Comparison of Traditional Optimization and Deep Learning (DL) techniques in ACOPF solvers for obtaining ACOPF Optimal Solutions

Alam Ali [#1], Prof. Chinmay Hedge [#2]
[#1] PhD EE Degree Student in NYU Tandon, USA
[#2] Course Paper submitted to Course Instructor in NYU Tandon, USA
[#1] aa8007@nyu.edu (Alam Ali), [#2] ch3773@nyu.edu (Prof. Chinmay Hedge)

***Abstract*—This paper is based on the comparison of traditional Optimization techniques and Deep Learning based techniques for usage in ACOPF solvers for computing ACOPF optimal solutions. This paper develops Neural Network based models and compare their computing performance in terms of time taken to get optimal solution with traditional optimization based methods. We will research upon the idea that deep learning based models aims to emulate an ACOPF solver, and we observe positive results on the chosen networks in terms of optimality gap, speed, and convergence success. High percentage generations and penetrations of renewable energy introduces significant uncertainty into the existing power systems and the accompanying power grid. It requires grid operators to solve alternative current optimal power flow (AC-OPF) problems more frequently for economical and reliable operation in both transmission and distribution grids. This paper studies the development of a Deep Neural Network approach for solving ACOPF problems in a fraction of the time as compared to the conventional optimization based solvers.**

***Keywords*—ACOPF, DCOPF, Deep Learning, Neural Networks, Optimization techniques, Optimal Solutions.**

I. Introduction

Use of machine learning to obtain solutions to AC optimal power flow has recently been a very active area of research due to the astounding speedups that result from bypassing traditional optimization techniques. In this project, we train a neural network to emulate an iterative solver in order to approximately iterate towards the optimum and obtain an overall AC-feasible solution. Results shown for networks up to IEEE500 bus network indicates that the proposed method is capable of finding feasible, near-optimal solutions to ACOPF problems in milliseconds on a laptop computer [1].

AC optimal power flow (OPF) problems are solved by grid operators in order to achieve the most economic generation dispatch to meet the power network power demands while adhering to the physical constraints of the network. With the increasing integration of fluctuating renewable energy sources, system operators are required to perform more frequent adjustments of the generator set-points. Currently, real-time adjustments are realized using the automatic generation control (AGC) which may lead to sub-optimal operational points due to the use of restrictive affine control policies. Various linearizations or approximations of the ACOPF problem have been developed throughout the years to address the computational burden of solving the ACOPF in real time, but in many cases cannot guarantee feasibility of the approximate solution, and still require solving an optimization problem, which can be prohibitive as the power network size grows [1].

The emergence of deep learning techniques can be attributed to the availability of increased computing power and an abundance of data collection and storage. Within the domains of power systems, a large quantity of data is generated by grid operators by repeatedly solving ACOPF problems throughout the day. This data is utilized for other tasks and often is not stored for very long periods of time, because grid operators struggle to see the additional use cases for the data **[**2**]**. This paper demonstrates an additional purpose for this data by using it to train a neural network in an offline method which can produce optimal solutions to ACOPF problem in an online manner.

To achieve this, we learning a mapping from the neural network loads to the optimal generator set-points, where a novel parameterization of the ACOPF solution is used to enforce power generation limits. Moreover, strictly feasible solutions are used to train the learning model and the output of the model is constrained to adhere to active power generation and voltage magnitude constraints. In order to circumvent solutions that violate the system requirements, the solution feasibility is ensured by solving a set of power flow equations. By using this neural network based framework, we can determine optimal generation set-points on timescales that are good for balancing the fast fluctuations in renewable energy generation and load.

II. Background Study

A. Literature Review

ACOPF is a canonical power systems operation problem that is at the heart of optimizing very large-scale power networks. Solving this problem quickly and efficiently has been the subject of decades of research. One particularly interesting development in solving these problems is the use of machine learning techniques, such as deep learning to obtain ACOPF solutions [3, 4]. We will propose a deep learning model which aims to emulate an ACOPF solver, and we observe positive results on the chosen networks in terms of optimality gap, speed, and convergence success. Deep Learning approaches have been proposed to recover the power systems states and to enhance solving optimal power flow problems such as ACOPF and DCOPF [5]. Deep Learning takes a large amount of data to compute the system operating conditions and solving parameters.

Neural Networks have the ability to learn by data and examples as depicted in figure 1 below, and is inspired by the biological neuron system. There are two main types of artificial neural networks: Feedforward and feedback artificial neural networks. Feedforward neural network is a network which is not recursive. Feedback neural networks contain cycles. Signals travel in both directions by introducing loops in the network. The feedback cycles can cause the network's behavior change over time based on its input.

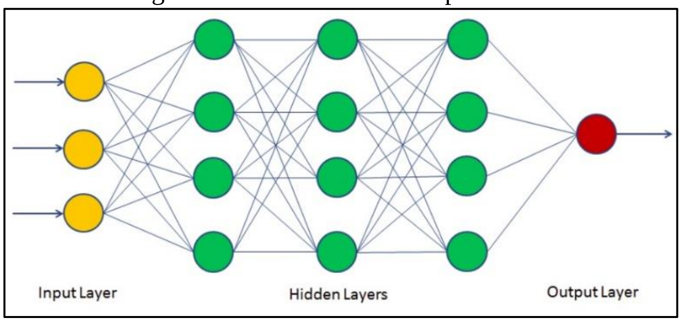

Figure 1: Layers of a Neural Network Model.

Figure 2 below shows the architecture of proposed NN method in operation phase. By using this framework, we can determine the optimal power generation set-points on timescales that are appropriate for balancing fluctuations in renewable power generation and load. For this NN model, we will have a neural network with three hidden layers with ReLU, tanh and linear activation functions as shown in this figure below. It describes ACOPF problem and data-driven approach to formulate this type of problem.

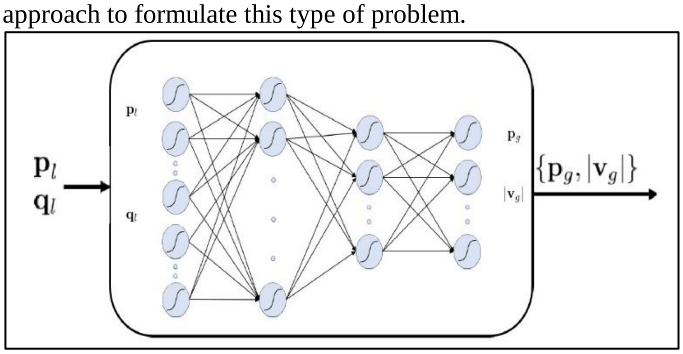

Figure 2: Our proposed Deep Learning Neural Network model for this ACOPF problem [1].

For achieving this task, we learn a mapping from the neural network loads to the optimal generator set-points, where a novel parameterization of the ACOPF solutions is used to enforce power generation limits. Strictly feasible solutions are used to train this learning model, and the output of this model is constrained to adhere to active power generation and voltage magnitude constraints.

Figure 3 below shows the architecture of the proposed approach in the operation phases. By using this framework, we can determine optimal generation set-points on timescales that are appropriate for balancing fast fluctuations in renewable energy generation and load.

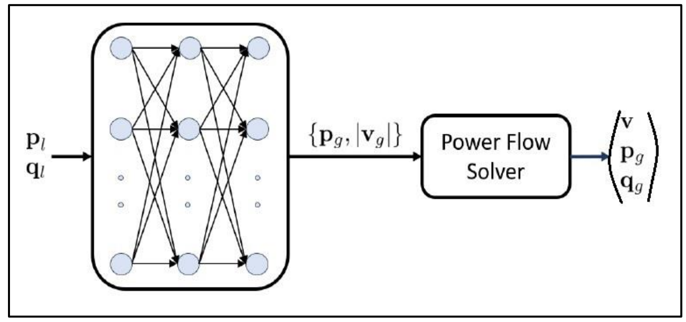

Figure 3: Flow of the proposed learning based approach [1].

B. Motivation for this study

In this paper, we present a framework for solving ACOPF problems which utilizes Deep learning mechanisms to learn a mapping between system loading and optimal power generation values. We provide a fast learning method to guarantee feasibility of the solution in the original ACOPF problem by training the model on strictly feasible solutions and then solving the power flow equations to recover the remainder of the complex network voltages and the reactive power generation values.

We present a machine learning based method to obtain feasible solutions to ACOPF problem with negligible optimality gaps with fast timescales bypassing solving an ACOPF altogether. This is motivated by the fact that as power grids experiences increasing amounts of renewable power generation, controllable loads, and other inverter interfaced devices, fast system dynamics and quick fluctuations in power supply are likely to occur [6]. Many efforts have focused on linearizing the problem in order to solve ACOPF on faster timescales. We leverage historical data to learn a mapping between system loads and optimal generation values, enabling us to find near-optimal and feasible ACOPF solutions on faster timescales without actually solving an optimization problem.

The outcomes obtained in this paper prove encouraging for using deep learning as a tool for quickly finding solutions to nonconvex optimization problems in areas outside of power systems. For applications which involve learning the optimal solution where ensuring feasibility is not possible, we ask the reader to consider developing learning-assisted or learning-enhanced optimization techniques, wherein machine learning and optimization work together to find the optimal solution of the original problem. This is done by either using deep learning to quickly finding a starting point and solution trajectory for optimization, or by learning the solution of a subset of optimized variables [7].

