Hypothesis testing framework

Friday, October 18, 2024 2:28 PM

* Example- Courtroom Trial

Person-'P' is accussed of murder

Natural assumption of the judge:

The person is innocent = $\boxed{\text{Null Hypothesis} = H_0}$

The claim:-

The person is guilty = $\boxed{\text{Alternate Hypothesis} = H_1 = H_a}$

The person who is accussing (or making claim that the person is guilty) have to proove their claim.

Q1. Frame work for weight loss

A group of people have volunteered to try out a diet for weight loss for 3 months.
How should the **null** and **alternate hypotheses** be set up?

A. H0: Diet increases weight; Ha: Diet has no impact on weight
B. H0: Diet reduces weight; Ha: Diet has no impact on weight
C. H0: Diet has no impact on weight; Ha: Diet reduces weight
D. H0: Diet has no impact on weight; Ha: Diet increases weight

Q2. Framework for Virus test

A test is done to detect if a person has a virus.
What is the null (H_0) and alternate (H_a) hypothesis?

A. H0: Patient has no virus; Ha: Patient has virus
B. H0: Patient has virus; Ha: Patient has no virus
C. H0: Patient has no virus; Ha: Patient may or may not have virus
D. Cannot determine

Q3. Frame work for GRE verbal reasoning

The verbal reasoning section in the GRE exam, has an **average** score of **150** and a **standard deviation** of **8.5**.
A coaching centre claims to improve these numbers for their students. How should the **null** and **alternate** hypotheses be set up?

A. H0: Coaching improves score; Ha: Coaching does not improve score
B. H0: Coaching reduces score; Ha: Coaching improves score
C. H0: Coaching does not improve score; Ha: Coaching reduces score
D. H0: Coaching does not improve score; Ha: Coaching improves score

Q4. Marketing the shampoo brand

Weekly sales of shampoo bottles has an average of **1800** . A marketing company feels that this can be improved with right advertisement and promotions.
What should be the **null** and **alternate hypothesis,** in order to validate their claim?
Let *u* denote the average sales after marketing.

i). H_0: $u <= 1800$ and H_a: $u = 1800$
ii). H_0: $u <= 1800$ and H_a: $u >= 1800$
iii). H_0: $u = 1800$ and H_a: $u = 1800$
iv). H_0: $u = 1800$ and H_a: $u > 1800$

* Types of errors:

Reality Test Result

* Types of Errors:

person →

Reality	Test Result	
Not Infected (H_0)	Negative	True Negative
Not Infected (H_0)	Positive	False Positive (Type-1 Error)
Infected (H_a)	Negative	Type-2 Error (False Negative)
Infected (H_a)	Positive	True Positive

Evidences:

1. The person had a knife in his/her pocket
2. The knife had blood stains on it.
3. The dna test of hair and other bilogical evidences at the site suggests that there was fight between the victim & the accussed.
4. The blood matches with the blood of the victim
5. The victim called his/her friend that the accussed threaten him/her to kill.

The Process:

→ We keep on calculating $P(\text{Evidence} \mid H_0)$

evi.-1: How much is the probability that an innocent person has knife in his/her pocket? $P(\text{Evidence} \mid H_0) = 0.10$

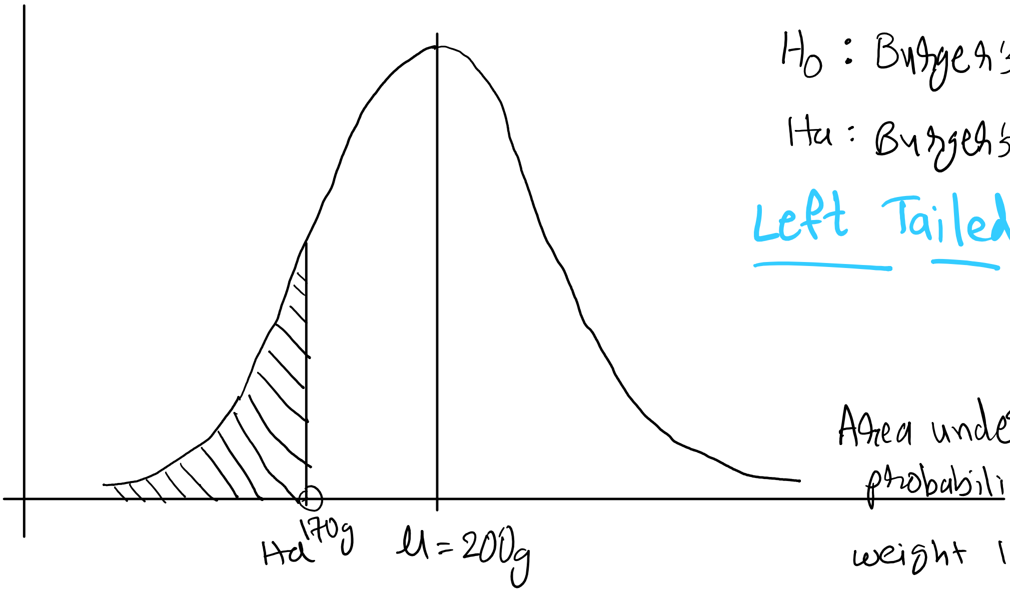
evi-2: How much is the probability that an innocent evi-2: How much is the probability that an innocent person has a knife with blood stain? $P(E|H_0) = 0.08$

evi-3: How much is the probability that an innocent person having knife with blood on it had a fight with the victim? $P(E|H_0)$ even lesser.

