Bangladesh University of Engineering and Technology

Electrical and Electronic Engineering Department

EEE 318 : Control System I Laboratory

Experiment No. 6

Part A: Sketching Bode Plot with MATLAB's sisotool

Objective: To sketch Bode plot of a system and find out important parameters from the plot using MATLAB

Minimum required software packages: MATLAB, Simulink, and the Control System Toolbox.

Theory:

Frequency response is a representation of the system's response to sinusoidal inputs at varying frequencies. The output of a linear system to a sinusoidal input is a sinusoid of the same frequency but with a different magnitude and phase. The frequency response is defined as the magnitude and phase differences between the input and output sinusoids.

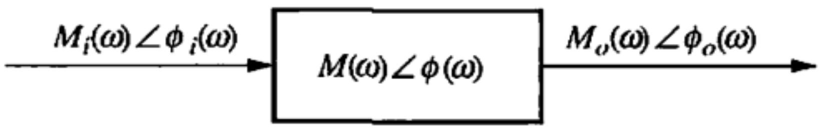


Figure 1: A linear time invariant system

The steady state output sinusoidal is,

$$M_0(\omega)\angle\Phi_0(\omega) = M_i(\omega)M(\omega)\angle|\Phi_i(\omega) + \Phi(\omega)|$$

Magnitude frequency response is,

$$M(\omega) = \frac{M_0(\omega)}{M_i(\omega)}$$

And phase frequency response is,

$$\Phi(\omega) = \Phi_0(\omega) - \Phi_i(\omega)$$

The combination of magnitude and phase frequency response is called frequency response

And is given by $M(\omega)\angle\Phi(\omega)$.

Bode Plot

The log-magnitude and phase frequency response curves as functions of $\log \omega$ are called Bode plots. The magnitude is plotted in decibels (dB), and the phase in degrees. The decibel calculation for magnitude is computed as $20\log_{10}(|M(j\omega)|)$, where $M(j\omega)$ is the system's frequency response. Bode plots are used to analyze system properties such as the gain margin, phase margin, DC gain, bandwidth, disturbance rejection, and stability.

Bode plot of some common terms:

1. Constant gain: For $H(j\omega) = K$, where K is a constant:

$$M(\omega) = 20\log_{10}|K|$$

$$\Phi(\omega) = \begin{cases} 0^0 \; if \; K \; is \; positive \\ 180^0 \; if \; K \; is \; negative \end{cases}$$

2. Poles and zeros in the origin:

$i.$ *Zero:* For $H(j\omega) = s = j\omega$,

$$M(\omega) = 20\log_{10}\omega$$

$$\Phi(\omega) = 90^0$$

If we increase the frequency ten times, magnitude increases by 20 times. So, the slope of the magnitude curve is 20dB/decade.

$ii.$ *Pole:* For $H(j\omega) = \frac{1}{s} = \frac{1}{j\omega}$,

$$M(\omega) = 20\log_{10}\left(\frac{1}{\omega}\right) = -20\log_{10}\omega$$

$$\Phi(\omega) = -90^0$$

The slope of the magnitude curve of this transfer function is -20dB/decade.

3. Real poles and zeros:

$i.$ *Zero of first order:* For $H(j\omega) = 1 + \frac{s}{\omega_0} = 1 + j\omega/\omega_0$,

$$M(\omega) = \begin{cases} 20\log_{10}1 = 0 \; for \; \omega \ll \omega_0 \\ 20\log_{10}\omega = 20dB/decade \;\; for \; \omega \gg \omega_0 \end{cases}$$

$$\Phi(\omega) = \begin{cases} 0 \; for \; \omega \ll \omega_0 \\ 45^0/\; decade \; around \;\; \omega_0 \\ 90^0 for \; \omega \gg \omega_0 \end{cases}$$

