Primer on Analysis of Experimental Data and Design of Experiments

Lecture 9. DOE and Taguchi Experiments

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Three representations of full factorial design

(All levels repeated equal times)

$$F = 2, L = 4;$$
$$R = L^F = 2^4 = 16$$

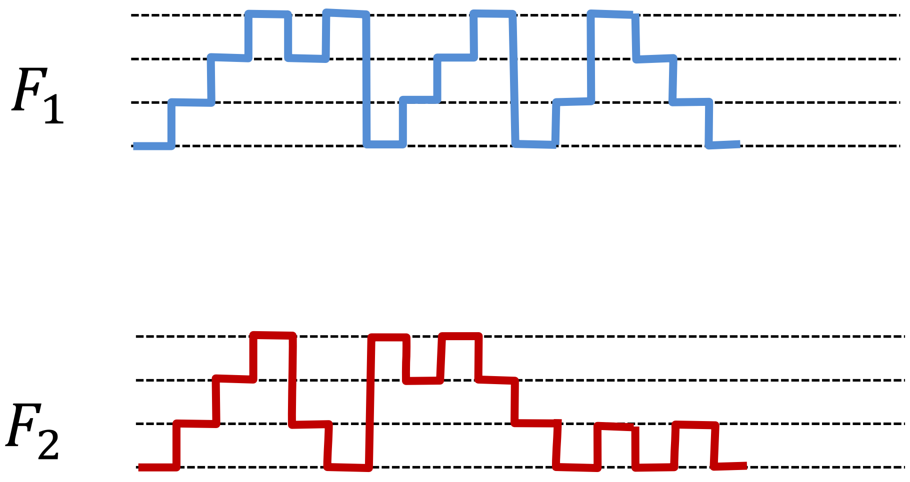
Aa (1)	Bc (8)	Cd (9)	Db (13)
Bb (2)	Ad (7)	Dc(10)	Ca (14)
Cc (3)	Da (6)	Ab (11)	Bd (15)
Dd (4)	Cb (5)	Ba (12)	Ac (16)

Field-view

Run	Name	Factors	
1	Aa	1	1
2	Bb	2	2
3	Cc	3	3
4	Dd	4	4
5	Cb	3	2
6	Da	4	1
7	Ad	1	4
8	Bc	2	3
9	Cd	3	4
10	Dc	4	3
11	Ab	1	2
12	Ba	2	1
13	Db	4	2
14	Ca	3	1
15	Bd	2	4
16	Ac	1	3

Run-view

4-level random code

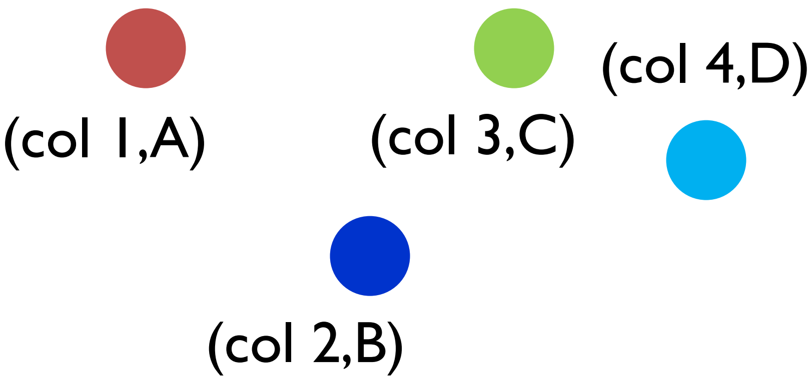


Codes are orthogonal if their product is zero.

Code-view

Uncorrelated linear graph (Field and Run views)

		A_1		A_2	
		B_1	B_2	B_1	B_2
C_1	D_1	x	x	x	x
	D_2	x	x	x	x
C_2	D_1	x	x	x	x
	D_2	x	x	x	x



A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

A	B	C	D
1	1	1	1
1	1	0	2
1	1	2	0
1	1	2	2
1	2	1	1
1	2	1	2
1	2	2	1
1	2	2	2
2	1	1	1
2	1	1	2
2	1	2	1
2	1	2	2
2	2	1	1
2	2	1	2
2	2	2	1
2	2	2	2

L=2
F=4

$$L_n(L^F) = L_n(2^4)$$

What is n?

Taguchi table: How to determine n (Run view)

$L_4(2^3)$

Run	Columns		
	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

$L_8(2^7)$

Run	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

$L_{12}(2^{11})$

Run	Columns										
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1

$$DOF = \ 1 + F(L-1)$$

Consider 2 level tables … because we are interested in $L_n(2^4)$

$L_n(L^F)$	2^3	2^7	2^{11}
DOF	$1 + 3 \times \ (2-1) = 4$	$1 + 7 \times \ (2-1) = 8$	$1 + 11 \times \ (2-1) = 12$
n	Closed multiple of 2 =4	Closed multiple of 2 =8	Closed multiple of 2 =12
Each col.	Two 1's, Two 2's	Four 1's, Four 2's	six 1's, six 2's

Aside: Taguchi Orthogonal Columns

$L_4(2^3)$

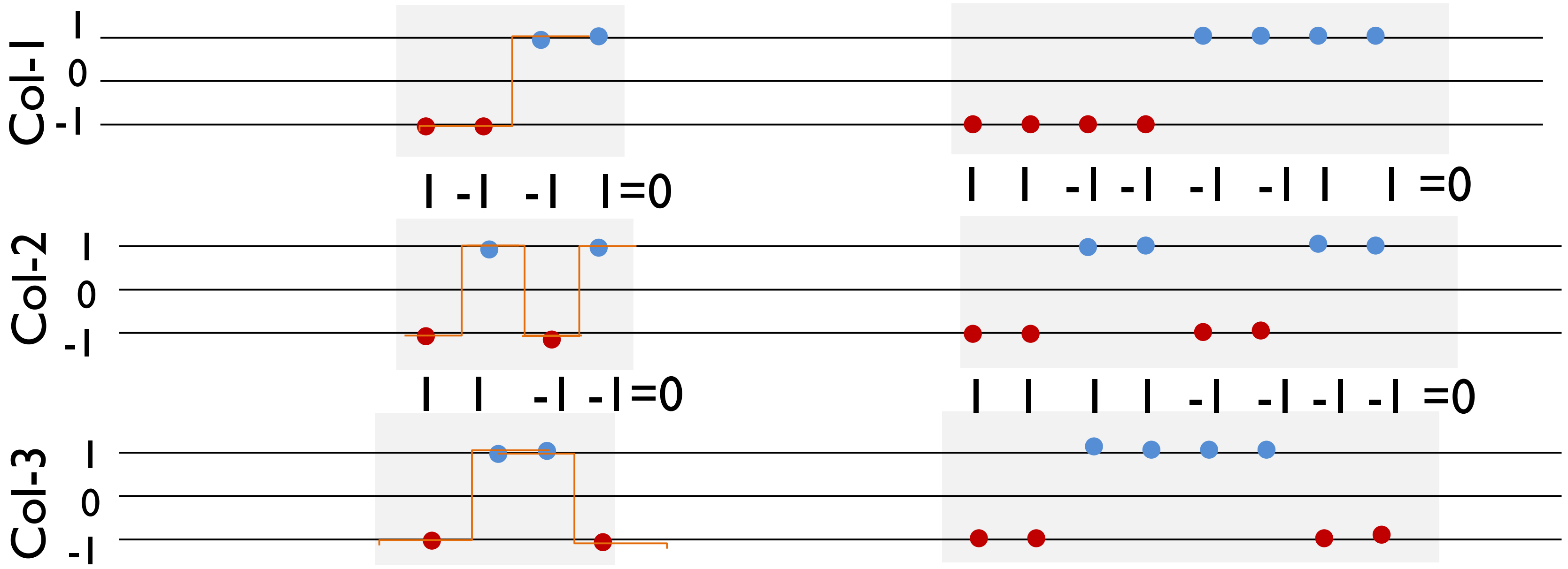
Run	Columns		
	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

