Financial Markets Volatility: Empirical Studies in the Volatility of Financial Assets

Master’s Thesis

MSc in Economics and Business Administration (Finance)

Student Name: Alamgir Hossain (20201823)

Supervisor: Dr. Douglas Eduardo Turatti

Date: 14/10/2022

Aalborg University Business School, Aalborg University

Financial Markets Volatility: Empirical Studies in the Volatility of Financial Assets

Alamgir Hossain
Aalborg University Business School, Aalborg University

14th October, 2022

Abstract

The thesis examined the forecast performance of the several stocks index with various GARCH models, VaR, and trading analysis. The study examines the Realized volatility index relative to the GARCH family of models in forecasting volatility for the S&P 500, Dow Jones Industry Average and Nasdaq 100.The research examined the forecasting volatility after Covid-19 and try to find the best model of the GARCH-type model. This investigation also usages GARCH, E-GARCH, GJR-GARCH, I-GARCH, APARCH, and FI-GARCH models to forecast volatility and stock index performance. To investigate the in-sample information content as well as the conditional variance equations of the GARCH family of models are enhanced with the realized volatility indices from Oxford Realized library. However, to appraise the out-of-sample forecasts, we generate one-day ahead GARCH rolling forecasts and apply the Value-at-Risk with 1% and 5% confidence interval. The comparative performance has been evaluated based on MSE, RMSE, and MAE. Furthermore, a statistical significance test was conducted to evaluate the data from each stock index. The empirical findings of the research shows mixed evaluation when selecting the best model in terms of accuracy and forecast. The experiment also shown there was a significant impact of volatility clustering and conditional variance due to the spread of Covid-19.

Preface

This thesis represents the conclusion of our Master of Science in Economics and Business Administration (Finance) at the Aalborg University Business School, Aalborg University. It is prepared from June to October 2022 and represents a term's worth of work.

We would like to thank our advisor, Dr. Douglas Eduardo Turatti, of the Department of Finance for his tremendous support, encouragement, and constructive criticism during the course of this thesis' development.

Alamgir Hossain
October 14, 2022

Table of Contents

List of Tables

List of Figures

1. Introduction

1.1 Background of the research

Proper assessment and forecasting of volatility in stock markets are essential for various reasons, including the financial system, investment selections, risk evaluation, debt securities, and many others. Economic regulatory agencies emphasize stock market volatility to prevent the volatility spillover impact of significant reforms in the international financial markets [1]. Stock market analysts monitor volatility on a real-time basis to smooth investment portfolios and mitigate market volatility. Since this is the scenario, previous researchers have explored how best to forecast volatility in the stock market [2]. The volatility observed in financial markets is a matter of significant concern. Financial market volatility is a major cause for alarm. Volatility is a critical dynamic factor for the stock market and investment because of its applicability as a risk measurement. Furthermore, investment rotation, risk analysis, hedging, and derivatives valuation correlate to the volatility phenomena [3]–[6]. Accordingly, higher volatility generates extensive uncertainty and leads to a turbulent financial environment, which lessens financial stability and impedes corporate financial expansion and investment [7]. Therefore, it is crucial to comprehend the process of financial stability and generate accurate forecasting volatility.

Furthermore, financial forecasting tools have evolved in response to the massive frequency of data flow within financial assets (Clerk & Savel, 2022). For accurate real-time forecasting of statistical features of financial sequences, this massive amount of data necessitates adjacent adaptation of regression models to empirical evidence [9]. This implies a prior understanding of the statistical characteristics that can be employed in adapting empirical designs to statistical analysis. Consequently, it is carefully addressed by financial experts, authorities, and financial professionals. Despite this, it is difficult to identify the primary financial factors that influence volatility before prediction. The key research of SCHWERT and WILLIAM (1989), in which the authors propose a time-varying relationship between stock market uncertainty and financial predictors [10]. In addition, the extant literature investigates the linearity of financial market factors and stock market volatility [11]. Conceptual and empirical research both agree that systemic and financial indicators are possible sources of financial market volatility [12]–[14]. The extant literature demonstrates an inherent connection between financial market volatility and econometric predictors [7].

Financial market volatility is typically explained using a standard econometric technique (like multiple regression or time series assessment) that assumes the volatility does not fluctuate over time. However, numerous empirical research proves that this proposition is insufficient to provide a neutral description of the financial data (Xu & Chen, 2010). The volatility segmentation feature describes the uncertain nature of economic time series records, where larger volatilities combine in the same duration of time and smaller variations constellations in a different timeframe [16]. Apparently, the homoscedasticity implication is not fulfilled for time series that exhibit features such as a "volatility clustered". According to Liu and Morley (2009), typical econometric techniques lead to significant deviations. Financial experts improved classic econometric techniques to increase predictive performance [17]. Professor Engle initially developed the ARCH method to explain variation in 1982. ARCH paradigm undermines standard reasoning and contradicts risk-return linearity. This model employs shifting volatility to construct a factor connected to prior volatilities and better characterizes financial data. Scholars prefer it because it gives a robust research technique for heteroscedasticity. A large order is required to predict transitive variances better when modeling detailed time-series data with the ARCH approach. Bollerslev (1986) added a latent period to the ARCH model's conditional volatility, creating the GARCH (generalized autoregressive conditional heteroscedasticity) approach [18]. After that, researchers extended comparable models including IGARCH, EGARCH, GARCH-M, and VGARCH, building a GARCH model family.

Despite prior research, our findings propose understanding what factors might be used to forecast financial market volatility. Firstly, the empirical attribute of volatility endures significant changes due to various circumstances, including financial recession, policy differences, and economic regulations. Due to these concerns, the study intends to describe the excessive volatility impact in GARCH models by considering both the short-term and long-term aspects. This paper presents a novel approach to forecasting and measuring stock market volatility by adopting the volatility thresholds in GARCH models for positive and negative outcomes. The thresholds are established by the quantitative percentile of the return [19], [20]. McNeil and Frey (2000) highlight that the suggested models can strengthen forecast reliability by addressing abnormal occurrences, especially in financial stock markets, where substantial volatility disruptions are typical since stock market volatility has different characteristics across diverse economic situations [21].

Financial time-series methods like ARCH, GARCH, and EGARCH use several features of financial time-series results to forecast financial market volatility. These features include leveraging impacts, surplus variance, and volatility segmentation. Accordingly, this study recommends merging the data gained from various financial time approaches with a GARCH family model, as opposed to the previously employed method, which was to integrate the results of a single econometric model with a single GARCH model. A volatility forecast is proposed in this research that uses the metrics from two or more GARCH models. The study hypothesizes that this would be a more accurate method of forecasting market volatility in the financial system. Therefore, financial market volatility could be better forecasted by integrating data from multiple econometric models with a GARCH family model rather than just one, as has been conducted in earlier research.

The remainder of the study is constructed accordingly following this introduction chapter. The study outlines the literature review in Chapter 2, and the significance test of the dataset is outlined in Chapter 3. Chapters 4, 5, and 6 consecutively consolidate data assessment, results in discussion, key findings, conclusions, and implications.

1.2 COVID-19 and Stock Market Volatility

The research on the effect of COVID-19 on the stock market is still comparatively limited but expanding rapidly) [22]. The extant scholars investigate how many incidences can affect the stock market framework during COVID-19 [23]. Prior research has been using systematic review to determine the effect of COVID-19 implications on the stock market's volatility, revealing that the outbreak has had a far more significant impact on the economy than other infectious epidemics like Ebola [24]. Accordingly, transformation in COVID-19 is associated with adverse stock market outcomes in 64 regions, as exhibited by findings [25]. The prior researchers revealed that U.S. stock market volatility negatively correlates with the frequency at which forecasts of COVID-19 infections fluctuate in real-time [26]. Consistently, according to empirical studies of significant COVID-19 stages, the global stock markets were adversely affected [27], [28].

The functioning of stock markets emphasizes volatility. Moreover, stock market volatility naturally captivates the attention of lenders, finance experts, authorities of the finance system, and decision-makers because of its role as an indicator of financial volatility around expenditures in financial assets [29]. According to stock market forecasts, high volatility in the financial industry is connected to regulatory barriers on financial enterprises and clients [30]. They also reveal that when authorities adopt drastic precautions, including informational initiatives and postponing public discussions, to avoid the expansion of COVID-19, volatility in stock markets rises substantially. In addition, Onali (2020) revealed that the notification of

COVID-19 incidents and fatalities across several regions triggered substantial growth in volatility for US financial markets [31]. The volatility of stock market investments may differ from industry to industry; for instance, Ecological and Social companies with better ratings tend to have less volatility [32]. In particular, prior researchers examine the connection between COVID-19 media exposure and changes in volatility [33]. Automobile, transportation, power, and tourism are the sectors most adversely affected by the addressed variations in volatility. Therefore, this research contributes to the limited body of research on how forecasting volatility after the COVID-19 epidemic might affect the volatility of financial assets through various GARCH, VaR and trading models. The study suggests mixed evaluations of the financial stock market when determining the optimal model regarding reliability and forecasting.

2.0 Literature review

2.1 Related work on financial market volatility

Previous studies have a keen concern in trying to forecast and quantify the volatility of the financial market returns. The rationale is that volatility has significant effects on many fields, including financial asset returns, asset management, and asset valuation (Liu & Pan, 2020). The long-term expenses of financial assets are also significantly affected by the volatility of stocks [35]. Some financial predictors may have an impact on financial stock volatility in-sample; however, extant researchers argue that this does not mean that adding such predictors to the criterion of a GARCH model will result in more appropriate out-of-sample predictions [36]. In addition, Paye (2012) discovers that integrating financial models can accomplish small but considerable market volatility forecasting [11]. Furthermore, financial market volatility can be responsible for the analytical indicators' prediction performance to volatility. Since asset managers tend to react appropriately to events at irregular intervals and respond to a repeat performance of significant accomplishment of forecasting (Liu & Pan, 2020), the financial market is inadequate in such the present stock value does not accurately portray all available raw data of the financial market.

Financial market Volatility, which is the contingent benchmark variation of the financial asset, is a crucial parameter in several economic contexts. Predictions of future volatility are used in many areas of finance, including financial planning, capital market, and stock valuation [37]. The phrase “volatility" refers to the extent of variation in an asset or portfolio and has generally been regarded as an indicator of risks [38]. When assessing the risk level of an asset, increased volatility is a negative indicator. As a result, the principle of volatility has become fundamental in modern financial assets. For this purpose, volatility modelling has been a popular area of study for academic research. Since volatility has been recognized for a long time as a critical aspect of financial markets, scholars and professionals in the field of finance consistently contribute much emphasis to its modernization [39]. Nevertheless, volatility has seen notable trends in assessing its complexities over the past few centuries. Significant financial markets in the United States, Europa, and Asians have revealed that volatility is associated with financial assets and performance (Brunnermeier & Pedersen, 2009). The research on the impact of volatility fluctuation on financial market forecasting has been expanding in recent years in light of volatility's importance to asset valuation and return forecasting and its fundamental uncertainty.

The financial and economic systems both benefit from less volatile financial markets. The economy's success and the financial system's stability are both impacted by the volatility of financial markets. Notably, creating risk criteria relies significantly on the formulation of implied volatility [41]. Moreover, as the necessity for analyzing volatility as a mechanism of measuring economic threat has emerged, time-varying volatility forecasting has been frequently implemented in the research on financial markets. The AutoRegressive Conditional Heteroscedasticity (ARCH) modelling approach and the Generalized ARCH (GARCH) models established, a more empirically version of ARCH, are two of the most significant modelling techniques predicting the mechanisms of volatility [18], [42]. Many dimensions of financial asset yields have been recently acquired by the benchmark GARCH model, which provides the foundation for various formulations employed to represent different quantitative advantages of yielding financial information. Furthermore, the volatility mechanisms of several financial series have experienced significant transformations. Forecasts of volatility based on GARCH methods decompose under such scenarios because they cannot comprise the interrelation of the possible factors (Arellano & Rodríguez, 2020). This research framework makes it feasible to represent the distinct variability trends during high and low-volatility durations with distinct GARCH practices in the extant literature.

2.2 Systematic literature review

According to the existing research, many time series in finance display fundamental gaps in their implied volatility, and overlooking these gaps could significantly impact the reliability of volatility forecasting. Accordingly, considering fundamental adjustments may enhance the forecasting accuracy of financial markets volatility [44]. However, Hamilton and Susmel (1994) added the volatility of financial assets to the GARCH model to represent the prospect of financial markets forecasting [45]. The GARCH model is adaptable and adequate to allow for a variation in the possibility process across volatility of financial assets based on the financial markets. Rapid adaptation of such models to adjustments in the degree of inherent volatility yields more accurate forecasts of threat [46]. This research studied several recent studies to investigate the accurate model for the volatility of financial assets, as shown in Table 1.

Table 1: Systematic literature review matrix

Author’s	Sample examined	Period examination	Method	Key Findings
[47]	Brazilian stock index	January 2006 to May 2017	GARCH Model	Monetary regulation and market fluctuations are quantitatively substantial as anticipated, excluding the Brazilian stock market index, which experienced volatility impacts and modifications after the recession.
[48]	NEI index in China	November 1, 2009, to March 31, 2020	GARCH Model	The study uses three types of statistical features to generate "great insights" into predicting China's renewable technology volatility.
[49]	Apple financial stock market	Jan 2000- June 2018	GARCH approach	In most cases, the forecasting efficiency of matrix factorization models is better than that of the equation used by most businesses today, i.e., dynamic modeling.
[7]	Pakistan stock market	August 2010 to December 2020	GARCH Model	This study looks at the potential of the economic complexity index and other economic indicators to forecast swings in the Pakistani capital market.
(Junior et al., 2022)	European and global financial crisis period	January 2000- to April 2018	GARCH Model	This research is significant for risk control modeling and legislative supervision and could increase credibility among international investment firms.
[51]	United States financial institutions	Period 1997-2019	GARCH Model	The analysis demonstrates financial amplitude connecting property investment and financial institutions during the global economic meltdown but not during the tech stock catastrophe.
[52]	29 stock market indices	3 January 2001 to 16 April 2018	GARCH Model	The symmetric GARCH approach based on actual correlation outperforms the BEKK, DCC, and DECO GARCH Family approaches predicated on measured datasets.

[53]	BitMex, (CBOE), (CME) and (ICE)	December 18, 2017 to December 4, 2020	GARCH Model	VaR estimations based on the GARCH approach for the Derivatives trading data period were shown to be realistic. Regarding favourable events, Bitcoin derivatives' return implied volatility does not spike like Bitcoin cash's, but they do jump regarding financial instability.
[54]	MSCI Emerging Market (MSCIEM) index	January 3rd, 2000 to May 7th, 2020	GARCH model	According to an essential connection between the actual operation and the supposed statistical technique, plus a significant adequate sampling quantity to reduce estimations, enhancement can generate reliability increases as anticipated, with the improvements becoming more apparent the higher the number of error terms.
(Wang et al., 2022)	Chinese stock market	4 January 2000 to 4 March 2020	GARCH Model	Forecasting the China Market Stock return sequence with Paired - samples t is most straightforward with a GARCH approach. The China Stock Market has become less sensitive to market movements than the Guangdong Stock Index.
[56]	Jakarta Composite Index (JCI)	27 September 2006 to 31 August 2021	GARCH Model	The current COVID-19 epidemic has worse share market results than the global economic meltdown in developing and creating emerging markets.
[57]	Romanian stock market	January 2020 and April 2021	GARCH Model	The everyday average earnings for the Romanian equity sector are platykurtic, not identically dispersed, and time-dependent.
[58]	18 stock market indices	2000 to June 2020	GARCH models	Volatility in stock systems appears to execute better for quick predictions. High volatility modeling forecasts do not surpass low-frequency ones.

[59]	United states financial stock market	January 2, 2004 and April 30, 2019	GARCH Model	The forecasting efficiency is greatly enhanced by including choice variations in the GARCH model.
[60]	S&P 500 index volatility	August 2007 to late June 2009	GARCH Models	When using effective methods, predictive factors are substantial, even though GARCH models.
[61]	China and Southeast Asian stock markets	1 January 1994 to 30 August 2019	multivariate GARCH models	Chinese financial stock markets had a positive direct provisional relationship during the economic crisis through the GARCH family models.
[37]	Composite index SP	July 1926 to January 2018	SMC & GARCH-type models	The statistical attributes of simulation results require a Statistical technique and an effective subsequent research instrument like the GARCH model.
[62]	S&P500 in the U.S. market	January 3, 2010 to December 31, 2015 & January 2, 2016 to July 31, 2020.	GARCH models	The implemented GARCH approach requires the best significance levels. Started to recognize that EGARCH predicts volatility best.
[63]	German Day-Ahead electricity market (EPEX SPOT)	1/1/2009 to 31/12/2018	GARCH model	Beneficial crises have a more extensive influence on inflationary energy pressures than adverse crises of similar intensity. The presence of average in regular energy price rising trends because historical averages forecast present expansion trends adversely through the 21-century lapse.
[64]	S&P 500 index	1st July 2013 to 13th August 2020	Hybrid GARCH models	Using composite GARCH approaches increases the system phase to a higher number of results in more accurate volatility estimates.

[65]	Mexican peso exchange rate	August 2017 to July 2019	GARCH models	The forecast accuracy of the GARCH family approach.
[66]	DJIA and GE market	Year 1990-2016	GARCH model	This research presents portfolio management techniques with smaller ratio, downturn, and risk-adjusted yields gross of financing expenses during Covid-19 through the GARCH model.
[67]	US stock market	January 2003 to May 2018	GARCH models	If fluctuation movements are disregarded, the united States share price conveys volatility to the United States Dollar's value but rarely inversely, according to research.
(Liu & Pan, 2020)	S&P 500 index	January 1950 through December 2015.	GARCH-Arch Type Models	Include analytical factors that can optimize the GARCH metric. Technological aspects execute superior financial indicators in advance, but economic indicators produce more reliable predictions in a recession.
(Liu & Gupta, 2022)	US stock market	24/07/1987 to 15/04/2004	GARCH Model	According to the forecast inclusion experiment, investment fund uncertainty can improve stock exchange volatility forecasts through the GARCH family model.
[69]	Tehran Stock Exchange	2013–2019	GARCH Model	The GARCH model investigates the stock market volatility uncertainty that performs the best to forecast the disruptive events during Covid-19.
[70]	US and Latin American stock market	January 1992 to November 2007	GARCH Model	The systemic model generates volatility modulation features clarified in a financial perspective than those from symmetrical ad-hoc variance degradation.
[71]	United States stock markets	1 January 2007 to 18 September 2015	GARCH Model	Adverse profits have more influence on derivatives volatility than increased profits, and 'poor' increases influence petroleum products' price volatility more than 'excellent' swings through the GARCH family model.

(Wang et al., 2020)	S&P 500 index	1993 to 2016	GARCH-MIDAS model	Severe disruptions affect stock market volatility, but the asymmetrical effect has a more significant long-term and short-term impact. The asymmetrical impact is much more detrimental than the extreme volatility effect, meaning it forecasts stock market volatility greater.
[72]	Nasdaq-100 stock index	January 4, 2000 to March 19, 2019	GARCH	Volatility has a more extensive influence on uncertainty than economic growth of similar intensity.
[73]	China stock market	2002 to 2017	EGARCH and DCC EGARCH models	Chinese stocks are better tied to Singaporean, Japanese, Australian, and Particular favorites than America, France, and the United Kingdom. The multivariable GARCH model demonstrated transmission in yield connections and volatility overflow for all Chinese marketplace.
(Arellano & Rodríguez, 2020)	Stock and Forex market of HI countries	1990 to 2019	Markov-switching GARCH models	In Foreign exchange markets, the massive statistic is lower than in Currency and other financial stocks. Financial stock market indexes show significant volatility continues.
(Wang et al., 2020)	Chinese Stock Market	January 2000 to March 2019	VAR model	Both the U.S and Chinese Volatilities affect China's financial market volatility, but the transmission is unequal.
[75]	DJIA, FTSE, CSI stock markets	January 1, 2008 to December 31, 2018	LSTM-ANN networks with GARCH model	GARCH predictions can boost the forecasting capacity of synaptic system models, and integrating network systems is an excellent way to build convolutional neural designs.