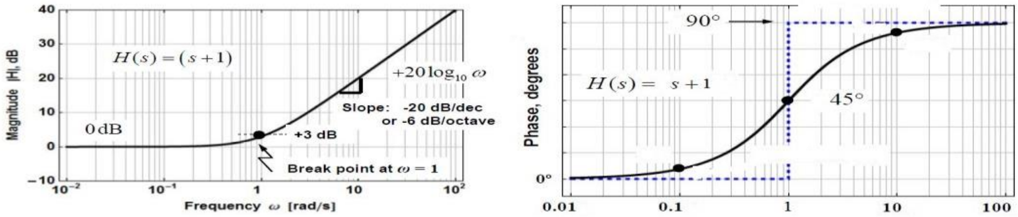


Figure 2: Bode plot of $H(s) = s + 1$

ii. *Pole of first order:* For $H(j\omega) = \frac{1}{1+\frac{s}{\omega_0}} = \frac{1}{1+\frac{j\omega}{\omega_0}}$,

$$M(\omega) = \begin{cases} 20\log_{10} 1 = 0 \; for \; \omega \ll \omega_0 \\ 20\log_{10}\frac{1}{\omega} = -20dB/decade \;\; for \; \omega \gg \omega_0 \end{cases}$$

$$\Phi(\omega) = \begin{cases} 0 \; for \; \omega \ll \omega_0 \\ -45^0/\; decade \; around \;\; \omega_0 \\ -90^0 for \; \omega \gg \omega_0 \end{cases}$$

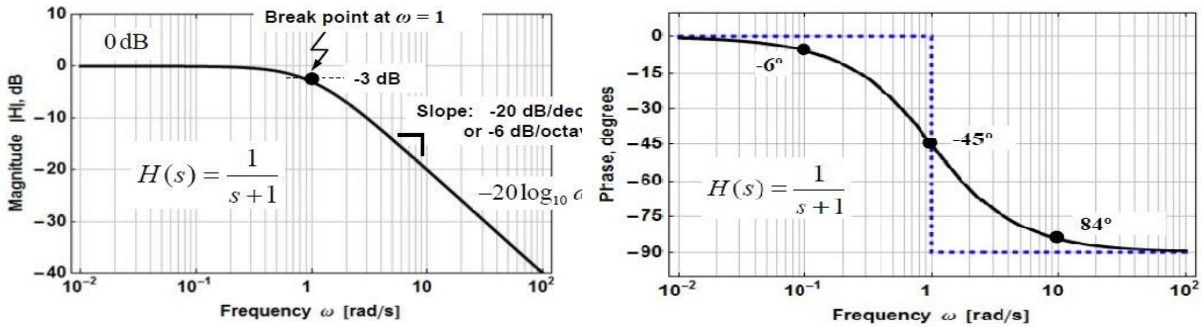


Figure 3: Bode plot of $H(s) = \frac{1}{s+1}$

*** *Figure 2 and 3 shows that the bode plot of a zero is reflected with respect to frequency axis of the bode plot of a pole of the same type.*

*** *A zero or pole of any other form can be converted to this standard form e.g.* $s + 20 = 20\left(1 + \frac{s}{20}\right)$. *This is called normalization.*

iii. *Zeros and poles of higher order:*

If $H(j\omega) = \left(1 + \frac{s}{\omega_0}\right)^n = \left(1 + \frac{j\omega}{\omega_0}\right)^n$,

$$M(\omega) = \begin{cases} 20\log_{10} 1 = 0 \; for \; \omega \ll \omega_0 \\ n * 20\log_{10}\omega = n * 20dB/decade \;\; for \; \omega \gg \omega_0 \end{cases}$$

$$\Phi(\omega) = \begin{cases} 0 \; for \; \omega \ll \omega_0 \\ n * 45^0/\; decade \; around \;\; \omega_0 \\ n * 90^0 for \; \omega \gg \omega_0 \end{cases}$$

Similarly, the bode plot of poles of n^{th} order can be found.

4. <u>Complex poles and zeros:</u>

i. *Complex poles*:

For

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$M(\omega) = \begin{cases} 0 \; for \; \omega \ll \omega_n \\ -40dB/decade \;\; for \; \omega \gg \omega_n \end{cases}$$

$$\Phi(\omega) = \begin{cases} 0 \; for \; \omega \ll \omega_n \\ -90^0 \; at \; \omega = \; \omega_n \\ -180^0 for \; \omega \gg \omega_n \end{cases}$$

Near the corner frequency ω_n, the shape of both the magnitude and phase plot depend on the value of ξ.