III. OPTIMIZATION PROBLEM FORMULATION

A. ACOPF Problem Formulation

Given the active and reactive power demand at the load buses, active power generated by the generators, and voltage magnitudes at the generator buses, the vector v can be recovered by solving the power flow equations.

ACOPF optimization problem can be formulated as follows.

$$\min_{v,\{p_g,n,q_g,n\},n\in G} \quad c\left(p_g\right) = \sum_{n\in G} c_n\left(p_{g,n}\right)$$

$$\textit{subject to:} \quad \underline{p_{g,n}} \le p_{g,n} \le \overline{p_{g,n}}$$

$$\underline{q_{g,n}} \le q_{g,n} \le \overline{q_{g,n}}$$

$$\underline{|v|} \le \mathrm{v} \le \overline{|v|}$$

$$\mathrm{h(v,p,q)} = 0$$

where, optimization variables $\underline{p_{g,n}}$ and $\underline{q_{g,n}}$ denote active and reactive power injections from the generator installed at bus $n \in G$, and v represents the voltage magnitude. Active power $\underline{p_{g,n}}$ is a vector which collects active power injections from all generators and cost of power generation from generator at bus n is given by $c_n\left(p_{g,n}\right)$. All non-linear power flow equations are collected in this non-linear vector of equations $h(v, p, q)$.

The objective of this proposed data-driven approach is to learn an underlying ACOPF mapping between the load demands at all load buses and all generators active power and voltage magnitude set points. There are also multiple drawbacks of this learning-based method that should be discussed here and in future papers. These results show an effective speed gain, traditional optimization has multiple upsides that are not yet covered by the machine-learning-for-OPF models in literature.

DCOPF as compared to ACOPF takes about the same amount of time to solve a much simpler convex problem, but whose solution does not satisfy the AC power flow equations. For smaller networks, the benefit of using a ML-based method is negligible and sub-second solution times are already achieved by standard ACOPF methods. Another interesting idea is to use a ML model to warm-start the ACOPF as in or the ACPF.

B. Approximate mapping between Loads and the Optimal Generation Solution

Because the solution of ACOPF is on boundary of feasibility set, we aim at ensuring that NN outputs are in the interior of ACOPF feasibility set [9]. We generate training by solving the following restricted R-ACOPF problem.

$$\min_{v,\{p_g,n,q_g,n\},n\in G} \quad c\left(p_g\right) = \sum_{n\in G} c_n\left(p_{g,n}\right)$$

$$\textit{subject to:} \quad \underline{p_{g,n}} \le p_{g,n} \le \overline{p_{g,n}}$$

$$\underline{q_{g,n}} \le q_{g,n} \le \overline{q_{g,n}}$$

$$\underline{|v|} + \lambda \le \mathrm{v} \le \overline{|v|} - \lambda$$

$$\mathrm{h(v,p,q)} = 0$$

Training samples generated by solving above problem are strictly in the interior of the voltage magnitudes feasibility set. Any bounded deviations in the learned mapping are expected to remain within the voltage limits. Parameter λ is an algorithmic tuning parameter that addresses the optimality and feasibility trade-off.

The generated solutions are guaranteed to be in the interior of the feasibility region with respect to the voltage magnitudes. The training samples generated by solving are strictly in the interior of the voltage magnitudes feasibility set. Any bounded deviations in the learned mapping are expected to remain within the voltage limits. Notice that large values of λ may render the original problem infeasible, and the parameter λ has to be tuned in order to obtain strictly feasible solutions.

C. Description of Dataset

For this project, we have generated IEEE 14-bus data, IEEE 30-bus data and IEEE-500 bus data which are known as big data files and they are prepared in excel sheet using MATPOWER MIPS power solver. We loaded these excel sheets in Jupiter Notebook to test our neural network (NN) algorithm and generate results and plot test and train graphs. Main answer computed is the average problem solving time for ACOPF problem for different starting algorithms. Power dataset is being accessed by MATPOWER software using the command .mpc struct, and it is comprised of huge amount of data of more than 16,000 data points. These data points are attached in the final report being generated. MATPOWER Interior Point Solver also known as MIPS solver was used to generate this data and was used as the baseline for comparison with our neural network model. A single training sample consists of the pair [input x^k, and output $x^{(k+1)}$] obtained from MIPS based solver.

D. Description of Neural Network Model

Neural Network training model is shown in the figure below. Figure 4 below shows a Neural Network model layout and an overview of the testing phase of algorithm.

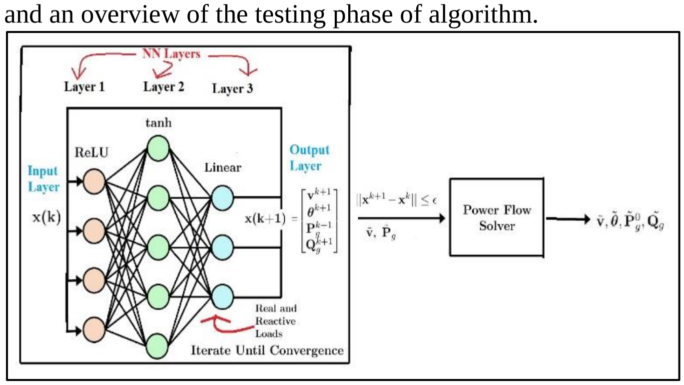

Figure 4: This figure shows the model of Neural Network for approximating fast iterations towards obtaining the optimum power flow solution.

We will use Deep Learning based model $\boldsymbol{F_R}(.)$ that takes $\boldsymbol{x^k}$ as an input and returns $\boldsymbol{x^{k+1}}$ as an output, such that

$$\boldsymbol{x^{k+1}} = \boldsymbol{F_R}(\boldsymbol{x^k}),$$

$$\textbf{\textit{where}}, \quad \textbf{\textit{vector}}\ \boldsymbol{x^k} = [\boldsymbol{v^k}, \boldsymbol{\theta^k}, \boldsymbol{P_g^k}, \boldsymbol{Q_g^k}]^{\boldsymbol{T}}$$

$$\textbf{\textit{vector}}\ \boldsymbol{x^{k+1}} = [\boldsymbol{v^{k+1}}, \boldsymbol{\theta^{k+1}}, \boldsymbol{P_g^{k+1}}, \boldsymbol{Q_g^{k+1}}]^{\boldsymbol{T}}$$

IV. NEURAL NETWORK PROBLEM FORMULATION

A. Deep Learning Model

This problem is a general non-convex optimization problem with n-dimensional optimization variable vector x, cost function $f(.) : R^n \rightarrow R$, M equality constraints $g_i(x) = 0$ and P inequality constraints $h_j(x) \leq 0$ given as follows:

$$\boldsymbol{min_x \ \ f(x)}$$

$$\boldsymbol{s.t \qquad g_i(x) = 0, \qquad for\ i = 1, 2, \ldots, M}$$

$$\boldsymbol{h_j(x) \leq 0, \qquad for\ j = 1, 2, \ldots, P}$$

Instead of using Lagrangian functions or forming Hessian matrices, learning-based method uses a deep learning model $F_R(.) : R^n \rightarrow R^n$ that takes in x^k as an input and provides x^{k+1} as an output, written below as where F defines the deep learning model:

$$\boldsymbol{x^{k+1} = F(x^k)}$$

Proposed model in this project is comprised of a fully connected three-layer neural network (NN) with feedback, where input x^k is the candidate optimal solution vector at iteration k. This model iteratively uses feedback from the output layer x^{k+1} to the input layer x^k until convergence occurs $|||x^{k+1} - x^k||| \leq \varepsilon$.

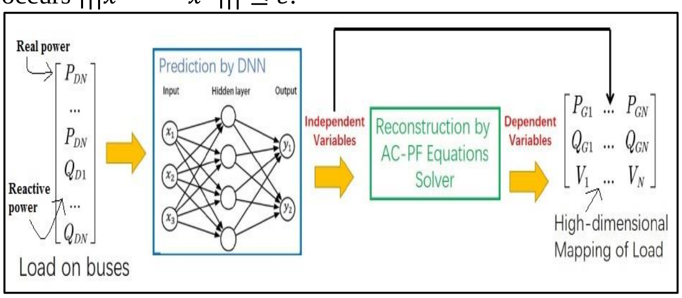

Figure 5: Predict and reconstruct framework for designing Deep NN solvers for ACOPF problems [9].

B. Description of Loss Function

For our model, we will train and test IEEE-14, IEEE-30 and IEEE 500 power bus data using two optimizers and the loss function used will be Mean Square Error loss function (MSE). One optimizer is Adam and the second optimizer one is RMSprop.

These optimizers are used to generate data plots and they are illustrated and compared below.

