Expert Systems with Applications 40 (2013) 139–149

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

An attitude-driven web consensus support system for heterogeneous group decision making

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ARTICLE INFO

Keywords:
Group decision making
Consensus support system
Consensus reaching
Heterogeneous information
Attitude

ABSTRACT

Consensus reaching processes are applied in group decision making problems to reach a mutual agreement among a group of decision makers before making a common decision. Different consensus models have been developed to facilitate consensus reaching processes. However, new trends bring diverse challenges in group decision making, such as the modelling of different types of information and of large groups of decision makers, together with their attitude to achieve agreements. These challenges require the capacity to deal with heterogenous frameworks, and the automation of consensus reaching processes by means of consensus support systems. In this paper, we propose a consensus model in which decision makers can express their opinions by using different types of information, capable of dealing with large groups of decision makers. The model incorporates the management of the group's attitude towards consensus by means of an extension of OWA aggregation operators aimed to optimize the overall consensus process. Eventually, a novel Web-based consensus support system that automates the proposed consensus model is presented.

1. Introduction

Decision making processes are one of the most frequent mankind activities in daily life. In group decision making (GDM) problems, a group of decision makers try to achieve a common solution to a problem consisting of two or more possible solutions or alternatives (Kacprzyk, 1986). Classically, GDM-based approaches are aimed to make decisions where few decision makers participate. However, nowadays technologies and societal models could imply the participation of large groups of decision makers in GDM problems.

A key aspect in GDM problems is to achieve a solution which is accepted by all decision makers in the group. Usually, GDM problems have been solved applying classic approaches, such as the majority rule, minority rule or total agreement (Butler & Rothstein, 2006; Kacprzyk, 1986; Martínez & Montero, 2007). However, these approaches do not guarantee achieving a solution accepted by all decision makers. Therefore, Consensus Reaching Processes (CRPs) are becoming increasingly necessary (Saint & Lawson, 1994) as part of GDM problems resolution. A number of theoretical consensus models have been proposed in the literature to conduct CRPs (Herrera-Viedma, Martínez, Mata, & Chiclana, 2005; Kacprzyk, Fedrizzi, & Nurmi, 1992; Parreiras, Ekel, Martini, & Palhares, 2010; Pedrycz, Ekel, & Parreiras, 2011; Saint & Lawson, 1994).

Some aspects in recent research for consensus have attained much attention, such as: (i) the need for ubiquitous CRPs, so that they can be conducted anywhere and anytime without physical meetings, which could be achieved by developing Consensus Support Systems (CSSs) that automate the CRP to a high extent; and (ii) the necessity of improving the static behavior present in most consensus models, irrespective of the changing complexity found in each particular problem, which may be addressed by developing adaptive consensus models (Mata, Martínez, & Herrera-Viedma, 2009).

Most classical consensus models and recent ones assumed that the group of decision makers were formed by a low number of decision makers. However, nowadays new trends like social networks (Sueur, Deneubourg, & Petit, 2012; Yager, 2008) and e-democracy (Kim, 2008), imply larger groups of decision makers in GDM problems, thus bringing new challenges to this research area:

(i) Dealing with heterogeneous information: A large number of decision makers implies many different profiles. Therefore, each decision maker may express his/her preferences in different information domains, depending on the level of knowledge, experience or the nature of alternatives. In such a case the GDM problem is defined in a heterogeneous framework, and an approach to deal with heterogeneous

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http://dx.doi.org/10.1016/j.eswa.2012.07.029

information is required (Herrera, Martínez, & Sánchez, 2005; Li, Huang, & Chen, 2010; Zhang & Lu, 2003).

(ii) Group's attitude towards consensus: The *attitude* of decision makers towards consensus is the capacity to modify preferences that they present during the CRP, which may affect to its performance significantly (Palomares, Liu, Xu, & Martínez, in press).

(iii) Automation: The management of large groups increases the complexity of physical meetings, therefore the development of a consensus model that allows a certain degree of automation on CRPs by implementing a CSS upon it becomes compulsory (Herrera, Herrera-Viedma, & Verdegay, 1996; Herrera-Viedma et al., 2005; Mata et al., 2009).