[38]	US equity market	January 2007 to April 2016	VOV Measures	Financial Stock markets with stronger GARCH model sensitivity from Volatility derivatives have more robust expected performance.
(Aziz et al., 2019)	Eight stocks and four bond indices	January 1985 to December 2014	Multivariate GARCH models	Simulators provide superior effectiveness than estimation methods. GARCH functions do not enhance the prediction error over the dynamic similarity approach.
[5]	Asian European, American financial markets	January 12, 2000, to December 04, 2018.	MSGARCH models	GARCH simulations can generate reliable Valuation estimates, which can help with financial time series, convertible hedging, and risk mitigation.
[77]	China stock markets	4/01/2008–5/09/2016 & 31/12/2008-22/09/2016	GARCH Model	GARCH had the quickest average reversion variability technique; therefore, volatility immediately fell.
[78]	New York SE	Year 1997-2001	GARCH models	The suggested GARCH models outperform industry standards in both step and composition forecasting.
[79]	XETRA market, German Stock Exchange	January 2009 to December 2012	GARCH models	When employing both the ZAF and DZAF statistics for in-sample prediction, the GARCH model yields excellent results.
[80]	S&P 500	September 13th, 2011 to August 26th, 2017	ANN GARCH Model	The global investment in all cryptocurrencies is exceptionally substantial, although little attempt has been undertaken to analyze its volatility.
[1]	China's stock market	July 26, 2013 to July 28, 2017	GARCH type models	From a statistical standpoint, the SSE Market Index is time-varying and clumping.

(Kim & Won, 2018)	KOSPI 200 stock index	January 1, 2001, to September 30, 2011	(GARCH)-type models	GEW-LSTM, a blended approach incorporating GARCH, exhibits the lowest prediction errors.
[82]	North America stock markets	8 January 1995 to 28 June 2016	GARCH Model	Financial economies are much less related to advanced economies regarding earnings and financial sector co-movement.
[83]	Asia-Pacific stock markets	2 January 2008–29 July 2016.	GARCH Model	Integrating the volatility scale in GARCH's volatility decomposition formula minimizes fluctuation endurance and enhances forecast performance.
(Li & Kang, 2018)	Stock index S&P 100-600	The year 1989-2015	GARCH Model	The research highlights the benefits of using existing attributes to comprehend multivariate regression dependency, strengthen tail-dependence and forecast densities modelling, and assess risk valuation.
[85]	Eleven stock market indices	2005 to 2016	GARCH Model	GARCH approaches are more reliable than separate modelling for everyday, weekend, and yearly equities for the financial stock market.
[86]	UK stock market	3 January 1950 to 31 December 2015	GARCH Model	The UK share market demonstrates no connection between technological reliance and forecasting accuracy.
[87]	Three equity indexes	01/01/2000 to 10/29/2015	GARCH Model	The actual volatility indicators can be described as linear equations of just a few parameters, retaining the identical periodic cascading pattern as the ARCH components of the multivariate GARCH model.
[88]	USA & Brazil index	Year 2000-2011	GARCH Model	Volatility forecasting uses in the investment portfolio, trading, compelling valuation, and risk evaluation through GARCH Model.
[89]	Asian & US financial stock	Year 2003-2012	GARCH Model	This research reveals that the GARCH model predicted the forecasting financial market

				volatility during economic crises of investment firms.
[90]	S &P500 index	Jan. 3, 2000 to July 1, 2011 & Jan 7, 2002 to July 3, 2013	GARCH model	The transition across domains is more likely between medium and high volatility ranges.

3. Preliminary Discussion about Volatility

Volatility sometimes refers to the degree of uncertainty or risk associated with the magnitude of movements in the value of a security [91]. Nevertheless, enormous volatility specifies that a security's value can vary over a broader range of prices. Investors and traders quantify the volatility of security due to assessing previous price variations and forecasting future price fluctuations. This indicates that the worth of the asset can fluctuate drastically within a small period. In contrast, lesser volatility shows that a security's price oscillates less frequently and is likely to be steadier. A model of volatility should be capable of predicting volatility. Moreover, all financial applications of volatility models effectively involve estimating upcoming return components [91]. Using either the standard deviation or beta, volatility can be determined. Furthermore, beta measures the market's volatility, whereas the standard deviation computes the value dispersal of a security or stock. As we know, beta can be calculated by regression analysis; somehow, we are not covering beta in our study; we will cover only standard deviation. There are many kinds of volatility, but implied and historical volatility are the most used volatility methods.

3.1 Volatility definition and measurement

The volatility models are well known as heteroscedastic conditional models. Financial economics is primarily concerned with the variance of returns over a definite period. Utilizing the variance of returns over a specific interval is a method for measuring volatility. Volatility is an essential aspect of options trading. Let represent the stock's logarithmic return at the specific time index *t*. Volatility analysis is based on the premise that the series is serially uncorrelated or has negligible lower-order serial correlations; on the other hand, volatility is a dependent series. Furthermore, let's assume the conditional mean and variance to put the volatility models in the appropriate context. Given these data, it is possible to calculate the sample variance using equation 3.1.1. [92].

$$\sigma^2 = \frac{1}{T-1}\sum_{t=1}^{T} (r_t - \mu)^2 \qquad 3.1.1$$

Nonetheless, one of the most prevalent statistical methods for calculating volatility involves calculating the returns and their standard deviation over a certain period. Here the standard deviation of returns is computed by the following equation (3.1.2) [92], [93].

$$\sigma = \frac{\sqrt{1}}{T-1}\sum_{t=1}^{T} (r_t - \mu)^2 \qquad 3.1.2$$

According to [92], there are four steps comprise the construction of a volatility model for a series of asset return data:

1. The first step is to identify a mean equation by checking the significance test for serial dependency in the data and, if required, developing a statistical model (such as an ARMA model) aimed at the asset return series to eliminate any linear reliance.
2. Now apply the residuals from the mean equation by the ARCH effect test.
3. If ARCH effects are statistically significant, determine a volatility model, and the next stage is to estimate the mean and volatility equations jointly.
4. Evaluate the fitted model meticulously, and if necessary, refine it.

3.2 Stylized fact about the volatility:

Numerous research has confirmed stylized truths regarding the price volatility of financial assets that have arisen throughout the years. Consequently, an effective volatility model must capture and reflect these stylized facts. Volatility fluctuates over time and tends to cluster. Furthermore, the return data shows that a significant (slight) return at present (regardless of its sign) is an excellent indicator of future significant (small) returns. However, the significance of such a volatility clustering effect is that volatility shocks in the present will affect the anticipation of volatility in many upcoming periods [94], [95].

Typically, volatility changes have a long-lasting effect on the subsequent evolution. We claim that volatility possesses a prolonged memory. The clustering of volatility suggests that volatility fluctuates. Consequently, a time of excessive volatility will ultimately give way to volatility closer to normal, and similarly, a period of small volatility will increase afterward. Generally, another stylized fact of volatility is mean reversion in volatility, which indicates that volatility will, in the extended run, return to a typical level [91]. Furthermore, there is a fascinating association between the amplitude of returns as well as the returns themselves: once prices decline, volatility rises, and at the same time, when prices rise, volatility drops to a smaller extent. This phenomenon is familiar as a leverage effect or asymmetric volatility [91]. Equities' volatility consists of intraday and overnight volatility, where volatility at the end refers to changing through trading days. The intraday returns with high frequency convey relatively little information about the overnight volatility. Furthermore, it is challenging to appraise conditional heteroscedastic models' forecasting usefulness due to the volatility's unobservability [92].

3.3 Volatility Proxy:

Volatility is likely the unique pattern in the financial time series. Numerous financial operations, for instance, portfolio selection, risk managing as well, and options pricing, necessitate accurate methodologies for predicting volatility. There is a substantial body of study on this subject [93]. Some issue are finding the conditional variance; it is challenging to observe, which creates complexities in assessing and comparing forecasts; moreover, it is a prevalent difficulty in forecasting volatility [96]. If an unbiased estimator of variance is available, there are, fortunately, some techniques to partially alleviate this issue [49]. Research studies offer multiple proxies for conditional variance, including the squared return, realized variance, and range-based variance, also known as the high-low range. We will briefly explore the squared return before moving on to these subsections.

3.3.1 Squared Return:

For investors and researchers, the daily squared return of a financial asset is regarded as a straightforward, generally accessible, and unbiased volatility proxy. However, this proxy measure is not very complex, where the squared return is likewise regarded as an identical noisy predictor of volatility. However, squared return is still considered an unbiased estimator of the hidden variable of interest, and the idiosyncratic error term makes this proxy inaccurate [96]. The squared return method frequently results in forecasts of extreme volatility. As a result, this approach will not be utilized in this study, as it is anticipated to be inferior to alternative variance proxies.

3.3.2 Realized Variance

Realized volatility is considered proxy volatility, whereas assessing, modeling, and forecasting conditional volatility, as well as correlation by high-frequency intra-day data, is a fascinating new area of study. Recent research demonstrates that daily conditional volatility and correlation are effectively approximated by realized volatility and correlation measures derived from the summation of high-frequency squared returns [97]. However, realized variance, or realized volatility (henceforth RV), created on intraday returns, is an alternative measure of volatility. [98] defines the realized variance as the simple sum of all intraday squared logarithmic returns. Poon and Granger (2005) stated that many realized variances are available, but the most widely used 5-minute and 15-minute interval yield; moreover, out of all the available Realized Variance estimates, a 5-minute or 15-minute interval yields the most precise outcomes [99]. Furthermore, for this study, we considered 5-minute realized volatilities since these time series are retrievable from the realized library of the Oxford-Man Institute. There are many realized variances in the realized library of the Oxford-Man Institute, but due to our limitation, we will focus on a 5-min subsample.

4 Forecasting volatility model

Measuring and predicting asset return volatility is crucial for managing risk, investment strategy, and options pricing. However, asset return volatility is well-known; volatility also exhibits certain features frequently observed [96]. However, there are some significant distinguished stylized facts about volatility that comprise the following: the first one is that volatility is persistent, the second one is volatility is mean reverting, and the third one is innovations that can play an asymmetric influence on volatility (Engle & Patton, 2001). According to Black, 1976 the asymmetric heavy reliance of volatility on positive and negative shocks is popularly known as the leverage effect. Furthermore, the leverage effect implies that adverse shocks can significantly impact rather than play the impact equal.

Additionally, due to the significant demand for precise volatility forecasts, practitioners and scholars have shown great interest in modeling time-varying volatility. Because of the widespread attention in volatility forecasting, a variety of models attempt to replicate the evolution and features of financial asset volatility. To effectively represent conditional variance, any model seeking to forecast volatility should contain as many properties as possible.

In the preliminary part of this section, we define extensive time series volatility models that use historical data to generate volatility forecasts. There are a large number of volatility models that expended gradually with time. Incorporating prior volatility errors yields the ARMA model for volatility. In an auto-regressive model, the target variable is forecast using a linear combination of its initial values. A model is called autoregressive when it's statistically auto-regression lag-1 autocorrelation incorporating that the lagged return can support predicting. The word autoregression denotes that the variable is regressed against itself. Here is an example of the autoregressive model of predictive power:

$$r_t = \mu_0 + \phi_1 r_{t-1} + a_t \tag{4.1}$$

A substantial body of theoretical literature indicates that ARMA models are effective at forecasting volatility.

First, let us consider a simple first-order autoregressive model AR (1), as defined in equation (4.2). According to equation (4.2), the variable is generated by its history and an error term. Here the error term is used to denote the consequence of all other variables, which should be comprised in the model, whereas, on the other hand, they are not included. The error term is called 'white noise process,' and in the time series analysis, the white noise process is the crucial assumption. A first-order autoregressive model depends on the current and prior values of error terms [100].

The following equation is shows the AR(1) model:

$$\sigma_t = \mu + \phi_1 \sigma_{t-1} + \mu_t, \qquad \text{where } |\phi_1| < 1 \text{ for stationary} \tag{4.2}$$

Equation (4.3) below shows the dependence on its past error terms as a moving average (MA) process. As a result of the MA(1) structure, observations separated by more than two lags are uncorrelated.

Below is a moving average model which is dependent on its past error term:

$$\sigma_t = \mu + \theta_1 \mu_{t-1} + \mu_t \qquad \text{Where } |\theta| < 1 \tag{4.3}$$

Hence, combining autoregressive models and moving average processes results in the time series ARMA model. Below is the given full ARMA model:

$$\sigma_t = \phi_1 \sigma_{t-1} + \cdots + \phi_p \sigma_{t-p} + \mu_t + \theta_1 \mu_{t-1} + \cdots + \theta_q \mu_{t-q} + \mu_t, \quad |\phi| < 1, |\theta| < 1 \tag{4.4}$$

4.1 Autoregressive Conditionally Heteroskedasticity Model (ARCH):

The Autoregressive conditionally heteroskedasticity model, widely known as ARCH, is advantageous when the data being examined is non-linear. Since their introduction, the autoregressive conditional heteroscedastic [42] started to be familiar with predicting the non-linear return; later, generalized ARCH known as GARCH [18] models have been widely used. The ARCH family of models uses only historical return data and incorporates volatility into the return behavior. However, ARCH and GARCH models were gradually established to capture volatility clustering, and unconditional return distributions with fatter tails frequently observed in macroeconomic series such as stock market returns. Heteroskedasticity may occur in financial time series, which explains why and how the variance error term is inconsistent over time. The ARCH model can be the appropriate model in terms of its characteristics as it does not use constant variance, which explains better volatility clustering. Additionally, volatility may manifest itself in clusters and a phenomenon acknowledged as volatility clustering. The ARCH-family models capture these effects.

The primary characteristic of an ARCH model, asset return, is serially uncorrelated, somehow dependent on its shock of, and at the same time dependent on its lagged values which can be explained by the simple quadratic function (Tsay, 2010).

As we are focusing one lag order, an ARCH (1) model assumes that

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 \tag{4.1.1}$$

Where is a series of independent and identically distributed (iid) random variables with zero mean and one variance; the parameter has to follow certain regularity conditions in order for it to have a finite unconditional variance.

The model's structure demonstrates that the extensive past squared shocks infer considerable conditional variance for the innovation. Furthermore, the ARCH model indicates significant shock followed by another large shock by its structure.

4.1.1 Forecasting:

Forecasts for the ARCH model in Eq. (4.1.1) can be obtained recursively in the same way forecasts for an AR model can be obtained. Take an ARCH(m) model as an example.

$$\sigma_h^2(1) = \alpha_0 + \alpha_1 a_h^2 + \cdots + \alpha_m a_{h+1-m}^2 \tag{4.1.2}$$

Here forecast origin is *h*

The *2*-step ahead forecast is

$$\sigma_h^2(2) = \alpha_0 + \alpha_1 \sigma_h^2(1) + \alpha_2 a_h^2 + \cdots + \alpha_m a_{h+2-m}^2 \tag{4.1.3}$$

And the step ahead forecast is

$$\sigma_h^2(\ell) = \alpha_0 + \sum_{i=1}^{m} \alpha_i \sigma_h^2(\ell - i) \tag{4.1.4}$$

Where $\sigma_h^2(\ell - i) = a_{h+\ell-i}^2$ if $\ell - i \leq 0$. This study mainly focuses on the ARCH (1) model from equation (4.1.2).

4.2 The GARCH Model

While the ARCH model is straightforward, it frequently requires many parameters to describe the volatility process associated with an asset return adequately. However, the GARCH model expanded from the ARCH model [18]. Furthermore, Generalized Autoregressive Conditional Heteroscedasticity (GARCH) shared many of the ARCH's fundamental properties. Moreover, it needs fewer parameters to complete the volatility process. The process enables a variable's conditional variance to be reliant on prior lags; the first lag is the squared residual from the mean equation, which is used to present information regarding the volatility from the preceding period. Then a_t follows a GARCH(p, q) model if

$$a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{i=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (4.2.1)$$

Where, once more $\{\epsilon_t\}$ is a sequence of iid random variables having a mean of zero and a variance of one. Here, $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $\sum_{i=1}^{\max(p,q)} \left(\alpha_i + \beta_{\mathrm{j}}\right) < 1$. Where $\alpha_i \equiv 0$ for $i > p$ and $\beta_j \equiv 0$ for $j > q$.

The variance equation requires that all parameters be positive and $\alpha + \beta$ is likely to be less than one but close to one. When the summation of the coefficients equivalents one, the process is denoted as an Integrated GARCH (I-GARCH). This study follows GARCH(1,1) model for further investigation. Here is given GARCH (1, 1) model:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta \sigma_{t-1}^2 \qquad (4.2.2)$$

Which validates GARCH (1, 1) model as equivalent to an ARCH ()) a model with a particular structure for the values of the lagged returns$a_{t-\mathrm{i}}^2$. Additionally, just as the ARCH (1) model can be rewritten as an ARCH (1) model on the squared returns, whether the GARCH (1, 1) model is expressed similarly as an ARMA (1, 1) model on the squared returns.

Forecasts for a GARCH model can be obtained using techniques similar to those described previously ARCH model. Take the GARCH (1, 1) model from Eq. (4.2.2), and suppose the origin of the forecast is *h*, and for *a 1*-step ahead forecast, it demonstrates.

$$\sigma_{h+1}^2 = \alpha_0 + \alpha_1 a_h^2 + \beta_1 \sigma_h^2 \qquad (4.2.3)$$

Where a_h and σ_h^2 are known at the time index h. However, to find the volatility clustering and volatility effect we mainly focus on one day ahead GARCH (1,1) forecast.

4.2.1 Asymmetric GARCH Models:

In the financial time series data, the returns and volatility always change whether ARCH and GARCH model covers only symmetric characteristics. While negative shocks can have a significantly negative impact, it makes greater volatility. In contrast, positive shock doesn't play a significant effect in volatility clustering. Hence, there are many asymmetric GARCH models expended such as exponential GARCH known as E-GARCH and another popular one is GJR-GARCH.

4.3 The Integrated GARCH model

In 1986, Engle and Bollerslev presented a new model known as the integrated GARCH model (I-GARCH), which is persistent in variance due to the fact that current information is still essential for forecasts at all-time horizons. The integrated GARCH model is asymmetric GARCH model. I-GARCH models are therefore unit-root GARCH models. Similar to ARIMA models, the impact of past squared shocks is a defining characteristic of IGARCH models $\eta_{t-i} = a_{t-i}^2 - \sigma_{t-i}^2$ for $i > 0$ on a_t^2 is persistent [92]. An I-GARCH *(p,q)* model assumes the form:

$$a_t = \sigma_t \epsilon_t$$
$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \left(1 - \beta_j\right) \alpha_{t-j}^2 \tag{4.3.1}$$

Then, an I-GARCH (1, 1) model could be expressed as:

$$a_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2 \tag{4.3.2}$$

Where, $1 > \beta_1 > 0$ and $\{\epsilon_t\}$ is a series of independent, also identically distributed random variables where mean 0 and variance 1. Nevertheless, the I-GARCH(1, 1) is equivalent to a GARCH(1, 1) model when $\alpha_1 + \beta_1 = 1$

4.4 Exponential GARCH:

There are many symmetric models weakness, most notably the leverage effect. However, in 1991 Nelson introduced another asymmetric model known as Exponential GARCH. Nelson mandated that conditional variance be expressed as the natural logarithm σ_t^2 to confirm the conditional variance is not negative, rather than imposing restrictions. Additionally, he allows for asymmetric effects between positive and negative returns through the use of a weighted innovation $g(\varepsilon_{t-i})$. The E-GARCH(*p, q*) is given by:

$$\ln\left(\sigma_t^2\right) = \alpha_0 + \sum_{i=1}^{p} \alpha_i g(\varepsilon_{t-i}) + \sum_{j=1}^{q} \beta_j \ln\left(\sigma_{t-j}^2\right) \quad (4.4.1)$$

where, $$g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - E|\varepsilon_t|]$$

Z_t is supposed to have the identical illustration as earlier, whatever see equation (4.4.2) with the conditional variance follow by

$$\ln\left(\sigma_t^2\right) = \alpha_0 + \sum_{i=1}^{p} \left[\propto_i Z_{t-i} + \gamma_i\left(|Z_{t-i}| - E(|Z_{t-i}|)\right)\right] + \sum_{j=1}^{q} \beta_j \ln\left(\sigma_{t-j}^2\right). \quad (4.4.2)$$

It is clearly shown that it doesn't have any constraints on the parameters in this case to make sure a positive conditional variance. Below is given the E-GARCH(1, 1) model:

$$\log\left(\sigma_t^2\right) = \alpha_0 + \alpha_1 Z_{t-1} + \gamma_1\left(|Z_{t-1}| - E(|Z_{t-1}|)\right) + \beta_1 \log\left(\sigma_{t-1}^2\right) \quad (4.4.3)$$

The E-GARCH model does not require any parameter constraints to ensure that the conditional variance is not negative. The E-GARCH model can model the persistence of volatility, mean reversion, and asymmetrical effect. Furthermore, allowing for different properties of positive and negative shocks on volatility expresses the primary benefit of the E-GARCH model over the symmetric GARCH model.

