Linear Algebra

27 December 2024 16:11

✯ Content

1. Linear Algebra
2. Calculus
3. Co-ordinate Geometry
4. Optimization

✯ Flow

How this topic is related to AI/ML?

↓

Understanding the concept

↓

Visualization

↓

Maths

↓

Coding

✯ Two types of problems in ML:

1. Classification problem

↳ classifying an e-mail as spam or not spam

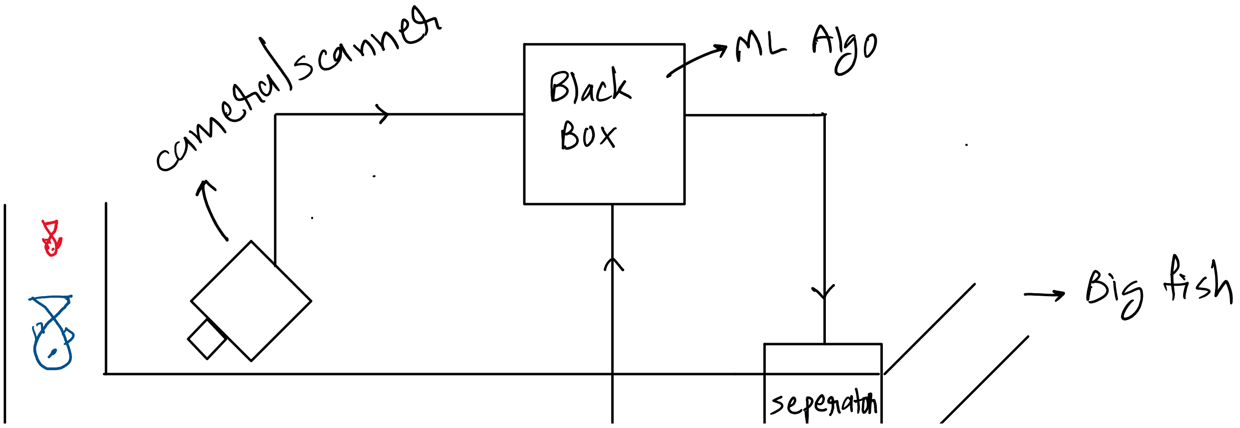
$\boxed{\text{email}}$ → inbox, promotional, social, info, spam

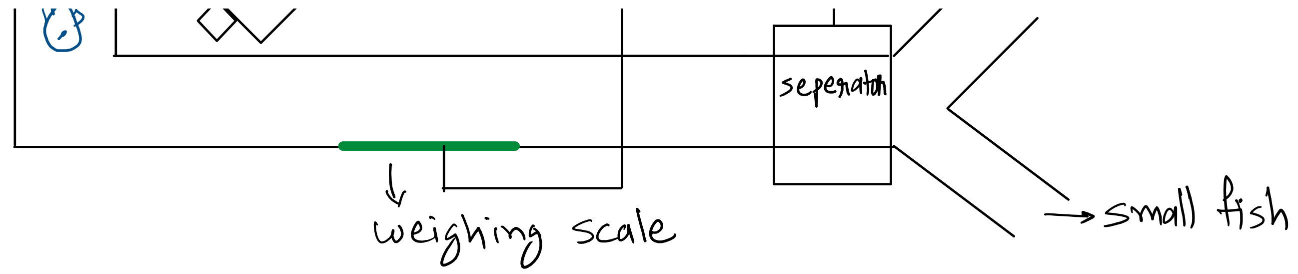
2. Regression Problem

↳ 10,000 Real estate deals (Residential)

features of a residential property → $\boxed{\text{predicting price}}$

✯ Our example problem (Classification)





→ If a human is doing this job and if he/she is very new, how will he/she learn which is a big fish and which one is small

→ Process of induction of an employee:

① Training : Train on a data that also has solution (A data is fed which also tells if a particular fish is a big or small)

Training Data

② Testing : He/she will be provided a data **without solution**. He/she is expected to provide the solution. This solution will be tested and if it is not satisfactory then send him/her to the training.

Testing Data

③ Deploy : Once the employee exhibits satisfactory results, deploy him/her.

The exact same procedure is applied to deploy an ML model.