C. Training Details

For the final results obtained in this paper, deep learning or Neural Network (NN) based method will be used to solve OPF problem and the result obtained will be compared with conventional DCOPF method. DCOPF does not produce an AC feasible solution, but it gives a good approximation for ACOPF and is used in many LMP-based markets to calculate prices and hence it can provide an interesting comparison between learning methods and traditional optimization techniques [1]. A single training sample consists of the input and output pair $\boldsymbol{x^k\ and\ x^{k+1}}$ obtained from this MIPS based solver. Tolerance $\boldsymbol{\varepsilon}$ of learning-based solver was set to $\boldsymbol{10^{-4}}$, where convergence is reached when $\left|\left|\boldsymbol{x^{k+1} - x^k}\right|\right| \leq \boldsymbol{\epsilon.}$

A very small value of tolerance was used for data generation to promote smoother convergence and basins of attraction within the ML model. For a fair comparison, the same convergence criteria was used for the neural network (NN) model and MIPS based solver during testing.

D. Initial Testing Results

For this research, we tested the given data as attached on GitHub repository with IEEE 14-bus and IEEE 30-bus power system models. Model Loss functions with Adam Optimizer and RMSprop Optimizer have been applied for this loss function, and the results obtained are plotted below. Model loss is lesser with Adam Optimizer, keeping in mind that same hyperparamters have been selected for tuning both networks.

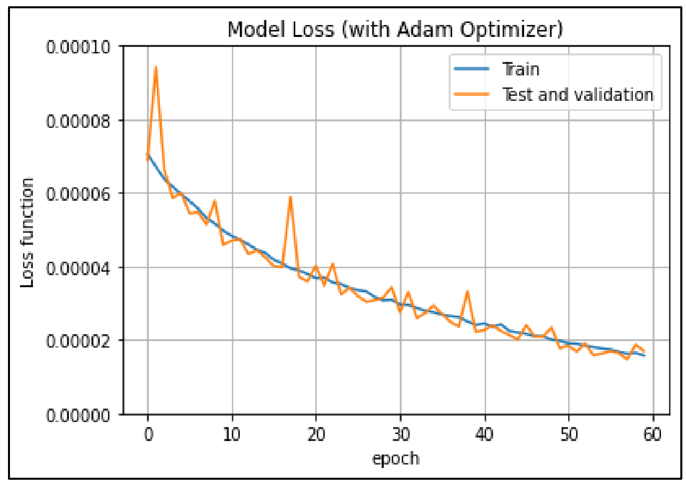

Figure 6: Plot of Model loss function for train and test data with Adam Optimizer.

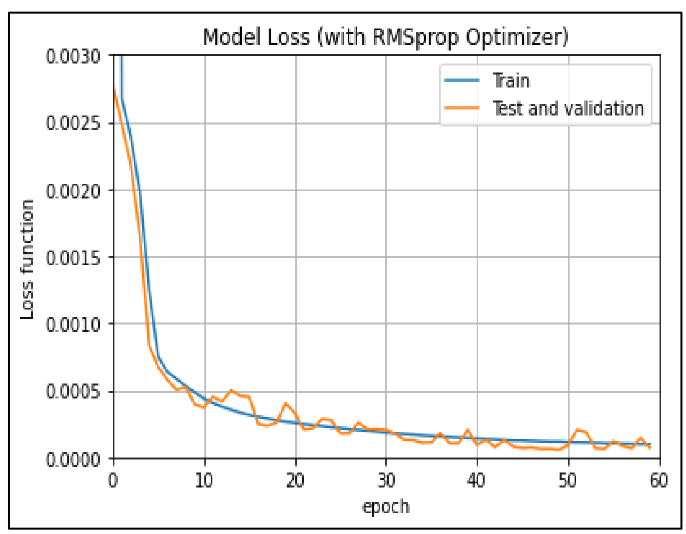

Figure 7: Plot of Model loss function for train and test data with RMSprop Optimizer.

The model emulates an iterative algorithm, meaning that each model run is a small step towards the optimum, instead of directly predicting the ACOPF solution. ML model utilizes information about the entire OPF solution, better informing the model as it iterates towards the optimum.

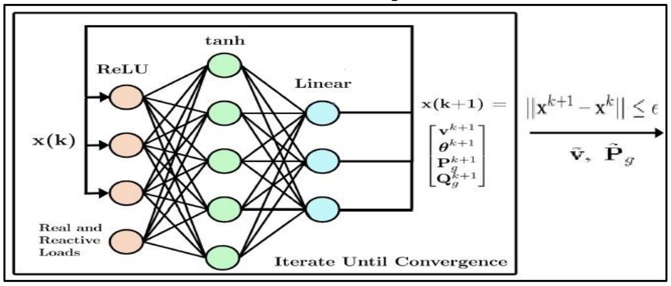

Figure 8: Overview of DL based algorithm.

E. Simulation Results

Solution times for finding OPF are computed below for three different types of power networks, IEEE-14 bus, IEEE-30 bus and IEEE-500 bus. Table 1 below shows the comparison of average solve time of Deep Learning-based method (known as "NN") with two other cases (ACOPF flat started and DCOPC) across the 500-sample training dataset.

Type of Power Network	Type of Optimal Power Flow (OPF)	Average Problem Solve Time (seconds)
IEEE 14-bus	ACOPF with flat start (traditional method)	0.41s
	DCOPF (traditional method)	0.29s
	NN with PF (deep learning-based)	0.08s
IEEE 30-bus	ACOPF with flat start (traditional method)	0.68s
	DCOPF (traditional method)	0.51s
	NN with PF (deep learning-based)	0.12s
IEEE 500-bus	ACOPF with flat start (traditional method)	3.24s
	DCOPF (traditional method)	1.61s
	NN with PF (deep learning-based)	0.24s

Table 1: Comparison of average solution finding time of ML method and traditional DCOPF method.

These methods are very robust and hard-coded and they can be very time consuming and may not be suitable for real-time operation. The learning-based method provides an alternative to these methods, as issues with singular Jacobian matrices and solutions close to voltage instability do not affect inference as much. These solutions may still prove challenging for the post-processing power flow step, which is perhaps why a few failures are still being encountered when using the learning-based method.

Now, we compare the difference in cost function value for DL and DCOPF solution methods. Table 2 below shows the optimality gap (difference in cost function value) for the deep-learning ("NN") based OPF solution method and the traditional DCOPF solution.

Power network type	Average Gap for Neural Network case	Average Gap for DCOPF case
IEEE 14-bus	0.12%	1.53%
IEEE 30-bus	0.23%	2.08%
IEEE 500-bus	2.61%	10.05%

Table 2: Optimality Gaps for Deep Learning based method and DCOPF method.

F. Project Outcomes

Results show extremely fast solution convergence times for Deep Learning (NN) based method as compared to traditional DCOPC method. The table comparison performed shows that learning-based method ("NN") is compared with two other cases across the 500-sample training dataset. The performance of solution convergence times increased by manifolds. We showed that for larger networks, the learning-based solution finds approximate solutions to ACOPF faster than it takes to find a solution to DCOPF, with a smaller optimality gap than DCOPF provides and without the AC infeasibility of DCOPF.

I will like to discuss some of the challenges faced in doing this research project. Generating the code in Tensor flow was quite a challenge, because we had to learn OPF data modelling in python to generate this optimization for NN and DCOPF power flow models. It is observed that generating a diverse and representative dataset is an important thrust of research within learning-based OPF methods [10].

ACOPF is one of the most commonly solved problems in power systems. The current emphasis on integration of renewable sources and power electronics into modern grids, has increased the complexity of power flow calculations. Obtaining a feasible ACOPF solution in large-scale power systems is a very big challenge. Existence of multiple solutions causes certain initial points to lead to non-feasible solutions in which some operating network constraints are violated, hence NN network solves this problem.

G. Reflections on Results

We learn a model that can predict an accurate solution over a fixed grid topology and constraints set, which further gives some measure of consistency in the solution space, meaning similar load distributions should correspond to similar generator settings. Neural networks have demonstrated the ability to model extremely complicated non-convex functions, making them highly attractive for this setting. A model can be trained off-line on historic data and used in real-time to make predictions on an optimal power setting [9].

Only a subset of variables was sent to the power flow solver because of huge amount of training data, but the ML model utilized information about the entire OPF solution for better informing the neural network model for iterating it towards the optimum solution [1]. Directions of future work include development of datasets or dataset generation methods for learning-based OPF, inclusion of additional constraints such as line flow limits, and speed or accuracy comparisons with other relaxations or convexifications.

V. CONCLUSION

This research work provides a learning-based approximation for solving ACOPF problem. Initial results show that this method can achieve very fast convergence speeds with minimal optimality gaps, even converging faster and with more accuracy than DCOPF on large neural networks. Hence, a larger neural network with more training data and more hyperparamters tuning may improve these results even further. Black-box models (such as NN) do not offer grid operators much insight in decision-making. A combination of greater understanding of these models and increased failsafes can help expedite the potential use of these NN models in actual operation. It is of great interest to explore Deep NN designs that are robust to changes in system typologies and network parameters.

The results in this paper prove encouraging for using machine learning as a tool for quickly finding solutions to nonconvex optimization problems in areas outside of power systems. In potential applications where learning the optimal solution while ensuring feasibility is not possible or is very difficult, we ask the reader to consider developing or using learning-assisted or learning-enhanced optimization techniques. It is shown that the proposed method can find very difficult ACOPF solutions that cause flat start or DC-warm start algorithms to diverge. Lastly, we can show that for larger networks the learning-based solution finds approximate solutions to ACOPF faster than it takes to find a solution to DCOPF and with a smaller optimality gap than DCOPF provides and without the AC infeasibility of DCOPF.