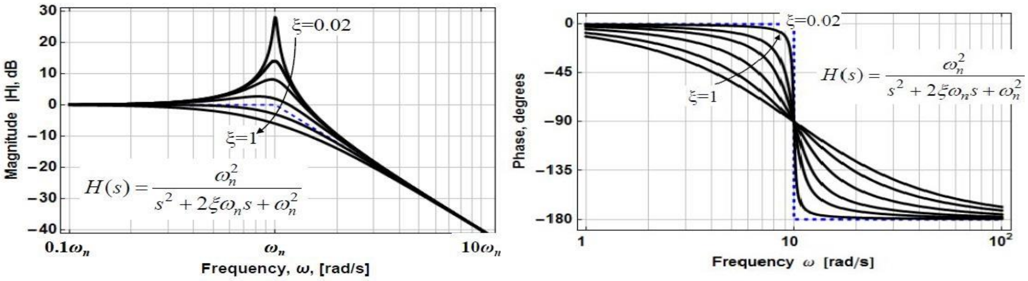


Figure 4: Bode plot of complex poles

**** Bode plot of complex zeros is just reflection of bode plot of complex poles w.r.t. frequency axis.*

Bode plot of combination of any terms:

Bode plot of any type of transfer function can be sketched easily from the aforementioned common terms. Let's say we have a frequency response defined as a fraction with numerator and denominator polynomials defined as,

$$M(j\omega) = \frac{\prod_n (j\omega + Z_n)}{\prod_m (j\omega + P_m)}$$

If we convert both sides to decibels, the logarithms from the decibel calculations convert multiplication of the arguments into additions, and the divisions into subtractions then,

$$Gain = \sum_n 20log\,|j\omega + Z_n| - \sum_m 20log\,|j\omega + P_m|$$

The phase frequency response is the sum of phase frequency response curves of the zero terms minus the sum of the phase frequency response curves of the pole terms.

$$phase = \sum_n \Phi(j\omega + Z_n) - \sum_m \Phi(j\omega + P_m)$$

Gain Margin, Phase Margin and Bandwidth Frequency

Gain margin G_M: The gain margin is the change in open-loop gain, expressed in decibels (dB), required at 180° of phase shift to make the closed-loop system unstable.

Phase margin Φ_M: The phase margin is the change in open-loop phase shift required at unity gain to make the closed-loop system unstable.

Bandwidth frequency ω_{BW}: It is the frequency at which the magnitude response curve is 3 dB down from its value at zero frequency.

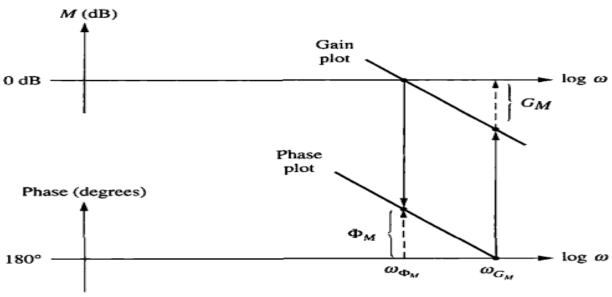


Figure 5: Gain and phase margin calculations from the Bode plots

Figure 5 shows how to evaluate the gain and phase margins by using Bode plots. The gain margin is found by using the phase plot to find the frequency ω_{GM}, where the phase angle is 180°. At this frequency, we look at the magnitude plot to determine the gain margin, G_M, which is the gain required to raise the magnitude curve to 0 dB.
The phase margin is found by using the magnitude curve to find the frequency $\omega_{\Phi M}$, where the gain is 0 dB. On the phase curve at that frequency, the phase margin, Φ_M, is the difference between the phase value and 180°.
There is a relationship between phase margin and percent overshoot,
Phase margin,

$$\Phi_M = tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}} \text{..........................(1)}$$

Percent Overshoot,

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100\% \text{ ....(2)}$$

And there is a relationship between bandwidth and settling time or peak time,
Band width,

$$\omega_{BW} = \frac{4}{T_s\zeta}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}} \text{ ....(3)}$$

Or,

$$\omega_{BW} = \frac{\pi}{T_p\sqrt{1-\zeta^2}}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}} \text{...............(4)}$$