★ Let's have a look at a sample training data for our problem: features

Label

features | Label

length	width	color	weight	Type
5	3	black	20	0
15	8	orange	40	1
20	10	black	40	1

0 - small fish
1 - Big fish

Features = independent variables, Datapoints, records

Label = dependent variable = target

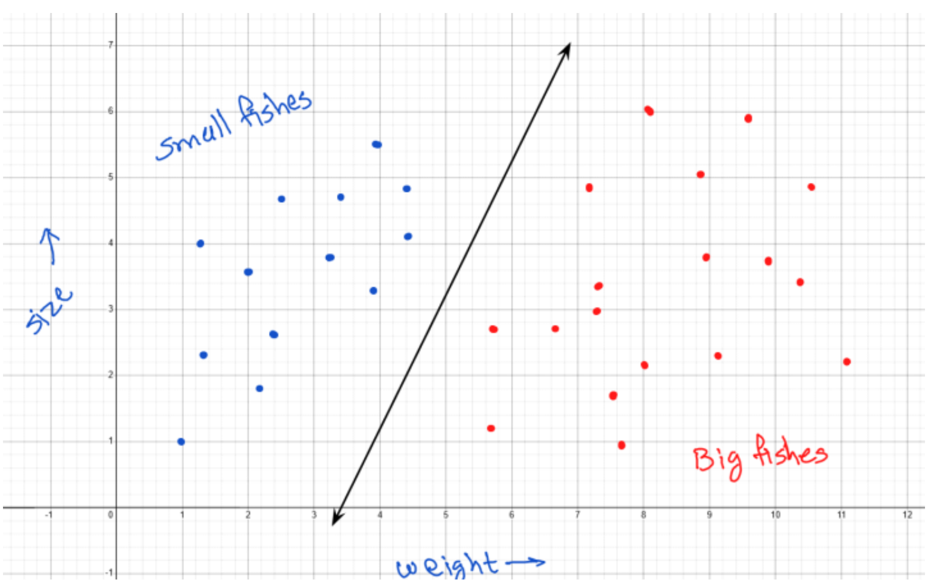
Predictions of our Black-box (ML Algo) must be in-line with the training data.

☆ Training Data Vs. Testing Data

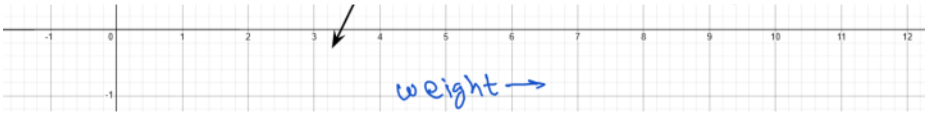
We split our data into two parts (proportion may be 60:40/70:30/80:20 or any else combination based on the case)

☆ Let's visualize this problem.

For simplicity, let's take only two features - weight & size (where the size may be $l \times w$)



→ If we plot all the datapoints on a graph, we can see that if our ML Algo (model) can somehow draw a straight line (Find

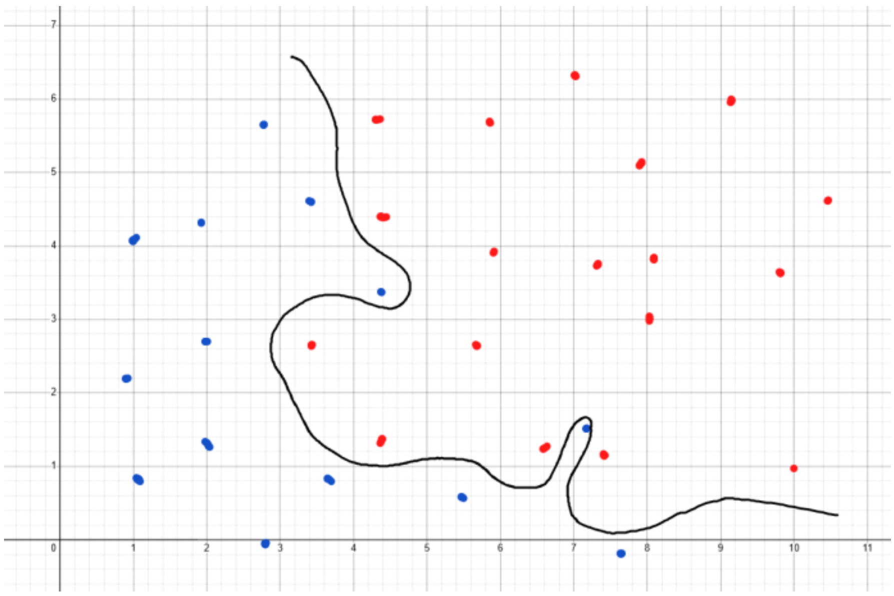


straight line (Find equation of such a line) that can act as a boundary then we can say that our machine has "learned".