The benefit of using a ML model instead of the MIPS based solver directly is that no matrix inverses or factorizations are needed, and inference is extremely fast which results in overall faster rate of convergence. While we do not claim that feasibility can be guaranteed for every single output of the learning-based model, we have observed very positive results on the chosen networks in terms of gap for optimality, speed of obtaining solutions, and the rate of convergence of solution.

ACKNOWLEDGMENTS

This paper has been written with the help of Prof. Chinmay Hedge and Prof. Yury Dvorkin at NYU Tandon School of Engineering. This is a student paper submitted for fulfilling the course requirements of Deep Learning. This paper has possible directions in which future improvements can be made to address this problem later. The weblink for the python code is available at this weblink with full comments and coding structures https://github.com/alamali88/.

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```
# Name: Alam Ali,
# Net ID: aa8007,
# Course: DL Project,

# Project Code for ACOPF solver,

from pandas import read_csv
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split
from sklearn.model_selection import cross_val_score
from sklearn.model_selection import KFold
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import MinMaxScaler
import tensorflow as tf
from tensorflow.python.keras.models import Sequential
from tensorflow.python.keras.layers import Dense
from tensorflow.python.keras.layers import LeakyReLU
from tensorflow.python.keras.layers import BatchNormalization
from tensorflow.python.keras.wrappers.scikit_learn import KerasRegressor
from keras.constraints import max_norm

# load dataset

system = "14"

dataframeX = read_csv("/content/drive/MyDrive/14result_overall_in.csv", delimiter=',', header=None)
X = dataframeX.values[0:]
dataframeY = read_csv("/content/drive/MyDrive/14result_overall_out.csv", delimiter=',', header=None)
Y = dataframeY.values

# split into input (X) and output (Y) variables
print (len(X))
print (X.shape)

print (len(Y))
print (Y.shape)

nsamples = X.shape[0]
npredictors = X.shape[1]
noutvars = Y.shape[1]
print (npredictors)
print (noutvars)
print (Y.shape[1])
```

```
70677
(70677, 60)
70677
(70677, 38)
60
38
38
```

```
from google.colab import drive
drive.mount('/content/drive')
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/cont
```

```
from google.colab import drive
drive.mount('/content/drive', force_remount=True)
```

```
Mounted at /content/drive
```

```
from keras.models import Sequential
from keras.layers import Dense

model = Sequential()
model.add(Dense(npredictors, input_dim=npredictors, kernel_initializer='normal', activation='tanh'))
model.add(Dense(10000, activation='relu'))
model.add(Dense(noutvars, activation='linear'))

model.summary()

opt = tf.keras.optimizers.Adam(learning_rate=0.00005)
model.compile(loss='mse', optimizer=opt, metrics=['mse','mae'])
```

```
Model: "sequential"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense (Dense)               (None, 60)                3660

 dense_1 (Dense)             (None, 10000)             610000

 dense_2 (Dense)             (None, 38)                380038

=================================================================
Total params: 993,698
Trainable params: 993,698
Non-trainable params: 0
_________________________________________________________________
```

```
history = model.fit(X, Y, epochs=50, batch_size=64,  verbose=1, validation_split=0.01)
```

```
model.save(system+'busNN.h5')
```

```
print(history.history.keys())

plt.plot(history.history['loss'])
plt.plot(history.history['val_loss'])
plt.title('Model Loss (with Adam Optimizer)')
plt.xlim([20,50]);
plt.ylim([0.00002,0.00018]);
plt.ylabel('Loss function')
plt.xlabel('epoch')
plt.legend(['Train', 'Test and validation'], loc='upper right')
plt.grid()
plt.show()
```

```
dict_keys(['loss', 'mse', 'mae', 'val_loss', 'val_mse', 'val_mae'])
```


```
from keras.models import Sequential
from keras.layers import Dense

model = Sequential() # do this every time to reset the model!
model.add(Dense(npredictors, input_dim=npredictors, kernel_initializer='normal', activation='tanh'))
model.add(Dense(10000, activation='relu'))
model.add(Dense(noutvars, activation='linear'))#,kernel_constraint=max_norm(3), bias_constraint=max_n

model.summary()
opt = tf.keras.optimizers.RMSprop(learning_rate=0.00005)
model.compile(loss='mse', optimizer=opt, metrics=['mse','mae'])
```

```
Model: "sequential_1"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_3 (Dense)             (None, 60)                3660

 dense_4 (Dense)             (None, 10000)             610000

 dense_5 (Dense)             (None, 38)                380038

=================================================================
Total params: 993,698
Trainable params: 993,698
Non-trainable params: 0
_________________________________________________________________
```

```
history = model.fit(X, Y, epochs=50, batch_size=64,  verbose=1, validation_split=0.01)
```

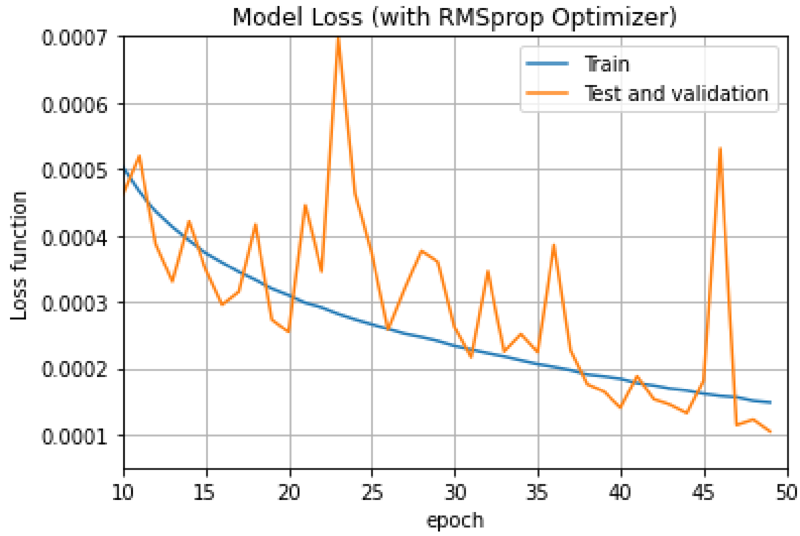
```
model.save(system+'busNN.h5')
```

```
print(history.history.keys())

plt.plot(history.history['loss'])
plt.plot(history.history['val_loss'])
plt.title('Model Loss (with RMSprop Optimizer)')
plt.xlim([10,50]);
plt.ylim([0.00005,0.00070]);
plt.ylabel('Loss function')
```

```
plt.xlabel('epoch')
plt.legend(['Train', 'Test and validation'], loc='upper right')
plt.grid()
plt.show()
```

```
dict_keys(['loss', 'mse', 'mae', 'val_loss', 'val_mse', 'val_mae'])
```




```
from keras.models import Sequential
from keras.layers import Dense

model = Sequential() # do this every time to reset the model!
model.add(Dense(npredictors, input_dim=npredictors, kernel_initializer='normal', activation='tanh'))
model.add(Dense(10000, activation='relu'))
model.add(Dense(noutvars, activation='linear'))#,kernel_constraint=max_norm(3), bias_constraint=max_n

model.summary()
opt = tf.keras.optimizers.SGD(learning_rate=0.005)
model.compile(loss='mse', optimizer=opt, metrics=['mse','mae'])
```

```
Model: "sequential_13"
_________________________________________________________________
 Layer (type)                Output Shape              Param #   
=================================================================
 dense_39 (Dense)            (None, 60)                3660      
                                                                 
 dense_40 (Dense)            (None, 10000)             610000    
                                                                 
 dense_41 (Dense)            (None, 38)                380038    
                                                                 
=================================================================
Total params: 993,698
Trainable params: 993,698
Non-trainable params: 0
_________________________________________________________________
```

```
history = model.fit(X, Y, epochs=50, batch_size=64,  verbose=1, validation_split=0.01)
```

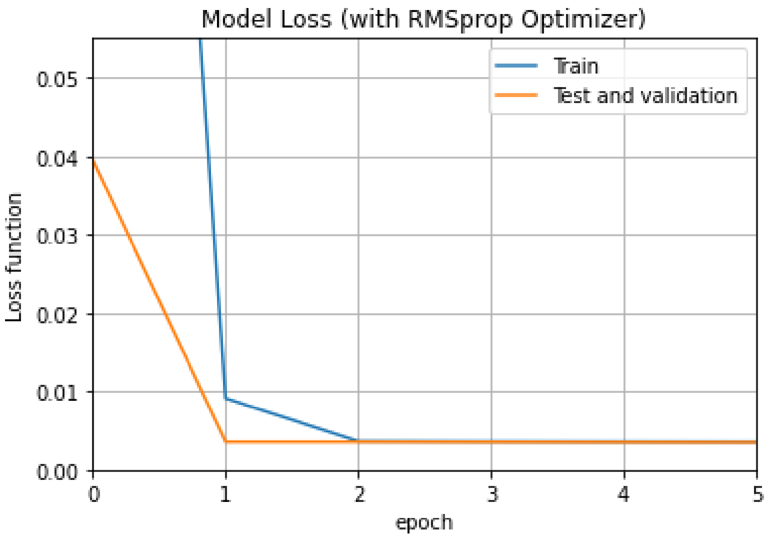
```
model.save(system+'busNN.h5')
```

