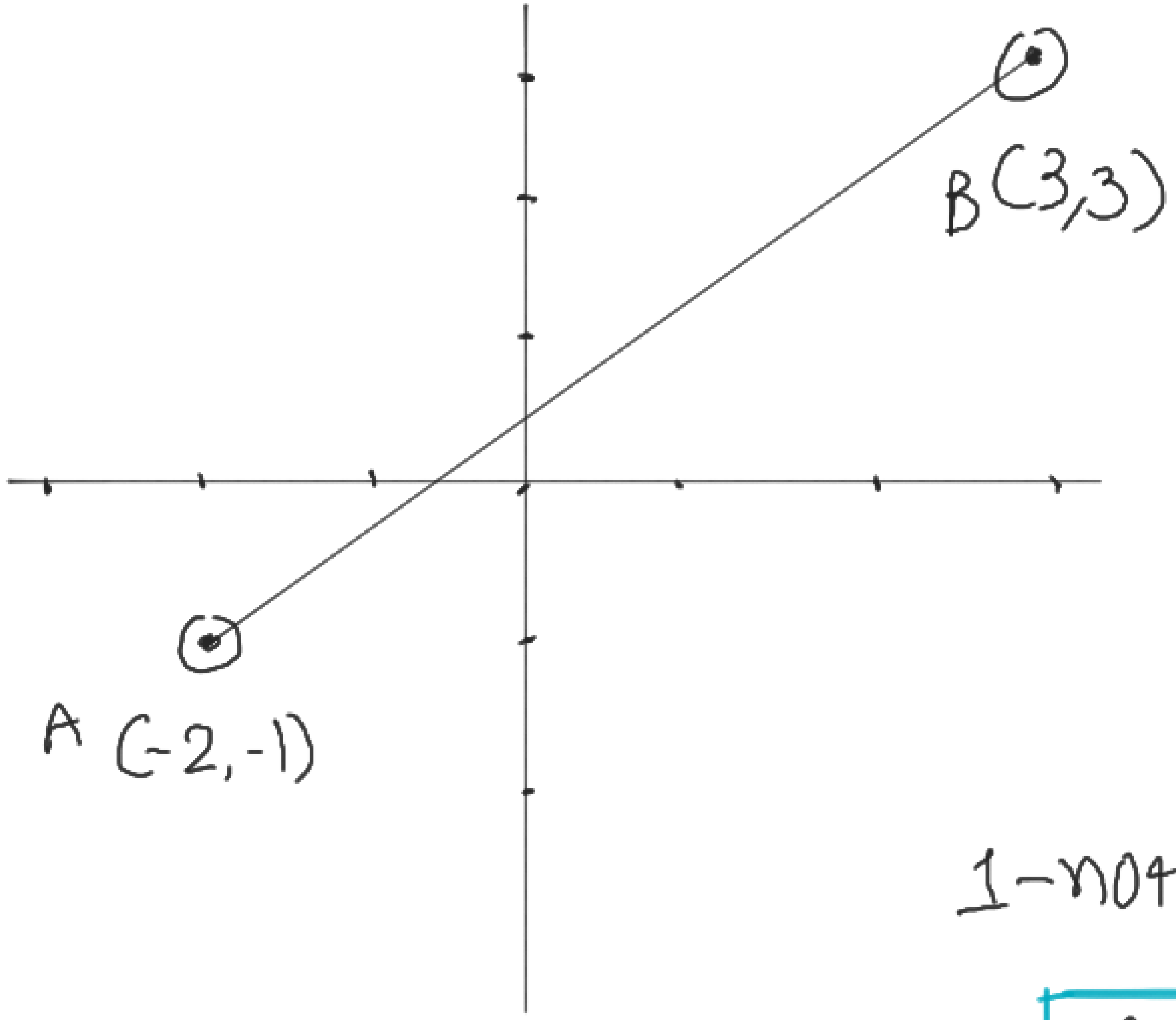
☆ Angle between two vectors



☆ 'norm' of a vector:

Eucledian Distance

magnitude of $\vec{v}(a, b) = \sqrt{a^2 + b^2} \Rightarrow$ 2-norm $= \|\vec{v}\|$