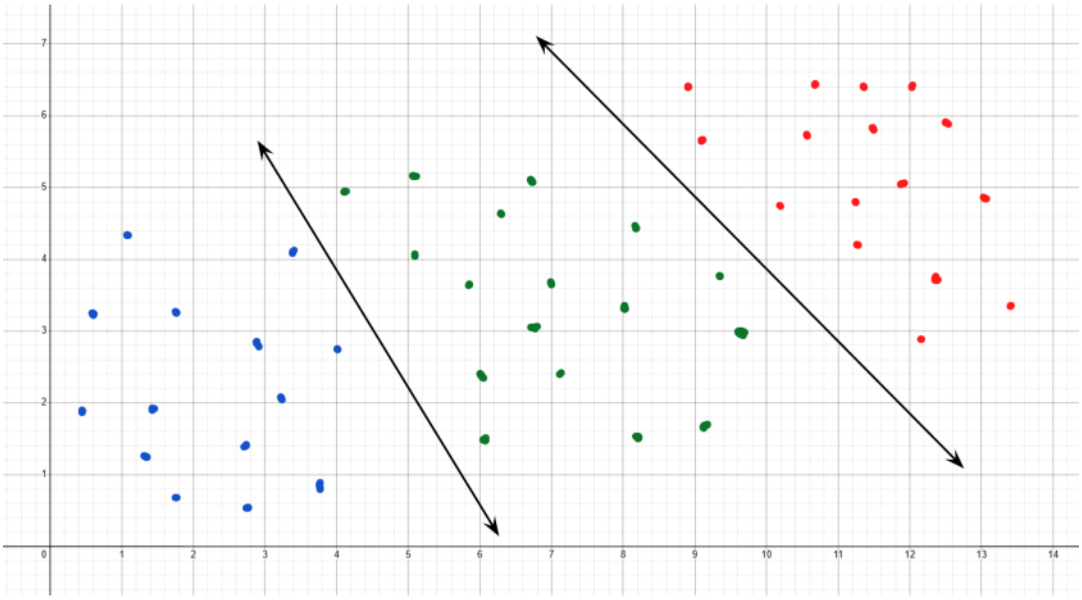
→ So our problem has changed to find a straight line.

→ This line is also called "boundry", "seperator" or "classifier"

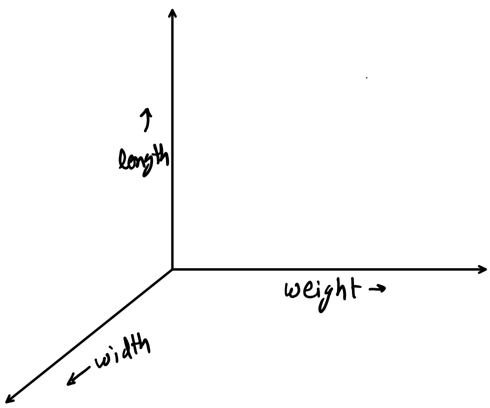
☆ **This may not be as simple.** What if our points are not seperable by a line? In such case, we might also need to draw a curve!



→ What if we have to sort more than two types of fishes – Big fish, medium fish, small fish