```
print(history.history.keys())

plt.plot(history.history['loss'])
plt.plot(history.history['val_loss'])
```

```
plt.plot(history.history['val_loss'])
plt.title('Model Loss (with RMSprop Optimizer)')
plt.xlim([0,5]);
plt.ylim([0.00000,0.055]);
plt.ylabel('Loss function')
plt.xlabel('epoch')
plt.legend(['Train', 'Test and validation'], loc='upper right')
plt.grid()
plt.show()
```

```
dict_keys(['loss', 'mse', 'mae', 'val_loss', 'val_mse', 'val_mae'])
```



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Name: Alam Ali, Net ID: aa8007, DL course project (Final report)
DL dataset (x(k) and x(k+1)) is given s.t x(k+1)=F(x(k))
This is a huge amount of data (more than 16,000 data points). So, I have just attached first two pages of data.

(1) x(k) is an input data to Deep Learning program.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974
2	0.00218	0.0018	0.001	0.00139	0.00151	0.00123	0.00133	0.00162	0.00112	0.0011	0.00115	0.00111	0.00108	0.00097	0.0097	0.00958	0.0093	0.00948	0.00951	0.00989	0.00977	0.00976	0.00984	0.00982	0.00984	0.0098
3	0.00218	0.00179	0.00094	0.00137	0.0015	0.00124	0.00134	0.00167	0.00113	0.00111	0.00115	0.00112	0.00109	0.00097	0.00974	0.00961	0.00933	0.0095	0.00953	0.00994	0.00981	0.00983	0.00988	0.00986	0.00989	0.00985
4	0.00218	0.00176	0.00089	0.00133	0.00146	0.00116	0.00127	0.00157	0.00106	0.00104	0.00108	0.00104	0.00102	0.0009	0.00976	0.00962	0.00935	0.00951	0.00953	0.00994	0.00981	0.00983	0.00988	0.00986	0.00989	0.00985
5	0.00218	0.00166	0.0007	0.00114	0.0013	0.00085	0.00096	0.00112	0.00076	0.00074	0.00077	0.00073	0.0007	0.00059	0.0098	0.00965	0.00935	0.00951	0.00953	0.00994	0.00982	0.00985	0.00989	0.00987	0.00989	0.00985
6	0.00218	0.00163	0.00063	0.00107	0.00123	0.00069	0.00081	0.00091	0.00062	0.0006	0.00062	0.00057	0.00055	0.00045	0.00986	0.0097	0.00939	0.00953	0.00956	0.00996	0.00984	0.00986	0.00991	0.00989	0.00991	0.00987
7	0.00218	0.00164	0.00069	0.00109	0.00125	0.00067	0.00079	0.00084	0.00061	0.00058	0.0006	0.00055	0.00053	0.00044	0.01004	0.00988	0.00959	0.00968	0.00971	0.01005	0.00995	0.00991	0.01002	0.01	0.01001	0.00997
8	0.00218	0.00165	0.00074	0.00112	0.00127	0.00069	0.00081	0.00082	0.00063	0.00061	0.00063	0.00057	0.00056	0.00047	0.0102	0.01004	0.00977	0.00983	0.00986	0.01018	0.01008	0.01001	0.01016	0.01014	0.01014	0.0101
9	0.00218	0.00165	0.00074	0.00111	0.00127	0.00067	0.00079	0.0008	0.00062	0.00059	0.00061	0.00056	0.00054	0.00045	0.01019	0.01004	0.0098	0.00984	0.00987	0.01019	0.01009	0.01003	0.01017	0.01015	0.01016	0.01011
10	0.00218	0.00165	0.00075	0.00111	0.00127	0.00067	0.00079	0.00079	0.00062	0.00059	0.00061	0.00056	0.00054	0.00045	0.0102	0.01005	0.00981	0.00985	0.00987	0.0102	0.0101	0.01005	0.01018	0.01016	0.01016	0.01011
11	0.00218	0.00165	0.00075	0.00111	0.00126	0.00067	0.00078	0.00078	0.00061	0.00059	0.00061	0.00056	0.00054	0.00045	0.0102	0.01005	0.00982	0.00986	0.00988	0.0102	0.01012	0.01009	0.01019	0.01017	0.01017	0.01011
12	0.00218	0.00165	0.00075	0.00111	0.00126	0.00067	0.00078	0.00078	0.00061	0.00059	0.00061	0.00056	0.00054	0.00045	0.0102	0.01005	0.00982	0.00986	0.00988	0.0102	0.01012	0.01009	0.01019	0.01017	0.01017	0.01011
13	0.00218	0.00165	0.00075	0.00111	0.00126	0.00067	0.00078	0.00078	0.00061	0.00059	0.00061	0.00056	0.00054	0.00045	0.0102	0.01005	0.00982	0.00986	0.00988	0.0102	0.01012	0.01009	0.01019	0.01017	0.01017	0.01011
14	0.00218	0.00165	0.00075	0.00111	0.00126	0.00067	0.00078	0.00078	0.00061	0.00059	0.00061	0.00056	0.00054	0.00045	0.0102	0.01005	0.00982	0.00986	0.00988	0.0102	0.01012	0.01009	0.01019	0.01017	0.01017	0.01011
15	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974
16	0.00218	0.00179	0.0012	0.00128	0.00143	0.00102	0.00115	0.0015	0.00087	0.00085	0.00092	0.00088	0.00084	0.00065	0.00976	0.00964	0.00948	0.00948	0.00952	0.00989	0.00975	0.00976	0.0098	0.00979	0.00982	0.0098
17	0.00218	0.00179	0.00119	0.00127	0.00142	0.00104	0.00117	0.00154	0.00088	0.00086	0.00093	0.0009	0.00085	0.00065	0.00979	0.00967	0.00953	0.00949	0.00953	0.00993	0.00978	0.00982	0.00982	0.00981	0.00985	0.00983
18	0.00218	0.00176	0.00114	0.00123	0.00138	0.00097	0.00111	0.00147	0.00082	0.0008	0.00086	0.00083	0.00078	0.00059	0.00981	0.00968	0.00954	0.0095	0.00953	0.00993	0.00978	0.00982	0.00983	0.00981	0.00985	0.00983
19	0.00218	0.00166	0.00094	0.00105	0.00123	0.00066	0.00082	0.00107	0.00054	0.00051	0.00057	0.00053	0.00048	0.0003	0.00985	0.0097	0.00954	0.00949	0.00953	0.00992	0.00979	0.00984	0.00984	0.00982	0.00986	0.00983
20	0.00218	0.00162	0.00087	0.00098	0.00116	0.00051	0.00069	0.00088	0.00042	0.00038	0.00043	0.00038	0.00034	0.00017	0.00992	0.00977	0.00958	0.00953	0.00957	0.00995	0.00982	0.00987	0.00987	0.00985	0.00989	0.00986
21	0.00218	0.00164	0.00093	0.00103	0.0012	0.00053	0.00072	0.00089	0.00046	0.00042	0.00046	0.0004	0.00036	0.00022	0.01017	0.01002	0.00982	0.00976	0.0098	0.01012	0.01001	0.01	0.01007	0.01004	0.01007	0.01003
22	0.00218	0.00165	0.00093	0.00103	0.0012	0.00052	0.00072	0.00089	0.00046	0.00043	0.00046	0.0004	0.00036	0.00022	0.0102	0.01005	0.00985	0.00978	0.00982	0.01015	0.01004	0.01004	0.0101	0.01008	0.0101	0.01006
23	0.00218	0.00164	0.00094	0.00102	0.00119	0.00049	0.00072	0.00089	0.00045	0.00041	0.00044	0.00037	0.00034	0.0002	0.0102	0.01005	0.00986	0.00981	0.00984	0.0102	0.01011	0.01015	0.01016	0.01013	0.01015	0.01011
24	0.00218	0.00164	0.00094	0.00102	0.00119	0.00048	0.00071	0.00089	0.00045	0.00041	0.00043	0.00036	0.00032	0.0002	0.0102	0.01006	0.00987	0.00981	0.00985	0.0102	0.01013	0.0102	0.01018	0.01015	0.01016	0.01011
25	0.00218	0.00164	0.00094	0.00102	0.00119	0.00047	0.00072	0.0009	0.00045	0.0004	0.00042	0.00035	0.00031	0.00019	0.0102	0.01006	0.00987	0.00982	0.00985	0.0102	0.01013	0.0102	0.01018	0.01015	0.01016	0.01011
26	0.00218	0.00164	0.00094	0.00102	0.00119	0.00047	0.00072	0.0009	0.00045	0.0004	0.00042	0.00035	0.00031	0.00019	0.0102	0.01006	0.00987	0.00982	0.00985	0.0102	0.01013	0.0102	0.01018	0.01015	0.01016	0.01011