Sketching bode plot using MATLAB:

i. Using code:

```
s=tf('s');
sys=100/(s^3+12*s^2+20*s);  %system transfer function
bode(sys);       %drawing the bode
grid on;
[gm,pm,wgc,wpc]=margin(sys); %check the matlab margin function
gm_db=20*log10(gm);
```

One can also find the parameters directly from the plot by right clicking on the plot and then *Characteristics > All Stability Margins*

ii. Using SISOTOOL:

1. Write the following code:

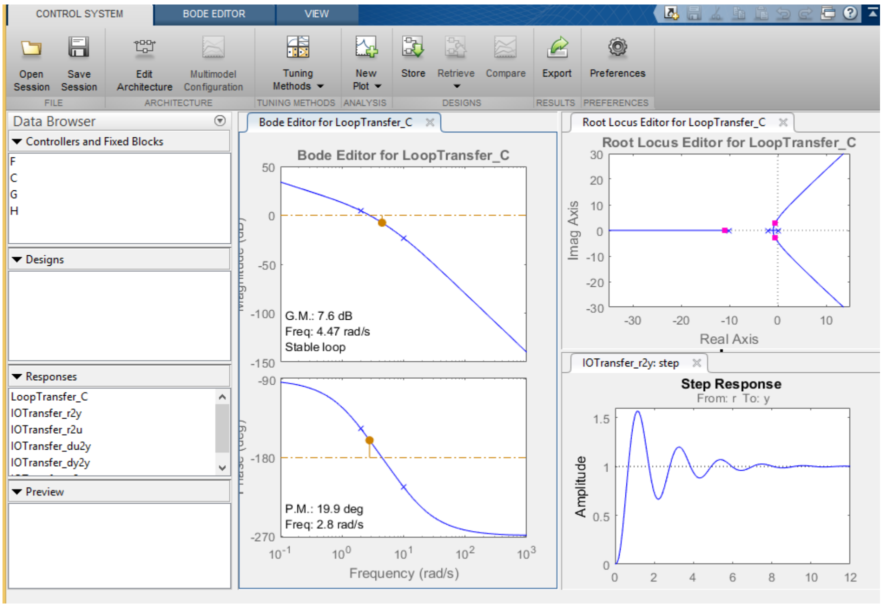
```
s=tf('s');
sys=100/(s^3+12*s^2+20*s);  %system transfer function
sisotool(sys)
```

2. A new window will open which contains bode plot, root locus, step response etc.

3. You can edit the values of F, C, G and H from 'Controllers and Fixed Blocks' where

   G: The main system

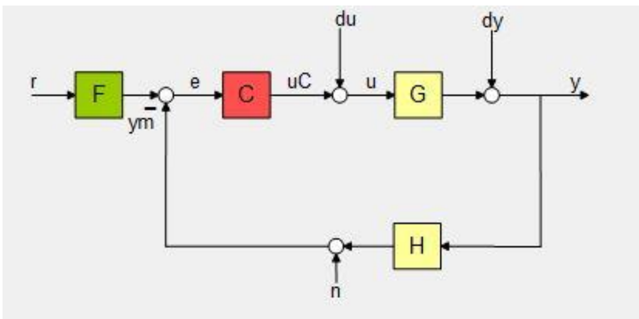
   C: Controller



H: Feedback Gain

F: Can be used to find impulse response or ramp response etc.

4. From root locus or Bode plot, one can drag the 'dot' to change the value of the proportional constant K and see the change in Bode plot, root locus, step response etc.
5. You can watch a video on https://www.youtube.com/watch?v=Ta4eh7gCBDE to get more knowledge about the sisotool.



Prelab:

1. Draw the approximate bode plot for transfer functions $100, s, \frac{1}{s}, s + 10$ *and* $1/(s + 10)$ by hands.
2. Draw the approximate bode plot of $H(s) = \frac{100(s+10)}{s(s+2)(s+5)(s+8)}$.