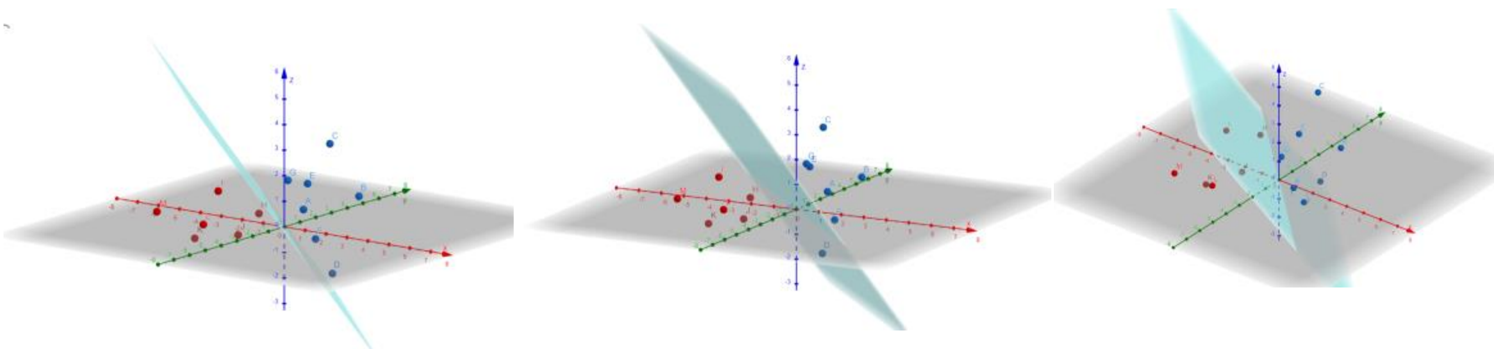


✯ Why did we consider only two features size & weight in the name of simplicity? What will happen if a new feature is added?



Ans: A new dimension will be added to our graph.

As a result, our classifier/boundary may not remain a simle straight line but will become a plane:



✯ What will happen if we have 4-features?

Ans: We will have 4-D graph which will go beyond the scope of visualization.

☆ But what will happen to our boundry?

Ans:

No. of features	Dimensions of the graph	Dimension of the seperator
2	2	1 (line)
3	3	2 (plane)
4	4	3
⋮	⋮	⋮
n	n	(n-1)

(3 to (n-1): Hyperplane)

In general **All** the seperators (irrespective of their dimensions) are also known as **Hyperplanes**.

☆ **But we figured out that all of them are essentially higher-dimension versions of a straight line. ∴ It is important to know about straight lines.**

→ Different forms of equation of straight line:

$\boxed{1}$ $y = mx + c$ ———— slope-intercept form

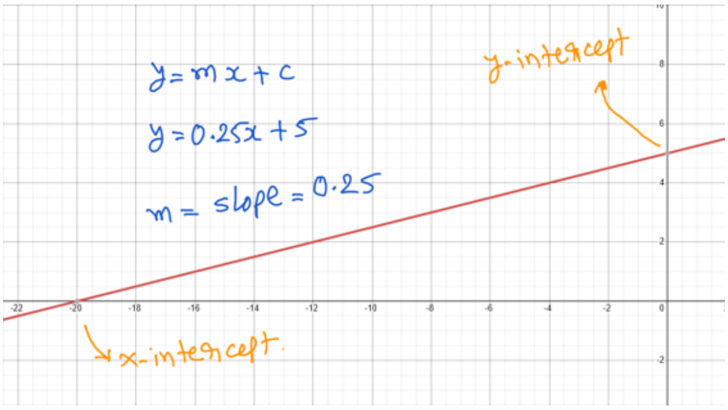
where m = slope

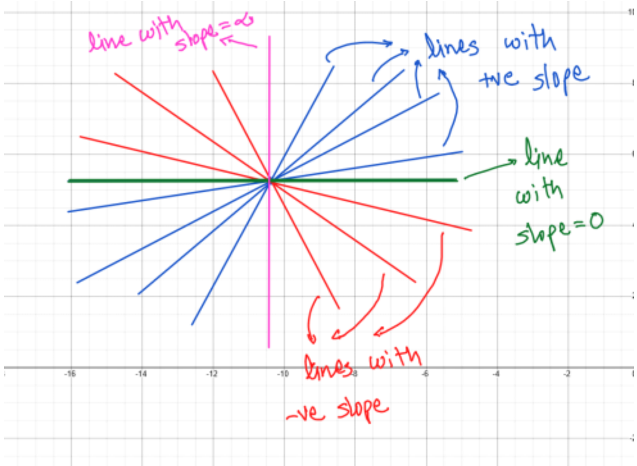
c = y-intercept

Understanding a straight line formula

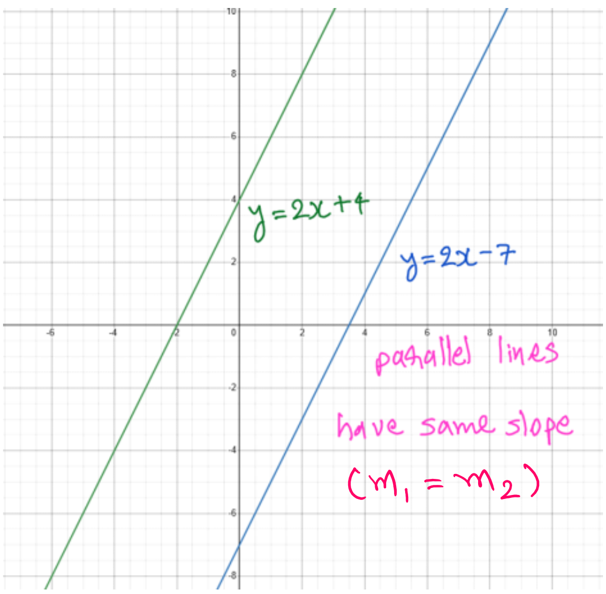
Example: $y = 4x + 5$. In this formula, if we put different values of x, we will get corresponding values of y and these pairs of (x, y) will give us varius points but, all these points will be linear. Means, they can but, all these points will be linear. Means, they can be connected using a straight line.

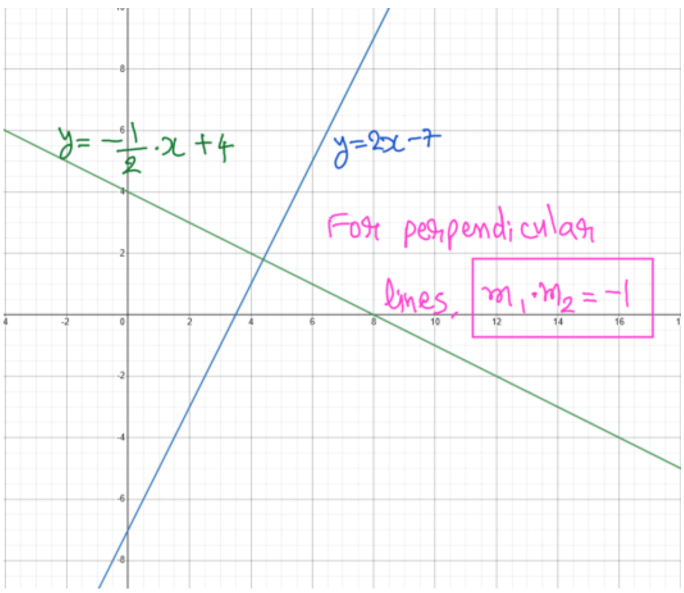
Understanding slope & intercepts





slopes of parallel & perpendicular lines:





$\therefore$ If we know slope & y-intercept of a line & if we want to generate its formula then we use slope-intercept formula

$\rightarrow$ **Slope in context of trigonometry**

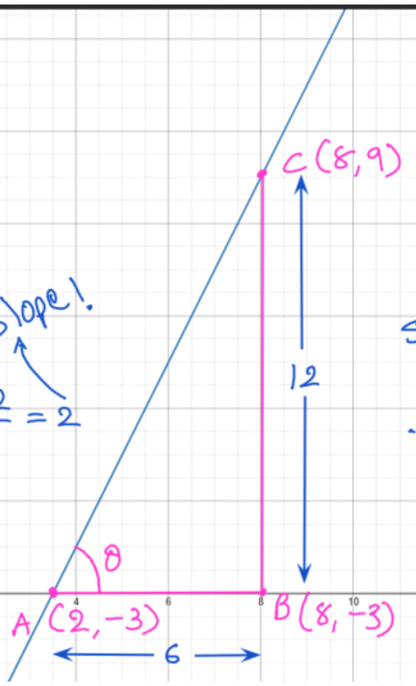
$$\sin\theta = \frac{\text{Opp. Side}}{\text{Hypo.}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{\text{Adj side}}{\text{Hyp.}} = \frac{AB}{AC}$$

$$\tan\theta = \frac{\text{Opp. Side}}{\text{Adj. side}} = \frac{BC}{AB} = \frac{12}{6} = 2$$

slope!