27	0.00218	0.00164	0.00094	0.00102	0.00119	0.00046	0.00072	0.0009	0.00045	0.0004	0.00042	0.00035	0.00031	0.00019	0.0102	0.01006	0.00987	0.00982	0.00985	0.0102	0.01013	0.0102	0.01018	0.01015	0.01016	0.01011

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
28	0.00218	0.00164	0.00094	0.00102	0.00119	0.00046	0.00072	0.0009	0.00045	0.0004	0.00042	0.00035	0.00031	0.00019	0.0102	0.01006	0.00987	0.00982	0.00985	0.0102	0.01013	0.0102	0.01018	0.01015	0.01016	0.01011
29	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974
30	0.00218	0.00177	0.00096	0.00126	0.00142	0.00121	0.00128	0.00166	0.00105	0.00103	0.00109	0.00108	0.00105	0.00084	0.00974	0.00961	0.00934	0.00946	0.0095	0.00991	0.00976	0.00977	0.00983	0.00981	0.00984	0.00983
31	0.00218	0.00177	0.00093	0.00125	0.00142	0.00123	0.0013	0.00171	0.00106	0.00105	0.00111	0.00111	0.00107	0.00086	0.00978	0.00965	0.00939	0.00948	0.00952	0.00996	0.00979	0.00983	0.00986	0.00984	0.00988	0.00988
32	0.00218	0.00174	0.0009	0.00122	0.00139	0.00117	0.00125	0.00165	0.00102	0.001	0.00106	0.00105	0.00101	0.00081	0.00979	0.00966	0.0094	0.00949	0.00953	0.00996	0.0098	0.00983	0.00986	0.00985	0.00988	0.00987
33	0.00218	0.00164	0.00073	0.00104	0.00123	0.00085	0.00095	0.00122	0.00073	0.00071	0.00075	0.00074	0.0007	0.0005	0.00984	0.00968	0.00941	0.00948	0.00953	0.00995	0.00981	0.00985	0.00987	0.00985	0.00988	0.00987
34	0.00218	0.00161	0.0007	0.00098	0.00117	0.00069	0.00081	0.00102	0.00059	0.00057	0.0006	0.00057	0.00054	0.00036	0.00989	0.00973	0.00946	0.00951	0.00956	0.00997	0.00983	0.00987	0.00989	0.00988	0.0099	0.00989
35	0.00218	0.00163	0.00077	0.001	0.00119	0.00065	0.00079	0.00095	0.00058	0.00055	0.00057	0.00054	0.00051	0.00035	0.01007	0.00991	0.00966	0.00966	0.0097	0.01006	0.00994	0.00993	0.01001	0.00999	0.01	0.00999
36	0.00218	0.00164	0.00082	0.00103	0.00121	0.00066	0.00081	0.00095	0.0006	0.00057	0.00059	0.00056	0.00052	0.00037	0.0102	0.01004	0.0098	0.00978	0.00982	0.01016	0.01004	0.01001	0.01012	0.0101	0.01011	0.01008
37	0.00218	0.00164	0.00082	0.00103	0.0012	0.00063	0.0008	0.00094	0.00059	0.00056	0.00057	0.00053	0.0005	0.00035	0.0102	0.01005	0.00983	0.0098	0.00984	0.01019	0.01008	0.01006	0.01015	0.01013	0.01014	0.01011
38	0.00218	0.00164	0.00082	0.00102	0.0012	0.00063	0.00079	0.00094	0.00059	0.00055	0.00056	0.00052	0.00049	0.00035	0.0102	0.01005	0.00984	0.00981	0.00985	0.0102	0.01011	0.01013	0.01018	0.01016	0.01015	0.01012
39	0.00218	0.00164	0.00082	0.00102	0.0012	0.00062	0.00079	0.00094	0.00058	0.00055	0.00056	0.00051	0.00048	0.00034	0.0102	0.01005	0.00984	0.00982	0.00985	0.0102	0.01014	0.01018	0.0102	0.01017	0.01016	0.01012
40	0.00218	0.00164	0.00082	0.00102	0.0012	0.00061	0.00079	0.00094	0.00058	0.00055	0.00055	0.00051	0.00048	0.00034	0.0102	0.01006	0.00984	0.00982	0.00985	0.0102	0.01014	0.01017	0.0102	0.01017	0.01016	0.01012
41	0.00218	0.00164	0.00082	0.00102	0.0012	0.00061	0.00079	0.00094	0.00058	0.00055	0.00055	0.00051	0.00048	0.00034	0.0102	0.01006	0.00984	0.00982	0.00985	0.0102	0.01014	0.01017	0.0102	0.01017	0.01016	0.01012
42	0.00218	0.00164	0.00082	0.00102	0.0012	0.00061	0.00079	0.00094	0.00058	0.00055	0.00055	0.00051	0.00048	0.00034	0.0102	0.01006	0.00984	0.00982	0.00985	0.0102	0.01014	0.01017	0.0102	0.01017	0.01016	0.01012
43	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00218	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974	0.00974
44	0.00218	0.00178	0.00099	0.00131	0.00145	0.00111	0.00126	0.00162	0.00101	0.00097	0.00101	0.00097	0.00096	0.0008	0.00974	0.00961	0.00934	0.00948	0.00951	0.00989	0.00977	0.00978	0.00984	0.00981	0.00982	0.00979
45	0.00218	0.00177	0.00095	0.0013	0.00144	0.00113	0.00128	0.00167	0.00103	0.00098	0.00102	0.00099	0.00098	0.00082	0.00977	0.00965	0.00939	0.0095	0.00953	0.00994	0.00981	0.00984	0.00987	0.00984	0.00986	0.00983
46	0.00218	0.00175	0.00092	0.00127	0.00141	0.00107	0.00123	0.0016	0.00098	0.00093	0.00097	0.00093	0.00092	0.00076	0.00979	0.00966	0.0094	0.00951	0.00954	0.00993	0.00981	0.00984	0.00987	0.00984	0.00986	0.00983
47	0.00218	0.00165	0.00074	0.00109	0.00125	0.00075	0.00092	0.00116	0.00068	0.00063	0.00066	0.00061	0.0006	0.00046	0.00984	0.00968	0.00941	0.00951	0.00954	0.00993	0.00982	0.00986	0.00988	0.00984	0.00986	0.00983
48	0.00218	0.00162	0.00069	0.00102	0.00119	0.00058	0.00077	0.00094	0.00054	0.00048	0.0005	0.00045	0.00044	0.00031	0.00989	0.00973	0.00946	0.00954	0.00956	0.00995	0.00984	0.00988	0.0099	0.00987	0.00988	0.00984
49	0.00218	0.00163	0.00077	0.00104	0.0012	0.00055	0.00074	0.00085	0.00052	0.00047	0.00048	0.00042	0.00041	0.00029	0.01007	0.00991	0.00966	0.00969	0.00971	0.01004	0.00995	0.00994	0.01002	0.00998	0.00999	0.00994
50	0.00218	0.00164	0.00082	0.00107	0.00122	0.00056	0.00075	0.00084	0.00054	0.00048	0.00049	0.00044	0.00043	0.00031	0.0102	0.01004	0.0098	0.00981	0.00983	0.01014	0.01005	0.01001	0.01013	0.01009	0.01009	0.01004
51	0.00218	0.00164	0.00082	0.00106	0.00121	0.00053	0.00073	0.00079	0.00052	0.00046	0.00046	0.0004	0.0004	0.00029	0.0102	0.01005	0.00983	0.00982	0.00985	0.01018	0.01009	0.01006	0.01016	0.01012	0.01013	0.01008
52	0.00218	0.00164	0.00083	0.00105	0.00121	0.00052	0.00072	0.00078	0.00051	0.00045	0.00045	0.00039	0.00039	0.00028	0.0102	0.01005	0.00984	0.00983	0.00985	0.0102	0.01011	0.0101	0.01018	0.01014	0.01015	0.0101
53	0.00218	0.00164	0.00083	0.00105	0.0012	0.00051	0.00071	0.00078	0.0005	0.00044	0.00045	0.00039	0.00038	0.00027	0.0102	0.01005	0.00984	0.00984	0.00986	0.0102	0.01013	0.01015	0.0102	0.01016	0.01015	0.0101
54	0.00218	0.00164	0.00083	0.00105	0.0012	0.0005	0.00071	0.00078	0.0005	0.00044	0.00044	0.00038	0.00038	0.00027	0.0102	0.01005	0.00984	0.00984	0.00986	0.0102	0.01013	0.01015	0.0102	0.01016	0.01015	0.0101

Name: Alam Ali, Net ID: aa8007, DL course project (Final report)
DL dataset (x(k) and x(k+1)) is given s.t x(k+1)=F(x(k))