$\sqrt[3]{a^3 + b^3} \Rightarrow$ 3-norm $= \|\vec{v}\|_3$

$|a| + |b| \Rightarrow$ 1-norm $= \|\vec{v}\|_1$

distance between $A(x_1, y_1)$ & $B(x_2, y_2)$

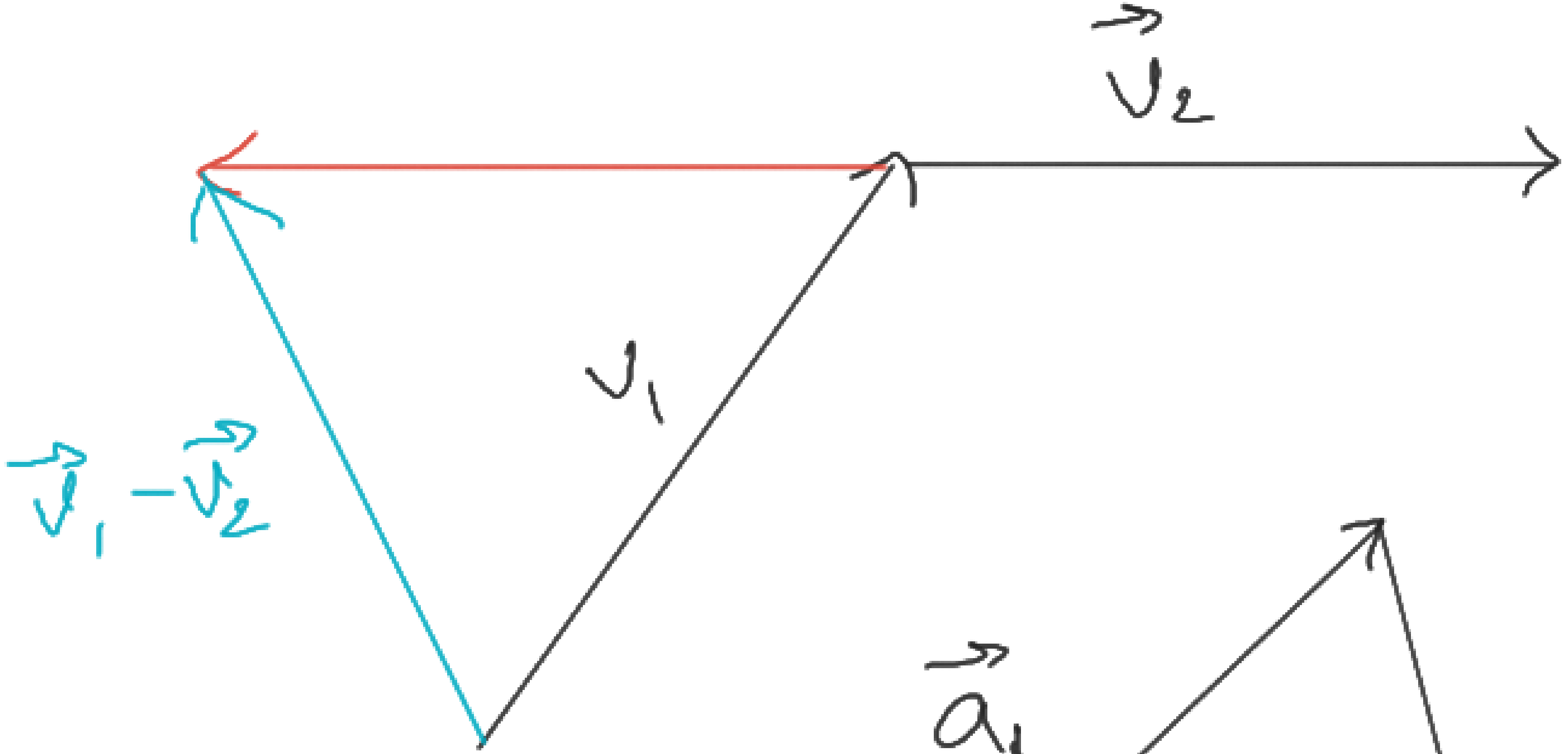
$$d_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = 6.41$$