$\therefore$ slope is nothing but $\tan\theta$



$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(8-2)^2+(-3-(-3))^2}$$
$$= \sqrt{(6)^2+0} = 6$$

Similarly, $BC = \sqrt{(8-8)^2+(9-(-3))^2}$

$\therefore BC = \sqrt{0+144} = 12$

$AC = \sqrt{AB^2+BC^2}$

$= \sqrt{36+144}$ $\quad AC = 13.41$

For a horizontal line, $\tan\theta = 0 \therefore$ slope $=0$,

For a verticle line, $\theta = 90^\circ \Rightarrow \tan 90 = \infty \therefore$ slope $=\infty$

But if we know only the slope & coordinates of a point that lies on the line [say, (x_1, y_1)] then we use Slope-Point Formula

Another Formula of slope is:

$$m = \frac{(y_2-y_1)}{(x_2-x_1)}$$ where $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$ are the points on the line.

eg. in our previous example, $A = P_1(2,-3)$ & $C = P_2(8,9)$

$\therefore$ slope can also be calculated as:

$$m = \frac{9-(-3)}{8-2} = \frac{12}{6} = 2$$

From $m = \frac{(y_2-y_1)}{(x_2-x_1)} \Rightarrow y_2 - y_1 = m(x_2-x_1)$

Replacing x_2 by x & y_2 by y:

Replacing x_2 by x & y_2 by y.

$\boxed{2}$ $(y-y_1) = m(x-x_1)$ ——— Slope-Point Formula

Replacing $m = \dfrac{y_2-y_1}{x_2-x_1}$ in the above formula

$\boxed{3}$ $(y-y_1) = \dfrac{(x-x_1)\cdot(y_2-y_1)}{(x_2-x_1)}$ ——— Two-point formula

We use this formula when only two points are known to us.

$\boxed{4}$ $\dfrac{x}{a} + \dfrac{y}{b} = 1$ ——— Intercept Formula

Here a is x-intercept & b is y-intercept

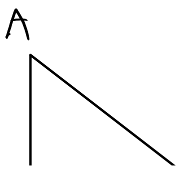
$\boxed{5}$ $ax + by + c = 0$ ——— General Form

Here a & b are **NOT x & y intercepts.**

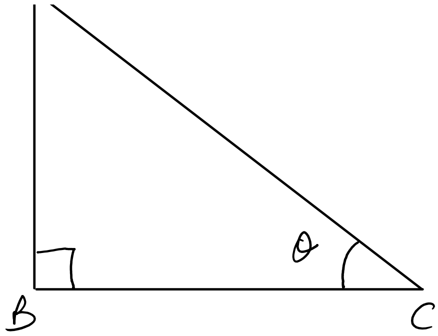
x-intercept $= \dfrac{-c}{a}$ $\quad$ y-intercept $= \dfrac{-c}{b}$ &

slope $= \dfrac{-b}{a}$

☆ Basics of trigonometry



$\sin\theta = \dfrac{\text{Opp. side}}{\text{Hyp.}}$



Hyp.

$$\boxed{\sin\theta = \frac{AB}{AC}}$$

$$\boxed{\operatorname{cosec}\theta = \frac{AC}{AB}}$$

$$\cos\theta = \frac{\text{Adj. side}}{\text{Hyp.}}$$

$$\boxed{\cos\theta = \frac{BC}{AC}}$$

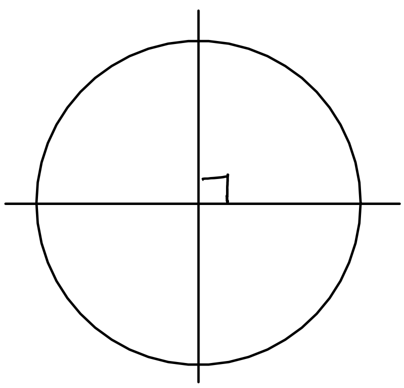
$$\boxed{\sec\theta = \frac{AC}{BC}}$$

$$\tan\theta = \frac{\text{Opp. Side}}{\text{Adj. side}} = \frac{\sin\theta}{\cos\theta}$$

$$\boxed{\tan\theta = \frac{AB}{BC}}$$

$$\boxed{\cot\theta = \frac{BC}{AB}}$$

✯ Units to measure an angle – Degree & Radian



degree	radian
180	π
360	2π
90	$\pi/2$
60	$\pi/3$
30	$\pi/6$
45	$\pi/4$

✯ $(\pi \pm \theta)$ Rules & $(\frac{\pi}{2} \pm \theta)$ Rules (All | s | T | c)

any-trig-fn $(\pi \pm \theta)$ = same-func( )