In this paper, we propose a novel large-scale oriented consensus model for GDM problems defined in heterogeneous contexts that is able to integrate decision makers' attitude regarding consensus. Once defined the model, and due to the necessity of automation, a Web-based CSS that integrates such a consensus model is presented.

It is remarkable that the proposed consensus model is able to integrate the group's attitude towards consensus in the measurement of the level of agreement, by means of an extension of OWA operators (Yager, 1988), so-called Attitude-OWA.

This paper is organized as follows: Section 2 reviews some preliminaries related to GDM problems in heterogeneous contexts, consensus processes and CSSs. Section 3 introduces the consensus model that integrates the group's attitude towards consensus and provides an approach to deal with heterogeneous information. Section 4 presents the Web-based CSS that uses the previous model and an illustrative example of its performance. Finally, some concluding remarks are pointed out in Section 5.

2. Preliminaries

This section reviews the formalization and management of GDM problems defined in heterogeneous contexts, and revises basic concepts about CRPs to understand the proposed consensus model. Because of the need for automating the proposed model, different CSSs are also revised.

2.1. GDM Problems with heterogeneous information

A GDM problem can be defined as a decision situation where a group of *decision makers* or *experts*, $E = \{e_1, \ldots, e_m\}$ $(m \geqslant 2)$, express their preferences over a set of feasible *alternatives*, $X = \{x_1, \ldots, x_n\}$ $(n \geqslant 2)$ (Kacprzyk, 1986). Each decision maker, e_i, provides his/her opinions on X by means of a preference relation P_i, $\mu_{P_i} : X \times X \to D$,

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

where each *assessment*, $p_i^{lk} = \mu_{P_i}(x_l, x_k)$, represents the preference degree of alternative x_l over x_k according to decision maker e_i, expressed in an information domain D, $i \in \{1, \ldots, m\}$ and $l,k \in \{1, \ldots, n\}$, $l \neq k$. In complex GDM problems usually defined with a high degree of uncertainty, decision makers might have different backgrounds and different levels of knowledge about a specific problem. Therefore, they could prefer to provide their preferences by using different domains according to their own characteristics. In such a case, the GDM problem is defined in an heterogeneous context. In this paper, we focus on this type of problems, so-called *heterogeneous GDM problems*, in which each decision maker e_i, may express his/her opinions on X by using different information domains $D_i \in \{numerical,\ interval-valued,\ linguistic\}$ (Herrera et al., 2005; Li et al., 2010; Zhang & Lu, 2003). Therefore, preferences could be assessed as:

- *Numerical*: Assessments p_i^{lk} are represented as values in $[0,1]$.
- *Interval-valued*: Assessments p_i^{lk} are represented as intervals, $I([0,1])$.
- *Linguistic*: Assessments p_i^{lk} are represented as linguistic labels $s_j \in S$, where $S = \{s_0, \ldots, s_g\}$ is a set of labels.

2.2. Consensus reaching processes (CRPs)

GDM problems have been usually solved by performing a *selection process* where the best alternative or subset of alternatives is obtained from decision makers' preferences (Roubens, 1997), which does not always guarantee that the decision would be accepted by all decision makers in the group, since some of them might consider that their opinions have not been sufficiently considered. In order to overcome this drawback and attempt to achieve a solution to the GDM problem which is accepted by the whole group, CRPs have attained a great attention as part of the decision process. *Consensus* can be understood as a state of mutual agreement among members of a group, where the decision made satisfies all of them (Butler & Rothstein, 2006; Saint & Lawson, 1994). Reaching a consensus usually requires that decision makers modify their initial opinions, making them closer to each other and towards a collective opinion which must be satisfactory for all of them. Furthermore, in many real CRPs decision makers might present different attitudes towards consensus, regarding the capacity they present to modify their own preferences to achieve an agreement, as will be further studied in this paper.