Labwork:

1. Draw the bode plot of Prelab 1 using MATLAB code.
2. Draw the bode plot of $H(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$ for $\omega_n = 5$ *and* $\xi = 0{:}\,0.1{:}10$ and see the change in bode plot for different values of ξ.
3. Draw the bode plot of Prelab 2 using 'sisotool'. Determine the gain margin, phase margin and bandwidth. Change the value of the proportional constant by dragging the bode plot and observe the changes

Part B: Compensator Design via Frequency Response

Objective: To design lag, lead and lag-lead compensator via frequency response and to see the effect these compensators upon the magnitude and phase responses at each step of the design

Minimum required software packages: MATLAB, Simulink, and the Control System Toolbox.

Theory:

Gain Adjustment:

By increasing the gain of the open loop, one can shift the magnitude plot upwards without changing the phase plot. Thus, it can provide the desired phase margin.

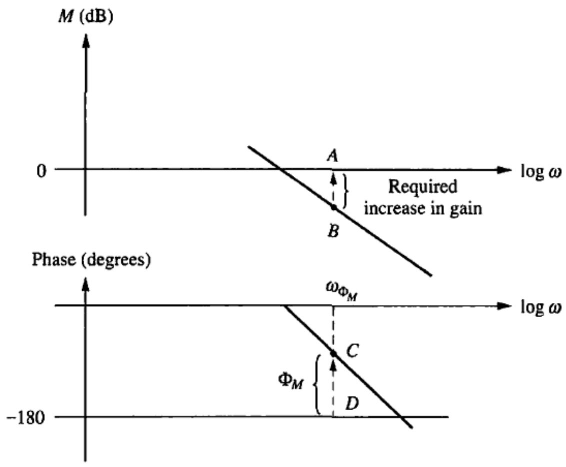


Figure 6: Bode plots showing gain adjustment for a desired phase margin

Procedure:

- Draw the Bode magnitude and phase plot.
- Determine the required phase margin from the percent overshoot by using Eqs. (1) and (2).
- By code: Determine the frequency (ω_{Φ_M}) that yields the desired phase margin. Determine the gain at that frequency (AB in figure 6). The magnitude plot must be shifted upward by AB. Find the gain that yields this shift by using $20\log(K_p) = AB.$ Draw the compensated Bode plot and check if the design is right.
- By sisotool: Shift the magnitude plot upwards by dragging and check when the phase margin is met in the phase plot.

Lag Compensator:

Frequency response method can be used to design lead lag compensation. The function of the lag compensator is to (1) improve the static error constant by increasing only the low-frequency gain without any resulting instability, and (2) increase the phase margin of the system to yield the desired transient response. These concepts are illustrated in Figure 7.

The lag compensator, while not changing the low-frequency gain, does reduce the high frequency gain. Thus, the low-frequency gain of the system can be made high to yield a large K_v without creating instability. This stabilizing effect of the lag network comes about because the gain at 180° of phase is reduced below 0 dB. Through judicious design, the magnitude curve can be reshaped, as shown in Figure 2, to go through 0 dB at the desired phase margin. Thus, both K_v and the desired transient response can be obtained.
The transfer function of lag compensator is

$$Gc(s) = \frac{1}{\alpha}\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \quad , where\ \alpha > 1$$

So, the pole is closer to the origin than the zero.