(1) x(k+1) is an output data to Deep Learning program.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	0	-0.05004	-0.15606	-0.10441	-0.08758	-0.12474	-0.11156	-0.07338	-0.13924	-0.14182	-0.13606	-0.1413	-0.14439	-0.16013	0.99439	0.97793	0.94167	0.96531	0.96883	1.01944	1.00287	1.00176	1.01222	1.01001	1.01252	1.00796
2	0	-0.05078	-0.16403	-0.10667	-0.08892	-0.12376	-0.11032	-0.06758	-0.1387	-0.14114	-0.13515	-0.14023	-0.14334	-0.15964	0.99942	0.9829	0.94578	0.9682	0.97157	1.02657	1.00858	1.01142	1.01757	1.01559	1.01879	1.01456
3	0	-0.05488	-0.16962	-0.11245	-0.09423	-0.13399	-0.11941	-0.07967	-0.14762	-0.1503	-0.14484	-0.15036	-0.15338	-0.16904	1.00165	0.98383	0.94764	0.96876	0.97209	1.02649	1.00905	1.01166	1.01807	1.016	1.01898	1.01452
4	0	-0.06798	-0.19522	-0.13665	-0.11574	-0.17557	-0.16115	-0.13955	-0.18713	-0.19017	-0.18556	-0.19179	-0.19467	-0.2093	1.00803	0.98732	0.94864	0.9686	0.97229	1.02633	1.01015	1.01349	1.01892	1.0167	1.01928	1.01441
5	0	-0.07265	-0.2044	-0.14656	-0.12472	-0.19654	-0.18032	-0.1677	-0.20542	-0.20891	-0.20538	-0.2125	-0.21519	-0.22846	1.01589	0.99452	0.95332	0.97214	0.97601	1.02828	1.01279	1.01517	1.0218	1.01946	1.02169	1.01643
6	0	-0.07113	-0.19711	-0.14402	-0.1228	-0.19954	-0.18321	-0.17714	-0.20707	-0.21068	-0.20771	-0.21499	-0.21757	-0.2299	1.03877	1.01784	0.97999	0.99198	0.99556	1.04084	1.02678	1.02248	1.03673	1.03404	1.03532	1.0293
7	0	-0.06896	-0.18949	-0.13996	-0.11945	-0.19685	-0.18117	-0.17887	-0.20383	-0.20737	-0.20464	-0.21177	-0.21425	-0.226	1.06	1.03927	1.00308	1.01197	1.01541	1.05788	1.04404	1.03584	1.05478	1.05198	1.05286	1.04661
8	0	-0.06914	-0.18925	-0.14074	-0.12022	-0.19878	-0.18329	-0.18255	-0.20569	-0.20924	-0.20653	-0.21366	-0.21614	-0.22783	1.0596	1.03955	1.00761	1.01335	1.01636	1.05942	1.04581	1.03805	1.05649	1.05366	1.05448	1.04817
9	0	-0.06929	-0.18917	-0.14095	-0.12039	-0.19903	-0.18384	-0.18353	-0.20614	-0.20965	-0.20687	-0.21389	-0.21639	-0.22817	1.05988	1.04042	1.00909	1.01439	1.01722	1.06	1.0475	1.04078	1.05787	1.05491	1.0554	1.04882
10	0	-0.06935	-0.18913	-0.14117	-0.12052	-0.19905	-0.18428	-0.18421	-0.20655	-0.20999	-0.20706	-0.21392	-0.21647	-0.2284	1.05997	1.04071	1.00963	1.01506	1.01768	1.05997	1.04965	1.04522	1.05936	1.05613	1.05601	1.0489
11	0	-0.06938	-0.18911	-0.1412	-0.12055	-0.19912	-0.18436	-0.18434	-0.20661	-0.21006	-0.20713	-0.21399	-0.21654	-0.22846	1.05999	1.0408	1.00954	1.01508	1.01771	1.06	1.04965	1.04521	1.05937	1.05615	1.05603	1.04893
12	0	-0.0694	-0.18911	-0.14121	-0.12056	-0.19913	-0.18438	-0.18438	-0.20663	-0.21008	-0.20715	-0.214	-0.21656	-0.22848	1.06	1.04087	1.00953	1.0151	1.01773	1.06	1.04964	1.04517	1.05937	1.05614	1.05603	1.04893
13	0	-0.0694	-0.18911	-0.14121	-0.12056	-0.19913	-0.18438	-0.18438	-0.20663	-0.21008	-0.20715	-0.214	-0.21656	-0.22848	1.06	1.04087	1.00954	1.0151	1.01773	1.06	1.04964	1.04516	1.05937	1.05614	1.05603	1.04893
14	0	-0.0694	-0.18911	-0.14121	-0.12056	-0.19913	-0.18438	-0.18438	-0.20663	-0.21008	-0.20715	-0.214	-0.21656	-0.22848	1.06	1.04087	1.00954	1.0151	1.01773	1.06	1.04964	1.04516	1.05937	1.05614	1.05603	1.04893
15	0	-0.05082	-0.12863	-0.1186	-0.09915	-0.15248	-0.13513	-0.09007	-0.17216	-0.17533	-0.16624	-0.17084	-0.1769	-0.20178	1.00194	0.98569	0.96511	0.96482	0.97022	1.01914	1.00084	1.00167	1.00813	1.00556	1.01038	1.00719
16	0	-0.0511	-0.13094	-0.12017	-0.09969	-0.15083	-0.13349	-0.08359	-0.17146	-0.17442	-0.16485	-0.16914	-0.17526	-0.20151	1.00625	0.99088	0.97225	0.96697	0.97196	1.02402	1.00428	1.00954	1.01074	1.00843	1.01413	1.01135
17	0	-0.05534	-0.13728	-0.12577	-0.10478	-0.15999	-0.14138	-0.09343	-0.17933	-0.18252	-0.17347	-0.17821	-0.18424	-0.20984	1.00816	0.99139	0.97277	0.96717	0.97219	1.02403	1.00469	1.01004	1.01115	1.00879	1.01433	1.01138
18	0	-0.06847	-0.16326	-0.14906	-0.12555	-0.20022	-0.17965	-0.14647	-0.21611	-0.21991	-0.21227	-0.21819	-0.22399	-0.24788	1.01413	0.99449	0.97304	0.96686	0.97214	1.0239	1.00596	1.01237	1.01222	1.0097	1.01478	1.01131
19	0	-0.07311	-0.17317	-0.15828	-0.13395	-0.22017	-0.19672	-0.17094	-0.23257	-0.23693	-0.23072	-0.23775	-0.24329	-0.26531	1.02369	1.00335	0.97892	0.97218	0.97758	1.02741	1.01064	1.01616	1.01713	1.01443	1.01897	1.01495
20	0	-0.07041	-0.16525	-0.15187	-0.12882	-0.21773	-0.19245	-0.17069	-0.22687	-0.23151	-0.2267	-0.2343	-0.23944	-0.25902	1.05694	1.03702	1.01059	1.00173	1.00706	1.04983	1.0349	1.03334	1.04268	1.03957	1.04286	1.03786
21	0	-0.07014	-0.16417	-0.15157	-0.12873	-0.2186	-0.19216	-0.17038	-0.22658	-0.23138	-0.22706	-0.23498	-0.24	-0.25889	1.06	1.04011	1.01408	1.00528	1.01046	1.05399	1.03941	1.03857	1.04721	1.04407	1.04722	1.04208
22	0	-0.07036	-0.16349	-0.15275	-0.13011	-0.22244	-0.19308	-0.17055	-0.22787	-0.23304	-0.22977	-0.23843	-0.24325	-0.26067	1.05995	1.04078	1.01602	1.00842	1.013	1.06	1.0482	1.05357	1.05509	1.05169	1.05409	1.04828
23	0	-0.07048	-0.16318	-0.15312	-0.13055	-0.22435	-0.19323	-0.16977	-0.2285	-0.23391	-0.23117	-0.24023	-0.24498	-0.26172	1.05998	1.04118	1.01684	1.00939	1.01357	1.05994	1.05141	1.06	1.05748	1.05367	1.0551	1.04839
24	0	-0.07052	-0.16302	-0.1532	-0.13076	-0.22573	-0.19302	-0.16872	-0.22867	-0.23428	-0.23204	-0.24152	-0.24619	-0.26232	1.06	1.04137	1.01729	1.00955	1.01365	1.06	1.05153	1.05995	1.05771	1.05388	1.05525	1.04845
25	0	-0.07053	-0.16295	-0.15324	-0.13084	-0.2262	-0.19299	-0.16845	-0.22875	-0.23443	-0.23234	-0.24196	-0.2466	-0.26255	1.06	1.04138	1.01734	1.00956	1.01364	1.06	1.05158	1.06	1.05778	1.05394	1.05528	1.04845
26	0	-0.07053	-0.16293	-0.15324	-0.13085	-0.2263	-0.19298	-0.16839	-0.22877	-0.23447	-0.23241	-0.24206	-0.24669	-0.2626	1.06	1.04139	1.01735	1.00957	1.01364	1.06	1.05158	1.06	1.05779	1.05395	1.05529	1.04845
27	0	-0.07053	-0.16293	-0.15325	-0.13085	-0.22631	-0.19298	-0.16839	-0.22877	-0.23447	-0.23241	-0.24206	-0.2467	-0.2626	1.06	1.04139	1.01735	1.00957	1.01364	1.06	1.05158	1.06	1.05779	1.05395	1.05529	1.04845