4.4.1 Forecasting

As we are considering only E-GARCH (1, 1) one day ahead forecast volatility where forecast origin is *h*, as well as the *1*-step ahead forecast of σ_{h+1}^2 is

$$\ln\left(\sigma_{h+1}^2\right) = \alpha_0 + \alpha_1 Z_h + \gamma_1\left(|Z_h| - E(|Z_h|)\right) + \beta_1 \ln\left(\sigma_h^2\right) \quad (4.4.4)$$

Equation (4.4.4) reveals that the parameters on the right are recognized at time h, and the forecast for the next day's volatility is simply $\sigma_h^2(1) = \sigma_{h+1}^2$.

4.5 The GJR-GARCH Model

[101] examined an alternative method for modeling the asymmetric properties of positive as well as negative stock returns, well known as a GJR-GARCH model. Equation (4.5.1) demonstrates GJR-GACRH (1, 1) model:

$$\sigma_{\mathrm{t}}^2 = \alpha_0 + \sum_{\mathrm{j=1}}^{\mathrm{p}} \beta_{\mathrm{j}}\sigma_{\mathrm{t-j}}^2 + \sum_{\mathrm{i=1}}^{\mathrm{q}} \alpha_{\mathrm{i}}\varepsilon_{\mathrm{t-i}}^2 + \gamma_{\mathrm{i}}\varepsilon_{\mathrm{t-1}}^2\, \mathrm{d}_{\mathrm{t-1}} \qquad (4.5.1)$$

Where, α_i and β_j are parameters under estimation and γ represents the asymmetric parameter.

Furthermore from the equation (4.5.2) shows the GJR-GARCH (1, 1) model is the one that is most frequently used for variance σ^2 estimation, and it takes the following form:

$$\sigma_{\mathrm{t}}^2 = \alpha_0 + \beta_1\sigma_{\mathrm{t-1}}^2 + \alpha_1\varepsilon_{\mathrm{t-1}}^2 + \gamma_1\varepsilon_{\mathrm{t-1}}^2\, \mathrm{d}_{\mathrm{t-1}} \qquad (4.5.2)$$

Where,

$$d_{t-1} = \begin{cases} 1 & \text{when} \quad \varepsilon_{t-1} < 0 \\ 0 & \text{when} \quad \varepsilon_{t-1} \geq 0 \end{cases}$$

By including the indicator, the model captures the asymmetrical nature of a time series. When applied to financial return data, the indicator function returns a value of one if there is a loss and zero if there are profits.

The following equation (4.5.3) is the GJR-GARCH model for one day ahead forecast volatility model:

$$\sigma_{\mathrm{h+1}}^2 = \alpha_0 + \beta_1\sigma_{\mathrm{h}}^2 + \alpha_1\varepsilon_{\mathrm{h}}^2 + \gamma_1\varepsilon_{\mathrm{h}}^2\, \mathrm{d}_{\mathrm{h}} \qquad (4.5.3)$$

The GJR-GARCH model also shows similar properties to the E-GARCH model, both of which are capable of capturing the asymmetric effect of positive and negative shocks.

4.6 The Asymmetric Power ARCH (APARCH) Model:

Ding, Granger, and Engle (1993) developed the Asymmetric Power ARCH (APARCH) model, which is one of the most promising ARCH-type models. The term APARCH refers to a group of models that includes both ARCH and GARCH models. The following equation (4.6.1) shows Ding, Granger, and Engle (1993) introduced model and defined by APARCH (p, q) model:

$$\varepsilon_t = z_t \sigma_t;$$

$$\sigma^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} \tag{4.6.1}$$

Where $\alpha_0, \alpha_i, \gamma_i, \beta_j$ and δ those the parameters that must be estimated. And γ_i which illustrates the leverage effect. Negative information is represented by a positiveγ_i, has which a greater impact on price volatility than positive information. And δ demonstrates the effect of leverage.

This model can be used to develop or estimate certain other models. This model incorporates the ARCH and GARCH models; we can obtain multiple models by modifying the parameters. Here are the given some examples,

Assume $\delta = 2,\ \beta_{\mathrm{j}} = 0(j = 1, \dots, p),\ \gamma_{\mathrm{i}} = 0(i = 1, \dots, q)$, then Asymmetric ARCH (well known as APACH) model converted to ARCH model [42].

- At the same time when we change parameters $\delta = 2,\ \gamma_i = 0(i = 1, \dots, q)$ Asymmetric ARCH model will be established as GARCH model [18].
- By changing parameters δ = 1 and γ_{i} = 0, TS-GARCH of Taylor and Stuart [102].
- Furthermore, GJR-GARCH when δ = 2 (Glosten, Jagannathan, and Runkle, 1993) and T-ARCH of Zakoian when δ = 1 [103].
- Moreover, Logarithmic ARCH when δ = 0 [104]
- N-ARCH of Higgins and Bera when δ = 0 and β_{j} = 0 [105]

Furthermore, for our analysis, we consider an APARCH (1, 1) specification with one day ahead forecast h. Equation (4.6.2) shows one day ahead forecast volatility of the APACH(1, 1) model:

$$y_t = \mu + \varepsilon_t$$
$$\sigma_{h+1}^{\delta} = \alpha_0 + \alpha_1(|\varepsilon_h| - \gamma\varepsilon_h)^{\delta} + \beta_1\sigma_h^{\delta} \quad (4.6.2)$$
$$z_t \sim \text{ i.i.d. } N(0,1).$$

Initially, estimation was performed using numerical gradients. In the second stage (with the same initial values as in the first scenario), the estimate was performed using the analytical gradients provided in the preceding sections.

4.7 FI-GARCH

Baillie et al. (1996) familiarized the Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) method. The FIGARCH model generated considerable interest and was applied to numerous instances of financial volatility due to its capacity to reflect the volatility's persistence. The fundamental objective of establishing the FIGARCH model was to create a more adaptable class of processes for the conditional variance that may explain and represent the observed temporal dependencies volatility in financial markets. Expressly, the FIGARCH model permits only a moderate hyperbolic decay rate for the lagged squared or absolute innovations in the conditional variance function. The persistence of a time series defines how quickly the influence of a current shock would dissipate and has been extensively observed and studied concerning numerous financial series data. This model may accommodate the time dependency of variance, a leptokurtic distribution for unconditional returns, and a long-memory behavior for conditional variances.

As we conducted only 1-step ahead forecast of FI-GARCH(1, 1), here is the given one step ahead forecast of h is by

$$\sigma_{h+1} = \alpha_0(1 - \beta(1))^{-1} + \lambda_1\epsilon_h^2 + \lambda_2\epsilon_h^2 + \text{ ……..} \quad (4.7.1)$$

4.8 Value-at-Risk

VaR is a financial statistic used to quantify investment risk. It is a statistical technique used to calculate the potential loss that might emerge in a portfolio of investments over a specified period. The value-at-risk is the likelihood that a portfolio will lose more than a particular sum.

Value-at-Risk (VaR), which corresponds to the least favorable expected outcome of a portfolio over a defined period and with a certain level of confidence, is the most common risk measure. VaR is an estimate of the tails of the empirical distribution. Numerous applications assume that asset returns are normally distributed, despite such a well-established fact that they demonstrate skewness and excess kurtosis, resulting in an underestimated or exaggeration of the actual VaR [106]. It is common knowledge that there are several sources of error in VaR estimations, such as sampling errors, data difficulties, insufficient specification, model defects, etc. These variables will lead to frequent estimating errors. Additionally, we must consider the inherent, intrinsic property of VaR values. Hence, we must first monitor VaR values and then evaluate models based on their compatibility with subsequent realized returns, given the confidence interval from which VaR values were produced [106].

Firstly, the GARCH(*p*, *q*) model's one-step-ahead conditional variance forecast, $\hat{\sigma}^2_{t+1|t}$ what we measured from different GARCH models. It is simple to generate one-step ahead VaR forecasts under all distributional assumptions and for zero mean observations as

$$\mathrm{VaR}_{t+1|t} = F(\alpha)\hat{\sigma}_{t+1|t} \qquad (4.8.1)$$

with $F(\alpha)$ representing the matching quantile (95th or 99th) of the expected distribution and $\hat{\sigma}_{t+1|t}$ representing the forecast of the conditional standard deviation at time $t + 1$ given the information at time t**.**

As VaR is considered only for the standard normal distribution then the below formula will be used to calculate VaR:

$$\mathrm{VaR}\ (\alpha) = -\sigma\Phi^{-1}(\alpha) \qquad (4.8.2)$$

Here σ is the volatility forecast for the following day, and $\Phi(\cdot)$ is considered as a cumulative disturbance function associated with standard normal variable. In this study VaR will be used to forecast the time-varying volatility with different GARCH-type model with 5% and 1% confidence interval. Furthermore, for GARCH-type models one-day-ahead volatility forecast estimated with the following formula:

$$\hat{\sigma}_{t+1} = \sqrt{\hat{a}_0 + \hat{a}_1 y_t^2 + \hat{b}_1 \hat{\sigma}_t^2} \qquad (4.8.3)$$

Here $\hat{a}_0$, $\hat{a}_{1,}$ and $\hat{b}_1$ are estimated parameters or coefficients. Moreover, parameters will be re-estimated for each volatility forecast.

5. Model Fitting and Forecasting Evaluation

Several tests for conditional variance predictions should be considered to evaluate the model's accuracy and predictive power of forecast. Furthermore, once the model fitting is complete, it is necessary to compare the fit quality of the various models. Moreover, whether the forecasted models are nested and how well each model fits the in-sample data can be measured using different measures by comparison. This section focuses on forecasting short- and medium-term volatility.

Consequently, we will investigate goodness-of-fit and forecast evaluation tests in the subsequent subsections.

5.1 Information Criteria Tests:

Information criterion is a standard method for comparing models of different orders *p* and *q*. A standard goodness-of-fit test tells us when a forecasting model is statistically significant; nonetheless, it does not inform us how to choose among these models [92]. Fortunately, tests like the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are better fitted to prevent such overfitting or underfitting models [92]. The fundamental concept underlying information criteria tests is the maximum likelihood of each model, which tells us how fitted the model is. How the penalty is assessed varies depending on the information criteria test.

Furthermore, AIC and BIC are considered comparative quality measures to identify the best-fitting model among several. The forecasted model is selected based on the minor information criteria coefficient because it fits in-sample data. In this thesis, we will use the Akaike information criterion (AIC) as well as the Bayesian information criterion (BIC) will be applied to find the in-sample fitted model.

Equation (5.1.1) represents the Akaike information criterion:

$$\mathrm{AIC} = -2\log\, L(\hat{\theta}) + 2k \tag{5.1.1}$$

Where the maximal logarithmic likelihood function for a volatility model with k parameters is represented by $L(\hat{\theta})$.

From the equation (5.1.2) the Bayesian information criterion is defined by

$$\mathrm{BIC} = -2\log L(\hat{\theta}) + k\log n \tag{5.1.2}$$

Equation (5.1.1) and Equation (5.1.2) utilize the same syntax as the AIC, adding the additional parameter *n*, which represents the total quantity of data points covered for the in-sample period. However, the BIC tends to select a more parsimonious model since it penalizes the number of parameters more than the AIC. The smaller the value of the criterion, the better when comparing the in-sample fit of various models based on AIC as well BIC information criteria.

5.2 Mean Squared Error (MSE)

We will evaluate our conditional variance forecasts using the mean squared error, well-known as MSE, which is a quadratic loss function [107]. Prior author asserts that MSE is a standard method for evaluating forecast models for in-sample and out-of-sample cross-validation. However, equation (5.2.1) demonstrates that MSE is the average squared difference between the actual and predicted conditional variance. Furthermore, it tends to penalize significant forecast errors more strongly than other standard accuracy measures and is therefore considered the best measurement for determining which methods avoid huge errors.

$$\text{MSE} = \frac{1}{T}\sum_{t=1}^{t=T} (\hat{\sigma}_t^2 - \sigma_t^2)^2 \qquad (5.2.1)$$

A significant deviation is assigned a significantly greater weight than an accumulation of tiny deviations, even if the combined deviations equal one significant deviance, as MSE is measured in squared error terms. Furthermore, a smaller MSE indicates better accuracy for the forecast model.

5.3 Root Mean Squared Error (RMSE)

The Root mean square error, well-known as RMSE, is analyzed when attempting to measure the predictive capacity of volatility models. In addition, the calculation of RMSE is the square root of the sample's average squared error term from the forecasted model. That means RMSE shows the difference between actual observations and predicted results by the forecast model. When a forecast model shows tiny or smaller error terms, it indicates more accurate cross-validation [92], [102]. The following equation (5.3.1) shows the RMSE:

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{t=T} (\hat{\sigma}_t^2 - \sigma_t^2)^2} \qquad (5.3.1)$$

5.4 Mean Absolute Error (MAE):

The next forecast accuracy measure is MAE. In the bargain, Mean Absolute Error (MAE) is a tool for forecasting error assessment utilized in this study for comparison purposes. Sometimes, the evaluation of forecast performance is highly sensitive to error measures. As a result, models may be ranked differently according to the accuracy test metrics used, but consistent estimations should be maintained in accordance with previous research. The Mean Absolute Error (MAE) is defined by

$$\text{MSE} = \frac{1}{T}\sum_{i=1}^{T} \left|\hat{\sigma}_i^2 - \sigma_i^2\right| \qquad (5.4.1)$$

Equation (5.4.1) shows MSE formula

6. Significance test of the dataset

6.1 Jarque-Bera test:

The Jarque-Bera test is a normality test for the time series data developed in 1987 [108]. Due to its simplicity, it becomes an extensive test for normalcy. JB test identifies the dataset's statistical observations, and its standardized residual distribution follows a Gaussian distribution [108]. However, the null hypothesis of the JB test states that the sample data are typically distributed, including skewness and kurtosis coefficients. In contrast, according to the alternative hypothesis H_a,, the sample time series data are not normally distributed. The formula for the Jarque-Bera test statistic or the Lagrange multiplier (LM) test statistic is given:

$$JB = \frac{n}{6}\left[s^2 + \frac{1}{4}(K-3)^2\right] \tag{6.1.1}$$

Where JB represents the Jarque-Bera test value, *n* represents the whole number of actual observations, *S* represents skewness, and *K* represents kurtosis. Finally, the greater the JB-value, the less likely it follows by the normal distribution.

6.2 Augmented Dickey-Fuller Test

In the time series, data stationarity of the series is crucial for the ARCH-type models. In static data, shocks gradually diminish, diminishing their impact over time. On the other hand, non-stationary series shocks are permanent, which means that the shock's persistence has an infinite effect. Many financial data show unit roots, which means data is not stationary.

However, whether a time series is stationary or not, the presence of a unit root can be determined through the statistical tests in our dataset; we have conducted an ADF test to find the unit root of the data. When it is not stationary, case asset prices need to generate differencing returns that confirm that data is stationary [109]. Equation (6.2.1) shows the Dickey-Fuller test formula:

$$y_t = \alpha + \phi y_{t-1} + \varepsilon_t \tag{6.2.1}$$

Here ϕ is the autoregressive coefficient, α is the constant, and ε_t is the random disturbance term what explain the stationary. Equation (6.2.2) is the changing of auto-correlation Brooks (2014), however, it is given following equation:

$$\Delta_{y_t} = \alpha + \Psi y_{t-1} + \varepsilon_t \tag{6.2.2}$$

as it is more intuitive and simpler to compute in practice and Ψ is *($\phi - 1$).* The test only holds true if ε_t is white noise process. Finally, the Dickey-Fuller test is enhanced by adding q lags of the dependent variables to its equation. Equation (6.2.3) is given the final version of Dickey-Fuller test:

$$\Delta_{y_t} = \Psi_{y_{t-1}} + \sum_{i=1}^{q} \alpha_i \Delta_{y_{t-1}+\varepsilon_t} \tag{6.2.3}$$

The null hypothesis and alternate hypotheses are stated as follows:

$$\mathrm{H}_0: \Psi = 0$$
$$\mathrm{H}_1: \Psi < 0$$

The null hypothesis express non-stationary that means there is unit root.

6.3 Ljung-Box test

The next significance test is the Ljung-Box test, which is used to decide the data points in a time series are associated with their initial values. Based on the Box-Pierce test, which performed poorly with small sample numbers, Ljung and Box (1978) developed the Ljung-Box test. Equation (6.3.1) represents the Ljung-Box test:

$$Q(m) = T(T+2)\sum_{\ell=1}^{m} \frac{\hat{\rho}_\ell^2}{T-\ell} \tag{6.3.1}$$

Here *m* is the longest possible lag, *T* is define as sample size, $\hat{\rho}_\ell$ is define by autocorrelation coefficient, and *k* is define by number of lags being examined (Tsay, 2010). According to the null hypothesis of the Ljung-Box test, there is no autocorrelation. Ljung-Box test assists to find the ARCH effect of the time series data.

7. Forecasting procedure

Prior to implementing these GARCH-type models, the forecasting process must be established. The data splitting is based on covid-19, and we divide the data sample into two distinct periods for estimation and forecasting. The sample time begins on January 1, 2015, and ends on February 28, 2020, as an in-sample, while the out-of-sample period begins on March 1, 2020, and ends on June 30, 2022. The in-sample consists of 1,297 daily data covering periods of relatively big and modest fluctuations in volatility, whereas the out-of-sample consists of 588 observations outside the sample. Thus, there are a total of 1,886 daily observations (see Table 2)**.** However, In-sample projections paint an overly optimistic view of the predictive ability of a model. Out-of-sample will be set as a test to validate the model forecast through accurate prediction. To avoid over fitting difficulties, the sample will be separated into two periods. Consequently, it is usual practice to utilize the out-of-sample forecast to determine which model is best for volatility forecast [96]. **Table 2** shows the data sample of a total of 1886 observations.

Table 2: Summary of the different sample periods used in this thesis

Data Sample	Start date	End date	Observations
Full sample	01/01/2015	06/30/2022	1,886
In-sample	01/01/2015	02/28/2020	1,297
Out-of-sample	03/01/2020	06/30/2022	588

Following forecasting models will be considered for the study: ARCH(1), GARCH(1,1), E-GARCH(1,1), GJR-GARCH(1,1), I-GARCH(1,1), APARCH(1,1), and FI-GARCH(1,1) in R using the "rugarch" package.

The study also focuses on parsimonious forecast models due to their computational efficiency and simplicity, as the order structure of these conditional variance equations offers many possible combinations. Notably, this work investigates what makes a particular GARCH model so effective in predicting volatility instead of merely identifying a model with the best specifications by luck. Therefore, just one lag order will be used for all forecast models.

The elementary construction is as follows: if the complete set consists of T number of data points $\{p_1, p_2, \ldots, p_n\}$ the data is divided into the subset $\{p1, p2, \ldots, pn\}$ and $\{p_{n+1}, p_{n+1}, \ldots, p_T\}$.

Let h represent the largest forecast horizon of interest, that is, one is interested in projections ranging from one step to h steps in the future. Finally, the out-of-sample forecasting evaluation procedure, utilizing a so-called recursive scheme, is carried out. The following steps have done to conduct forecast:

1. Set $m = n$ as the first origin of the forecast. Then, each model is fitted using the data$\{p_1, p_2, \ldots, p_m\}$.
2. Calculate the forecast errors for each 1- to h-step ahead forecast for each model $e(i, m) = \sigma^2_{\text{actual},1} - \sigma^2_{\text{Forecaste},1}, \quad i = 1,2..,h$
3. progress the origin by 1, $m = m + 1$, and begin again at step 1. And repeat this procedure until the origin of the forecast m equals the final data point T.

Finally, just the once all forecasts and related forecast errors for each model have been generated, the sole remaining step is to assess the 1-step to h-step ahead forecasts for each model using a loss function.

7.1 Data Description

We will employ three different main equity indices in this thesis paper, all of them from US stock market. Furthermore, for the sake of our study, we will employ the S&P 500, Nasdaq 100 and Dow Jones indices, respectively. However, the Oxford-Man Institute's realized library is used to obtain time series data of realized volatility estimates. The database provides examples of numerous realized volatility estimations based on various sampling techniques and Yahoo Finance is used to collect the data of three indices. The 5-minute and 10-minute subsampled realized variances are used as proxies in this study. The formula for calculating the daily log returns of stock indices is $r_t = \ln(P_t/P_{t-1})$. The sample period of this dataset begins on January 1, 2015 and ends on June 30, 2022 yielding a total of 1886 daily observations, while the Oxford-Man Institute database begins in January 2000. However, missing value omitted from the dataset through the R function.