$L_8(2^7)$

Run	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

$L_{12}(2^{11})$

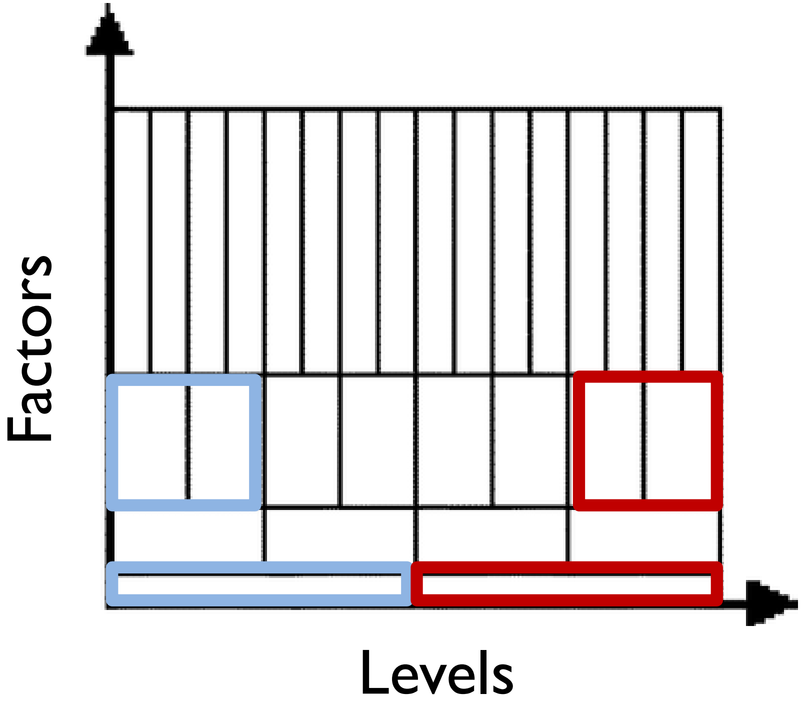
Run	Columns										
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1

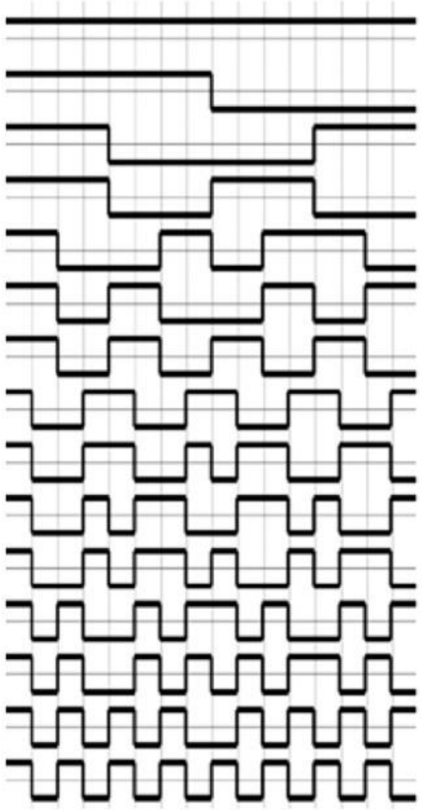


Take any two columns (i.e. factors), set 2 to 1 And 1 to -1, then take Inner product and sum. The result is always zero.

Generating Taguchi (orthogonal) Arrays

Run	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2





CDMA Coding and Wavelet Transform

Full Factorial to Taguchi Table

A	B	C	D
1	1	1	1
1	1	0	2
1	1	2	0
1	1	2	2
1	2	1	1
1	2	1	2
1	2	2	1
1	2	2	2
2	1	1	1
2	1	1	2
2	1	2	1
2	1	2	2
2	2	1	1
2	2	1	2
2	2	2	1
2	2	2	2

Null … 1
Single 4
Pair ….. 6
Triple … 6
Quad … 1

A	B	C	D
1	1	1	1
1	1	1	2
~~1~~	~~1~~	~~2~~	~~0~~
~~1~~	~~1~~	~~2~~	~~2~~
~~1~~	~~2~~	~~1~~	~~1~~
~~1~~	~~2~~	~~1~~	~~2~~
1	2	2	1
1	2	2	2
~~2~~	~~1~~	~~1~~	~~1~~
~~2~~	~~1~~	~~1~~	~~2~~
2	1	2	1
2	1	2	2
2	2	1	1
2	2	1	2
~~2~~	~~2~~	~~2~~	~~1~~
~~2~~	~~2~~	~~2~~	~~2~~

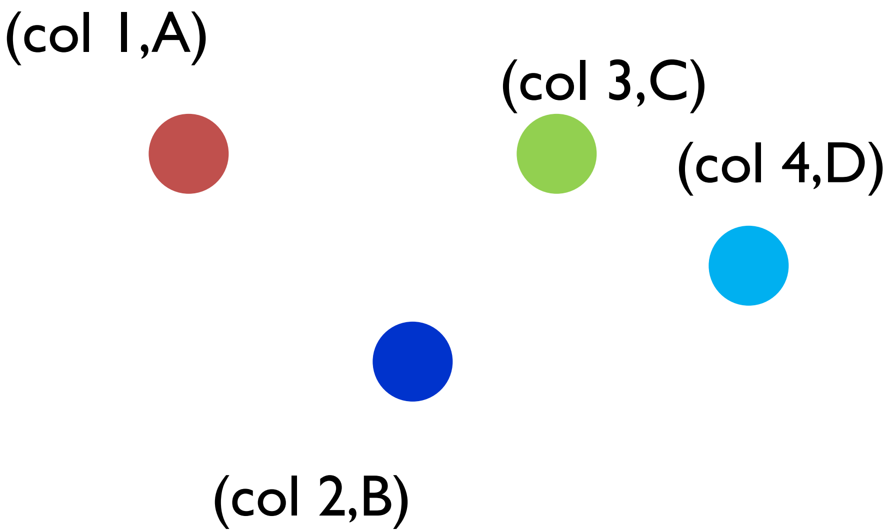
Null … 1
Single … 1
Pair ….. 3
Triple … 3
Quad … 0