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
28	0	-0.07053	-0.16293	-0.15325	-0.13085	-0.22631	-0.19298	-0.16839	-0.22877	-0.23447	-0.23241	-0.24206	-0.2467	-0.2626	1.06	1.04139	1.01735	1.00957	1.01364	1.06	1.05158	1.06	1.05779	1.05395	1.05529	1.04845
29	0	-0.05366	-0.16035	-0.12132	-0.10001	-0.12815	-0.1189	-0.06845	-0.14911	-0.15099	-0.14372	-0.1443	-0.14949	-0.17683	0.99971	0.98194	0.94616	0.96223	0.96834	1.02259	1.00214	1.00363	1.01099	1.00917	1.01261	1.01208
30	0	-0.05383	-0.16448	-0.12255	-0.10022	-0.12545	-0.11631	-0.06197	-0.14692	-0.14859	-0.14099	-0.14128	-0.14641	-0.1746	1.00449	0.98818	0.95329	0.96529	0.97111	1.02851	1.00647	1.01174	1.01484	1.01332	1.01753	1.01751
31	0	-0.05728	-0.16855	-0.1267	-0.10408	-0.13278	-0.12233	-0.06941	-0.15302	-0.15491	-0.14782	-0.14853	-0.15358	-0.18115	1.0067	0.98906	0.95474	0.9659	0.9717	1.02831	1.00683	1.01174	1.01524	1.01363	1.01761	1.01734
32	0	-0.07043	-0.19139	-0.15011	-0.12519	-0.17502	-0.16251	-0.12611	-0.19156	-0.1941	-0.18852	-0.19051	-0.19532	-0.22105	1.01236	0.99186	0.95583	0.96553	0.97151	1.02794	1.0082	1.01393	1.01638	1.01455	1.01796	1.01706
33	0	-0.07426	-0.19571	-0.15855	-0.13324	-0.19679	-0.18085	-0.15341	-0.20931	-0.21254	-0.20859	-0.21192	-0.21645	-0.24011	1.01982	0.99882	0.96223	0.9694	0.97529	1.02986	1.01153	1.01643	1.0199	1.01785	1.02064	1.01908
34	0	-0.07217	-0.18563	-0.15487	-0.1307	-0.20152	-0.18349	-0.16272	-0.211	-0.21463	-0.21191	-0.21605	-0.22031	-0.24193	1.0428	1.02236	0.9884	0.9894	0.99489	1.04236	1.02624	1.02489	1.03548	1.03298	1.03457	1.03193
35	0	-0.07037	-0.17971	-0.15112	-0.12773	-0.20017	-0.1812	-0.16267	-0.20809	-0.21187	-0.20975	-0.21423	-0.21831	-0.23869	1.06	1.03976	1.00687	1.00547	1.01077	1.05519	1.03983	1.03503	1.04973	1.04705	1.04809	1.04498
36	0	-0.07052	-0.17926	-0.15206	-0.12886	-0.20393	-0.18255	-0.16386	-0.20976	-0.21387	-0.2126	-0.21774	-0.22166	-0.24089	1.05972	1.04012	1.01122	1.00779	1.01249	1.05886	1.04464	1.04239	1.05427	1.05147	1.05218	1.04873
37	0	-0.07064	-0.17904	-0.15245	-0.12925	-0.20501	-0.18291	-0.1639	-0.21033	-0.21452	-0.21346	-0.21872	-0.22263	-0.24151	1.05991	1.04072	1.01216	1.00928	1.0136	1.06	1.04898	1.05059	1.05768	1.0545	1.0543	1.05004
38	0	-0.07072	-0.17885	-0.15275	-0.12955	-0.20609	-0.18311	-0.16358	-0.21083	-0.21512	-0.21432	-0.21973	-0.22362	-0.24215	1.05998	1.04108	1.01275	1.01022	1.01419	1.05998	1.05217	1.0571	1.06	1.05643	1.05529	1.05019
39	0	-0.07076	-0.17872	-0.15283	-0.12967	-0.20667	-0.18326	-0.16366	-0.21106	-0.21541	-0.21475	-0.22028	-0.22415	-0.24251	1.06	1.04118	1.01272	1.0102	1.01418	1.05999	1.0521	1.05689	1.05998	1.05642	1.05529	1.0502
40	0	-0.07076	-0.17871	-0.15284	-0.12968	-0.20675	-0.18325	-0.1636	-0.21107	-0.21544	-0.2148	-0.22036	-0.22422	-0.24255	1.06	1.04121	1.01276	1.01022	1.01419	1.06	1.05211	1.05688	1.06	1.05643	1.0553	1.05021
41	0	-0.07076	-0.17871	-0.15284	-0.12969	-0.20675	-0.18325	-0.1636	-0.21107	-0.21544	-0.2148	-0.22036	-0.22423	-0.24255	1.06	1.04121	1.01276	1.01022	1.0142	1.06	1.05211	1.05688	1.06	1.05644	1.05531	1.05021
42	0	-0.07076	-0.17871	-0.15284	-0.12969	-0.20675	-0.18325	-0.1636	-0.21107	-0.21544	-0.2148	-0.22036	-0.22423	-0.24255	1.06	1.04121	1.01276	1.01022	1.0142	1.06	1.05211	1.05688	1.06	1.05644	1.05531	1.05021
43	0	-0.05291	-0.15742	-0.11453	-0.09664	-0.14096	-0.12072	-0.07321	-0.15374	-0.15979	-0.15476	-0.15929	-0.16063	-0.18151	0.99937	0.98186	0.94697	0.96547	0.96954	1.01947	1.00406	1.0049	1.0126	1.0083	1.0105	1.0061
44	0	-0.05316	-0.16212	-0.11558	-0.09691	-0.13848	-0.11844	-0.06712	-0.15184	-0.15768	-0.15233	-0.15661	-0.15795	-0.17958	1.00408	0.98765	0.95321	0.96846	0.97221	1.02545	1.00833	1.01297	1.01633	1.01233	1.0154	1.01153
45	0	-0.05683	-0.16672	-0.12026	-0.10123	-0.1467	-0.1254	-0.07595	-0.15879	-0.16485	-0.16002	-0.16473	-0.166	-0.187	1.00623	0.98848	0.95463	0.96898	0.97271	1.02531	1.00869	1.01305	1.01673	1.01266	1.01552	1.01142
46	0	-0.07004	-0.19039	-0.14408	-0.12259	-0.18898	-0.16667	-0.13483	-0.1981	-0.20468	-0.20106	-0.20681	-0.20787	-0.22739	1.01214	0.99153	0.95571	0.96858	0.97255	1.02495	1.00991	1.01517	1.01767	1.01341	1.01576	1.01112
47	0	-0.07411	-0.19584	-0.15308	-0.13101	-0.21082	-0.18595	-0.16386	-0.21652	-0.22366	-0.22144	-0.22832	-0.22915	-0.24691	1.01991	0.99877	0.9621	0.97246	0.97641	1.027	1.01303	1.01739	1.02101	1.01657	1.01843	1.01327
48	0	-0.07212	-0.18593	-0.14994	-0.12877	-0.21526	-0.18954	-0.17516	-0.21877	-0.22608	-0.22476	-0.23214	-0.23285	-0.24907	1.04302	1.02245	0.98849	0.99235	0.996	1.03965	1.02748	1.02549	1.03636	1.03155	1.03235	1.02627
49	0	-0.07036	-0.17981	-0.14663	-0.12609	-0.21378	-0.18818	-0.1771	-0.21643	-0.22369	-0.22272	-0.23016	-0.23081	-0.24621	1.06	1.03962	1.00685	1.00818	1.0117	1.0528	1.04101	1.03577	1.05055	1.04564	1.04603	1.03967
50	0	-0.07058	-0.17884	-0.14808	-0.12767	-0.21822	-0.1916	-0.18265	-0.21955	-0.22697	-0.22653	-0.23437	-0.23491	-0.24955	1.05972	1.03996	1.01136	1.01038	1.01344	1.05771	1.04541	1.04198	1.05498	1.05019	1.05078	1.04459
51	0	-0.07069	-0.17846	-0.14859	-0.12821	-0.21954	-0.19254	-0.184	-0.22043	-0.22789	-0.22763	-0.23558	-0.2361	-0.25044	1.0599	1.04051	1.01233	1.01167	1.01454	1.06	1.04862	1.04734	1.05785	1.05297	1.05334	1.04695
52	0	-0.07078	-0.17814	-0.14903	-0.12861	-0.2207	-0.19335	-0.18492	-0.22136	-0.22886	-0.22871	-0.2367	-0.23723	-0.25141	1.05998	1.04081	1.01286	1.01248	1.01504	1.05994	1.05165	1.05366	1.06	1.05475	1.05423	1.04705
53	0	-0.07083	-0.17798	-0.14913	-0.12876	-0.22141	-0.19356	-0.18507	-0.22166	-0.22923	-0.22925	-0.23738	-0.23788	-0.25185	1.06	1.04095	1.01291	1.01248	1.01504	1.05999	1.05156	1.05337	1.05998	1.05475	1.05425	1.04709
54	0	-0.07084	-0.17797	-0.14914	-0.12877	-0.22149	-0.19356	-0.18503	-0.22167	-0.22925	-0.2293	-0.23746	-0.23795	-0.25189	1.06	1.04098	1.01294	1.0125	1.01505	1.06	1.05157	1.05335	1.06	1.05477	1.05427	1.0471