The following **Table 3** demonstrates the general information utilized for the equity indexes in this thesis paper.

Table 3: The Table shows the Index country, stock market, stock index, symbol and constituents respectively.

Country	Stock exchange	Stock Index	Trading Symbol	Constituents
USA	New York Stock Exchange NASDAQ Cboe BZX Exchange	The Standard and Poor's 500 (S&P 500)	GSPC	500
USA	New York Stock Exchange NASDAQ	Dow Jones Industrial Average Index	DJI	30
USA	NASDAQ	NASDAQ 100 Index	NDX	100

In the following part, summary statistics of the sample data will be further upon. Appendix sections provide additional details of particular time series and their accompanying tables and graphs.

8. Empirical Results and Analysis

8.1 Descriptive statistics:

Table 4 shows descriptive summary statistics of the daily logarithmic returns of our selected three stock indices: GSPC, DJI, and NDX. The table specifies the mean, minimum, maximum, standard deviation, skewness, kurtosis, and Jarque-Bera test for these stock indices. It is observed from the data that the return distributions of these three stock indices tend to follow a leptokurtic distribution. The kurtosis estimates for all the stock indices are more significant than three, which implies that all the stock indices have kurtosis return distributions. Another reflection finds that the average logarithmic return seems close to zero for all the indices, and the standard deviation for three indices tends to be close to one.

Table 4: This table provides a statistical overview of the daily logarithmic returns of several stock indices.

Indices	Mean	Std. dev.	Min.	Max.	Skewness	Kurtosis	Jarque-Bera
GSPC	0.0004697	0.01136228	-0.1276522	0.0896832	-1.038408	20.96746	32741
DJI	0.0004014	0.0117596	-0.1384181	0.1076433	-1.097802	25.09227	46790
NDX	0.0007482	0.0133871	-0.1300315	0.0959664	-0.7277528	10.99293	9068.5

Table- 5 demonstrates the existing correlation in our selected sample data of the three stock indices returns. It is observable from the data that all three stock return indices have a strong correlation with each other, whereas GSPC has a strong correlation with DJI, and NDX shows a weak correlation with DJI in our selected sample data returns. One reason for this strong correlation could be that all the stock indices are from the exact geographical location, US-Market.

Table 5: Correlation of stock indices of S&P500, Dow Jones Industry Average and Nasdaq 100

correlation of stock index			
GSPC	1		
DJI	0.96733	1	
NDX	0.92851	0.83723	1

Table 6 shows the correlation matrix among the indices 5-min subsample S&P 500, Dow Jones industrial average, and Nasdaq 100. However, it shows a negative correlation between S&P 500 and DJI. Similarly, DJI and NDX also correlated negatively, which means weak relations with each other. Furthermore, only S&P 500 and NDX correlated positively.

Table 6: Correlation of 5-min subsample of S&P500, Dow Jones Industry Average and Nasdaq 100

Correlation of realized volatility proxy (5- min)			
	GSPC	DJI	NDX
GSPC	1		
DJI	-0.039741043	1	
NDX	0.177339352	-0.07706	1

Table 7 shows the summary statistics of the realized volatility proxies. The features of the descriptive statistics are pretty similar for realized volatility proxies compared to stock return indices. For all three stock samples, kurtosis is more significant than three, which implies that the realized volatility proxies tend to follow a leptokurtic distribution. The average return of volatility proxies is close to zero, and the standard deviation is close to one for all the three stock indices realized volatility proxies.

Table 7: Summary statistics of volatility proxies of 5-min subsample of S&P 500, DJI and NDX

Indices	Mean	Std. dev.	Min.	Max.	Skewness	Kurtosis	Jarque-Bera
GSPC	0.01779	0.7724253	-3.93	3.93	-0.4708571	6.061428	757
DJI	0.009955	0.884278	-5.55	5.32	-0.4405584	9.295435	2 980
NDX	0.02242	0.9607463	-6.83	4.63	-0.6590391	7.639243	1 715

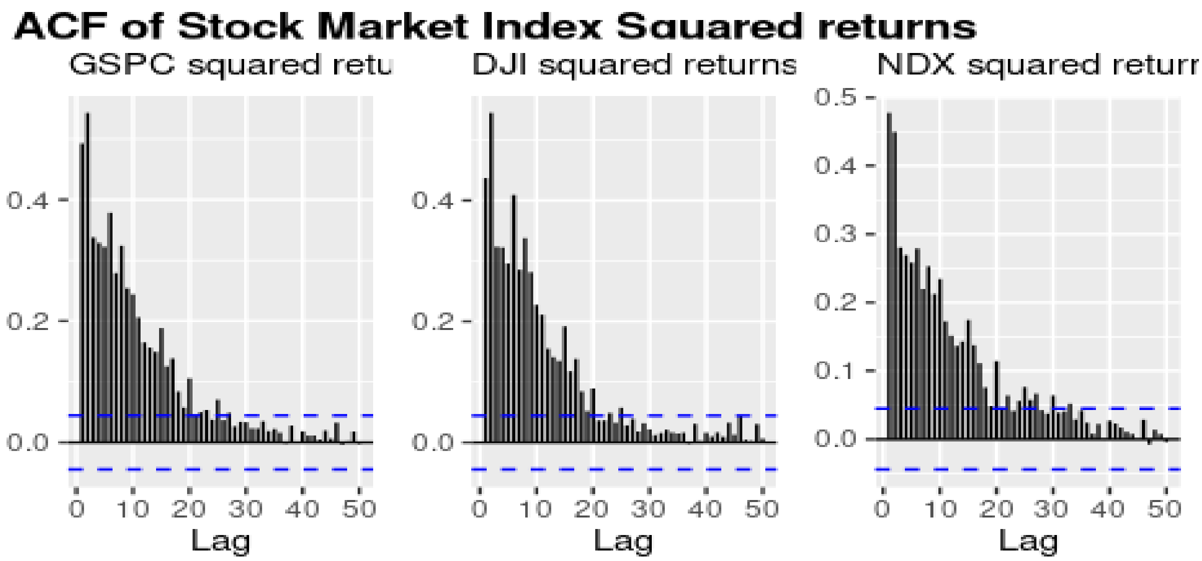


Figure 1: Squared Return ACF of S&P500, Dow Jones Industry Average and Nasdaq 100

Figure 1 explains autocorrelation plots of our selected three stock indices. We observed a significant decay in the autocorrelation coefficients of the variance proxies after 10th lags approximately for all the three stock indices. After some time, the autocorrelation of this volatility tends to meet zero in all three stock indices sample data. Furthermore, these correlograms recommend that asset return volatility for the S&P 500, DJI, and NDX indices tend to be persistent.

Furthermore, from the **figure-1,** Serial correlations for indices returns are less significant for most lags; however, autocorrelations for squared and absolute returns are significant, considerable, and persistent. Furthermore, the result is consistent with the output shown in **Table 3** for the Ljung-Box Q-statistics, and it also confirms by Engle's Lagrange multiplier ARCH effects test. Hence, the null hypothesis of no serial correlation up to lag ten cannot be rejected for returns, while non-linear serial correlation recommends the existence of conditional heteroscedasticity.

Figure 2: Squared Return PACF of S&P500, Dow Jones Industry Average, and Nasdaq 100

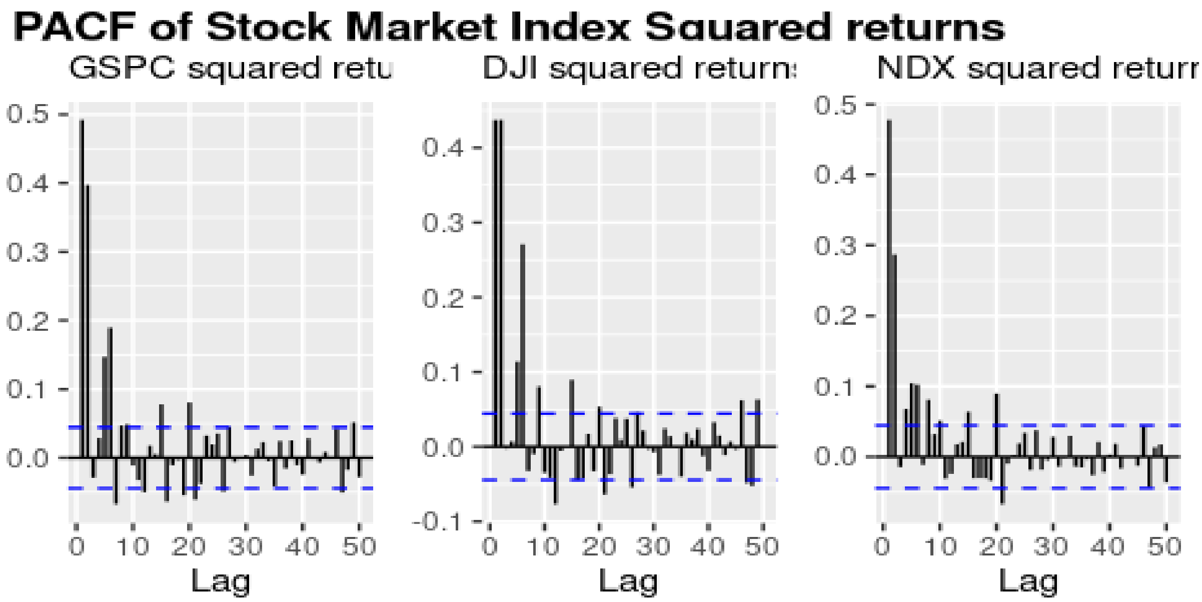


Figure 2 depicts the partial autocorrelation plots of the major stock indexes. Meanwhile, the graph panels reveal significant indications of serial correlation, although in contrast to the preceding figure, the partial autocorrelation function for these observations exhibits a rather rapid declining pattern. After a few lags, the serial correlation coefficients of these volatility proxy series approach zero and afterward remain rather constant. The partial autocorrelation graphs, compared to the preceding correlograms, demonstrate that the stability of asset return volatility is not especially long-lasting.

Normality Test:

In **table 4**, the Jarque-Bera test statistic indicates that the assumption of data normality is rejected at the 5% significance level for all three stock indexes. This result of the Jarque-Bera test implies that the sampled returns on the stock market are not distributed normally. Therefore, the empirical distribution of daily returns has significantly heavier tails than the normal distribution.

Unit Root Test:

The Augmented Dickey-Fuller (ADF) test is utilized to establish stationarity. **Table 8** displayed ADF test coefficients and test outcomes, indicating that the time series data is stationary. Each index's logarithmic return data is subjected to the ADF test. In addition, the null hypothesis is rejected by the ADF test statistics, indicating that the return data series has a unit root. In this instance, the test rejects the null hypothesis of a unit root in time series. We consequently deny that the time series data is stationary. Moreover, all indices' log returns are statistically significant at the 1% level, demonstrating that the logarithmic returns of these stock indexes adhere to a stationary process.

Table 8: Displays the coefficient of the ADF test for the logarithmic return of the 3 indices from the US stock market. The test examined in total 1886 observations. However, * indicates the significance level of 1%

Unit root test for stock index (ADF test)				
	coefficient	Std. error	*t*-stat	*p*-value
GSPC	-1.08434*	0.0366	-3.934	0.01
DJI	-1.05121*	0.01149	-1.451	0.093
NDX	-0.002*	0.001	0.001	2.20E-16

Table 9 illustrates the 5-min subsample realized volatility proxy of stock indices ADF test coefficients result, where the 1% significance level of Augmented Dickey-Fuller test result shows statistically significant for all the indices. In addition, the result also supports to rejection of the null hypothesis for the variance proxies in each of them. Hence, we can say the variance proxies will also follow the stationary process of the unit root test. There are no unit roots in these series.

Table 9: displays the coefficient of the ADF test for the 5-min subsample of S&P 500, DJI, and NDX. The test examined in total 1886 observations. However, indicate the significance level of 1%

Unit root test for proxy (5-min subsample) (ADF test)				
	coefficient	Std. error	*t*-stat	*p*-value
GSPC	-1.06475*	0.7717	0.0345	0.01
DJI	-1.06848*	0.877	-1.871	2.20E-16
NDX	0.03192*	0.9465	1.339	0.01

ARCH Effect Test:

The experiment was conducted using the lag order *q*=1, *q*=5, and *q*=10. **Table 10** displays the outcomes of these examinations. Following the null hypothesis of this ARCH LM test statistic, there is no ARCH effect. However, based on the data for all three stock indexes, the p-value for *lag-1, lag-5,* and *lag-10* of the ARCH LM test statistic is less than 5%. It indicates that the null hypothesis is rejected for the three lag periods and that the ARCH effect is present in all of our chosen stock indexes. These results indicate that the squared residuals equity indices be likely serially dependent, suggesting that forecast models that account for conditional heteroskedasticity may generate more precise volatility estimates.

Table 10: The ARCH LM test's results are shown in this table for S&P 500, Dow Jones Industry Average and Nasdaq 100. We employ one, five, and ten as the three different lag periods in this test. In total 1886 observations were tested for the ARCH LM test. . However, **** and * denotes 1% & 10% significance level.

ARCH LM test statistics for stock index						
	Lag = 1		Lag = 5		lag = 10	
	Test Statistics	*p*-value	Test Statistics	*p*-value	Test Statistics	*p*-value
GSPC	454***	2.20E-16	697.78***	2.20E-16	749.91***	2.20E-16
DJI	346.25***	2.20E-16	638.15***	2.20E-16	732.21***	2.20E-16
NDX	465.46***	2.20E-16	595.36***	2.20E-16	623.78***	2.20E-16

Furthermore, **Table -11** illustrates the ARCH LM test for proxy realized variance. The test was carried out using the lag order of ***q****=1*, ***q****=5*, and ***q****=10*. The null hypothesis of this ARCH LM test statistic represents no ARCH effect. However, from the data of all the stock indices proxy realized, we observe that *the p*-value is less than 5% for *lag 1*, *lag five*, and *lag 10* of ARCH LM test statistic of proxy realized variance. It means the null hypothesis is rejected for the three lag periods, and there is the presence of an ARCH effect in all proxy realized variance.

These results indicate that the squared residuals of selected equity indices have a tendency to be serially dependent, suggesting that forecast models that account for conditional heteroskedasticity may generate more precise volatility estimates.

Table 11: The ARCH LM test's results are shown in this table for S&P 500, Dow Jones Industry Average and Nasdaq 100 Proxy realized variance. We employ one, five, and ten as the three different lag periods in this test. In total 1886 observations were tested for the ARCH LM test. However, **** and * denotes 1% & 10% significance level.

ARCH LM test statistics for Proxy realized variance						
	Lag = 1		Lag = 5		lag = 10	
	Test Statistics	p-value	Test Statistics	p-value	Test Statistics	p-value
GSPC	171.25***	2.20E-16	254.14***	2.20E-16	278.87***	2.20E-16
DJI	258.75***	2.20E-16	556.54***	2.20E-16	619.28***	2.20E-16
NDX	145.44***	2.20E-16	272.88***	2.20E-16	303.27***	2.20E-16

Figure 3. – The graph illustrated the historical price of S&P 500 index, logarithmic return, and squared logarithmic return (volatility). The data starting on January 1, 2015, and ending on June 30, 2022, the sample period has 1886 observations.

Figure 3 above displays the historical price of the S&P 500 index, logarithmic return, and squared logarithmic return (volatility) from the beginning of 2015 to mid-2022. If we observe

the three graphs closely before Covid-19 and during Covid-19, it is evident that S&P 500 indices have volatility clustering. Especially, in the year 2020, during Covid-19, the volatility spike is very high for the price level, logarithmic return, and squared logarithmic return (volatility). It demonstrates the negative impact of Covid-19 on the volatility of stock returns across all US stock market indices. In addition, this graph illustrates that massive changes are likely to be followed by other significant changes, whereas tiny changes tend to be followed by additional minor changes. In addition, the standard deviation is a prominent indicator of stock market volatility. The standard deviation measures return dispersion; thus, the greater the deviation, the greater the likelihood of negative or positive returns. From 2015 through 2022, **Figure 3** depicts the standard deviations of the daily returns of S&P500 stocks.

Furthermore, it can be observed that the standard deviations of monthly stock returns for all three stock indices range between 2 and 4 percent. As a result of Covid-19, however, the S&P 500's standard deviation in 2020 will be substantially greater. Significant standard deviations characterize years with extraordinary returns. These standard deviation figures confirm that the volatility of stock indices was way higher during the Covid-19 period for all of the stock S&P 500. The rest of the indices' historical prices of the DJI index and NDX index, logarithmic return, and squared logarithmic return (volatility) are given in **Figure 4 and Figure 5.**

Figure 4. – The graph illustrated the historical price of Dow Jones Industry Average index, logarithmic return, and squared logarithmic return (volatility). The data starting on January 1, 2015, and ending on June 30, 2022, the sample period has 1886 observations.

