Bangladesh University of Engineering and Technology

Electrical and Electronic Engineering Department

EEE 318 : Control System I Laboratory

Experiment No. 3

a) Equivalency of block diagram
b) System stability and effect of pole location

Part-A: Equivalency of block diagram

Objective: To verify the equivalency of the basic forms, including cascade, parallel and feedback forms. To verify the equivalency of the basic moves, including moving blocks past summing junctions, and moving blocks past pickoff points.

Minimum required software packages: MATLAB, Simulink, and the Control System Toolbox.

Theory:
A block diagram is a diagram of a system in which the principal parts or functions are represented by blocks connected by lines that show the relationships of the blocks. Components of a block diagram for a time-invariant system and different equivalent forms are shown below.

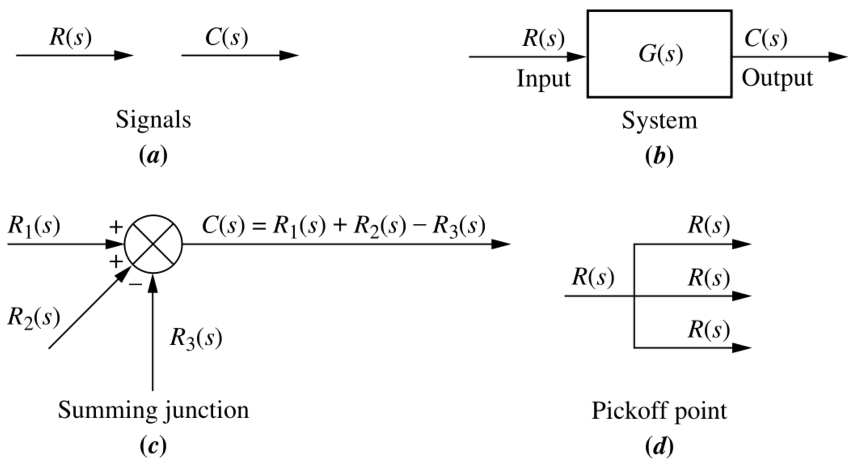


Figure 1: Components of block diagram

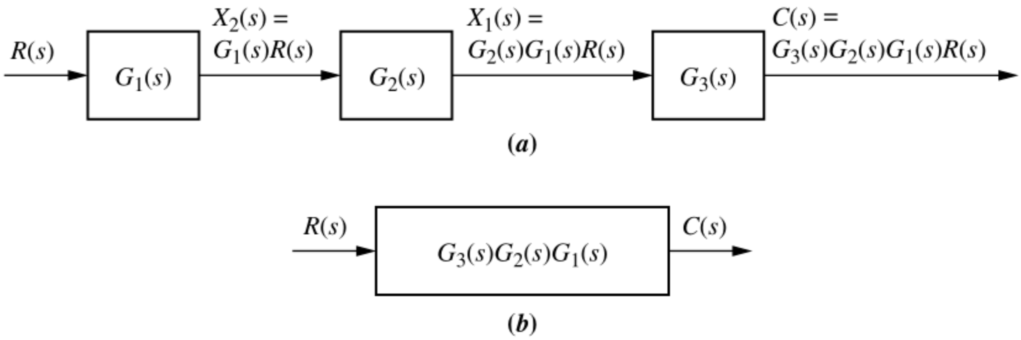


Figure 2: a) Cascaded subsystems; b) equivalent transfer function

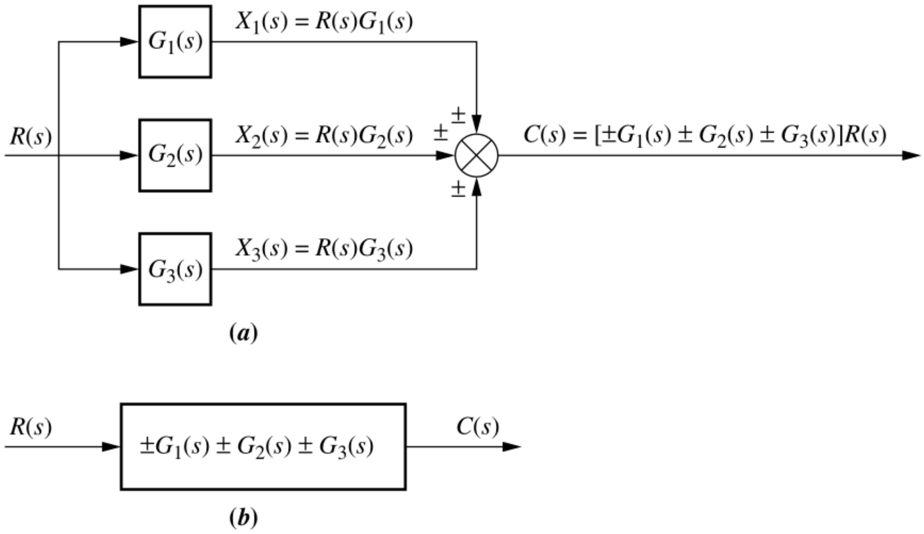


Figure 3: a) Parallel subsystems; b) equivalent transfer function

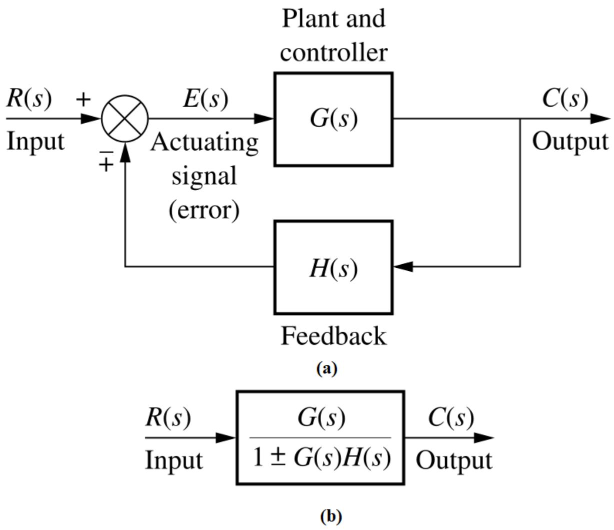


Figure 4: a) Feedback control system; b) equivalent transfer function

Moving Blocks to Create Familiar Forms

To create familiar form, sometimes it's necessary to move a block. There are some standard techniques to perform these operations. Several common techniques are shown below. In the diagrams, the symbol ≡ means ''equivalent to.''

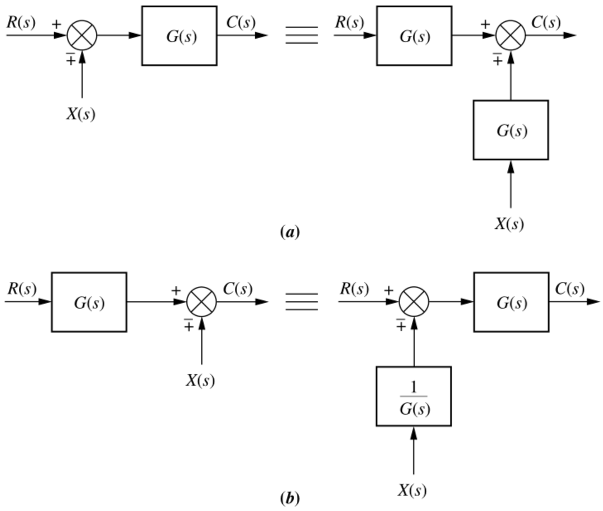


Figure 5: Block diagram algebra for summing junctions — equivalent forms for moving a block a) to the left past a summing junction; b) to the right past a summing junction