The notion of consensus has been interpreted in different ways, ranging from consensus as a total agreement to more flexible approaches (Kacprzyk & Fedrizzi, 1988; Kacprzyk et al., 1992). Consensus as a total agreement, where all decision makers are aimed to achieve a mutual agreement in all their opinions, may be quite difficult to achieve in practice, and in those cases that it could be achieved, the cost derived from the CRP is usually unacceptable. Subsequently, more flexible notions of consensus have been proposed to soften the strict view of consensus as a total agreement, considering different degrees of partial agreement among decision makers to decide about the existence of consensus. One of the most widely accepted approaches for a flexible measurement of consensus is the so-called notion of *soft consensus*, proposed in Kacprzyk (1986). This approach applies the concept of fuzzy linguistic majority, which establishes that consensus exists if *most decision makers participating in a problem agree with the most important alternatives*. *Soft consensus*-based approaches have been used in different GDM problems providing satisfactory results (Herrera et al., 1996; Kacprzyk & Zadrozny, 2010; Zadrozny & Kacprzyk, 2003).

CRPs are iterative and dynamic processes consisting of several rounds of discussion. These processes are frequently coordinated by a human figure known as *moderator*, who is responsible for supervising and guiding decision makers in the overall process, as well as giving them advice to modify their opinions (Martínez & Montero, 2007). A general scheme to conduct CRPs is depicted in Fig. 1 and briefly described below:

1. *Gathering preferences*: Each decision maker provides his/her preferences.
2. *Computing the level of agreement*: The moderator obtains the level of agreement in the group by means of *consensus measures* (Kacprzyk & Fedrizzi, 1988; Kuncheva & Krishnapuram, 1995), *similarity measures*, and *aggregation operators* (Beliakov, Pradera, & Calvo, 2007).

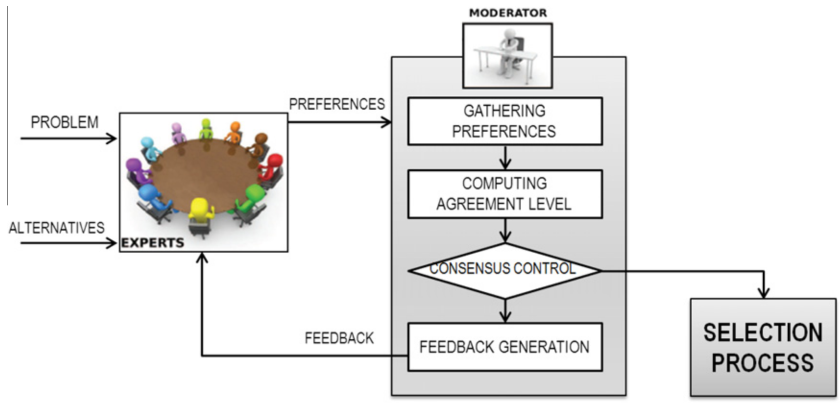

Fig. 1. General CRP scheme.

3. *Consensus control*: If the level of agreement is high enough, the group moves onto the alternatives selection process, otherwise more discussion rounds are required.
4. *Feedback generation*: The moderator identifies furthest preferences from consensus and gives decision makers some feedback, suggesting them how to modify their preferences and make them closer.

In order to deal with CRPs, a high number of theoretical consensus models have been proposed in the literature by different authors (Herrera-Viedma et al., 2005; Kacprzyk et al., 1992; Mata et al., 2009; Parreiras et al., 2010; Pedrycz et al., 2011; Saint & Lawson, 1994). Nevertheless, most consensus models do not consider the importance of replacing the human moderator (due to the subjectivity and lack of impartiality that he/she may sometimes present towards decision makers), which could make the CRP automatic to some extent. Since it would be interesting to implement the tasks defined in these consensus models into a CSS that achieves such an automation degree in practice by means of intelligent techniques, as well as eliminating the need for physical meetings and making ubiquitous CRPs possible, different CSSs have been proposed in the literature in the last few years.