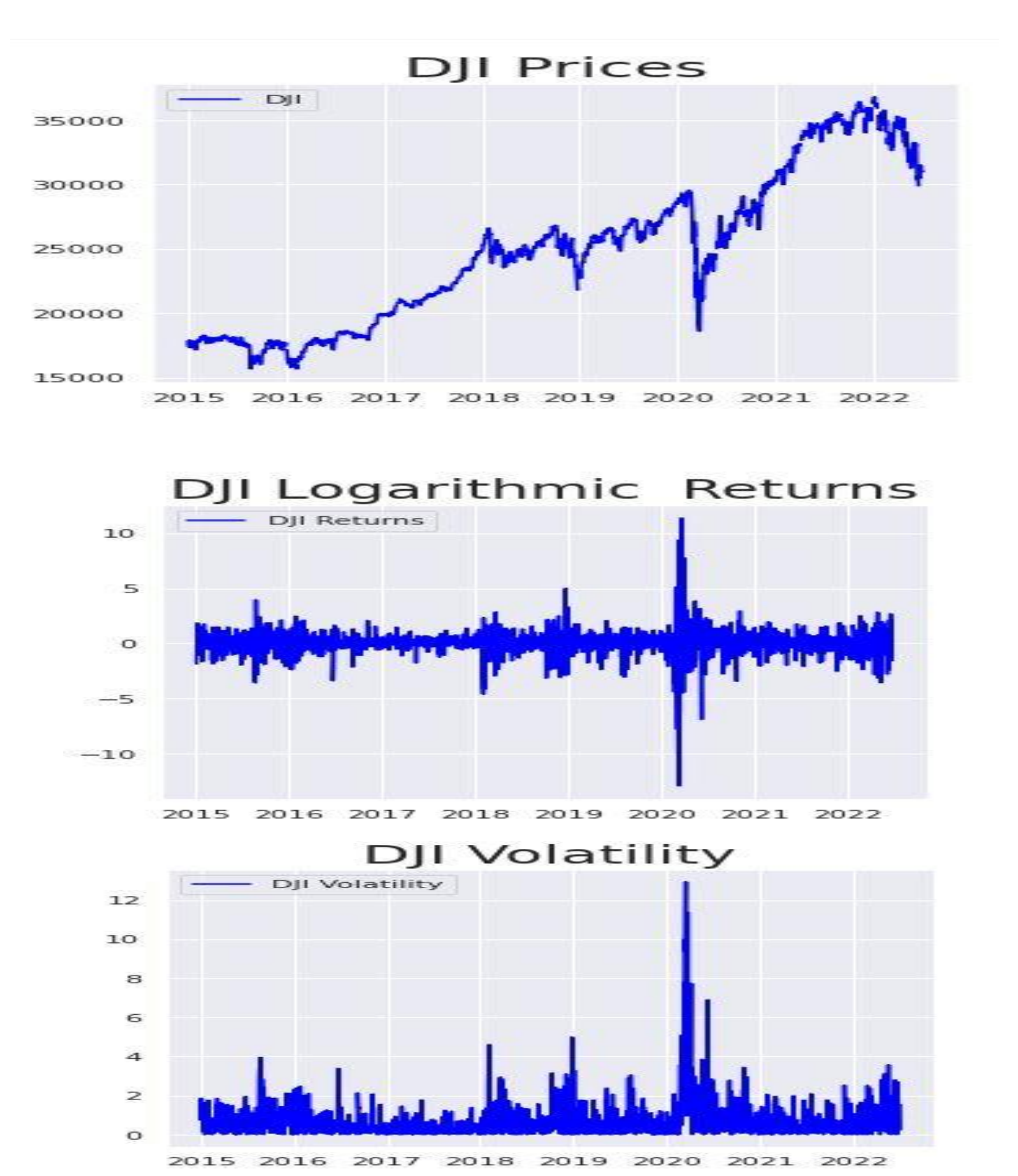
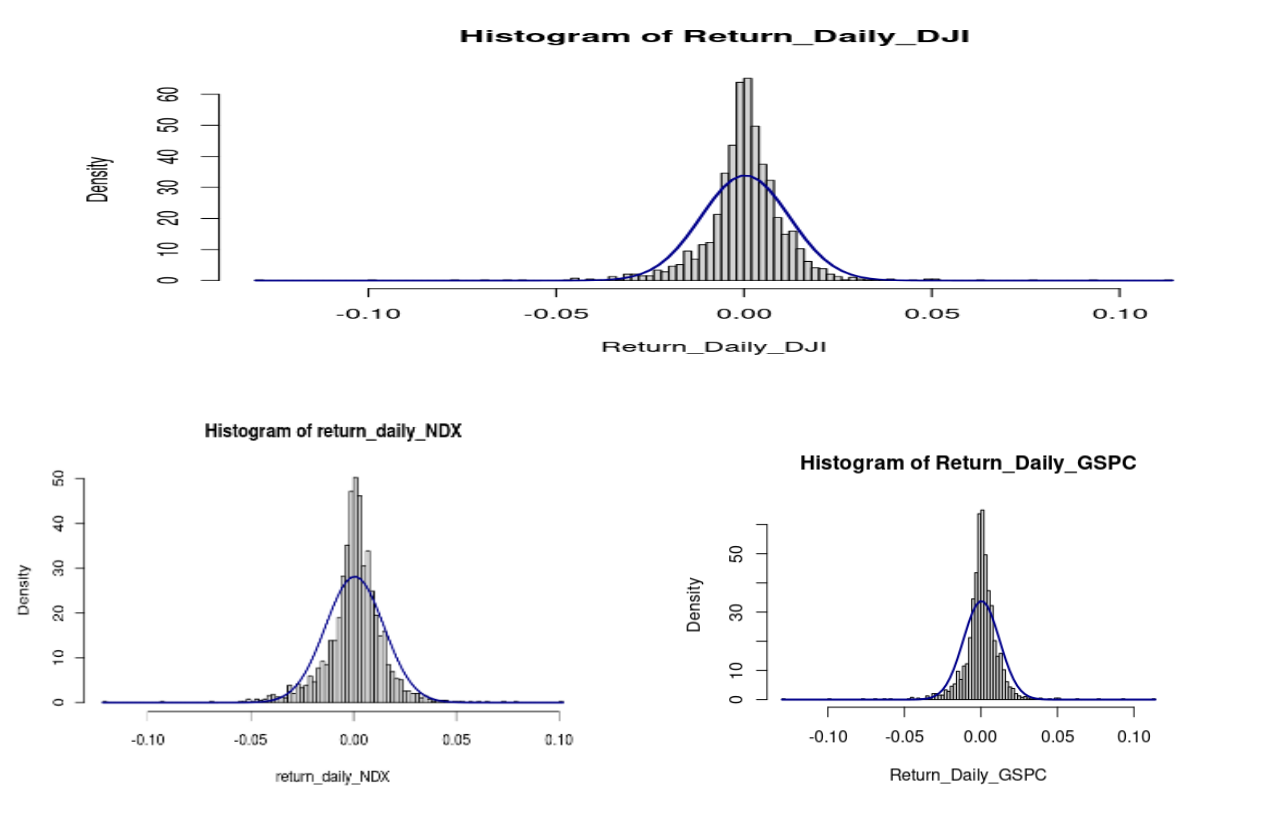


Figure-4 from above shows the historical price of the DJI index, logarithmic return, and squared logarithmic return (volatility) from January 1, 2015, to June 30, 2022. If we notice that the three graphs, meticulously before beginning the covid-19, behave abnormally, it is also undoubtedly evident that the DJI index has a volatility clustering effect. Especially, in the year 2020, during Covid-19, the volatility spike is very high for the price level, logarithmic return, and volatility. It shows the adverse effect of Covid-19 on the DJI index return volatility. However, the NDX index's historical price, log return, and volatility graph are presented in the appendix as a **Figure-6**.

Figure 5 - This histogram displays the return distribution of the S&P 500, Dow Jones Industry Average and Nasdaq 100. In each histogram, the dark-blue curve represents the standard normal distribution.



The **figure-5** shows that the histogram for the S&P 500 exhibits significantly more peaks in the center of the distribution. However, Nasdaq 100 appears to be experiencing a similar pattern; the lowest peak appears in the center of the distribution. Furthermore, Table 1 illustrates summary statistics of the S&P 500 and Nasdaq 100, where skewness and kurtosis show significantly more prominent figures. Moreover, the Jarque-Bera test statistics support this conclusion.

9. Empirical Result and Analysis:

9.1 In-sample fitting results of GARCH Models:

The in-sample fitting results of GARCH-type models are shown in **Table 12**. Based on the goodness-of-fit tests, the main outcome appears to be mixed since there is not one single model that fits the equity indices best in every instance**.** Moreover, from Panel A, we find that E-GARCH(1,1) is the best-fitting model in our subset of volatility forecasting models for the S&P 500 according to the information criteria of AIC and BIC, whereas log-likelihood implies that I-GARCH(1,1) is the best-fitting model in-sample. Furthermore, the E-GARCH(1,1) model fits the in-sample data estimation of the DJI best-fitting model from Panel B. Last but not least, from Panel C, Nasdaq 100 shows E-GARCH(1,1) model is preferred under the Information criteria of AIC and BIC. On the other hand, Log-likelihood indicating I-GARCH(1,1) is the best-fitting model in-sample.

Table 12. - illustrates the outcomes of in-sample goodness-of-fit tests for the previously stated conditional variance of forecasted model. The study phase started on January 1, 2015 and extends on February 28, 2020, giving 1,297 observations in total.*** represents the forecasting model with the best in-sample fit according to the log-likelihood, AIC, and BIC criteria.

Forecast model	Log-likelihood	AIC	BIC
Panel A: S&P 500			
ARCH(l)	4,231.24	-6458.47	-8441
GARCH(1,1)	4,418.80	-6.9965	-6.9802
E-GARCH(1,1)	4,462.28	-7.0638***	-7.0435***
GJR-CGARCH(1,1)	4,448.35	-7.0418	-7.0214
I-GARCH(1,1)	4,414.64***	-6.9915	-6.9793
APARCH(1,1)	4,439.47	-7.0261	-7.0017
FI-GARCH(1,1)	4417.941	-6.9936	-6.9732
Panel B: DJI			
ARCH(l)	4,220.95	-8437.9	-8420.48
GARCH(1,1)	4,405.52	-6.9755	-6.9592
E-GARCH(1,1)	4,001.42***	-7.0308***	-7.0104***
GJR-CGARCH(1,1)	4,434.26	-7.0194	-6.9991
I-GARCH(1,1)	4400.519	6.9691	-6.9569

APARCH(1,1)	4,425.86	-7.0045	-6.9801
FI-GARCH(1,1)	4408.233	-6.9782	-6.9578
Panel C: NDX			
ARCH(l)	3,924.21	-5844.43	-5827.01
GARCH(1,1)	4,068.76	-6.4418	-6.4255
E-GARCH(1,1)	4,111.06	-6.5072***	-6.4869***
GJR-CGARCH(1,1)	4,099.55	-6.489	-6.4686
I-GARCH(1,1)	4061.004***	-6.4311	-6.4188
APARCH(1,1)	4089.776	-6.4719	-6.4475
FI-GARCH(1,1)	4063.351	-6.4316	-6.4113

Table 13 – In-sample 1-period generalized autoregressive conditional coefficient estimates are presented in Table 13, along with the corresponding p-values. The study phase started on January 1, 2015 and concludes on February 28, 2020, generating a total of 1297 observations. Panel A illustrates the S&P 500, while Panels B and C depict DJI and NDX, respectively.

Table 13. - In- sample One period coefficients estimates of GARCH models of Stock indices

Forecast model	α_0	α_1	β_1	β_2	γ_1	γ_2
Panel A: S&P 500						
GARCH(1,1)	0.000161	0.000001	0.019845	0.019173		
	(0e+00)	(3e-06)	(0e+00)	(0e+00)		
E-	0.000146	0.006568	0.017945	0.000566	0.00406	
GARCH(1,1)	(0.014314)	(0.0000)	(0.0000)	(0.0000)	(0.000)	
GJR-	0.000157	0	0.009598	0.016335	0.042601	
CGARCH(1,1)	(0.009927)		(0.102018)	(0.0000)	(0.0000)	
I-GARCH(1,1)	0.000175	0.000001	0.027344			
	(0.000001)	(0.001249)	(0)			
APARCH(1,1)	0.000182	0	0.028725	0.034642	0.071116	0.035829
	(0.052062)	(0.933193)	(0.000009)	(0.0000)	(0.000006)	(0.000)
FI-	0.000176	0.000002	0.098959	0.124344	0.173355	
GARCH(1,1)	(0.000002)	(0.055547)	(0.269450)	(0.000138)	(0.000607)	
Panel B: DJI						

GARCH(1,1)	0.000181	0.000002	0.127238	0.162881	0.090434	
	(0.000013)	(0.140972)	(0.427482)	(0.031917)	(0.000001)	
E-GARCH(1,1)	0.000171	0.020404	0.012453	0.002124	0.022419	
	(0.008308)	(0)	(0)	(0)	(0)	
GJR-CGARCH(1,1)	0.000172		0.004558	0.013411	0.035169	
	(0.003306)		(0.427070)	(0.0000)	(0.0000)	
I-GARCH(1,1)	0.000179	0.000002	0.037992			
	(0.000003)	(0.034137)	(0)			
APARCH(1,1)	0.000157		0.01119	0.036542	0.144934	0.034188
	(0.000423)	(0.909180)	(0.0000)	(0.0000)	(0.001545)	(0.0000)
FI-GARCH(1,1)	0.000181	0.000002	0.127238	0.162881	0.090434	
	(0.000013)	(0.140972)	(0.427482)	(0.031917)	(0.000001)	
Panel C: NDX						
GARCH(1,1)	0.000237	0.000001	0.017804	0.017048		
	(7e-06)	(0e+00)	(0e+00)	(0e+00)		
E-GARCH(1,1)	0.000209	0.003621	0.015812	0.000224	0.024044	
	(0.023476)	(0.0000)	(0.0000)	(0.0000)	(0.000009)	
GJR-CGARCH(1,1)	0.000226		0.006001	0.013257	0.033254	
	(0.010233)		(0.999996)	(0)	(0)	
I-GARCH(1,1)	0.000261	0.000002	0.033358(0)			
	(0.00002)	(0.00184)				
APARCH(1,1)	0.000224		0.015866	0.019755	0.140895	
	(0.013905)	(0.671071)	(0.005366)	(0.0000)	(0.000015)	0.032891
						(0.0000)
FI-GARCH(1,1)	0.000236	0.000002	0.026321	0.035761	0.166713	
	(0.000009)	(0.000084)	(0.0000)	(0.0000)	(0.001320)	

Table 13 demonstrates estimated coefficients and associated p-values for the different GARCH models. The coefficients and the *p*-values indicate the GARCH models' impact on the heteroscedasticity in the stock indices or stock market. Furthermore, *the p*-value is associated with the hypothesis of whether its estimated coefficient is equal to zero. Different parameters estimate expressed information as an example of how the new information reacts with the volatility. Similarly, express how past volatility reacts with present volatility. However, based on the analysis of the parameters, we can also find a volatility clustering effect on each stock index. The leverage effect and power effect are also a present in the analysis. In the extended

GARCH family of models, the significant and positive coefficient of stock volatility indices indicates that it delivers appropriate information for explaining the volatility process.

9.2 Comparative Forecast Evaluation:

Table 14 demonstrates the out-of-sample forecasted evaluation; three loss functions are used to evaluate the model performance. The Mean squared error, root mean squared error and mean absolute error is loss function to evaluate the model performance and find the best model that explains with the highest accuracy. Nevertheless, stock indices out-of-sample forecast accuracy performs better than the 5-min sub-sample realized proxy volatility. Furthermore, the S&P 500 all the GARCH-type models explain better than the rest. From table 14, we can evaluate that I-GARCH and FI-GARCH is better performing model in terms of model predictions and accuracy rate.

Table 14 - Out-of-sample forecast S&P 500, DJI and NDX indices and realized variance 5-min sub-sample from based on in-sample 1297 observations and out-of-sample 588 observations.

Out-of-sample forecast of Stock indices				Out-of-sample forecast of realized volatility(5-min)			
Forecast model	MSE	RMSE	MAE	Forecast model	MSE	RMSE	MAE
Panel A: S&P 500				**Panel A: S&P 500**			
GARCH(1,1)	0.00028	0.01669	0.00992	GARCH(1,1)	0.7253	0.851645466	0.6044
E-GARCH(1,1)	0.00028	0.01670	0.00997	E-GARCH(1,1)	0.72400	0.850881895	0.60520
GJR-CGARCH(1,1)	0.00028	0.01670	0.00996	GJR-CGARCH(1,1)	0.72420	0.850999412	0.60510
I-GARCH(1,1)	0.00028	0.01670	0.00997	I-GARCH(1,1)	0.72520	0.851586754	0.60440
APARCH(1,1)	0.00028	0.01670	0.00995	APARCH(1,1)	0.72390	0.850823131	0.60580
FI-GARCH(1,1)	0.00028	0.01669	0.00992	FI-GARCH(1,1)	0.72530	0.851645466	0.60440
Panel B: DJI				**Panel B: DJI**			
GARCH(1,1)	0.00031	0.01757	0.01004	GARCH(1,1)	1.26900	1.126499001	0.75820

E-GARCH(1,1)	0.00031	0.01757	0.01006	E-GARCH(1,1)	1.26800	1.126055061	0.75910
GJR-CGARCH(1,1)	0.00031	0.01757	0.01006	GJR-CGARCH(1,1)	1.26800	1.126055061	0.75910
I-GARCH(1,1)	0.00031	0.01757	0.01006	I-GARCH(1,1)	1.26900	1.126499001	0.75820
APARCH(1,1)	0.00031	0.01757	0.01006	APARCH(1,1)	1.26800	1.126055061	0.75920
FI-GARCH(1,1)	0.00031	0.01757	0.01004	FI-GARCH(1,1)	1.26900	1.126499001	0.75830
Panel C: NDX				**Panel C: NDX**			
GARCH(1,1)	0.00034	0.01849	0.01214	GARCH(1,1)	1.47800	1.215730233	0.88000
E-GARCH(1,1)	0.00034	0.01850	0.01222	E-GARCH(1,1)	1.47900	1.216141439	0.88440
GJR-CGARCH(1,1)	0.00034	0.01850	0.01222	GJR-CGARCH(1,1)	1.47900	1.216141439	0.88340
I-GARCH(1,1)	0.00033	0.01849	0.01213	I-GARCH(1,1)	1.47800	1.215730233	0.88040
APARCH(1,1)	0.00034	0.01850	0.01220	APARCH(1,1)	1.47900	1.216141439	0.88470
FI-GARCH(1,1)	0.00034	0.01850	0.01120	FI-GARCH(1,1)	1.47800	1.215730233	0.88030

9.3 Value-at Risk

Value-at_Risk is forecasted through the one-day ahead various GARCH models, including standard normal distribution. Here VaR is estimated with the 5% and 1% confidence intervals. **Table 15** demonstrates the Panel A E-GARCH (1,1), representing a good model with a 95% daily VaR forecast. Moreover, Panel B I-GARCH (1,1) and APACH(1,1) indicate the best forecasting model. Lastly, Panel C for the NDX stock index GJR-GARCH (1,1) expresses the best forecasting model with 95% and 99% daily VaR forecasting methods.

Table 15: Value-at-Risk forecast of S&P 500, DJI and NDX indices with 5% and 1% significance level.

Forecast model	Unconditional Coverage **Pr. ()**	Conditional coverage **Pr. ()**	Unconditional Coverage **Pr. ()**	Conditional coverage **Pr. ()**
Panel A: S&P 500	*95% daily VaR forecasts*		*99% daily VaR forecasts*	

GARCH(1,1)	0.138	0.232	0	0
E-GARCH(1,1)	0.025	0.039	0.001	0.002
GJR-CGARCH(1,1)	0.12	0.13	0.03	0.03
I-GARCH(1,1)	0.025	0.039	0.001	0.002
APARCH(1,1)	0.062	0.171	0	0
FI-GARCH(1,1)	0.89	0.305	0.005	0.014
Panel B: DJI				
GARCH(1,1)	0.062	0.171	0.001	0.002
E-GARCH(1,1)	0.015	0.052	0.001	0.002
GJR-CGARCH(1,1)	0.015	0.052	0.001	0.002
I-GARCH(1,1)	0.015	0.052	0.001	0.002
APARCH(1,1)	0.015	0.052	0.001	0.002
FI-GARCH(1,1)	0.89	0.983	0.079	0.18
Panel C: NDX				
GARCH(1,1)	0.025	0.039	0	0
E-GARCH(1,1)	0.001	0	0	0
GJR-CGARCH(1,1)	0.001	0	0	0
I-GARCH(1,1)	0.372	0.566	0.002	0.005
APARCH(1,1)	0.062	0.103	0	0
FI-GARCH(1,1)	0.063	0.503	0	0

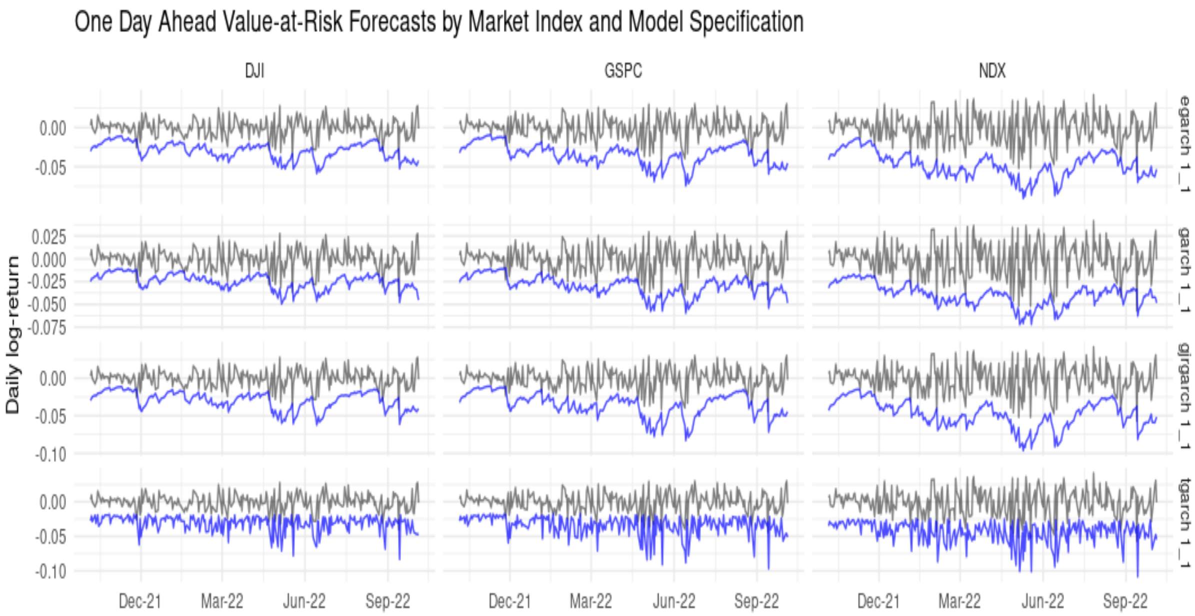


Figure 6.: The figure illustrated One day ahead VaR forecasts for S&P 500, DJI and NDX indices daily log return with the GARCH(1,1), T-GARCH(1,1), GJR-GARCH(1,1) and E-GARCH(1,1) with 5% significance and 5% probability.

10. Research Limitations:

This thesis study is an excellent starting to learn series analysis to predict volatility. However, many methods and techniques are still not covered due to the complexities of r programming. Furthermore, in GARCH forecast models, we only focus on GARCH(1,1) for all the GARCH-type models with one-day ahead forecasting rather than go through many complexities due to technicalities or advanced-level coding. This is the way we try to keep our focus on selecting the best models among all the GARCH-type models. In this study, we also use the python programming language to formulate graphs and predictions.