Run	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

$$S=(-1)*(-1)+(-1)*(-1)+(-1)*(+1)+(-1)*(+1)+(+1)*(-1)+(+1)*(-1)+(+1)*(+1)+(+1)*(+1)=0$$

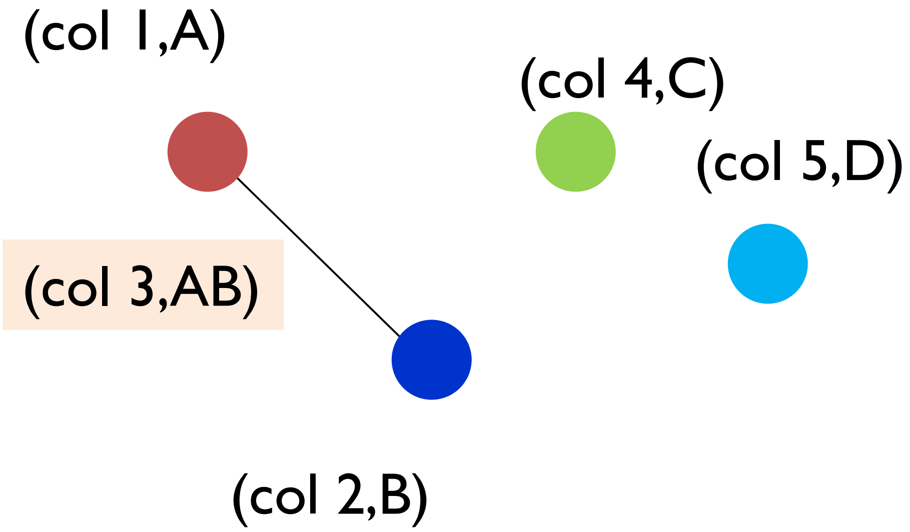
Main effect assuming **no** interactions (Run view)

	A	B	C	D	Y
R-1	1	1	1	1	
R-2	1	1	1	2	
R-3	1	2	2	1	
R-4	1	2	2	2	
R-5	2	1	2	1	
R-6	2	1	2	2	
R-7	2	2	1	1	
R-8	2	2	1	2	



Let us say that the analysis of Y indicates **AB interaction**, but nothing else.
We need to redo the experiment

Main effect with interactions (Run view)



	A	B	AxB	C	D
R-1	1	1	1	1	1
R-2	1	1	1	2	2
R-3	1	2	2	1	1
R-4	1	2	2	2	2
R-5	2	1	2	1	2
R-6	2	1	2	2	1
R-7	2	2	1	1	2
R-8	2	2	1	2	1

Run	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

$$L_n(2^5) = 32$$
$$DOF = 1 + F(L-1) = 1 + 5(2-1) = 6 \quad \ldots n = 8$$

(AB) is a dummy column, without it the C and D would have different arrangements …

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Null … 1
Single 4
Pair ….. 6
Triple … 6
Quad … 1

Run	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

A	B	C	D
0	0	0	0
0	0	0	1
~~0~~	~~0~~	~~1~~	~~0~~
~~0~~	~~0~~	~~1~~	~~1~~
~~0~~	~~1~~	~~0~~	~~0~~
~~0~~	~~1~~	~~0~~	~~1~~
0	1	1	0
0	1	1	1
~~1~~	~~0~~	~~0~~	~~0~~
~~1~~	~~0~~	~~0~~	~~1~~
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
~~1~~	~~1~~	~~1~~	~~0~~
~~1~~	~~1~~	~~1~~	~~1~~

Null … 1
Single 1
Pair ….. 3
Triple … 3
Quad … 0

Run	Columns						
	1	2	[illegible]	4	5	6	7
1	1	1	[illegible]	1	1	1	1
2	1	1	[illegible]	2	2	2	2
3	1	2	[illegible]	1	1	2	2
4	1	2	[illegible]	2	2	1	1
5	2	1	[illegible]	1	2	1	2
6	2	1	[illegible]	2	1	2	1
7	2	2	[illegible]	1	2	2	1
8	2	2	[illegible]	2	1	1	2

A	B	C	D
0	0	0	0
~~0~~	~~0~~	~~0~~	~~1~~
~~0~~	~~0~~	~~1~~	~~0~~
0	0	1	1
0	1	0	0
~~0~~	~~1~~	~~0~~	~~1~~
~~0~~	~~1~~	~~1~~	~~0~~
0	1	1	1
~~1~~	~~0~~	~~0~~	~~0~~
1	0	0	1
1	0	1	0
~~1~~	~~0~~	~~1~~	~~1~~
~~1~~	~~1~~	~~0~~	~~0~~
1	1	0	1
1	1	1	0
~~1~~	~~1~~	~~1~~	~~1~~

Null … 1
Single 1
Pair ….. 3
Triple … 3
Quad … 0

The effect of B is now understood

Web Design: 4 Factor, 5 level

4 factors (font, color, background, foreground) and 5 levels of each

$$L_n(L^F) = L_n(5^4)$$
$$= L_n(125), n = ?$$

$$DOF = 1 + F(L-1) =$$
$$4 \times 4 = 17.$$

Balanced design, multiple of 5

$$n = 20, 25$$

Partial factorial Design

Factors

Levels

X1	X2	X3	X4	X5	X6
				-	-
1	1	1	1	1	1
1	2	2	2	2	2
1	3	3	3	3	3
1	4	4	4	4	4
1	5	5	5	5	5
2	1	2	3	4	5
2	2	3	4	5	1
2	3	4	5	1	2
2	4	5	1	2	3
2	5	1	2	3	4
3	1	3	5	2	4
3	2	4	1	3	5
3	3	5	2	4	1
3	4	1	3	5	2
3	5	2	4	1	3
4	1	4	2	5	3
4	2	5	3	1	4
4	3	1	4	2	5
4	4	2	5	3	1
4	5	3	1	4	2
5	1	5	4	3	2
5	2	1	5	4	3
5	3	2	1	5	4
5	4	3	2	1	5
5	5	4	3	2	1

Randomization

fjords	jawbox	phlegm	qiviut	zincky
zincky	fjords	jawbox	phlegm	qiviut
qiviut	zincky	fjords	jawbox	phlegm
phlegm	qiviut	zincky	fjords	jawbox
jawbox	phlegm	qiviut	zincky	fjords

https://www.mne.psu.edu/cimbala/me345/Lectures/Taguchi_orthogonal_arrays.pdf

https://www.york.ac.uk/depts/maths/tables/orthogonal.htm

Conclusions

1. Design of experiment is a powerful technique universally used in industry and in large scale field trials.

2. Taguchi/Fisher methods replace the older one-factor-at-a-time experiments with experiments based on orthogonal arrays; In this approach, only the effect of main factors remain; others are cancelled.