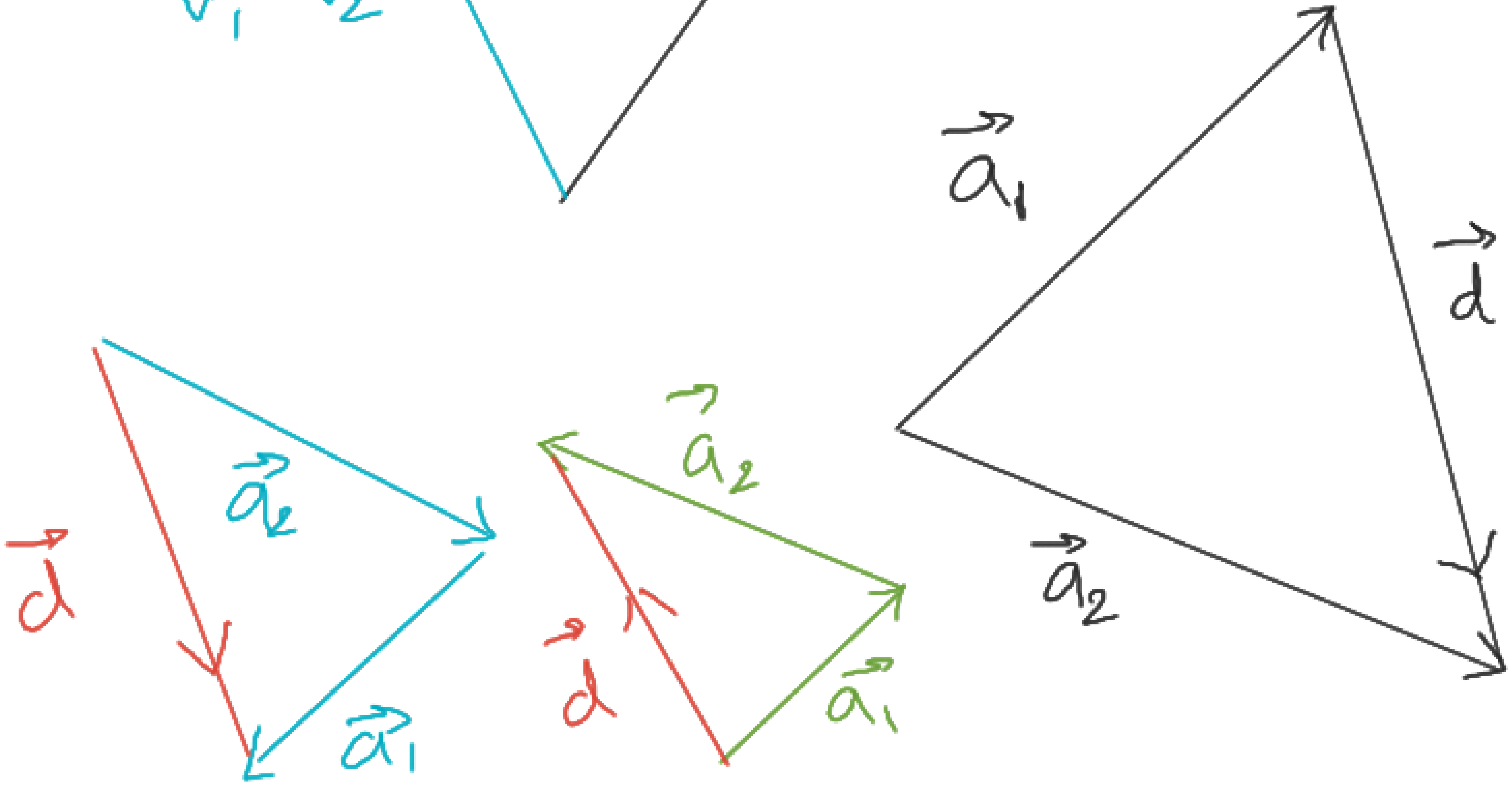
$$\text{1-norm} = |(x_1 - x_2)| + |y_1 - y_2| = 9$$

Manhattan Distance

☆ Subtraction of two vectors / Distance between two vectors:

$$\vec{v}_1 - \vec{v}_2 = \vec{v}_1 + (-\vec{v}_2)$$





$$\vec{a}_2 = \vec{a}_1 + \vec{d}$$

$$\boxed{\vec{d} = \vec{a}_2 - \vec{a}_1}$$

$$\boxed{\vec{a}_1 - \vec{a}_2 = -\vec{d}}$$

Let $\vec{A}_1 (a_1, b_1)$ & $\vec{A}_2 (a_2, b_2)$

$$\therefore \vec{A}_1 = \underline{a_1 \hat{i} + b_1 \hat{j}} \quad \& \quad \vec{A}_2 = \underline{\underline{a_2 \hat{i} + b_2 \hat{j}}}$$

$$\therefore \vec{A}_1 - \vec{A}_2 = a_1 \hat{i} - a_2 \hat{i} + b_1 \hat{j} - b_2 \hat{j}$$

$$\boxed{\vec{d} = (a_1 - a_2)\hat{i} + (b_1 - b_2)\hat{j}}$$

& magnitude of $\vec{d} = ||\vec{d}|| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