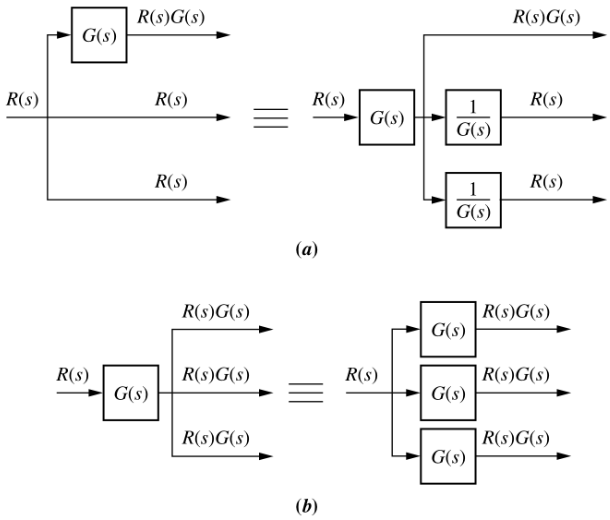
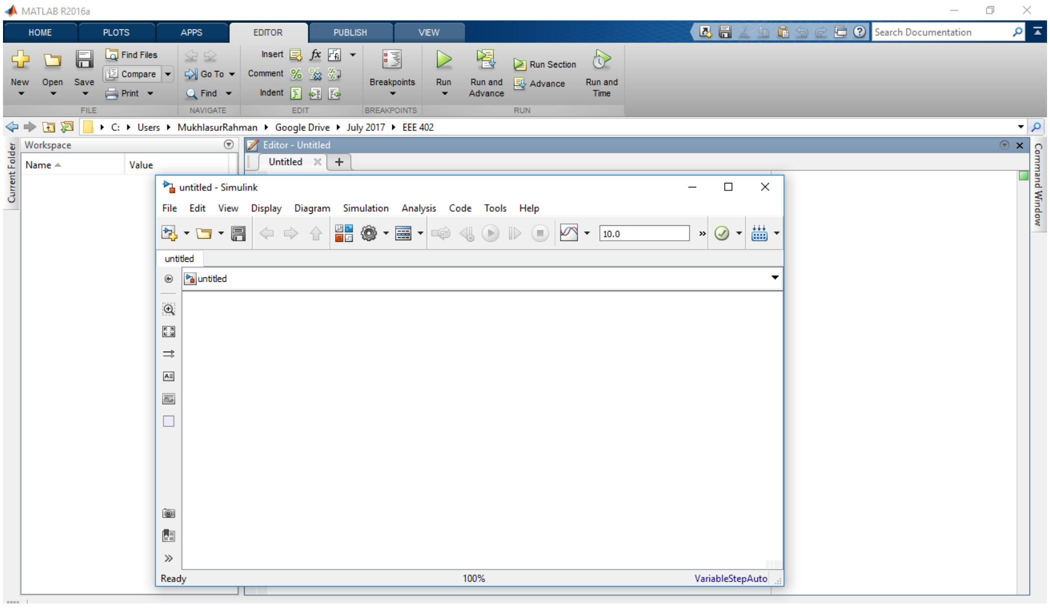


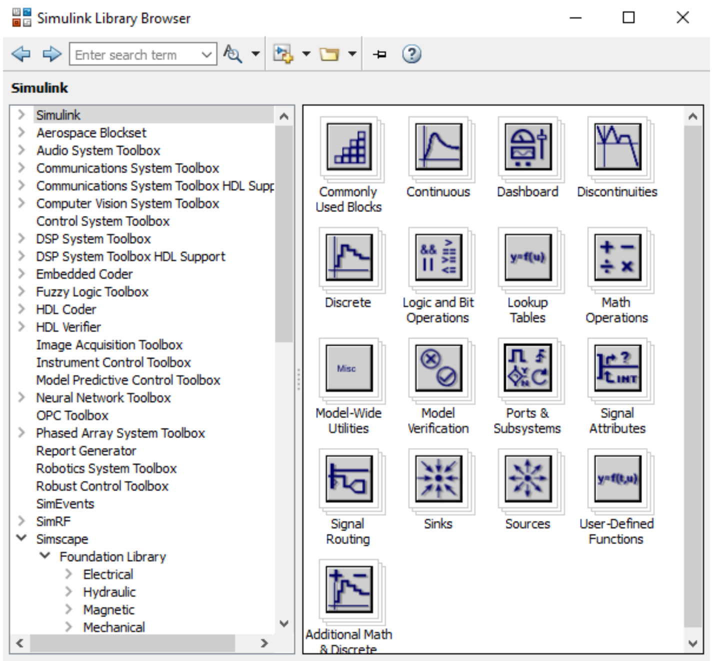
Figure 6: Block diagram algebra for pickoff points — equivalent forms for moving a block a) to the left past a pickoff point; b) to the right past a pickoff point

Block Diagram in Simulink:

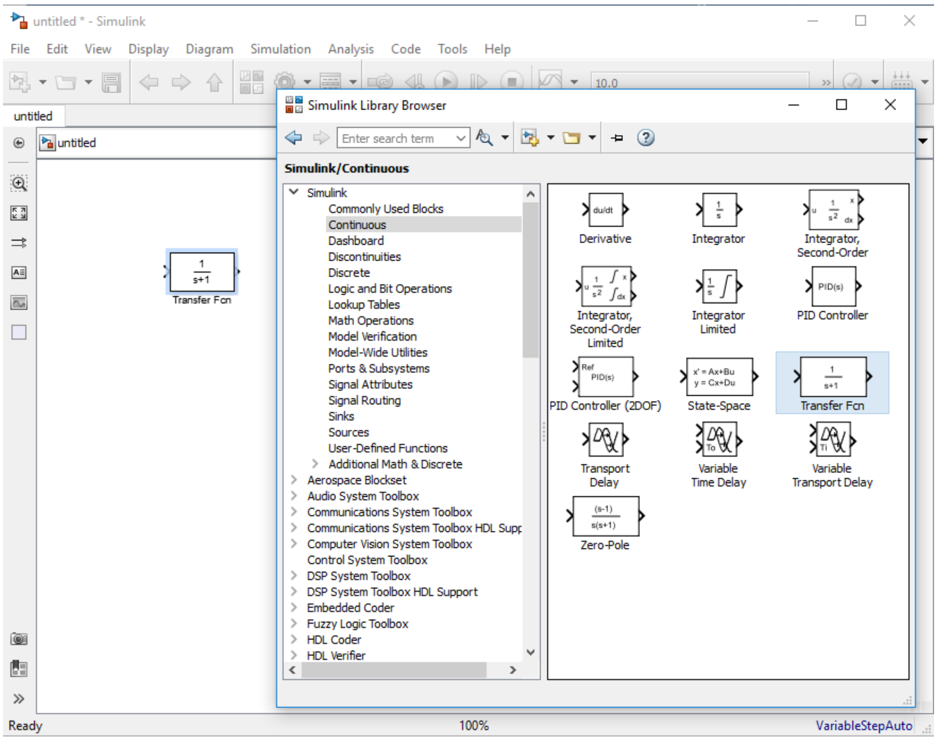
- Open MATLAB. (This example is based on MATLAB R2016a)
- Select New → Simulink model. A new window should appear and select ‘Blank Model’ from that window. It will create a window named ‘untitled’ as shown in the following figure.



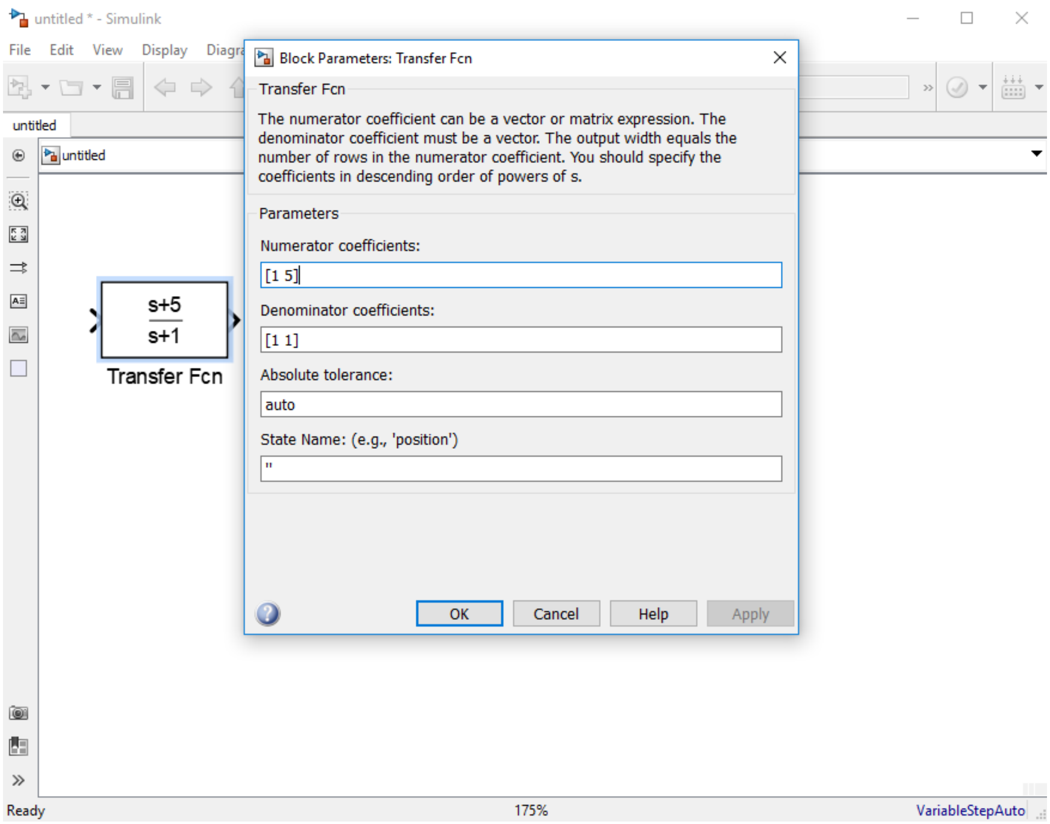
- Select View → Library Browser. Another window called 'Simulink Library Browser' should appear on screen as shown in the following figure.