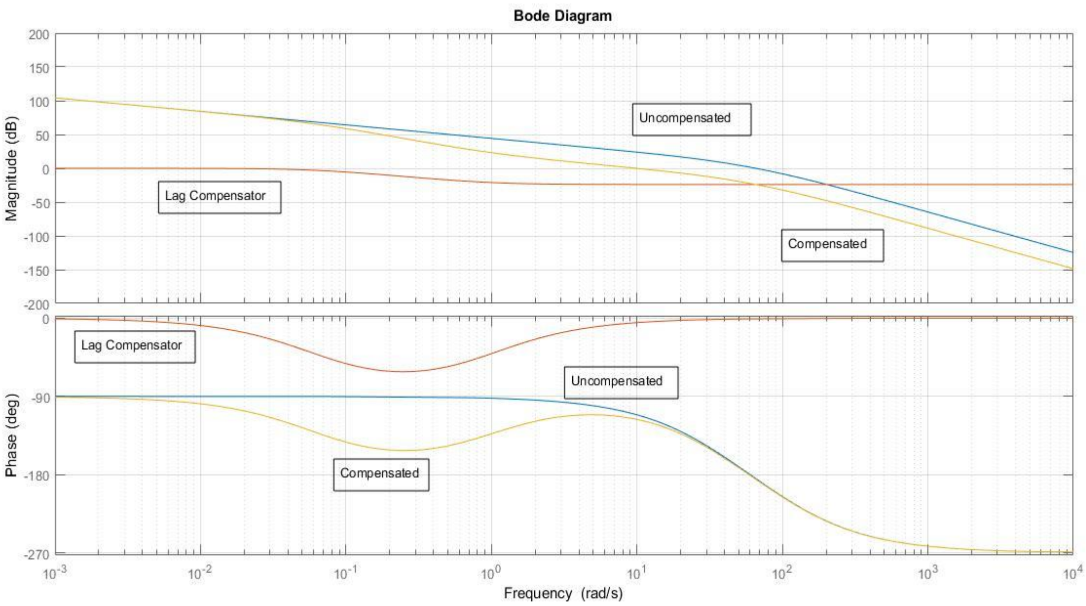


Figure 7: Lag Compensation

Procedure:

1. Draw the Bode magnitude and phase plot for the system which gives the desired error.
2. Determine the required phase margin from the percent overshoot by using Eqs. (1) and (2).

By Sisotool:

1. Right click on Bode plot area and select grid. Set proper limits for the axes.
2. Estimate the frequency at which the phase margin occurs. (Normal Bode is better for this purpose)
3. Right click on the Bode magnitude plot. Select *Add Pole/Zero> Real Zero*. Add a zero at a frequency that is 1/10 that found in step 2.

4. Right click on the Bode magnitude plot. Select *Add Pole/Zero> Real Pole*. Add a pole at the left side of the zero. Move it and check the phase margin from the phase plot. Stop if the desired phase margin is reached.

Lead Compensator:

The lead compensator increases the bandwidth by increasing the gain crossover frequency. At the same time, the phase diagram is raised at higher frequencies. The result is a larger phase margin and a higher phase-margin frequency. In the time domain, lower percent overshoots (larger phase margins) with smaller peak times (higher phase margin frequencies) are the results. The concepts are shown in Figure 4.

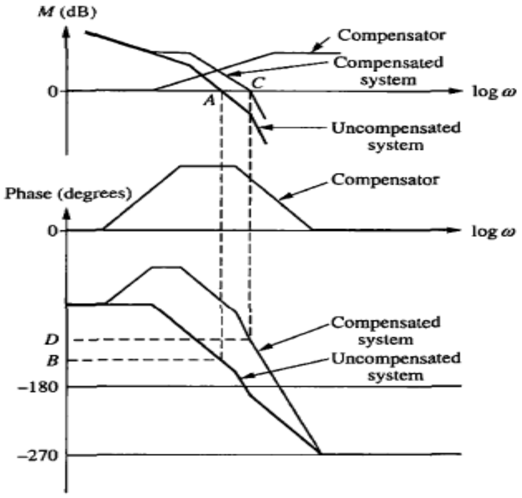


Figure 8: Lead Compensation

The uncompensated system has a small phase margin (B) and a low phase margin frequency (A). Using a phase lead compensator, the phase angle plot (compensated system) is raised for higher frequencies. At the same time, the gain crossover frequency in the magnitude plot is increased from A rad/s to C rad/s. These effects yield a larger phase margin (D), a higher phase-margin frequency (C), and a larger bandwidth.
The transfer function of lag compensator is

$$G_c(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \quad , where\ \beta < 1$$

So, the zero is closer to the origin than the pole.

Procedure:

1. Determine the required phase margin and bandwidth from the percent overshoot and peak/settling time by using Eqs. (1), (2), (3) and (4).
2. Draw the Bode magnitude and phase plot for the system which gives the desired error.
3. Determine the phase margin the lead system has to provide with a correction factor (around 10^o). This value is ϕ_{max}.