Moreover, we covered the Value-at-Risk to confirm the forecasting volatility's better accuracy and select the best index among them. Datasets are divided based on the covid-19 effect rather than following any standardization for the cross-validation to find better accuracy. Furthermore, the normal distribution is investigated on all GARCH models. Oxford realized library had shown many datasets as a proxy realized variance among them; we investigated only a 5-min subsample of S&P 500, DJI, and NDX indices.

11. Suggestions for further research

Time series analysis is a trendy topic in finance and economics studies; academia-industry and practitioners also use it for their business, product, sales, and many more forecasting areas. In this study, we keep our analysis with a GARCH-type model; a multi-step forecast also can be an excellent way to select the best-performing model with better accuracy. Many empirical analyses already existed for volatility forecasting; still, many experiments were incomplete due to complexities like the topic. This field is fast growing in terms of its demand to analysis and forecast of the investment, risk measurement, trading analysis and decision making. Adding more realized variances as a proxy also can play role to achieve better forecast volatility result and accuracy. Some other volatility forecast models such as exponential weighted moving average (EWMA), HAR, MIDAS and many more. Last but not least, there is no perfect way to achieve perfect volatility forecasting. Though some models predict better conditional variance than others. In addition, range based realized variance and univariate MZ regression also enhance the future research scope with this research area.

12. Discussion about the implications

This study can be a good resource for the theoretical implication for future study and research development in risk management, forecasting, and volatility management. In practical implication, it can be a good resource that people can use in risk management, trading analysis, forecasting of business or sales forecasting, time series data analysis, and so forth. In recent years financial data analysis has been a growing field of study in data analytics. Hence we can say this research can help to explore more financial data or time series data analysis. In the commodities market, forecasting is a vital task, and we believe trading analysts and data analysts can get a good idea for analyzing their data. As we know stock market volatility leads to the investment risk hence this research can help to investor to make investment decision.

13. Summary and conclusion:

In conclusion, we have examined many forecasting volatility tools and techniques in this thesis study. We have conducted experiments for the S&P 500, Dow Jones Industry Average, and Nasdaq 100. As a realized volatility, we considered a 5-min sub-sample of the S&P 500, DJI, and NDX. This study covers GARCH, E-GARCH, GJR-GARCH, I-GARCH, APARCH, and FI-GARCH for the in-sample and out-of-sample forecasting to find the accuracy through the MSE, RMSE, and MAE as loss function. Furthermore, we have investigated the VaR forecasting method to measure the risk, including backtesting. Moreover, Realized volatility is considered proxy volatility based on the out-of-sample test, and we found a mixture of results to select the best GARCH-type model. The addition of realized variance 5-min sub-sample to the GARCH family model improves the strength of the model. Additionally, the significant and favorable coefficient of realized volatility 5-min sub-sample in the expanded GARCH family of models shows that rpoxy volatility provides evidence relating to explaining the volatility process.Many GARCH model shows good performance in terms of accuracy. We construct out-of-sample 1-day forecast data using a GARCH rolling window method. And to assess the predictive accuracy of distinct volatility forecast including Value-at-Risk with 1% and 5% confidence interval to assess the maximum loss in forecasting model.

After observing descriptive statistics and various graphs and figures and performing some testing, we can conclude that our selected three stock indices sample data return series seems to have a non-normal distribution with excess kurtosis, have a clustering effect, and possess conditional heteroskedasticity. These characteristics make the GARCH-type models the ideal candidate for volatility forecast.

Whatever general market volatility does indeed have a long-term average that seems significant. The volatility will typically be close to the normal in the long-run. Today's volatility seems to be the best predictor of tomorrow's volatility with in short term and short-term volatility is typically relatively constant. In the study we observe due to the Covid-19 had significant impact on volatility clustering effect and conditional variance what make the financial market more volatile. Later on it impacts become the normal situation.

14. References

[1] Z. Lin, “Modelling and forecasting the stock market volatility of SSE Composite Index using GARCH models,” *Futur. Gener. Comput. Syst.*, vol. 79, pp. 960–972, 2018, doi: 10.1016/j.future.2017.08.033.

[2] A. J. Patton and K. Sheppard, “Good volatility, bad volatility: Signed jumps and the persistence of volatility,” *Rev. Econ. Stat.*, vol. 97, no. 3, pp. 683–697, 2015, doi: 10.1162/REST_a_00503.

[3] D. Chun, H. Cho, and D. Ryu, “Economic indicators and stock market volatility in an emerging economy,” *Econ. Syst.*, vol. 44, no. 2, 2020, doi: 10.1016/j.ecosys.2020.100788.

[4] Y. Zhang, F. Ma, and Y. Liao, “Forecasting global equity market volatilities,” *Int. J. Forecast.*, vol. 36, no. 4, pp. 1454–1475, 2020, doi: 10.1016/j.ijforecast.2020.02.007.

[5] M. Naeem, A. K. Tiwari, S. Mubashra, and M. Shahbaz, “Modeling volatility of precious metals markets by using regime-switching GARCH models,” *Resour. Policy*, vol. 64, 2019, doi: 10.1016/j.resourpol.2019.101497.

[6] D. Mei, J. Liu, F. Ma, and W. Chen, “Forecasting stock market volatility: Do realized skewness and kurtosis help?,” *Phys. A Stat. Mech. its Appl.*, vol. 481, pp. 153–159, 2017, doi: 10.1016/j.physa.2017.04.020.

[7] M. Ghani, Q. Guo, F. Ma, and T. Li, “Forecasting Pakistan stock market volatility: Evidence from economic variables and the uncertainty index,” *Int. Rev. Econ. Financ.*, vol. 80, pp. 1180–1189, 2022, doi: 10.1016/j.iref.2022.04.003.

[8] L. De Clerk and S. Savel’ev, “AI algorithms for fitting GARCH parameters to empirical financial data,” *Phys. A Stat. Mech. its Appl.*, vol. 603, 2022, doi: 10.1016/j.physa.2022.127869.

[9] M. S. Irtiza, S. Khan, N. Baig, S. M. A. Tirmizi, and I. Ahmad, “The turn-of-the-month effect in Pakistani stock market,” *Futur. Bus. J.*, vol. 7, no. 1, 2021, doi: 10.1186/s43093-021-00087-4.

[10] G. W. SCHWERT, “Why Does Stock Market Volatility Change Over Time?,” *J. Finance*, vol. 44, no. 5, pp. 1115–1153, 1989, doi: 10.1111/j.1540-6261.1989.tb02647.x.

[11] B. S. Paye, “‘Déjà vol’: Predictive regressions for aggregate stock market volatility using macroeconomic variables,” *J. financ. econ.*, vol. 106, no. 3, pp. 527–546, 2012, doi: 10.1016/j.jfineco.2012.06.005.

[12] S. Bahloul, M. Mroua, and N. Naifar, “The impact of macroeconomic and conventional stock market variables on Islamic index returns under regime switching,” *Borsa Istanbul Rev.*, vol. 17, no. 1, pp. 62–74, 2017, doi: 10.1016/j.bir.2016.09.003.

[13] L. Fang, B. Chen, H. Yu, and Y. Qian, “The importance of global economic policy uncertainty in predicting gold futures market volatility: A GARCH-MIDAS approach,” *J. Futur. Mark.*, vol. 38, no. 3, pp. 413–422, 2018, doi:

10.1002/fut.21897.

[14] C. H. Hsu, H. C. Lee, and D. Lien, “Stock market uncertainty, volatility connectedness of financial institutions, and stock-bond return correlations,” *Int. Rev. Econ. Financ.*, vol. 70, pp. 600–621, 2020, doi: 10.1016/j.iref.2020.08.002.

[15] Y. Xu, X. and Chen, “Empirical Study on Nonlinearity in China Stock Market,” *Quant. Tech. Econ.*, no. 18(3), pp. 110-113., 2010.

[16] B. Mandelbrot, “The Variation of Certain Speculative Prices,” *J. Bus.*, vol. 36, no. 4, p. 394, 1963, doi: 10.1086/294632.

[17] W. Liu and B. Morley, “Volatility forecasting in the hang seng index using the GARCH approach,” *Asia-Pacific Financ. Mark.*, vol. 16, no. 1, pp. 51–63, 2009, doi: 10.1007/s10690-009-9086-4.

[18] T. Bollerslev, “Generalized autoregressive conditional heteroskedasticity,” *J. Econom.*, vol. 31, no. 3, pp. 307–327, 1986, doi: 10.1016/0304-4076(86)90063-1.

[19] Y. E. Arisoy, A. Altay-Salih, and L. Akdeniz, “Aggregate volatility expectations and threshold CAPM,” *North Am. J. Econ. Financ.*, vol. 34, pp. 231–253, 2015, doi: 10.1016/j.najef.2015.09.013.

[20] A. Y. H. Huang, “Value at risk estimation by threshold stochastic volatility model,” *Appl. Econ.*, vol. 47, no. 45, pp. 4884–4900, 2015, doi: 10.1080/00036846.2015.1037439.

[21] A. J. McNeil and R. Frey, “Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach,” *J. Empir. Financ.*, vol. 7, no. 3–4, pp. 271–300, 2000, doi: 10.1016/S0927-5398(00)00012-8.

[22] N. J. Gormsen and R. S. J. Koijen, “Coronavirus: Impact on stock prices and growth expectations,” *Rev. Asset Pricing Stud.*, vol. 10, no. 4, pp. 574–597, 2020, doi: 10.1093/rapstu/raaa013.

[23] H. Yilmazkuday, “COVID-19 effects on the S&P 500 index,” *Appl. Econ. Lett.*, 2021, doi: 10.1080/13504851.2021.1971607.

[24] S. R. Baker, N. Bloom, S. J. Davis, K. Kost, M. Sammon, and T. Viratyosin, “The unprecedented stock market reaction to COVID-19,” *Rev. Asset Pricing Stud.*, vol. 10, no. 4, pp. 742–758, 2020, doi: 10.1093/rapstu/raaa008.

[25] B. N. Ashraf, “Stock markets’ reaction to COVID-19: Cases or fatalities?,” *Res. Int. Bus. Financ.*, vol. 54, 2020, doi: 10.1016/j.ribaf.2020.101249.

[26] J. W. Goodell, “COVID-19 and finance: Agendas for future research,” *Financ. Res. Lett.*, vol. 35, 2020, doi: 10.1016/j.frl.2020.101512.

[27] K. J. Heyden and T. Heyden, “Market reactions to the arrival and containment of COVID-19: An event study,” *Financ. Res. Lett.*, vol. 38, 2021, doi: 10.1016/j.frl.2020.101745.

[28] L. Wang, F. Ma, J. Liu, and L. Yang, “Forecasting stock price volatility: New evidence from the GARCH-MIDAS model,” *Int. J. Forecast.*, vol. 36, no. 2, pp. 684–694, 2020, doi: 10.1016/j.ijforecast.2019.08.005.

[29] S. Baek, S. K. Mohanty, and M. Glambosky, “COVID-19 and stock

market volatility: An industry level analysis," *Financ. Res. Lett.*, vol. 37, 2020, doi: 10.1016/j.frl.2020.101748.

[30] A. Zaremba, R. Kizys, D. Y. Aharon, and E. Demir, "Infected Markets: Novel Coronavirus, Government Interventions, and Stock Return Volatility around the Globe," *Financ. Res. Lett.*, vol. 35, 2020, doi: 10.1016/j.frl.2020.101597.

[31] E. Onali, "COVID-19 and Stock Market Volatility," *SSRN Electron. J.*, 2020, doi: 10.2139/ssrn.3571453.

[32] R. A. Albuquerque, Y. J. Koskinen, S. Yang, and C. Zhang, "Love in the Time of COVID-19: The Resiliency of Environmental and Social Stocks," *SSRN Electron. J.*, 2020, doi: 10.2139/ssrn.3583611.

[33] O. Haroon and S. A. R. Rizvi, "COVID-19: Media coverage and financial markets behavior—A sectoral inquiry," *J. Behav. Exp. Financ.*, vol. 27, 2020, doi: 10.1016/j.jbef.2020.100343.

[34] L. Liu and Z. Pan, "Forecasting stock market volatility: The role of technical variables," *Econ. Model.*, vol. 84, pp. 55–65, 2020, doi: 10.1016/j.econmod.2019.03.007.

[35] A. Kumar, S. Mallick, M. Mohanty, and F. Zampolli, "Market Volatility, Monetary Policy and the Term Premium," *Oxf. Bull. Econ. Stat.*, 2022, doi: 10.1111/obes.12518.

[36] C. Christiansen, M. Schmeling, and A. Schrimpf, "A comprehensive look at financial volatility prediction by economic variables," *J. Appl. Econom.*, vol. 27, no. 6, pp. 956–977, 2012, doi: 10.1002/jae.2298.

[37] D. Li, A. Clements, and C. Drovandi, "Efficient Bayesian estimation for GARCH-type models via Sequential Monte Carlo," *Econom. Stat.*, vol. 19, pp. 22–46, 2021, doi: 10.1016/j.ecosta.2020.02.002.

[38] R. Bu, X. Fu, and F. Jawadi, "Does the volatility of volatility risk forecast future stock returns?," *J. Int. Financ. Mark. Institutions Money*, vol. 61, pp. 16–36, 2019, doi: 10.1016/j.intfin.2019.02.001.

[39] T. Bollerslev, M. Gibson, and H. Zhou, "Dynamic estimation of volatility risk premia and investor risk aversion from option-implied and realized volatilities," *J. Econom.*, vol. 160, no. 1, pp. 235–245, 2011, doi: 10.1016/j.jeconom.2010.03.033.

[40] L. H. Brunnermeier, M., and Pedersen, "Market liquidity and funding liquidity," *Rev. Financ. Stud.*, vol. 22, pp. 2201–2238, 2009.

[41] D. Ardia, "Financial risk management with Bayesian estimation of GARCH models: Theory and applications," *Heidelb. Springer*, 2008, doi: 10.1007/978-3-540-78657-3.

[42] R. F. Engle, "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation," *Econom. J. Econom. Soc.*, pp. 987–1007, 1982, doi: https://doi.org/10.2307/1912773.

[43] M. Ataurima Arellano and G. Rodríguez, "Empirical modeling of high-income and emerging stock and Forex market return volatility using Markov-switching

GARCH models," *North Am. J. Econ. Financ.*, vol. 52, 2020, doi: 10.1016/j.najef.2020.101163.

[44] P. Teterin, R. Brooks, and W. Enders, "Smooth volatility shifts and spillovers in U.S. crude oil and corn futures markets," *J. Empir. Financ.*, vol. 38, pp. 22–36, 2016, doi: 10.1016/j.jempfin.2016.05.005.

[45] J. D. Hamilton and R. Susmel, "Autoregressive conditional heteroskedasticity and changes in regime," *J. Econom.*, vol. 64, no. 1–2, pp. 307–333, 1994, doi: 10.1016/0304-4076(94)90067-1.

[46] D. Ardia, K. Bluteau, K. Boudt, and D.-A. Trottier, "Markov-Switching GARCH Models in R: The MSGARCH Package," *SSRN Electron. J.*, 2017, doi: 10.2139/ssrn.2845809.

[47] T. G. da Silva, O. T. de Carvalho Guillén, G. A. N. Morcerf, and A. de Melo Modenesi, "Effects of monetary policy news on financial assets: Evidence from Brazil on a bivariate VAR-GARCH model (2006–17)," *Emerg. Mark. Rev.*, vol. 52, 2022, doi: 10.1016/j.ememar.2022.100916.

[48] L. Wang, C. Zhao, C. Liang, and S. Jiu, "Predicting the volatility of China's new energy stock market: Deep insight from the realized EGARCH-MIDAS model," *Financ. Res. Lett.*, vol. 48, 2022, doi: 10.1016/j.frl.2022.102981.

[49] T. Bollerslev, A. J. Patton, and R. Quaedvlieg, "Multivariate leverage effects and realized semicovariance GARCH models," *J. Econom.*, vol. 217, no. 2, pp. 411–430, 2020, doi: 10.1016/j.jeconom.2019.12.011.

[50] P. Owusu Junior, A. K. Tiwari, G. Tweneboah, and E. Asafo-Adjei, "GAS and GARCH based value-at-risk modeling of precious metals," *Resour. Policy*, vol. 75, 2022, doi: 10.1016/j.resourpol.2021.102456.

[51] H. Herwartz and J. Roestel, "Asset prices, financial amplification and monetary policy: Structural evidence from an identified multivariate GARCH model," *J. Int. Financ. Mark. Institutions Money*, vol. 78, 2022, doi: 10.1016/j.intfin.2022.101568.

[52] L. Bauwens and Y. Xu, "DCC- and DECO-HEAVY: Multivariate GARCH models based on realized variances and correlations," *Int. J. Forecast.*, 2022, doi: 10.1016/j.ijforecast.2022.03.005.

[53] Z. Y. Guo, "Risk management of Bitcoin futures with GARCH models," *Financ. Res. Lett.*, vol. 45, 2022, doi: 10.1016/j.frl.2021.102197.

[54] G. M. Martin, R. Loaiza-Maya, W. Maneesoonthorn, D. T. Frazier, and A. Ramírez-Hassan, "Optimal probabilistic forecasts: When do they work?," *Int. J. Forecast.*, vol. 38, no. 1, pp. 384–406, 2022, doi: 10.1016/j.ijforecast.2021.05.008.

[55] Y. Wang, Y. Xiang, X. Lei, and Y. Zhou, "Volatility analysis based on GARCH-type models: Evidence from the Chinese stock market," *Econ. Res. Istraz.* , vol. 35, no. 1, pp. 2530–2554, 2022, doi: 10.1080/1331677X.2021.1967771.

[56] B. Setiawan, M. Ben Abdallah, M. Fekete-Farkas, R. J. Nathan, and Z. Zeman, "GARCH (1,1) Models and Analysis of Stock Market Turmoil during COVID-19 Outbreak in an Emerging and Developed Economy," *J. Risk Financ. Manag.*, vol. 14, no. 12, p. 576, 2021, doi: 10.3390/jrfm14120576.

[57] Ștefan C. Gherghina, D. Ștefan Armeanu, and C. C. Joldeș, “COVID-19 Pandemic and Romanian Stock Market Volatility: A GARCH Approach,” *J. Risk Financ. Manag.*, vol. 14, no. 8, p. 341, 2021, doi: 10.3390/jrfm14080341.

[58] Š. Lyócsa, P. Molnár, and T. Výrost, “Stock market volatility forecasting: Do we need high-frequency data?,” *Int. J. Forecast.*, vol. 37, no. 3, pp. 1092–1110, 2021, doi: 10.1016/j.ijforecast.2020.12.001.

[59] I. Wilms, J. Rombouts, and C. Croux, “Multivariate volatility forecasts for stock market indices,” *Int. J. Forecast.*, vol. 37, no. 2, pp. 484–499, 2021, doi: 10.1016/j.ijforecast.2020.06.012.

[60] J. M. Kim, D. H. Kim, and H. Jung, “Estimating yield spreads volatility using GARCH-type models,” *North Am. J. Econ. Financ.*, vol. 57, 2021, doi: 10.1016/j.najef.2021.101396.

[61] Y. Zhong and J. Liu, “Correlations and volatility spillovers between China and Southeast Asian stock markets,” *Q. Rev. Econ. Financ.*, vol. 81, pp. 57–69, 2021, doi: 10.1016/j.qref.2021.04.001.

[62] C. W. S. Chen, T. Watanabe, and E. M. H. Lin, “Bayesian estimation of realized GARCH-type models with application to financial tail risk management,” *Econom. Stat.*, 2021, doi: 10.1016/j.ecosta.2021.03.006.

[63] F. Ioannidis, K. Kosmidou, C. Savva, and P. Theodossiou, “Electricity pricing using a periodic GARCH model with conditional skewness and kurtosis components,” *Energy Econ.*, vol. 95, 2021, doi: 10.1016/j.eneco.2021.105110.

[64] S. Aras, “Stacking hybrid GARCH models for forecasting Bitcoin volatility,” *Expert Syst. Appl.*, vol. 174, 2021, doi: 10.1016/j.eswa.2021.114747.

[65] M. Flores-Sosa, E. Avilés-Ochoa, J. M. Merigó, and R. R. Yager, “Volatility GARCH models with the ordered weighted average (OWA) operators,” *Inf. Sci. (Ny).*, vol. 565, pp. 46–61, 2021, doi: 10.1016/j.ins.2021.02.051.

[66] M. S. Paolella, P. Polak, and P. S. Walker, “A non-elliptical orthogonal GARCH model for portfolio selection under transaction costs,” *J. Bank. Financ.*, vol. 125, 2021, doi: 10.1016/j.jbankfin.2021.106046.