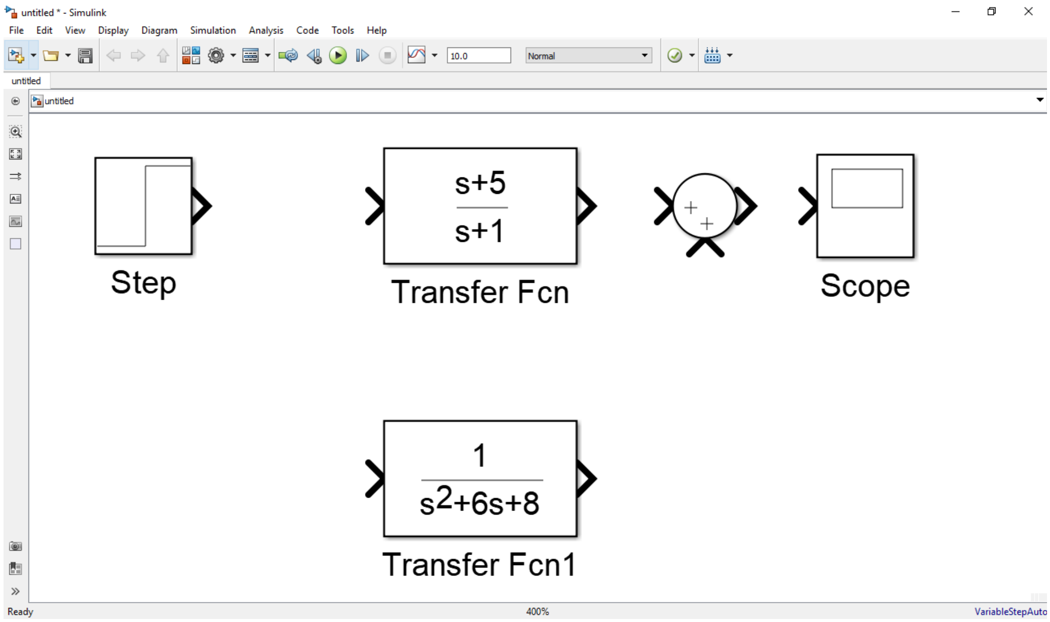
- Now you can drag and drop your desired blocks to the 'untitled' window and connect them as required. A simple transfer function block (Simulink → Continuous → Transfer Fcn) is dragged and dropped. Following figure shows how it appears on screen.



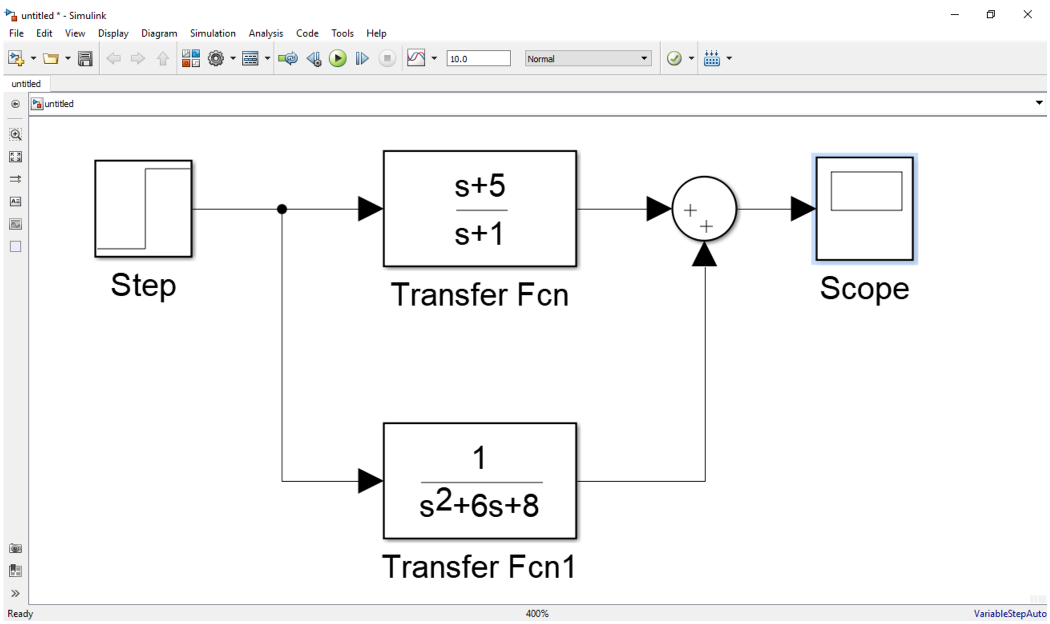
- Double click on the ‘Transfer Fcn’ block. It’ll enable you to change the transfer function by entering the numerator and denominator coefficients.