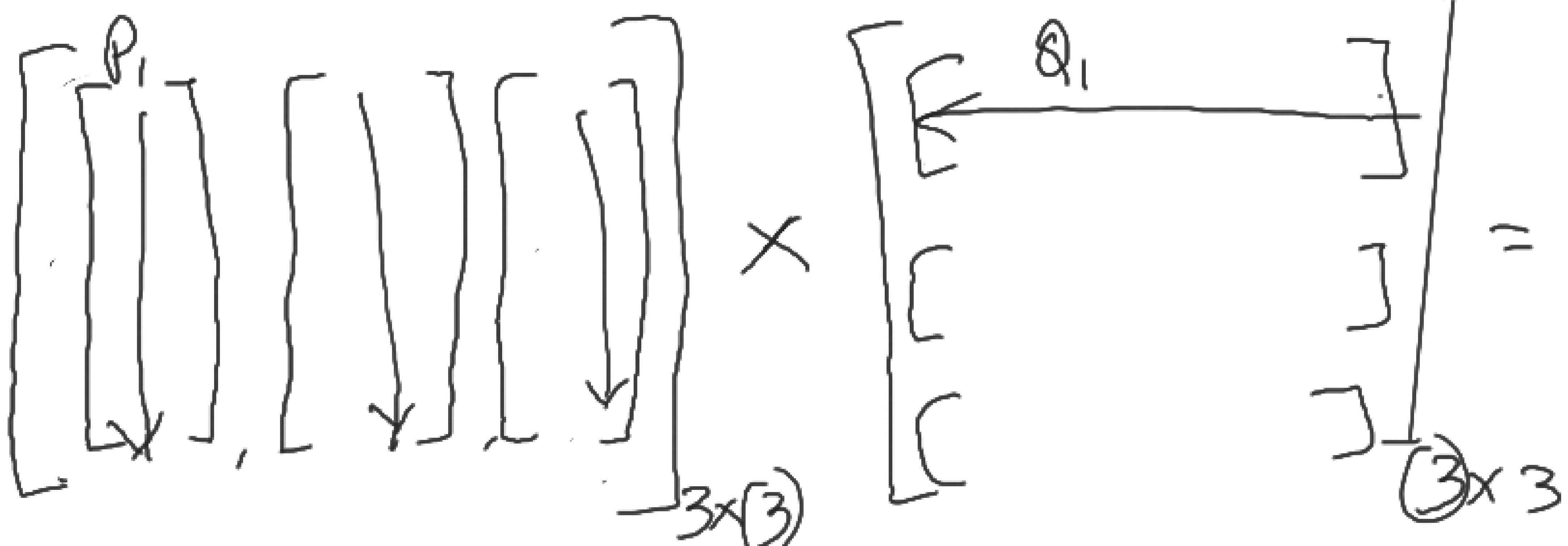
☆ Dot Product

$$\vec{v}_1 \cdot \vec{v}_2 = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cdot \cos\theta \Rightarrow \boxed{\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