[67] F. Malik, “Volatility spillover between exchange rate and stock returns under volatility shifts,” *Q. Rev. Econ. Financ.*, vol. 80, pp. 605–613, 2021, doi: 10.1016/j.qref.2021.04.011.

[68] R. Liu and R. Gupta, “Investors’ Uncertainty and Forecasting Stock Market Volatility,” *J. Behav. Financ.*, vol. 23, no. 3, pp. 327–337, 2022, doi: 10.1080/15427560.2020.1867551.

[69] M. Zolfaghari and S. Hoseinzade, “Impact of exchange rate on uncertainty in stock market: Evidence from Markov regime-switching GARCH family models,” *Cogent Econ. Financ.*, vol. 8, no. 1, 2020, doi: 10.1080/23322039.2020.1802806.

[70] C. M. Hafner, H. Herwartz, and S. Maxand, “Identification of structural multivariate GARCH models,” *J. Econom.*, vol. 227, no. 1, pp. 212–227, 2022, doi: 10.1016/j.jeconom.2020.07.019.

[71] Y. Tang, F. Ma, Y. Zhang, and Y. Wei, “Forecasting the oil price realized volatility: A multivariate heterogeneous autoregressive model,” *Int. J. Financ. Econ.*, 2020, doi: 10.1002/ijfe.2399.

[72] F. Aliyev, R. Ajayi, and N. Gasim, “Modelling asymmetric market volatility with univariate GARCH models: Evidence from Nasdaq-100,” *J. Econ. Asymmetries*, vol. 22, 2020, doi: 10.1016/j.jeca.2020.e00167.

[73] A. Do, R. Powell, J. Yong, and A. Singh, “Time-varying asymmetric volatility spillover between global markets and China’s A, B and H-shares using EGARCH and DCC-EGARCH models,” *North Am. J. Econ. Financ.*, vol. 54, 2020, doi: 10.1016/j.najef.2019.101096.

[74] Z. Wang, Y. Li, and F. He, “Asymmetric volatility spillovers between economic policy uncertainty and stock markets: Evidence from China,” *Res. Int. Bus. Financ.*, vol. 53, 2020, doi: 10.1016/j.ribaf.2020.101233.

[75] Y. Hu, J. Ni, and L. Wen, “A hybrid deep learning approach by integrating LSTM-ANN networks with GARCH model for copper price volatility prediction,” *Phys. A Stat. Mech. its Appl.*, vol. 557, 2020, doi: 10.1016/j.physa.2020.124907.

[76] N. S. Abdul Aziz, S. Vrontos, and H. M. Hasim, “Evaluation of multivariate GARCH models in an optimal asset allocation framework,” *North Am. J. Econ. Financ.*, vol. 47, pp. 568–596, 2019, doi: 10.1016/j.najef.2018.06.012.

[77] A. Atabani Adi, “Modeling exchange rate return volatility of RMB/USD using GARCH family models,” *J. Chinese Econ. Bus. Stud.*, vol. 17, no. 2, pp. 169–187, 2019, doi: 10.1080/14765284.2019.1600933.

[78] S. Dimitrakopoulos and M. Tsionas, “Ordinal-response GARCH models for transaction data: A forecasting exercise,” *Int. J. Forecast.*, vol. 35, no. 4, pp. 1273–1287, 2019, doi: 10.1016/j.ijforecast.2019.02.016.

[79] A. Naimoli and G. Storti, “Heterogeneous component multiplicative error models for forecasting trading volumes,” *Int. J. Forecast.*, vol. 35, no. 4, pp. 1332–1355, 2019, doi: 10.1016/j.ijforecast.2019.06.002.

[80] W. Kristjanpoller and M. C. Minutolo, “A hybrid volatility forecasting framework integrating GARCH, artificial neural network, technical analysis and principal components analysis,” *Expert Syst. Appl.*, vol. 109, pp. 1–11, 2018, doi: 10.1016/j.eswa.2018.05.011.

[81] H. Y. Kim and C. H. Won, “Forecasting the volatility of stock price index: A hybrid model integrating LSTM with multiple GARCH-type models,” *Expert Syst. Appl.*, vol. 103, pp. 25–37, 2018, doi: 10.1016/j.eswa.2018.03.002.

[82] A. K. Panda and S. Nanda, “A garch modelling of volatility and m-garch approach of stock market linkages of North America,” *Glob. Bus. Rev.*, vol. 19, no. 6, pp. 1538–1553, 2018, doi: 10.1177/0972150918793554.

[83] P. C. Pati, P. Barai, and P. Rajib, “Forecasting stock market volatility and information content of implied volatility index,” *Appl. Econ.*, vol. 50, no. 23, pp. 2552–2568, 2018, doi: 10.1080/00036846.2017.1403557.

[84] F. Li and Y. Kang, “Improving forecasting performance using covariate-dependent copula models,” *Int. J. Forecast.*, vol. 34, no. 3, pp. 456–476, 2018, doi: 10.1016/j.ijforecast.2018.01.007.

[85] D. Ardia, K. Bluteau, K. Boudt, and L. Catania, “Forecasting risk with Markov-switching GARCH models: A large-scale performance study,” *Int. J. Forecast.*, vol. 34, no. 4, pp. 733–747, 2018, doi: 10.1016/j.ijforecast.2018.05.004.

[86] H. Herwartz, “Stock return prediction under GARCH — An empirical assessment,” *Int. J. Forecast.*, vol. 33, no. 3, pp. 569–580, 2017, doi: 10.1016/j.ijforecast.2017.01.002.

[87] G. Cubadda, B. Guardabascio, and A. Hecq, “A vector heterogeneous autoregressive index model for realized volatility measures,” *Int. J. Forecast.*, vol. 33, no. 2, pp. 337–344, 2017, doi: 10.1016/j.ijforecast.2016.09.002.

[88] L. Maciel, F. Gomide, and R. Ballini, “Evolving Fuzzy-GARCH Approach for Financial Volatility Modeling and Forecasting,” *Comput. Econ.*, vol. 48, no. 3, pp. 379–398, 2016, doi: 10.1007/s10614-015-9535-2.

[89] S. R. Bentes, “A comparative analysis of the predictive power of implied volatility indices and GARCH forecasted volatility,” *Phys. A Stat. Mech. its Appl.*, vol. 424, pp. 105–112, 2015, doi: 10.1016/j.physa.2015.01.020.

[90] G. M. Gallo and E. Otranto, “Forecasting realized volatility with changing average levels,” *Int. J. Forecast.*, vol. 31, no. 3, pp. 620–634, 2015, doi: 10.1016/j.ijforecast.2014.09.005.

[91] R. F. Engle and A. J. Patton, “What good is a volatility model?,” *Forecast. Volatility Financ. Mark.*, pp. 47–63, 2007, doi: 10.1016/B978-075066942-9.50004-2.

[92] R. S. Tsay, “Analysis of financial time series,” *John Wiley Sons Ltd.*, 2005.

[93] S. H. Poon, “A practical guide to forecasting financial market volatility,” *John Wiley Sons.*, 2005.

[94] E. F. (Fama, “The behavior of stock-market prices,” *J. Bus.*, 1965.

[95] R. T. Baillie, T. Bollerslev, and H. O. Mikkelsen, “Fractionally integrated generalized autoregressive conditional heteroskedasticity,” *J. Econom.*, vol. 74, no. 1, pp. 3–30, 1996, doi: 10.1016/S0304-4076(95)01749-6.

[96] T. Andersen, T. G., & Bollerslev, “Answering the skeptics: Yes, standard volatility models do provide accurate forecasts,” *Int. Econ. Rev. (Philadelphia).*, 1988.

[97] E. Zivot, “Analysis of High Frequency Financial Data: Models, Methods and Software. Part II: Modeling and Forecasting Realized Variance Measures,” *washington*, 2005.

[98] P. Christoffersen, “Elements of financial risk management,” *Acad. Press*, 2011.

[99] C. Poon, S. H., & Granger, “Practical issues in forecasting volatility,” *Financ. Anal. J.*, vol. 61(1), pp. 45–56, 2005.

[100] M. Verbeek, “A guide to modern econometrics,” *John Wiley Sons.*, 2008.

[101] L. R. GLOSTEN, R. JAGANNATHAN, and D. E. RUNKLE, “On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks,” *J. Finance*, vol. 48, no. 5, pp. 1779–1801, 1993, doi: 10.1111/j.1540-6261.1993.tb05128.x.

[102] S. J. Taylor, “Modelling financial time series,” *world Sci.*, 2008.

[103] J. M. Zakoian, “Threshold heteroskedastic models,” *J. Econ. Dyn. Control*, vol. 18, no. 5, pp. 931–955, 1994, doi: 10.1016/0165-1889(94)90039-6.

[104] J. Geweke, “Commet.,” *Econom. Rev.*, 1986.

[105] A. K. Higgins, M. L., & Bera, “A class of nonlinear ARCH models,” *Int. Econ. Rev. (Philadelphia).*, 1992.

[106] T. Angelidis, A. Benos, and S. Degiannakis, “The use of GARCH models in VaR estimation,” *Stat. Methodol.*, vol. 1, no. 1–2, pp. 105–128, 2004, doi: 10.1016/j.stamet.2004.08.004.

[107] T. Bollerslev, A. J. Patton, and R. Quaedvlieg, “Exploiting the errors: A simple approach for improved volatility forecasting,” *J. Econom.*, vol. 192, no. 1, pp. 1–18, 2016, doi: 10.1016/j.jeconom.2015.10.007.

[108] A. K. Jarque, C. M., & Bera, “A test for normality of observations and regression residuals,” *Int. Stat. Rev. Int. Stat.*, pp. 163–172, 1987.

[109] D. A. Dickey and W. A. Fuller, “Distribution of the Estimators for Autoregressive Time Series with a Unit Root,” *J. Am. Stat. Assoc.*, vol. 74, no. 366a, pp. 427–431, 1979, doi: 10.1080/01621459.1979.10482531.

15. Appendixes:

Figure 7 .– The graph illustrated the historical price of Nasdaq 100 index, logarithmic return, and volatility clustering. The data starting on January 1, 2015, and ending on June 30, 2022, the sample period has 1886 observations. It shows volatility clustering effect starting of the Covid-19.

Figure-7: NDX index (Nasdaq 100)

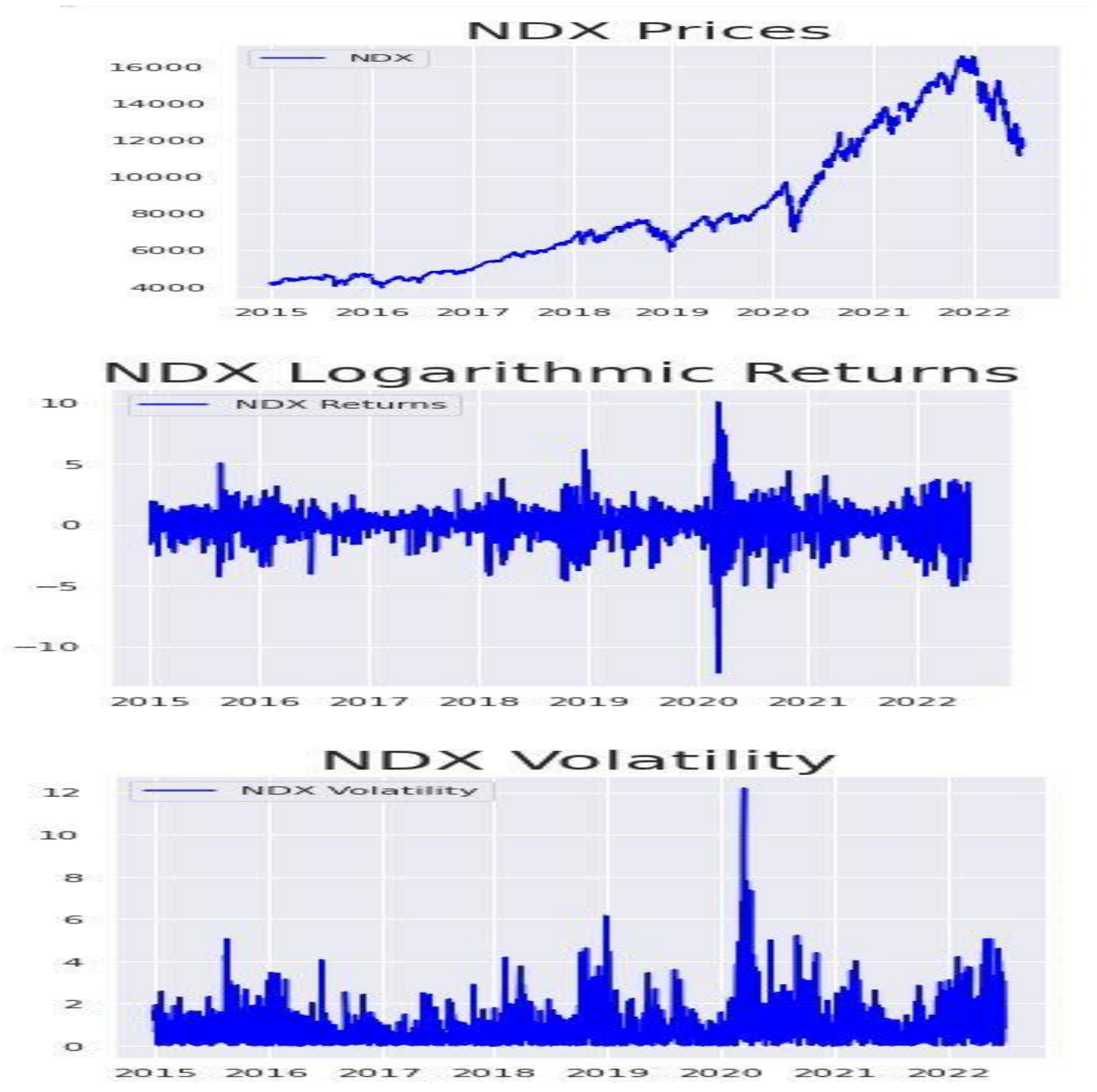


Figure 8- Correlation matrix of S&P500, Dow Jones Industry Average and Nasdaq 100. The figure illustrated strong positive relation among each other. It shows the 1897 observations beginning from January 1, 2015 and ending on June 30, 2022.

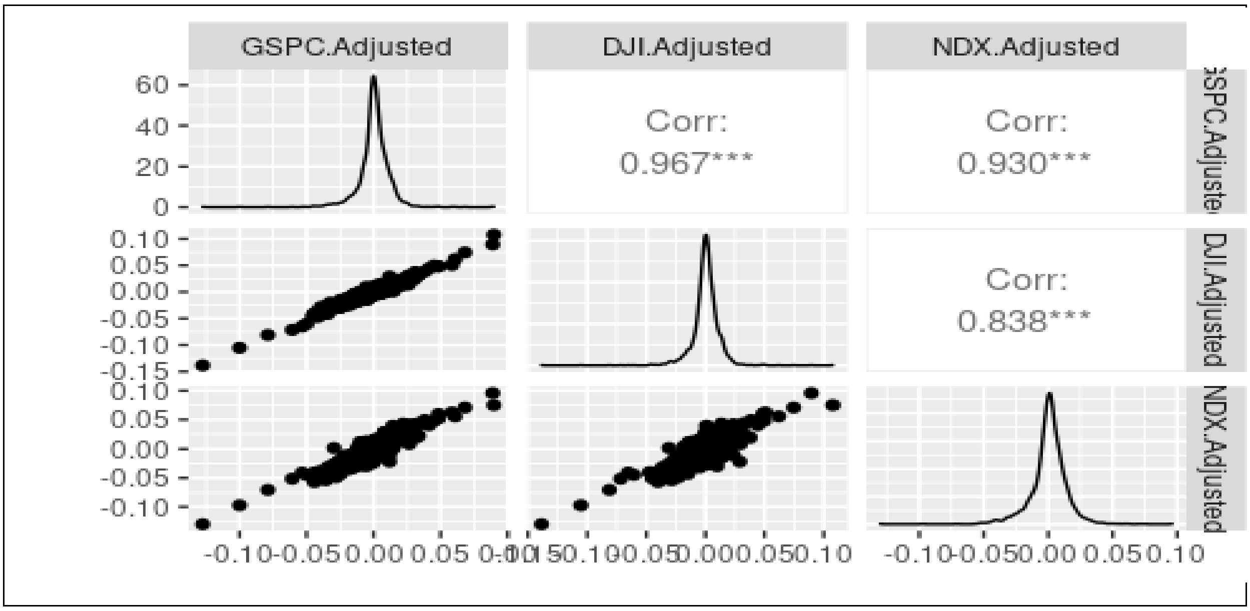


Figure 9- Realized variance 5-min subsample return of S&P 500, DJI and NDX Proxy. It illustrated volatility clustering effect on the 2020 beginning of Covid-19. Furthermore, it was calculated from January 1, 2015 to June 30, 2022.

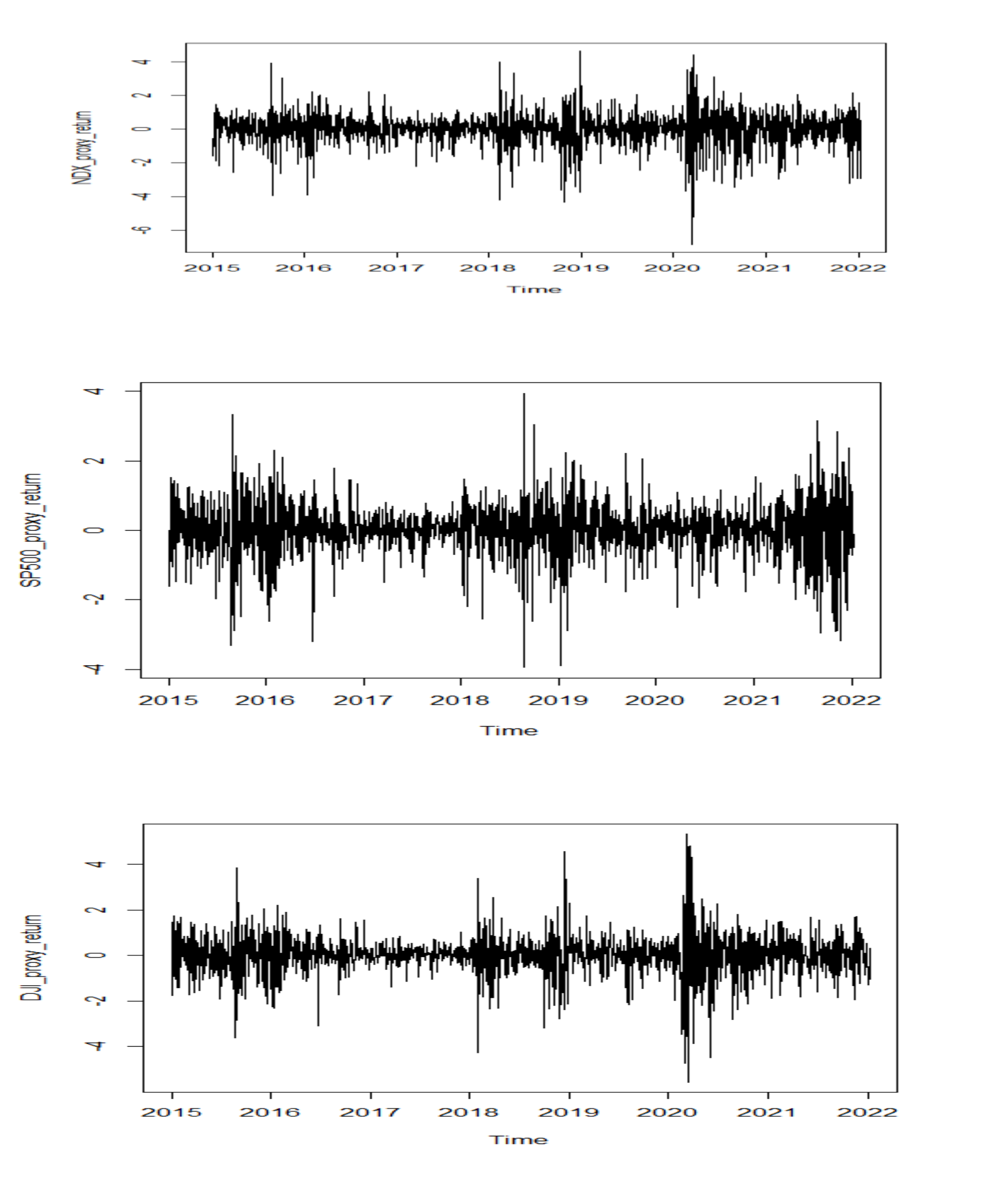


Figure 10- Auto-correlation Function of 5-min Subsample of Proxy Volatility of S&P 500, DJI and NDX. The auto-correlation graphs display with 12 lags window is used in all 3 indices of proxy data from 2015 to mid 2022.