3. Understanding and analyzing correlation is important in design of experiments. Unless the correlation is well understood and incorporated through dummy variables, the analysis may lead to faulty conclusions.

Review Questions

1. What role did Fisher play in developing the design of experiment?
2. If you have 3 variables at two levels, what Taguchi array would you choose?
3. How does one find correlation among variables in Full factorial method?
4. What is the role of linear graphs in Taguchi method?
5. In what ways Fisher philosophy of change the ways experiments are done? Is there a down side of such analysis?
6. What is dummy variable? What does dummy variable to do in DOE?
7. Can you have 3rd or higher order correlation, if you do not have second order correlation?

Primer on Analysis of Experimental Data and Design of Experiments

Lecture 9. DOE Analysis by ANOVA

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Course Outline

$$\bar{y} = f(\bar{x}) \quad \bar{x} = x_1, x_2, \dots x_n \quad \bar{y} = y_1, y_2, \dots y_m$$

Lecture 1: Introduction
Lecture 2: Collecting and plotting $x_1, x_2, \dots x_n$
Lecture 3: Physical and empirical $f, F, df/dx, \dots$
Lecture 4: Model selection among $f_1, f_2, \dots$
Lecture 5: Scaling theory with known f, $f(\bar{x}) = f(\overline{X})$
Lecture 6: Scaling theory with unknown f, $\bar{x} \to X$
Lecture 7: Design of experiments to determine $\bar{y}_{\max} = f(\bar{x})$
Lecture 8: Machine learning … Statistical approach to learn f
Lecture 9: Physics-based machine learning $f = f_{\text{physics}} + \Delta f$
Lecture 10: Principle component analysis for classifying $\{y\}$.
Lecture 11: Conclusions

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Outline

1. Introduction to Analysis of Variance (Anova)
2. Single factor Analysis of Variance
3. Two factor Anova
4. Generalized Anova
5. Conclusions

Another way to reduce the number of experiments

(A) Oxide Thickness

(B) Doping

(C) Anneal temp.

(D) Junction depth

(E) Gate Overlap

(F) Halo implant

(G) Supply voltage

MOSFET

Drain current

7 parameter optimization for a single objective function

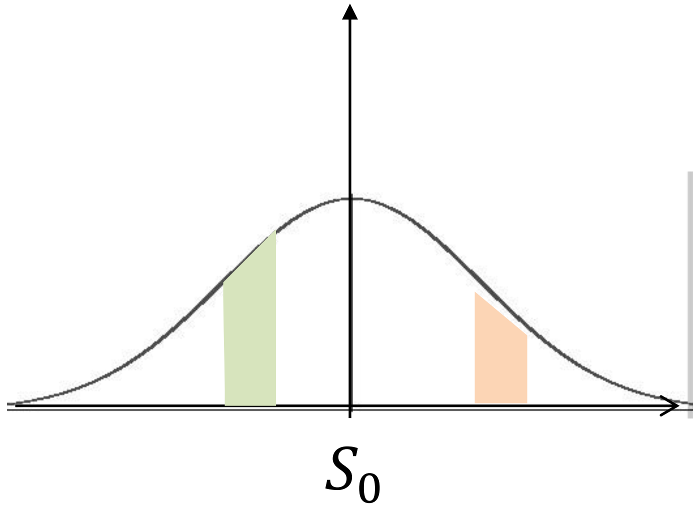
Could be car, farm, airport

Single factor ANOVA: Treatment

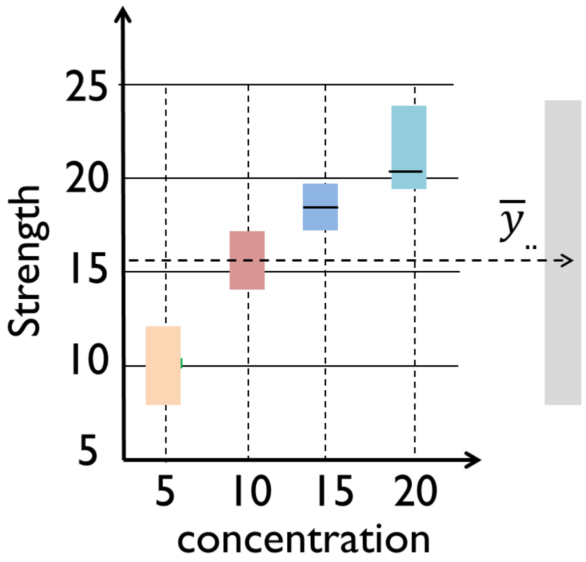
replicates

Treatments (levels)

	1	2	3	4	5	6
5	7	8	15	11	9	10
10	12	17	13	18	19	15
15	14	18	19	17	16	18
20	19	25	22	23	18	20