{\|\vec{v}_1\| \, \|\vec{v}_2\|}}$$

$$= \text{scalar}$$

let $\vec{v}_1 \, [2 \quad 5]$ & $\vec{v}_2 \, [7 \quad 3]$

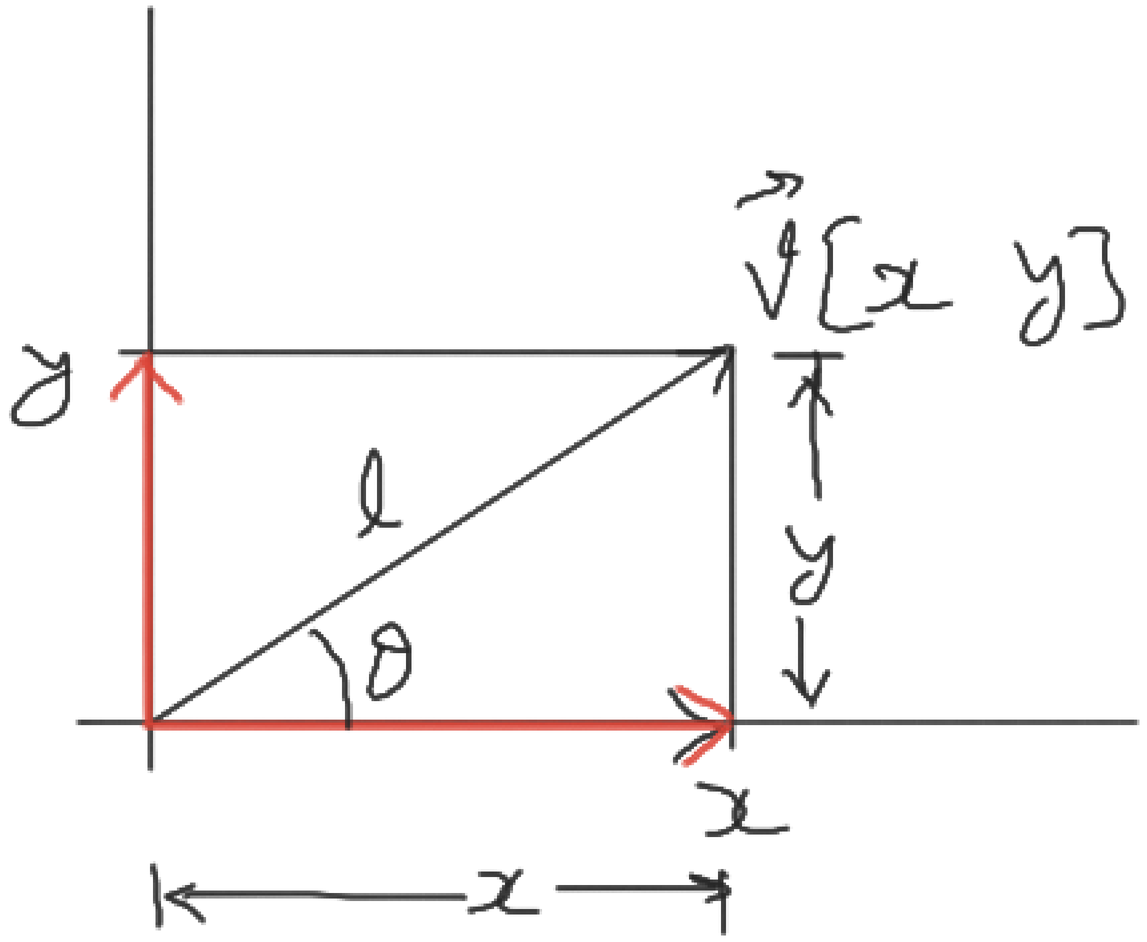
Matrix multiplication
☆ is a bunch of dot products