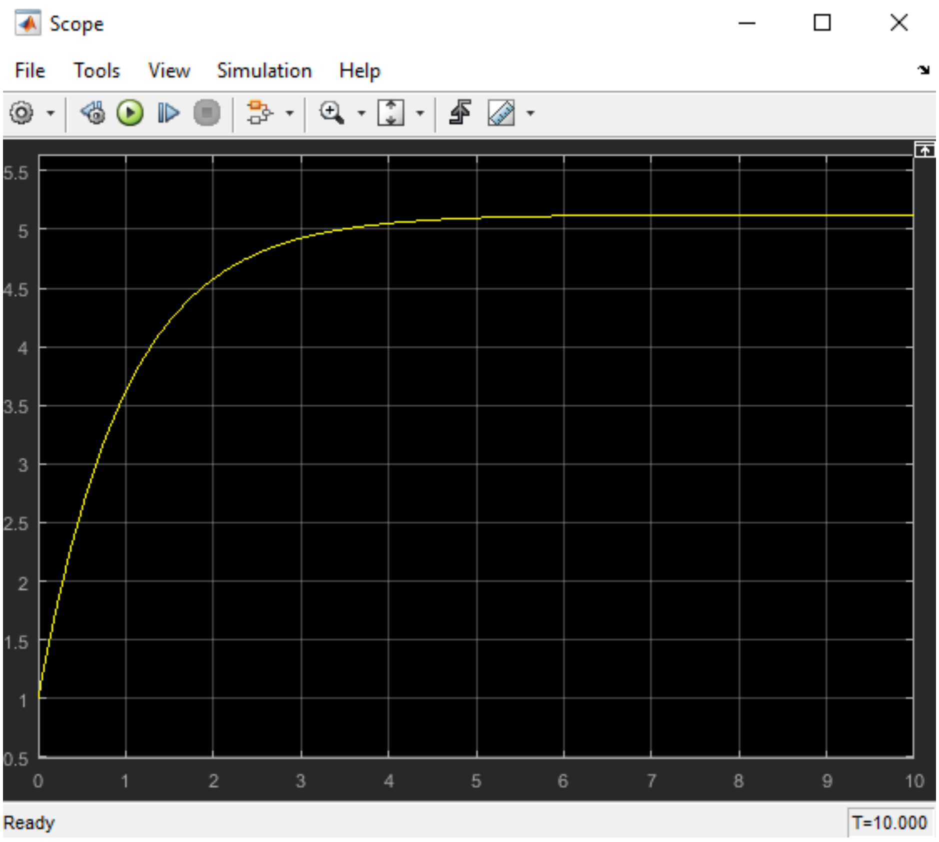
- Similarly add another transfer function, step input (Simulink → Sources → Step), summing junction (Simulink → Math Operations → Sum), and output monitor (Simulink → Sinks → Scope). Double click on the step input and set ‘step time’ to 0; it ensures that the step transition occurs at $t = 0$.



- Now the components are to be connected. To do this, place the mouse on any port of the symbol, it should change the mouse cursor from arrow ( ) to cross (+). Click and hold the mouse and place it to a port of another symbol to be wired. Now the ports are supposed to be connected with a line. Scroll to zoom-in and zoom-out, if necessary.



- Select Simulation → Run. Ignore warning, if shows.
- Double click on the Scope and a figure window should appear as shown in the following figure.



Congratulations! You have successfully completed first Simulink example.

Prelab:

1. Visit https://www.mathworks.com/videos/getting-started-with-simulink-118723.html to watch a short video and get yourself introduced with the MATLAB Simulink environment.
2. Read the theory discussed above and perform 'Block Diagram in Simulink' on your computer as discussed in the previous section.
3. Find the equivalent transfer function of three cascaded blocks, $G_1(s) = \frac{1}{s+1}$, $G_2(s) = \frac{1}{s+4}$, and $G_3(s) = \frac{s+3}{s+5}$.
4. Find the equivalent transfer function of three parallel blocks, $G_1(s) = \frac{1}{s+1}$, $G_2(s) = \frac{1}{s+4}$, and $G_3(s) = \frac{s+3}{s+5}$.
5. Find the equivalent transfer function of the negative feedback system of Fig. 7 if $G(s) = \frac{s+1}{s(s+2)}$ and $H(s) = \frac{s+3}{s+4}$.

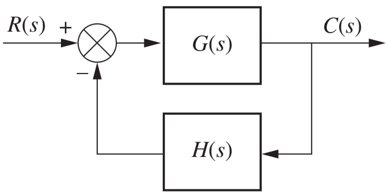


Figure 7: Negative feedback system

6. For the system of Prelab 5, push $H(s)$ to the left past the summing junction and draw the equivalent system.
7. For the system of Prelab 5, push $H(s)$ to the right past the pickoff point and draw the equivalent system.