2.3. Related work on CSSs

Due to the fact that in our proposal for a consensus model we consider GDM problems with a large number of decision makers who might express their preferences in different domains, it is convenient to automate such model by means of a CSS. Here, we review some CSSs presented in the literature to support decision makers in GDM problems. These CSSs designs are based on consensus models whose tasks are easily automated, therefore both the human moderator and the need for physical meetings disappear. Besides, they may facilitate dealing with large groups of decision makers, depending on the specific consensus model considered. Notice that some of the systems revised are denoted as CSS models, i.e. a proposal of a CSS scheme which may have not been implemented and put in practice yet.

Some CSSs have been developed based on the notion of *soft consensus* and fuzzy majority (Kacprzyk, 1986; Kacprzyk & Fedrizzi, 1989), such as the system presented in Zadrozny and Kacprzyk (2003), which is one of the earliest Web-based CSSs providing decision makers with a web user interface to let them insert and modify their preferences; and the one proposed in Kacprzyk and Zadrozny (2010), that applies additional techniques to manage knowledge, such as the use of ontologies.

The CSS model presented in Herrera-Viedma et al. (2005) is based on a consensus model that incorporates the use of multigranular linguistic preference relations. Considering that decision makers with different backgrounds and level of knowledge about each problem might be users of the system, they provide their preferences by means of linguistic term sets with different granularity. In addition, the system is able to generate pieces of advice for decision makers, suggesting them how to change their preferences.

In Mata et al. (2009), a CSS model based on an adaptive consensus model was presented. Such a model adapts its behavior throughout the overall discussion process by applying different procedures to identify decision makers preferences that should be changed according to the consensus degree achieved in each round. This way, the model attempts to minimize the number of discussion rounds required to achieve a consensus, compared to other non-adaptive models.

Recently, several CSSs operating in web and mobile environments have been presented. In Alonso, Herrera-Viedma, Chiclana, and Herrera (2010), a Web-based CSS to deal with incomplete preference relations was presented. The system provides a web user interface to the decision makers involved in the GDM problem.

3. Attitude-based consensus model for heterogeneous GDM problems

In this section, we propose a new large-scale oriented consensus model for GDM problems defined in heterogeneous contexts, that deals with large groups of decision makers and is able to integrate their attitude towards consensus. The model is able to deal with heterogeneous frameworks and allows decision makers to express their opinions by using different information domains. Besides, the group's attitude provides a new vision to the CRP, since the discussion process is adapted to achieve the level of consensus required according to the decision makers' attitude.

Before developing in further detail the consensus model, we are going to present the management of heterogeneous information that our model will use, as well as the way of integrating the attitude of decision makers in the CRP.

3.1. Dealing with heterogeneous information

As previously pointed out, our interest is focused on dealing with GDM problems defined in heterogeneous frameworks in which the information provided by decision makers can be numerical, interval-valued or linguistic.

- *Numerical domain*: $p_i^{lk} = v,\ \ v \in [0,1]$.
- *Interval-valued domain*: $p_i^{lk} = I([0,1]) = [d,f],\ (d,f \in [0,1] \wedge d \leqslant f)$.
- *Linguistic domain*: Linguistic variables (Zadeh, 1975) are assessed by linguistic terms, $p_i^{lk} = s_j \in S$, where semantics is

defined by a fuzzy membership function, denoted as $\mu_{s_j}(y),\ y \in [0,1]$.

In order to deal with such heterogeneous frameworks, different solutions have been proposed (Herrera et al., 2005; Li et al., 2010; Zhang & Lu, 2003). Here, we consider the method proposed in Herrera et al. (2005) to unify information expressed in different domains, p_i^{lk} (either numerical, interval-valued or linguistic), into fuzzy sets $F(S_T)$, in a common linguistic term set $S_T = \{s_0, \ldots, s_g\}$:

$$\begin{aligned}&\tau_{DS_T} : D \to F(S_T)\\ &\tau_{DS_T}(p_i^{lk}) = \sum_{j=0}^{g} s_j/\gamma_{ij}^{lk}\end{aligned} \tag{1}$$

where $g + 1$ is the granularity of S_T, γ_{ij}^{lk} is the membership degree of p_i^{lk} to s_j and at least $\exists \gamma_{ij}^{lk} \geqslant 0, j = 0, \ldots, g$.