$$= \begin{bmatrix} \vec{Q}_1 \cdot \vec{P}_1 & \vec{Q}_1 \cdot \vec{P}_2 & \vec{Q}_1 \cdot \vec{P}_3 \\ \vec{Q}_2 \cdot \vec{P}_1 & \vec{Q}_2 \cdot \vec{P}_2 & \vec{Q}_2 \cdot \vec{P}_3 \\ \vec{Q}_3 \cdot \vec{P}_1 & \vec{Q}_3 \cdot \vec{P}_2 & \vec{Q}_3 \cdot \vec{P}_3 \end{bmatrix}$$

$\vec{v}_1[a_1 \quad b_1]$ și $\vec{v}_2[a_2 \quad b_2] \Rightarrow \boxed{\vec{v}_1 \cdot \vec{v}_2 = a_1 \cdot a_2 + b_1 \cdot b_2}$

☆ Trigo-2



$$\tan\theta = \frac{y}{x}$$

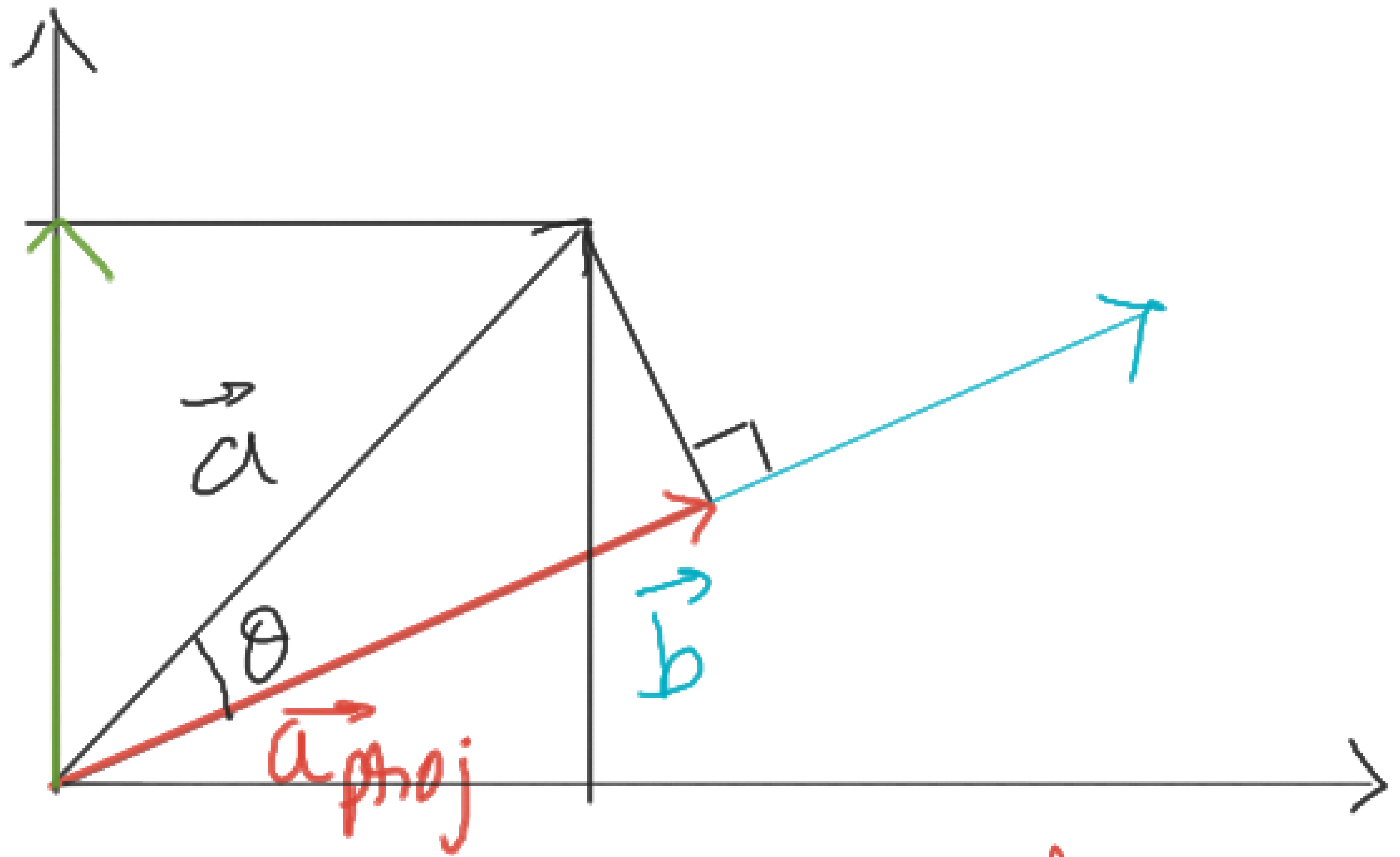
$$y = x\tan\theta$$

$$\|\vec{v}\| = l = \sqrt{x^2 + y^2}$$

$$= \sqrt{x^2 + x^2\tan^2\theta}$$

$$\boxed{\|\vec{v}\| = x\sqrt{1 + \tan^2\theta}}$$

☆ Projection of a vector



↳ Projection of $\vec{a}$ on $\vec{b}$

components of a vector are special cases of projection vectors.

$$\cos\theta = \frac{\|\vec{a}_{proj}\|}{\|\vec{a}\|}$$

$$\boxed{\|\vec{a}_{proj}\| = \|\vec{a}\| \cdot \cos\theta} \quad \text{—— (I)}$$

$\vec{a}_{proj}$ = magnitude of $\vec{a}_{proj}$ * unit vector in direction of $\vec{b}$ ($\hat{b}$)

$$\therefore \vec{a}_{proj} = \|\vec{a}_{proj}\| \cdot \hat{b}$$

$$\boxed{\vec{a}_{proj} = \|\vec{a}\| \cdot \cos\theta \cdot \frac{\vec{b}}{\|\vec{b}\|}}$$

★ An interesting result:

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos\theta \quad \Rightarrow \quad \|\vec{a}\| \cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$$

substituting this value into (I),

$$\boxed{\|\vec{a}_{proj}\| = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}}$$

$$\boxed{\vec{a}_{proj} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|} \cdot \frac{\vec{b}}{\|\vec{b}\|}}$$