In essence, no effect

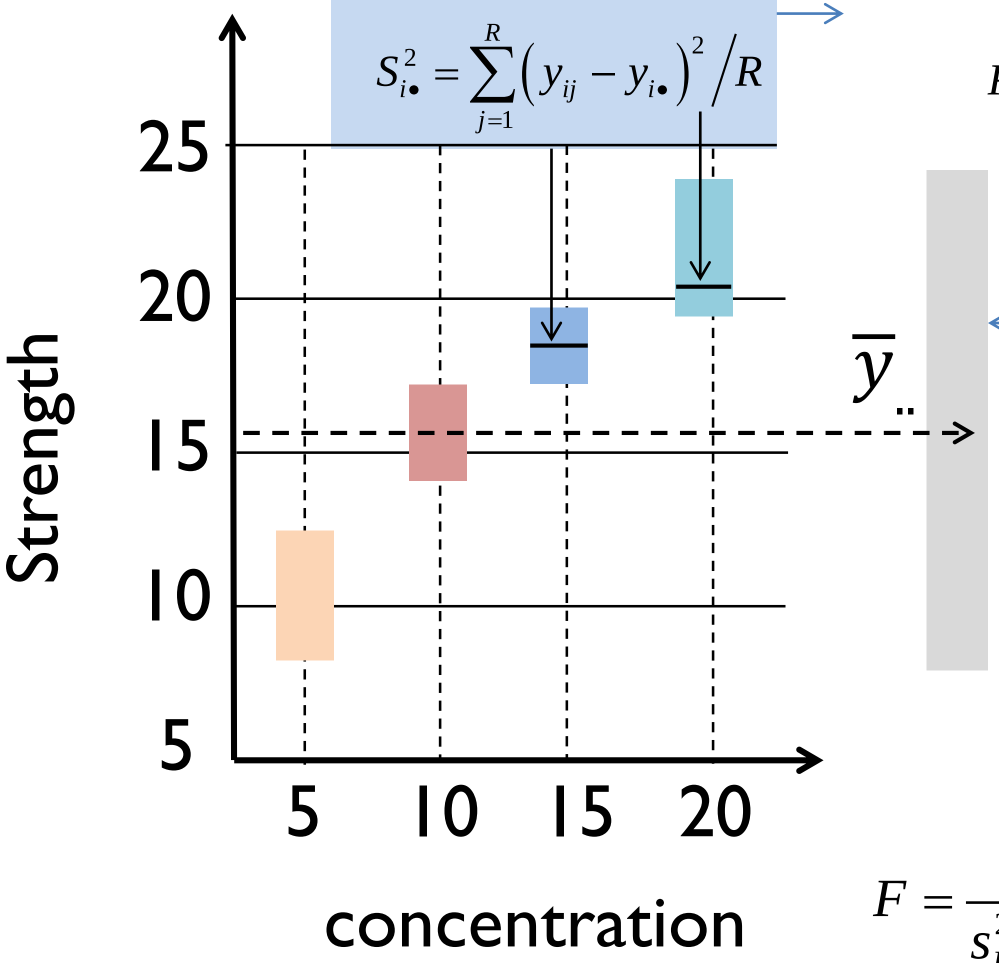


Single factor Anova: Treatment Analysis

R=6 (replicate),
L=4 (level)

$$\bar{y}_{i\bullet} = \sum_{j=1}^{R} y_{ij} / R$$

$$S_{i\bullet}^2 = \sum_{j=1}^{R} \left(y_{ij} - y_{i\bullet} \right)^2 / R$$



$$SS_T^2 = \sum_{j=1}^{L} \sum_{i=1}^{R} (y_{ij} - \bar{y}_{\bullet\bullet})^2$$

$$RL \times s_E^2 = SS_T^2 - SS_{Treatment}^2$$

$$\bar{y}_{\bullet\bullet} = \sum_{i=1}^{L} \sum_{j=1}^{R} y_{ij} / RL$$

$$SS_{Treatment} = L\sigma^2 = R\sum_{i=1}^{L} (\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2$$

Recall: χ^2, KS, etc.

$$F = \frac{\sigma^2 \times L/(L-1)}{s_E^2 \times RL/L(R-1)} : F_{crit}(L-1, L(R-1))$$

Single factor ANOVA (continued)

	1	2	3	4	5	6
5	7	8	15	11	9	10
10	12	17	13	18	19	15
15	14	18	19	17	16	18
20	19	25	22	23	18	20

- Treatment number, $a = 4;$, $dof_a = 3;$ Sample number: $n = 6$
- Global sample number: $a \times n = 24$, $dof_n = 23$, global AVG =15.96
- Total sum of square, $SS_T = \sum_{24}(data - AVG)^2 = 512.96$
- Treatment sum: $SS_{treatment} = n \times \sum_4 (treat.avg - AVG)^2$=*382.*
- $SS_{Error} = SS_T$ – $SS_{treatment}$=130.62
- $ME_{treatment} = SS_E/dof_a$, $ME_E = SS_{error}/(dof_n - dof_a)$
- Finally, $F = (ME_{treatment})/(ME_{error}) = 19.6$
- Compare: $f(0.01, dof_a, dof_n)$, or $P\left(F_{3,20} > 19.6\right) = 3.59 \times 10^{-6}$

Single factor ANOVA: Wood Treatment

treatments ↓ replicates →

	1	2	3	4	5	6
5	7	8	15	11	9	10
10	12	17	13	18	19	15
15	14	18	19	17	16	18
20	19	25	22	23	18	20

S_avg	(s-avg-AVG)^2
10.00	35.50174
15.67	0.085069
17.00	1.085069
21.17	27.12674
15.96	63.79861

$$\sum(data - AVG)^2 = 512$$

$$6 \times\ 63.8\ = 382.8$$

Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	382.7917	3	127.60	19.605	3.59E-06	4.94
Within Groups	130.1667	20	6.51			
Total	512.9583	23				

