

Homework Set 3, CPSC 8420, Fall 2023

Last Name, First Name

Due 11/17/2023, Friday, 11:59PM EST

Problem 1

Considering soft margin SVM, where we have the objective and constraints as follows:

$$\begin{aligned} \min \quad & \frac{1}{2} \|w\|_2^2 + C \sum_{i=1}^m \xi_i \\ \text{s.t.} \quad & y_i(w^T x_i + b) \geq 1 - \xi_i \quad (i = 1, 2, \dots, m) \\ & \xi_i \geq 0 \quad (i = 1, 2, \dots, m) \end{aligned} \tag{1}$$

Now we formulate another formulation as:

$$\begin{aligned} \min \quad & \frac{1}{2} \|w\|_2^2 + \frac{C}{2} \sum_{i=1}^m \xi_i^2 \\ \text{s.t.} \quad & y_i(w^T x_i + b) \geq 1 - \xi_i \quad (i = 1, 2, \dots, m) \end{aligned} \tag{2}$$

1. Different from Eq. (1), we now drop the non-negative constraint for ξ_i , please show that optimal value of the objective will be the same when ξ_i constraint is removed.
2. What's the generalized Lagrangian of the new soft margin SVM optimization problem?
3. Now please minimize the Lagrangian with respect to w, b , and ξ .
4. What is the dual of this version soft margin SVM optimization problem? (should be similar to Eq. (10) in the slides)

Problem 2

Recall vanilla SVM objective:

$$L(w, b, \alpha) = \frac{1}{2} \|w\|_2^2 - \sum_{i=1}^m \alpha_i [y_i(w^T x_i + b) - 1] \quad \text{s.t.} \quad \alpha_i \geq 0 \tag{3}$$

If we denote the margin as γ , and vector $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_m]$, now please show $\gamma^2 * \|\alpha\|_1 = 1$.