Figure 11.- Auto-correlation Function of 5-min Subsample of Proxy Volatility

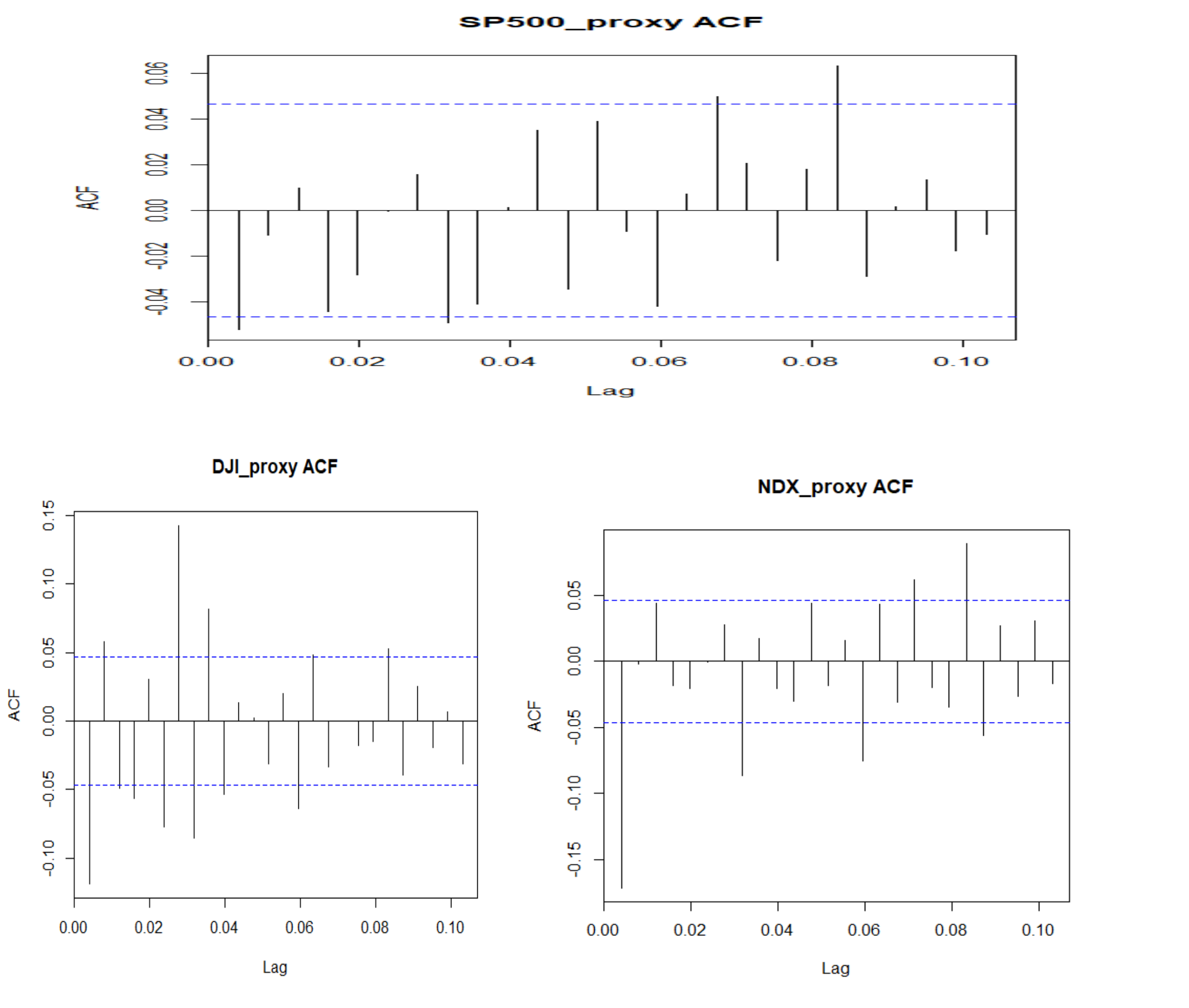


Figure 11– The figures explain Sample partial auto-correlation function of returns and squared returns associated with values of S&P 500, DJI and NDX 5-min subsample from January 1, 2015 to June 30, 2022. The horizontal dashed lines represent the two boundaries of standard error for the Partial ACF.

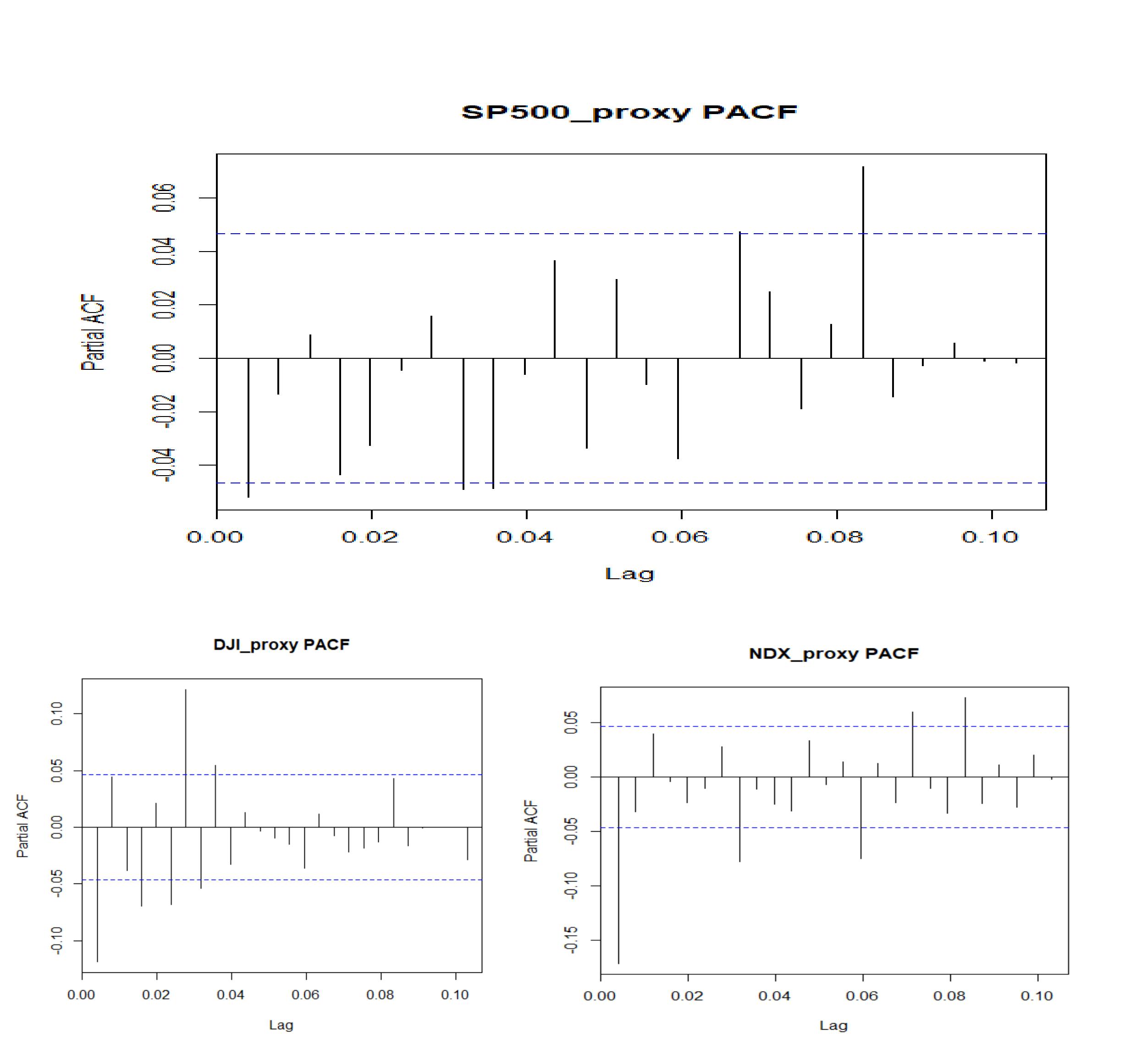


Table 16.- This table depicts the outcomes of in-sample goodness-of-fit tests for the volatility-forecasting models discussed previously. The sample period begins on January 1, 2015 and ends on February 28, 2020, yielding a total of 1,297 observations. *** represents the forecasting model with the best in-sample fit according to the log-likelihood, AIC, and BIC criteria. However, from the **Table-16** based on information criteria AIC and BIC **Panel A**, **Panel B and Panel C** shows **E-GARCH(1,1)** is the best fitting model in-sample. Furthermore, only log-likelihood for the **Panel C** indicating **I-GARCH(1,1)** is the best fitting model in-sample GARCH model selection.

Table 16. - Results of in-sample goodness-of-fit tests for the realized proxy variance

Forecast model	**Log-likelihood**	**AIC**	**BIC**
Panel A: S&P 500			
ARCH(l)	-1,422.44	2848.87	2866.3
GARCH(1,1)	-1,267.57	2.0152	2.0315
E-GARCH(1,1)	-1,231.16	1.959	1.9794
GJR-CGARCH(1,1)	-1,248.63	1.9867	2.0071
I-GARCH(1,1)	**-1,270.96*** **	2.0189	2.0312
APARCH(1,1)	-1,224.15	**1.9495*** **	**1.974*** **
FI-GARCH(1,1)	-1263.388	2.0101	2.0305
Panel B: DJI			
ARCH(l)	-1,452.95	2909.91	2927.33
GARCH(1,1)	-1,217.28	1.9355	1.9518
E-GARCH(1,1)	**-1,193.59*** **	**1.8995*** **	**1.9199*** **
GJR-CGARCH(1,1)	-1,202.06	1.9129	1.9333
I-GARCH(1,1)	-1220.116	1.9384	1.9506
APARCH(1,1)	-1,197.62	1.9075	1.9319
FI-GARCH(1,1)	-1212.105	1.9289	1.9492
Panel C: NDX			
ARCH(l)	-1,567.88	3139.77	3157.19
GARCH(1,1)	-1,427.68	2.2689	2.2852
E-GARCH(1,1)	-1,406.32	**2.237** **	**2.2573*** **
GJR-CGARCH(1,1)	-1,506.52	2.2384	2.2589
I-GARCH(1,1)	**-1,306.52*** **	2.2796	2.3588
APARCH(1,1)	-1399.535	2.2275	2.2519
FI-GARCH(1,1)	-1429.556	2.2735	2.2938

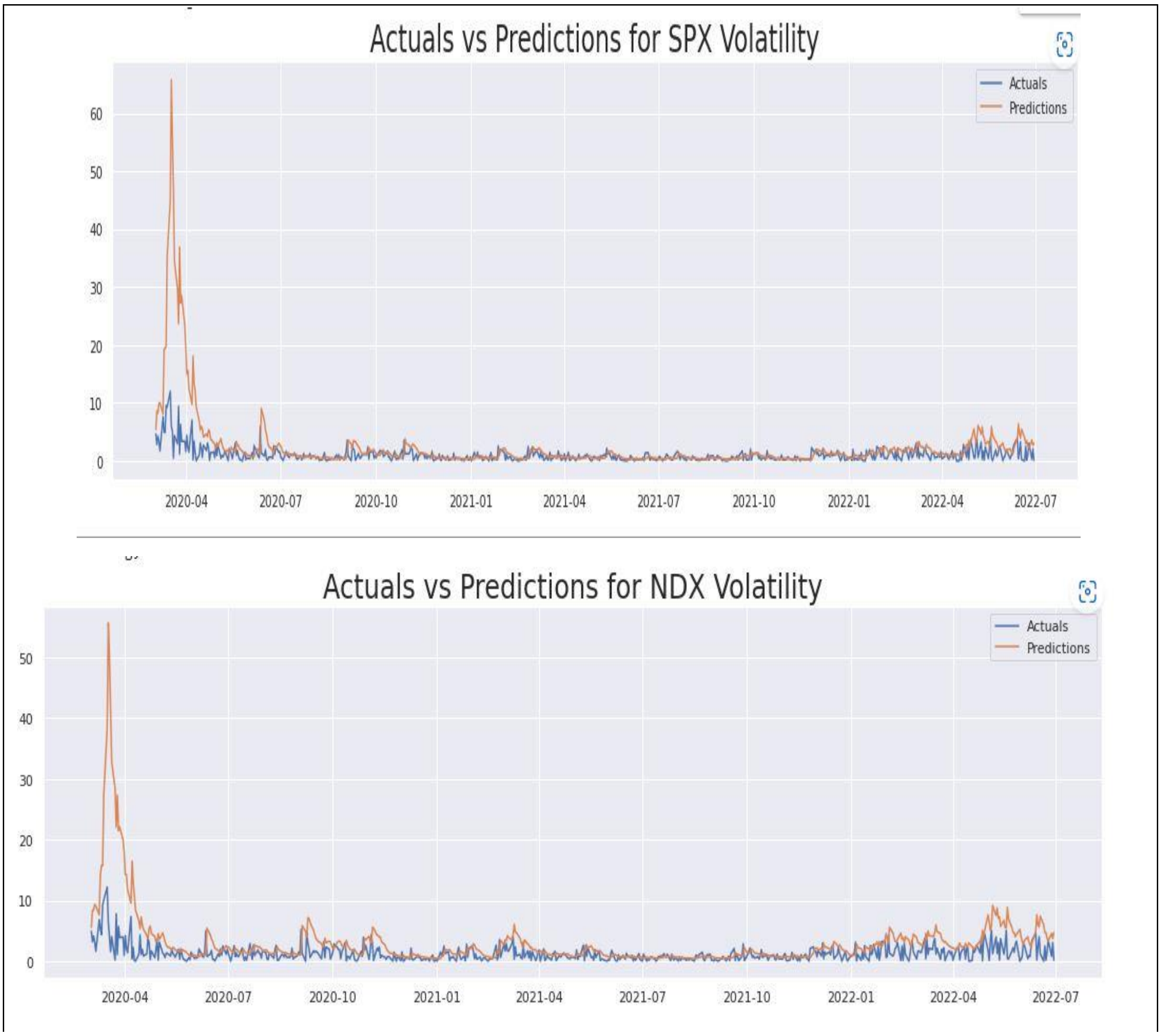
Table 17– In sample 1-period GARCH models estimated coefficients and p-values for realized variance or porxy volatility. The in-sample period starts from January 1, 2015, to February 28, 2020, resulting in 1286 observations. In the extended GARCH family of models, the significant and positive coefficient of realized volatility proxy indicates that it conveys appropriate information for explaining the volatility process.

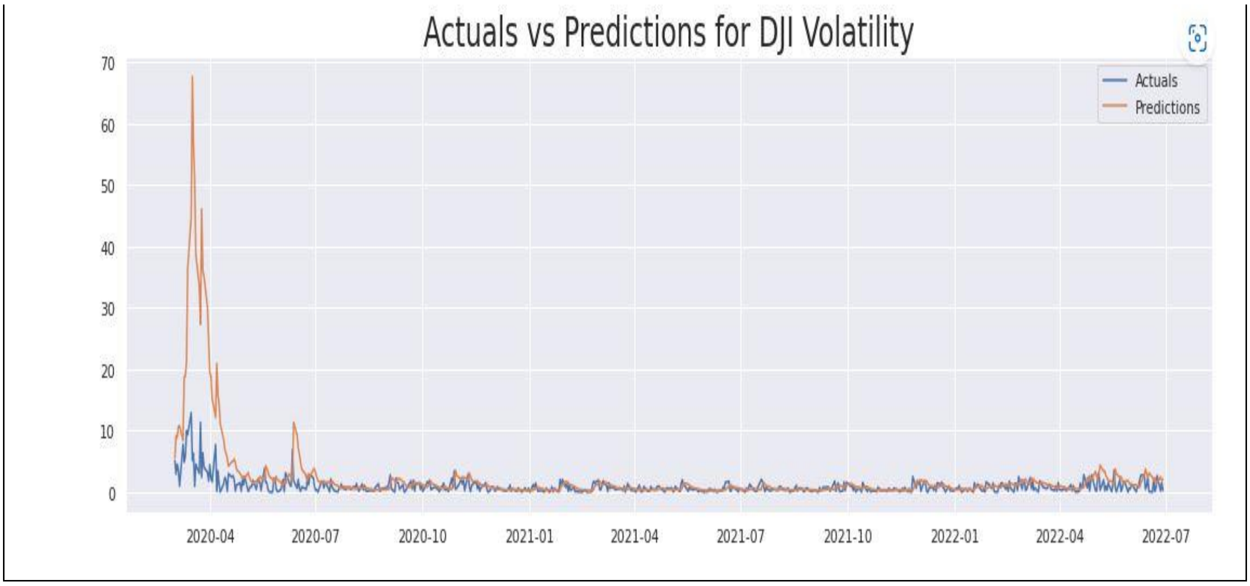
Table 17. in- sample One period Parameter estimates of GARCH models of Realized volatility proxy

Forecast model	α_0	α_1	β_1	β_2	γ_1	γ_2
Panel A: S&P 500						
GARCH(1,1)	0.016067 (0.073488)	0.006168 (0.000104)	0.027913 (0)	0.031498 (0)		
E-GARCH(1,1)	0.015879 (0.597609)	0.012396 (0.001068)	0.025928 (0)	0.013472 (0)	0.033027 (0)	
GJR-CGARCH(1,1)	0.01631 (0.516571)	0.006529 (0.000022)	0.021333 (0.241105)	0.03112 (0)	0.050261 (0.000001)	
I-GARCH(1,1)	0.016034 (0.079848)	0.004717 (0.000110)	0.031222 (0)			
APARCH(1,1)	0.000885 (0)	0.004642 (0)	0.007233 (0)	0.002707 (0)	0.000425 (0)	0.104563 (0)
FI-GARCH(1,1)	0.015337 (0.048661)	0.003406 (0.602869)	0.196236 (0.289172)	0.049155 (0)	0.185919 (0)	
Panel B: DJI						
GARCH(1,1)	0.014874 (0.058337)	0.010923 (0.000074)	0.021459 (0)	0.00985 (0)	0.030856 (0)	
E-GARCH(1,1)	0.014874 (0.058337)	0.010923 (0.000074)	0.021459 (0)	0.00985 (0)	0.030856 (0)	
GJR-CGARCH(1,1)	0.015187 (0.062895)	0.004345 (0)	0.022833 (0.024974)	0.0245 (0)	0.0400149 (0)	
I-GARCH(1,1)	0.014909 (0.001775)	0.003604 (0.000006)	0.02498 (0)			
APARCH(1,1)	0.015156 (0.098862)	0.006845 (0.000001)	0.020479 (0)	0.02063 (0)	0.126645 (0.000003)	0.190801 (0)
FI-GARCH(1,1)	0.014655 (0.002960)	0.010015 (0.027340)	0.132611 (0.876071)	0.158837 (0.065860)	0.079237 (0)	
Panel C: NDX						
GARCH(1,1)	0.018822 (0.018399)	0.01096 (0.000049)	0.027925 (0)	0.036394 (0)		
E-GARCH(1,1)	0.01883 (0.676659)	0.011015 (0.000237)	0.024557 (0)	0.015573 (0)	0.030967 (0)	
GJR-CGARCH(1,1)	0.018882 (0.470646)	0.01032 (0.000002)	0.02002 (0.355219)	0.034324 (0)	0.048405 (0)	
I-GARCH(1,1)	0.018874 (0.031719)	0.007633 (0.000129)	0.03391 (0)			

APARCH(1,1)	0.01842 (0.811395)	0.003851 (0)	0.003059 (0)	0.022819 (0)	0.000833 (0)	0.143255(0)
FI-GARCH(1,1)	0.019044 (0.035893)	0.017406 (0.002890)	0.129317 (0.382730)	0.151672 (0.037673)	0.093556 (0.000056)	

Figure 12.- The graph shows the actual and prediction of volatility from April 4, 2020 to July 2022 for the S&P 500, DJI and NDX. This also clearly indicates higher volatility what started from the initial stage of the Covid-19.




Actuals vs Predictions for DJI Volatility
Actuals
Predictions
70
60
50
40
30
20
10
0
2020-04
2020-07
2020-10
2021-01
2021-04
2021-07
2021-10
2022-01
2022-04
2022-07