Lab Work:

1. Using Simulink, set up the cascade system of Prelab 3 and the equivalent single block. Make separate plots of the step response of the cascaded system and its equivalent single block. Record the values of settling time and rise time for each step response.
2. Using Simulink, set up the parallel system of Prelab 4 and the equivalent single block. Make separate plots of the step response of the parallel system and its equivalent single block. Record the values of settling time and rise time for each step response.
3. Using Simulink, set up the negative feedback system of Prelab 5 and the equivalent single block. Make separate plots of the step response of the negative feedback system and its equivalent single block. Record the values of settling time and rise time for each step response.
4. Using Simulink, set up the negative feedback systems of Prelabs 5, 6, and 7. Make separate plots of the step responses of each of the systems. Record the values of settling time and rise time for each step response.

Report:

1. Using your lab data, verify the equivalent transfer function of blocks in cascade.
2. Using your lab data, verify the equivalent transfer function of blocks in parallel.
3. Using your lab data, verify the equivalent transfer function of negative feedback systems.
4. Using your lab data, verify the moving of blocks past summing junctions and pickoff points.
5. Discuss your results. Were the equivalencies verified?

Part B: System stability and effect of pole location

Objective: To verify the effect of pole location upon stability.

Minimum required software packages: MATLAB, Simulink, and the Control System Toolbox

Theory:
If the closed-loop system poles are in the left half of the plane and hence have a negative real part, the system is stable. To be more precise,

- *Stable systems have closed-loop transfer functions with poles only in the left half-plane.*
- *Unstable systems have closed-loop transfer functions with at least one pole in the right half-plane and/or poles of multiplicity greater than 1 on the imaginary axis.*
- *Marginally stable systems have closed-loop transfer functions with only imaginary axis poles of multiplicity 1 and poles in the left half-plane.*[1]

Some examples are shown below for better understanding.