☆ Type of tests:

Case – 1:

A customer usually feels satisfied after having a burger at a snacks parlor. Once, she becomes unhappy as she does not feel full even after having the burger and claims that the burger was lighter than usual. The shop owner claims that the average weight of their burger is 200g and assume that the burger weight is normally distributed around the mean.



H_0 : Burger's weight is 200g

H_a : Burger's weight < 200g

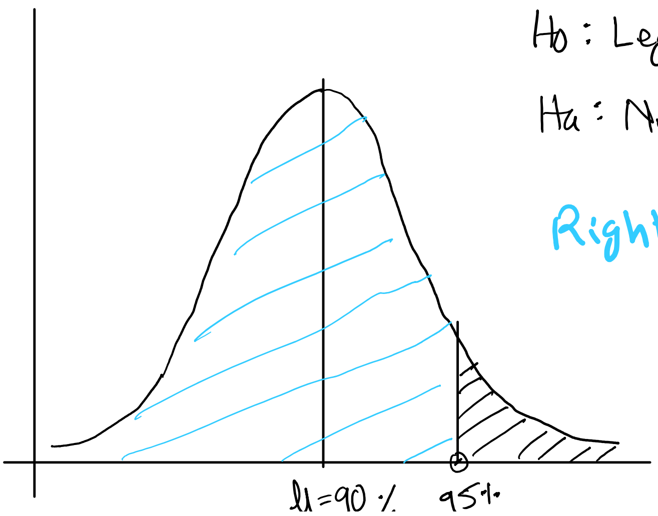
Left Tailed Test

p = norm.cdf

Area under the curve represents probability of having burger's weight 170g

Case – 2:

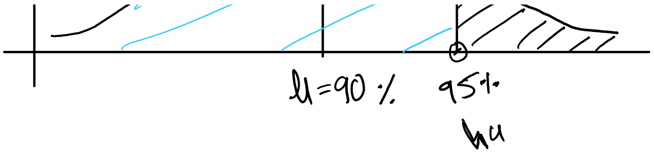
Legacy model at a company provides 90% accuracy. You are coming up with a new model claiming that it gives better accuracy (let's say 95%). Assume Gaussian.



H_0 : Legacy model is better

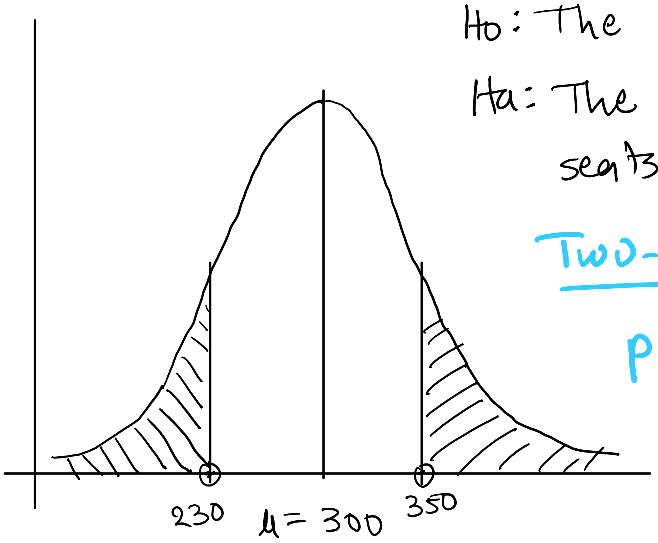
H_a : New model is better

Right-Tailed Test: p = 1 - norm.cdf



Case – 3:

The data suggests that a political party will win 300 seats but you believe that either it will win less than 230 seats or it will win more than 350 seats but not 300. Assume normal distribution.



Ho: The party will win 300 seats

Ha: The party will win either less than 230 seats or more than 350 seats.

Two-tailed test:

$p = \text{norm.cdf} + (1 - \text{norm.cdf})$

☆ The threshold:

If the p-value goes below a certain threshold, we say "We **Reject the Ho**."

But if this p-value doesn't go below that threshold then we say - "**We failed to Reject Ho**."

This threshold is also called "confidence" & denoted by α.

95% confidence $\Rightarrow \alpha = 0.05$

90% confidence $\Rightarrow \alpha = 0.1$

99% confidence $\Rightarrow \alpha = 0.01$

☆ The Framework:

Step-1: Identify Ho, Ha & α (if not given)

Step-2: Identify the type of the distribution step-2 : Identify the type of the distribution

step-3 : Determine the type of test - left | right | two tailed

step-4 : Calculate p-value.

step-5 : Conclude : If $p < \alpha$ then : Reject H_0

else : Failed to reject H_0 (accept H_0)