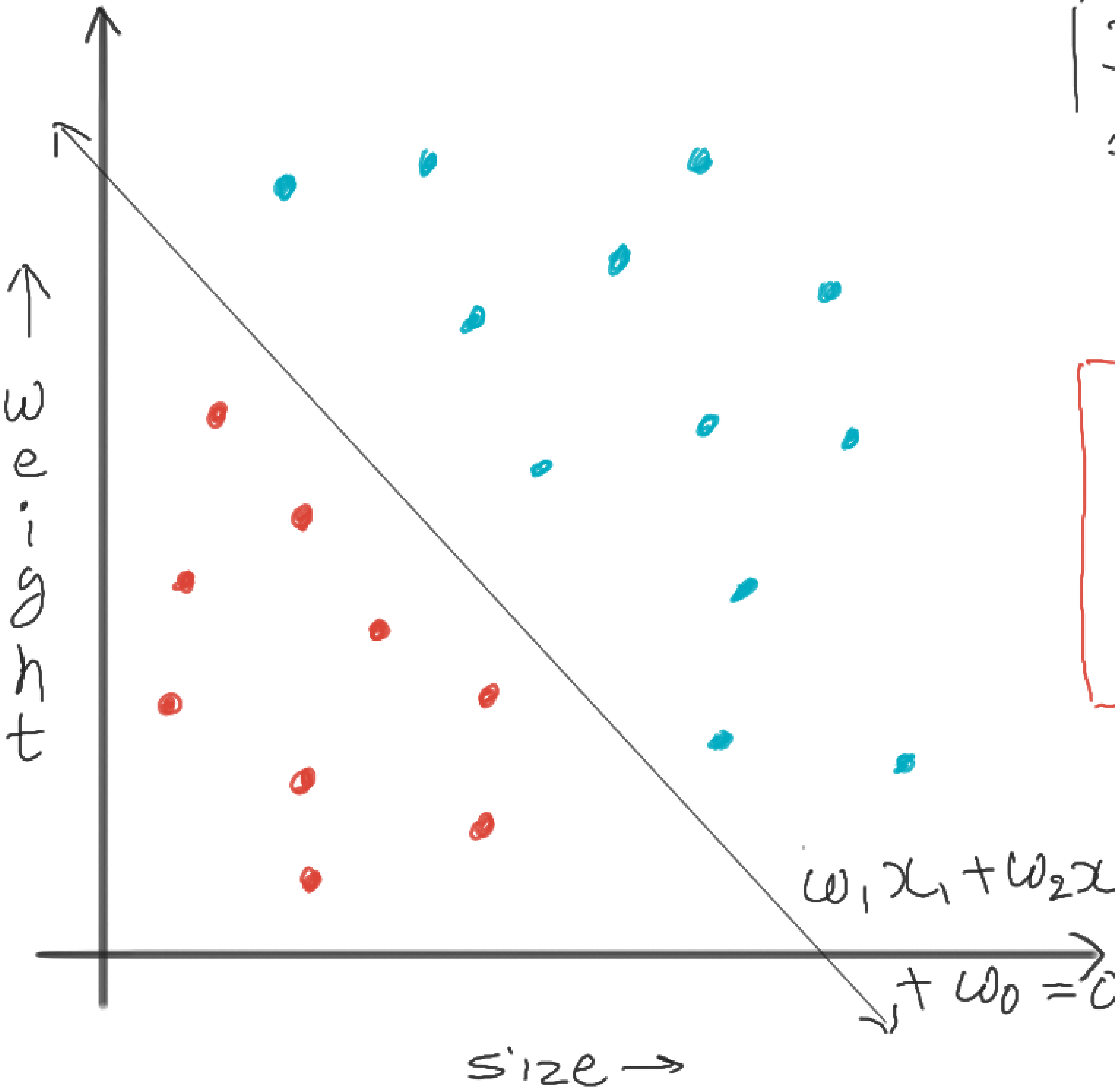
☆ Normal Formula of straight line equation:

General equation of a line: $\underline{ax + by} + c = 0$

Let $\vec{w}\,[a \quad b]$ & $\vec{x}\,[x \quad y]$ then $\vec{w} \cdot \vec{x} = ax + by$

$$\boxed{\vec{w} \cdot \vec{x} + c = 0 \qquad \text{OR} \qquad \vec{w} \cdot \vec{x} = -c}$$

☆ This is called 'normal' equation because $\vec{w}$ is alway perpandicular to the straight line.