Outline

1. Introduction to Analysis of Variance (Anova)

2. Single factor Analysis of Variance

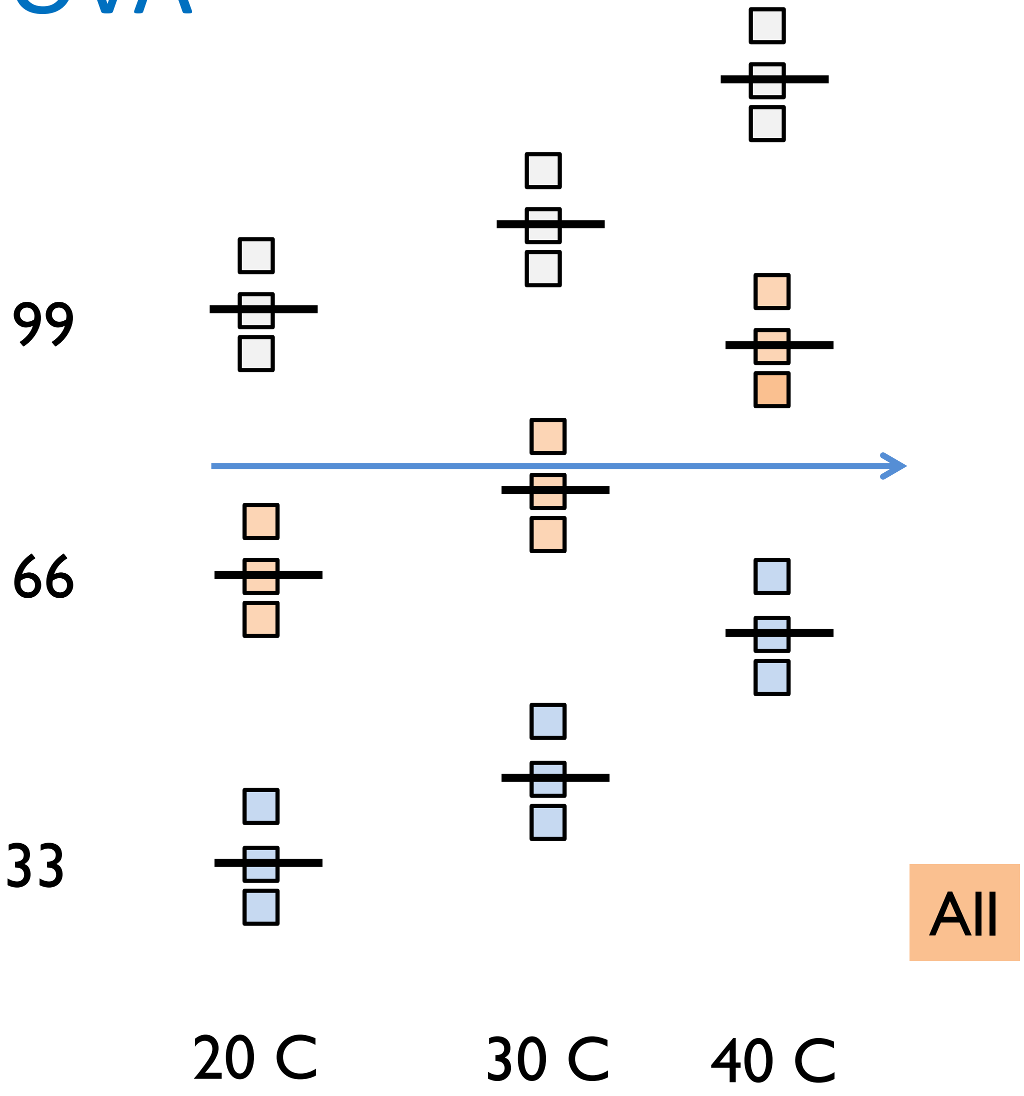
3. **Generalized Anova**

4. Conclusions

Two factor ANOVA

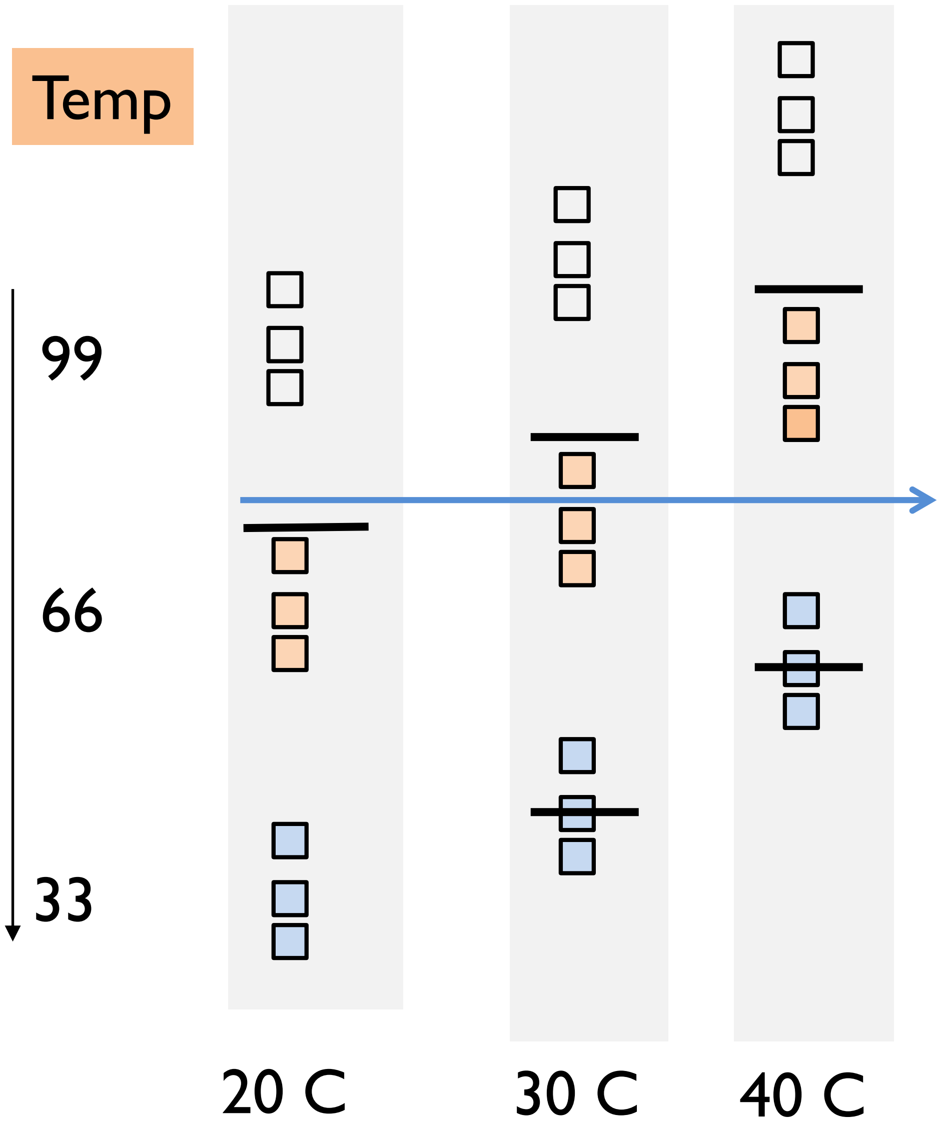
Full factorial:
2 factor, 3 level,
3 replicate experiment