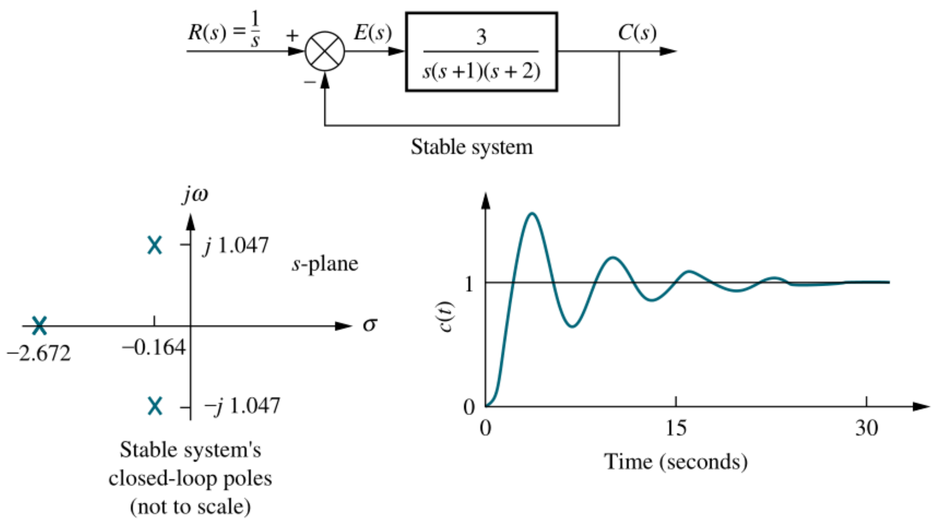


Figure 1: Closed-loop poles and response of a stable system

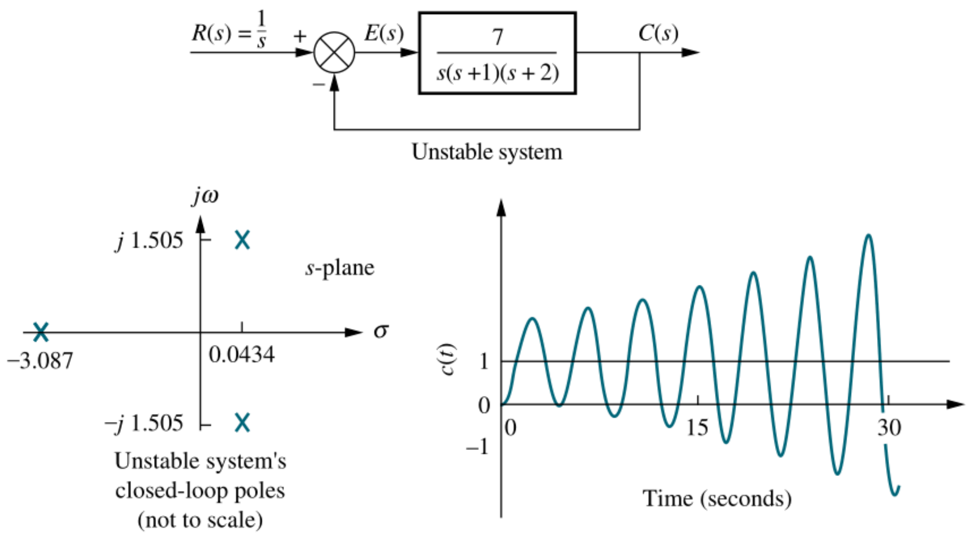
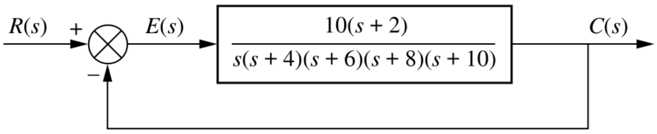


Figure 2: Closed-loop poles and response of an unstable system

[1] Control Systems Engineering by Norman S. Nise, 6th edition, Chapter 6, Page 303

It is not always a simple matter to determine if a feedback control system is stable. Unfortunately, a typical problem that arises is shown in Figure 3. Although we know the poles of the forward transfer function in Figure 3(a), we do not know the location of the poles of the equivalent closed-loop system of Figure 3(b) without factoring or otherwise solving for the roots.



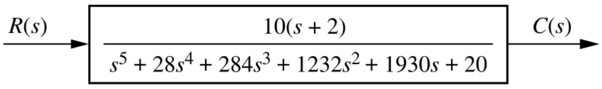


Figure 3: Common cause of problems in finding closed-loop poles: (a) original system; (b) equivalent system

However, under certain conditions, we can draw some conclusions about the stability of the system. First, if the closed-loop transfer function has only left–half-plane poles, then the factors of the denominator of the closed-loop system transfer function consist of products of terms such as $(s + a_i)$, where a_i is real and positive, or complex with a positive real part.[2] The product of such terms is a polynomial with all positive coefficients. No term of the polynomial can be missing, since that would imply cancellation between positive and negative coefficients or imaginary axis roots in the factors, which is not the case. Thus, a *sufficient* condition for a system to be unstable is that all signs of the coefficients of the denominator of the closed-loop transfer function *are not the same*. If powers of s are missing, the system is either unstable or, at best, marginally stable. *Unfortunately, if all coefficients of the denominator are positive and not missing, we do not have definitive information about the system's pole locations.*

Prelab:

1. Study the theory section discussed above.
2. Find the equivalent transfer function of the negative feedback system of Fig. 4 if $G(s) = \frac{K}{s(s+2)^2}$ and $H(s) = 1$.

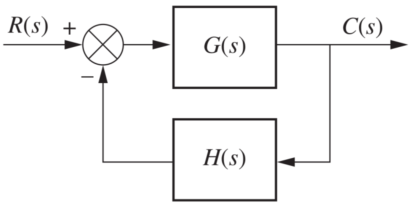


Figure 4: Negative feedback system

3. For the system of Prelab 2, find two values of gain, K, that will yield closed-loop overdamped second-order poles. Repeat for underdamped poles.
4. For the system of Prelab 2, find the value of gain, K, that will make the system critically damped.
5. For the system of Prelab 2, find the value of gain, K, that will make the system marginally stable. Also, find the frequency of oscillation at that value of K that makes the system marginally stable.
6. For each of Prelab 3 through 5, plot on one graph the pole locations for each case and write the corresponding value of gain, K, at each pole.

[2] The coefficients can also be made all negative by multiplying the polynomial by -1. This operation does not change the root location.

Lab Work:

1. Using Simulink, set up the negative feedback system of Prelab 2. Plot the step response of the system at each value of gain calculated to yield overdamped, underdamped, critically damped, and marginally stable responses.
2. Plot the step responses for two values of gain, K, above that calculated to yield marginal stability.
3. At the output of the negative feedback system cascade the transfer function, $G_1(s) = \frac{1}{s^2+4}$. Set the gain, K, at a value below that calculated for marginal stability and plot the step response. Repeat for K calculated to yield marginal stability.

Report:

1. From your plots, discuss the conditions that lead to unstable responses.
2. Discuss the effect of gain upon the nature of the step response of a closed-loop system.

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