$$3x+4y-6=0$$

$$ax+by+c=0$$

size	weight	Type
25	250	big
10	90	small

Features

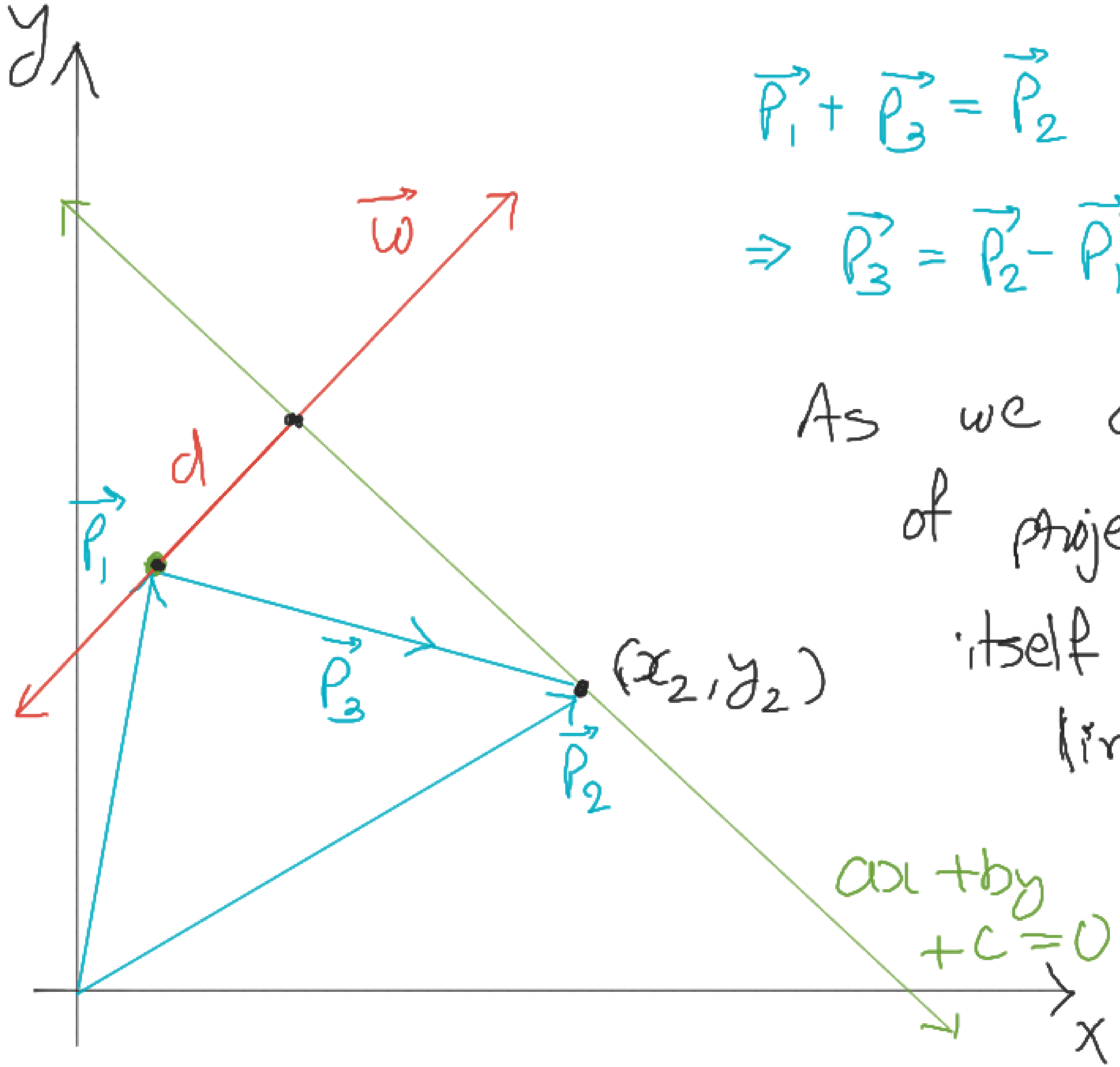
Label

$$w_1x_1+w_2x_2+w_0=0$$

x is our feature vector

w is weight vector

☆ Distance of a point from a line



$$\vec{P_1} + \vec{P_3} = \vec{P_2}$$

$$\Rightarrow \vec{P_3} = \vec{P_2} - \vec{P_1}$$

As we can see, if we find magnitude of projection vector of $\vec{P_3}$ on $\vec{w}$ then itself is the distance of P_1 from the line.

$$\boxed{\|\vec{d}\| = \frac{\vec{P_3} \cdot \vec{w}}{\|\vec{w}\|}} \quad \text{—— (I)}$$

Let $P_1(x_1, y_1)$ & $P_2(x_2, y_2) \Rightarrow \vec{P_1} = x_1 \hat{i} + y_1 \hat{j}$ & $\vec{P_2} = x_2 \hat{i} + y_2 \cdot \hat{j}$

& $\vec{P_3} = \vec{P_2} - \vec{P_1} = (x_2 - x_1) \cdot \hat{i} + (y_2 - y_1) \cdot \hat{j}$

$$\vec{w} = \begin{bmatrix} a \\ b \end{bmatrix} = a \cdot \hat{i} + b \hat{j} \Rightarrow \|\vec{w}\| = \sqrt{a^2 + b^2}$$

Substituting all of these into (I),

$$\boxed{\|\vec{d}\| = \frac{[(x_2 - x_1) \cdot \hat{i} + (y_2 - y_1) \cdot \hat{j}] \cdot [a \hat{i} + b \cdot \hat{j}]}{\sqrt{a^2 + b^2}}} \quad \text{— (II)}$$

From dot product of two vectors $v_1[a_1\ b_1]$ & $v_2[a_2\ b_2]$

$$\vec{v}_1 \cdot \vec{v}_2 = a_1 \cdot a_2 + b_1 b_2$$

Applying this result in the numerator of eqn II

$$\boxed{\|\vec{d}\| = \frac{a \cdot (x_2 - x_1) + b(y_2 - y_1)}{\sqrt{a^2 + b^2}}}$$