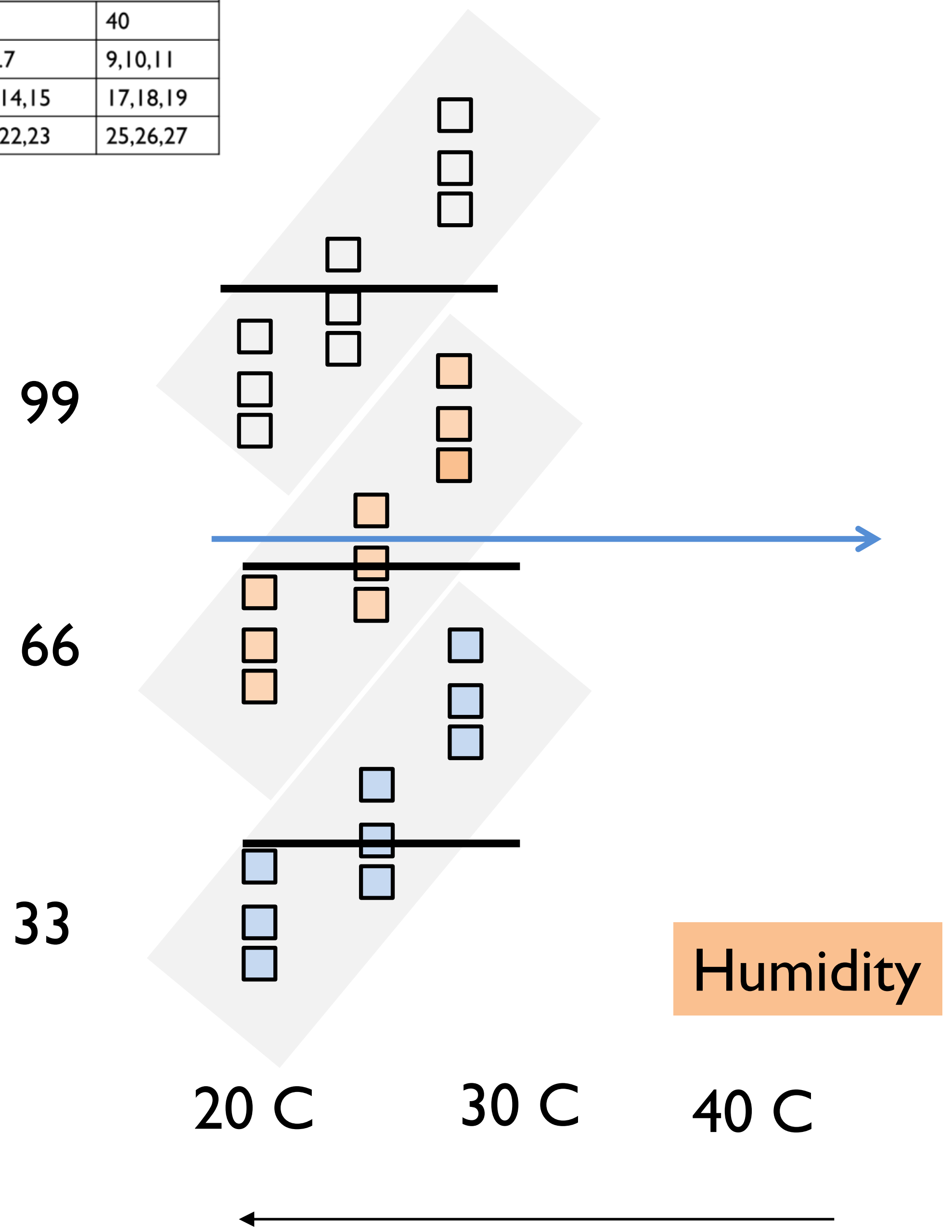
	Temperature (C)		
Humidity (%)	20	30	40
33	1,2,3	5.6.7	9,10,11
66	9,10,11	13,14,15	17,18,19
99	17,18,19	21,22,23	25,26,27



Two factor ANOVA

	Temperature (C)		
Humidity (%)	20	30	40
33	1,2,3	5.6.7	9,10,11
66	9,10,11	13,14,15	17,18,19
99	17,18,19	21,22,23	25,26,27





Two factor ANOVA

Full factorial:
2 factor, 3 level,
3 replicate experiment

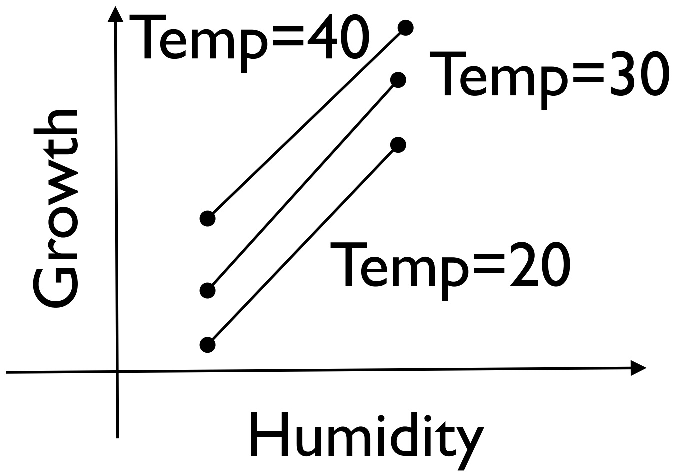
	Temperature (C)		
Humidity (%)	20	30	40
33	1,2,3	5.6.7	9,10,11
66	9,10,11	13,14,15	17,18,19
99	17,18,19	21,22,23	25,26,27

Excel/Minitab Analysis

	Sum of Squares	dof	Mean-square	F Ratio	Significance
Temp	312.66	3-1=2	312.66/2=156.33	156.33/1.0=156.33	0.000 (significant)
Humidity	1200.66	3-1=2	1200.66/2=600.33	600.66/1=600.33	0.000 (Significant)
Temp*Humidity	1.33	2x2=4	1.33/4=0.33	0.33/1.0=0.33	0.853 (insignificant)
Error	18.00	27-2-2-4=19	18.00/18=1		

Two factor ANOVA (Excel/Minitab Analysis)

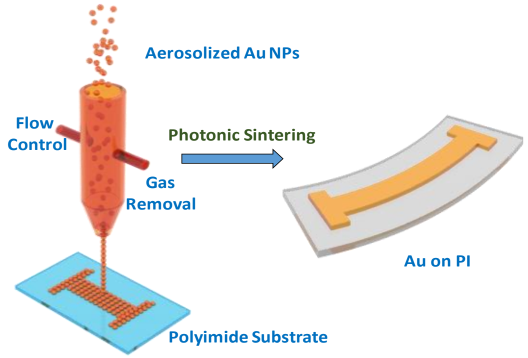
	Sum of Squares	dof	Mean-square	F Ratio	p-value
Temp	312.66	3-1=2	312.66/2=156.33	156.33/1.0=156.33	0.000 (significant)
Humidity	1200.66	3-1=2	1200.66/2=600.33	600.66/1=600.33	0.000 (Significant)
Temp*Humidity	1.33	2x2=4	1.33/4=0.33	0.33/1.0=0.33	0.853 (insignificant)
Error	18.00	27-2-2-4=19	18.00/18=1		

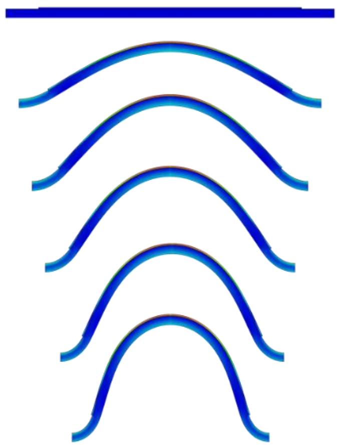


No mutual interaction

Ref. Statistics Explained,
S. Mckillup, Cambridge Press

Multiple Factor ANOVA





Thickness (µm)	BR	BF	Resistance Change (Percent)				
			50	100	150	200	250
10	Half	Slow	46.5	52.61	66.3	91.58	149.6
10	Half	Fast	67.7	69.4	97.34	124.4	139.3
10	Full	Slow	92.975	174.45	232.45	252.64	275.3
10	Full	Fast	85.6	135.47	162.59	157.53	208.6
25	Half	Slow	22.4	23.57	24.87	29.14	28.9
25	Half	Fast	25.2	35.22	22.14	24.5	29.65
25	Full	Slow	24.7	32.89	54.68	78.23	95.63
25	Full	Fast	45.23	51.29	65.26	61.4	78.95