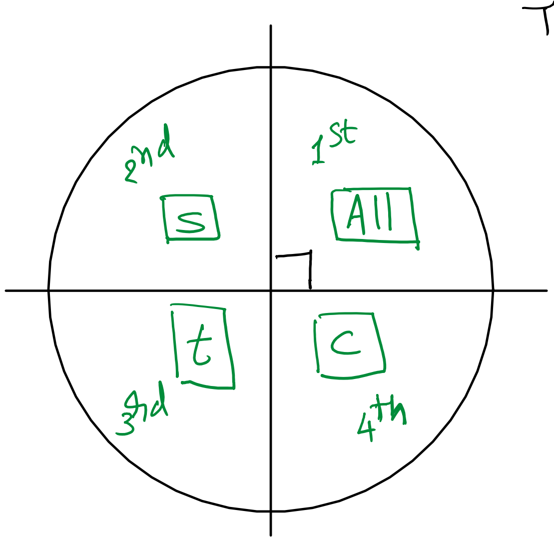
any trig-fn $(\frac{\pi}{2} + \theta)$ = function-changes →

sin → cos , cosec → sec
cos → sin , sec → cosec
tan → cot , cot → tan

any_trig_fn $\left(\frac{\pi}{2} \pm \theta\right)$ = function_changes → tan → cot, cot → tan

Ex: $\sin(\pi - \theta) = \sin\theta$ but $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

☆ Signs of trigonometric functions in each quadrant



This means if $0 \leq \theta \leq 90$,

$$\left.\begin{matrix} \sin\theta,\ \cos\theta,\ \tan\theta \\ \csc\theta,\ \sec\theta,\ \cot\theta \end{matrix}\right\} - \text{+ve}$$

But if $90 < \theta \leq 180$,
Only $\sin\theta$ & $\csc\theta$ are +ve.

If $180 < \theta \leq 270$, only $\tan\theta$ & $\cot\theta$ are +ve

And if $270 < \theta \leq 360$ then only $\cos\theta$ & $\sec\theta$ are +ve.

$\sin(\pi - \theta) \Rightarrow \pi - \theta$ will lie in 2nd quadrant ($\because$ sin is +ve)

$= \sin\theta$

$\sin(\pi + \theta)$ = The function will be the same (because of $\pi \pm \theta$ rule) but $\pi + \theta$ lies in 3rd quadrant where 'sin' is −ve

$\therefore \sin(\pi + \theta) = -\sin\theta$

$\cos(\pi/2 + \theta) = \cos \rightarrow \underline{\sin}$ ($\because \pi/2 \pm \theta$ rule)

$\pi/2 + \theta$ lies in 2nd quadrant where 'sin' is +ve

$\therefore \cos(\pi/2 + \theta) = +\sin\theta$

$\cos(\pi + \theta) = -\cos\theta$

	$0 = 0$	$30 = \pi/6$	$45 = \pi/4$	$60 = \pi/3$	$90 = \pi/2$
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\frac{\sqrt{3}}{2}$	$1/\sqrt{2}$	$\frac{1}{2}$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
cosec	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
cot	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0

☆ Hyperplane & Half spaces

→ Hyperplane is any boundary that acts as a seperator between datapoints of two different classes. It can be 1D, 2D, 3D, - - -

Hyperplane Dimension | Formula

1D (line)

$$\underbrace{w_1}_{a} \overset{x}{x_1} + \underbrace{w_2}_{b} \overset{y}{x_2} + \underbrace{w_0}_{c} = 0 \quad (ax + by + c = 0)$$

2D

$$(ax + by + cz + d = 0)$$

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$$

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$$

$$\underset{a}{w_1}\ \underset{x}{x_1} + \underset{b}{w_2}\ \underset{y}{x_2} + \underset{c}{w_3}\ \underset{z}{x_3} + \underset{d}{w_0} = 0$$

3D

(Difficult to generate $ax+by+\ldots$ type of form)

$$w_1 x_1 + w_2 + x_2 + w_3 x_3 + w_4 x_4 + w_0 = 0$$

⋮

n-D

$$w_1 x_1 + w_2 + x_2 + \ldots + w_n x_n + w_0 = 0$$

→ The two seperate areas created in our n-D space by the hyperplane are know as "Half-Spaces"