4. Determine β from $sin\phi_{max} = \frac{1-\beta}{1+\beta}$. Determine the compensator's magnitude at ω_{max} by $|G_c(j\omega_{max})| = \frac{1}{\sqrt{\beta}}$.
5. Using $|G_c(j\omega_{max})|$ find out ω_{max} from the graph. Now calculate T from the $\omega_{max} = \frac{1}{T\sqrt{\beta}}$.
6. Now complete the compensator transfer function and add it to the Bode plot.
7. Check if desired phase margin and bandwidth have been achieved. For bandwidth, use `bandwidth()` function of MATLAB. It takes the closed loop transfer function as input.

Lag-Lead Compensator:

Lag-lead compensation improves the transient response and steady-state error. One method is to design the lag compensation to lower the high-frequency gain, stabilize the system, and improve the steady-state error and then design a lead compensator to meet the phase-margin requirements.

The transfer function of a single, passive lag-lead network is

$$G_c(s) = G_{lead}(s)G_{lag}(s) = \left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\gamma T_2}}\right), where\ \gamma > 1\ and\ \gamma = \frac{1}{\beta}$$

Suppose, we want to design a passive lag-lead compensator for a unity feedback system where $G(s) = \frac{K}{s(s+1)(s+4)}$ using Bode diagrams to yield a 13.25% overshoot, a peak time of 2 seconds and $K_v = 12$.

1. Using Eqs. (1), (3) and (4), we get a bandwidth of 2.29 rad/s and $\Phi_M = 55^o$.
2. For $K_v = 12, K = 48$. So we have to plot the Bode plot of $G(s) = \frac{48}{s(s+1)(s+4)}$.
3. Let us select $\omega = 1.8\, rad/s$ as the new phase margin frequency. At this frequency, phase is -176^o from Bode plot. So, we need $\phi_{max} = 55^0 - 4^o + 5^o = 56^o$. From, $sin\phi_{max} = \frac{1-\beta}{1+\beta}$. and $\gamma = \frac{1}{\beta}$, β=0.094 and $\gamma = 10.6$. $\omega_{max} = \frac{1}{T\sqrt{\beta}}$ gives us $\frac{1}{T_1} = 0.56\frac{rad}{s}$.
4. For the lag compensator we select the higher break frequency to be 1 decade below the new phase margin frequency, at 0.18 rad/s. So, $\frac{1}{T_2} = 0.18$. So the transfer function becomes,

$$G_{lag}(s) = \frac{1}{\gamma}\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\gamma T_2}}\right) = \frac{\frac{1}{10.6}(s+0.183)}{s+0.0172}$$

And

$$G_{lag-comp} = \frac{4.53(s+0.183)}{s(s+1)(s+4)(s+0.0172)}$$

5. For the lead compensator we can use the data from step 3 for calculation.

$$G_{lead}(s) = \gamma \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\gamma T_2}} \right) = \frac{10.6(s + 0.56)}{s + 5.96}$$

So, the lag-lead compensated system is:

$$G_{lag-lead-comp} = \frac{48(s + 0.183)(s + 0.56)}{s(s + 1)(s + 4)(s + 0.0172)(s + 5.96)}$$

6. Check if desired phase margin and bandwidth have been achieved.

Labwork:

1. Using MATLAB's SISO Design Tool, find the value of K for a unity feedback system with $G(s) = \frac{360K}{s(s+36)(s+100)}$ to yield a 9.5% overshoot in the transient response for a step input. Use only gain adjustment.
2. Using MATLAB's SISO Design Tool, design a lag compensator for a unity feedback system with $G(s) = \frac{58390}{s(s+36)(s+100)}$ to yield a 9.5% overshoot and a tenfold improvement in the steady state error.
3. Design a lead compensator for a unity feedback system with $G(s) = \frac{100K}{s(s+36)(s+100)}$ to yield a 20% overshoot and a $K_v = 40$, with a peak time of 0.1 second.
4. Design the lag-lead compensator of the example and note down phase margin, phase margin frequency, closed loop bandwidth, percent overshoot and peak time for the lag-lead compensated system.

Report:

1. Describe the effect of gain adjustment, lag compensator, lead compensator and lag-lead compensator by comparing the phase margin, phase margin frequency, closed loop bandwidth, percent overshoot and peak time for the uncompensated and compensated system for each of the case.
2. Discuss the advantages and disadvantages for each of the design.