Field view

Thickness=2 (i.e. 10, 15), BR=2 (i.e. slow, fast)
BF=2 (Half and full), Cycles=5 (i.e. 50,100,150, 200, 250)

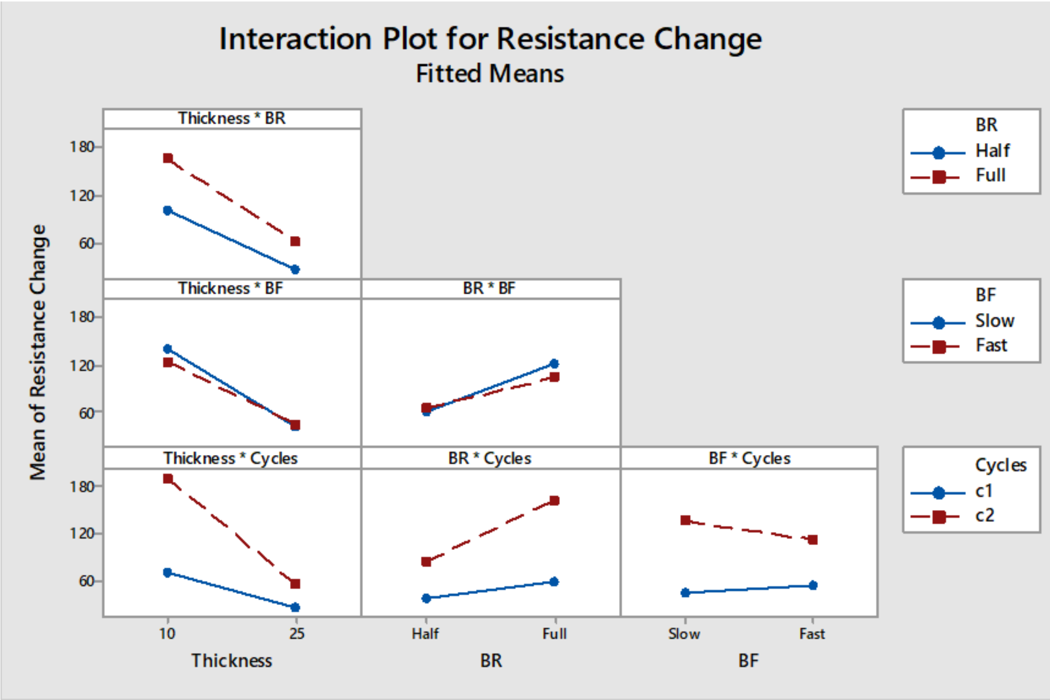
5 factors, (2,2,2,5) levels, single replicate DOE experiments

40 experiments (Need to use statistical package, e.g. Minitab)

Multiple Factor ANOVA (Continued)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	79330.2	5666.4	1685.12	0.019
Linear	4	64198.5	16049.6	4772.93	0.011
Thickness	1	31944.0	31944.0	9499.67	$\boxed{0.007}$
BR	1	9887.1	9887.1	2940.27	$\boxed{0.012}$
BF	1	194.4	194.4	57.82	0.083
Cycles	1	22173.1	22173.1	6593.95	$\boxed{0.008}$
2-Way Interactions	6	14177.8	2363.0	702.71	0.029
Thickness*BR	1	915.3	915.3	272.19	$\boxed{0.039}$
Thickness*BF	1	311.3	311.3	92.58	0.066
Thickness*Cycles	1	8300.3	8300.3	2468.40	$\boxed{0.013}$
BR*BF	1	448.1	448.1	133.26	0.055
BR*Cycles	1	3145.1	3145.1	935.31	$\boxed{0.021}$
BF*Cycles	1	1057.6	1057.6	314.52	$\boxed{0.036}$
3-Way Interactions	4	953.8	238.5	70.91	0.089
Thickness*BR*BF	1	454.5	454.5	135.16	0.055
Thickness*BR*Cycles	1	85.2	85.2	25.34	0.125
Thickness*BF*Cycles	1	166.2	166.2	49.42	0.090
BR*BF*Cycles	1	247.9	247.9	73.74	0.074
Error	1	3.4	3.4		
Total	15	79333.5			



Check for p-values < 0.05

One way: Thickness, BR, and Cycles (BF does not matter on its own)

Two-way: Thickness/BR, Thickness/cycles, BR/cycles, BF/cycles

Three-way: Thickness, BR, BF (may be, close to 0.05, more experiments)

Conclusions

1. Design of experiment results are analyzed by ANOVA test to make sure that the effect of variables on the final result is statistically significant. Insignificant variables can be dropped to simplify analysis.

2. ANOVA generalizes hypothesis testing to continuous variables.

3. A positive test from Anova says one of the treatment is different from others – it does not say which one. With a positive result, one can do pair-wise comparison.

4. Simple Anova tests are easily done by calculator. More complicated Anova tests are best done by statistical packages, such as S or Minitab, etc.

5. With results of Anova at hand, the new design of experiments based on new Taguchi table must be performed.

Review Questions

1. What does the word ANOVA stand for? Who developed the technique?
2. How does ANOVA compare with standard hypothesis testing?
3. If an ANOVA test identifies correlation among the variables, how should one redo the Taguchi tables?
4. Can ANOVA analysis include discrete variables?
5. If there are 7 replicates and 5 treatments, how many samples are tested?
6. For 7 replicates and 5 treatments, what is the degree of freedom for the treatments? What about the samples?
7. An experiment involving single factor ANOVA can be analyzed by Excel. Is this correct?

References

The classical AVONA method is discussed in great detail in Chapter 13 and 14 of "Applied Statistics and Probability for Engineers, 3rd Edison, D.C. Montgomery and G. C. Runger, Wiley, 2003.

Hunter's lectures on AVONA is also very enjoybale

http://www.youtube.com/watch?v=k3n9iSB6Cns

http://www.youtube.com/watch?v=F05zZL3uyRo

A slightly different approach that also reduces the number of experiments greatly is based on the response surface approach. It uses Newton-like algorithm to find the peaks/valleys of the response surface, see R. H. Myers and D.C. Montgomery, "Response Surface Methodology", Wiley Interscience, 2002. This book discusses design of experiment in great detail.

For general reference see

Joan Fisher Box, "R. A. Fisher and the Design of Experiments, 1922-1926", *The American Statistician,* vol. 34, no. 1, pp. 1-7, Feb. 1980.

F. Yates, "Sir Ronald Fisher and the Design of Experiments", *Biometrics,* vol. 20, no. 2, In Memoriam: Ronald Aylmer Fisher, 1890-1962., pp. 307-321, (Jun. 1964.