Remark 1. The unification of heterogeneous information is conducted into fuzzy sets in a common linguistic domain to facilitate computations (see (Herrera et al., 2005) for further detail).

Once applied this unification and assuming that each fuzzy set will be represented by its membership degrees $p_i^{lk} = (\gamma_{i0}^{lk}, \ldots, \gamma_{ig}^{lk})$, the preference relation $P_i\ i \in \{1, \ldots, m\}$ of decision maker e_i is represented as follows:

$$P_i = \begin{pmatrix} - & \cdots & p_i^{1n} = (\gamma_{i0}^{1n}, \ldots, \gamma_{ig}^{1n}) \\ \vdots & \ddots & \vdots \\ p_i^{n1} = (\gamma_{i0}^{n1}, \ldots, \gamma_{ig}^{n1}) & \cdots & - \end{pmatrix}$$

Subsequent computations on unified assessments p_i^{lk} are applied to a central value cv_i^{lk} computed upon them (Herrera-Viedma et al., 2005), as will be shown in Section 3.3.

3.2. Integrating attitude in consensus reaching process

The group attitude towards consensus refers to the importance given by decision makers to reach a consensus, compared to modifying their own preferences. If decision makers adopt an *optimistic attitude*, such that achieving an agreement is more important than their own preferences, then more importance is given to those positions in the group whose level of agreement is higher; on the other hand, if they adopt a *pessimistic attitude*, so that decision makers' preferences are considered more important than achieving an agreement, then those positions in the group whose level of agreement is lower are given more importance. Further detail about this concept can be found in Palomares et al. (in press).

The attitude will be integrated across the consensus process in the computation of agreement level (see Fig. 1) by means of OWA operators, due to their appropriateness to manage the attitudinal character of aggregation (Beliakov et al., 2007). To do so, we define the *Attitude-OWA*, an extension of OWA operators especially suitable for dealing with a high number of elements, h, in the aggregation process (i.e. large groups of decision makers). OWA (*Ordered Weighted Averaging*) operators are defined as follows:

Definition 1 (Yager (1988)). Let $A = \{a_1, \ldots, a_h\}$, $a_i \in R$, be a set of h values to aggregate. An *OWA operator* is a mapping $F{:}R^h \to R$, with an associated weighting vector $W = [w_1 \ldots w_h]^\top$ $(w_i \in [0,1], \sum_i w_i = 1)$:

$$F(a_1, \ldots, a_h) = \sum_{j=1}^{h} w_j b_j \tag{2}$$

where b_j is the jth largest of a_i values.

OWA operators can be classified according to their optimism degree, by means of a measure so-called *orness*, associated with W. This measure provides the attitudinal character of aggregation, by determining how close the operator is to the maximum (OR) function, and is defined as (Beliakov et al., 2007):

$$orness(W) = \frac{1}{h-1} \sum_{i=1}^{h} (h-i) w_i \tag{3}$$

While optimistic or OR-LIKE OWA operators are those whose $orness(W) > 0.5$, in pessimistic or AND-LIKE operators we have $orness(W) < 0.5$.

Different methods have been proposed to compute OWA weights. We consider the method proposed in Yager (1996) to compute them based on linguistic quantifiers (Zadeh, 1983), more specifically, *Regular Increasing Monotone* (RIM) quantifiers (Liu & Han, 2008), whose linear membership function $Q(r)$, $r \in [0,1]$, is defined by $\alpha, \beta \in [0,1]$ as:

$$Q(r) = \begin{cases} 0 & \text{if } r \leqslant \alpha, \\ \frac{r-\alpha}{\beta-\alpha} & \text{if } \alpha < r \leqslant \beta, \\ 1 & \text{if } r > \beta. \end{cases} \tag{4}$$

Yager proposed the following method to compute OWA weights, w_i, upon $Q(r)$ (Yager, 1988; Yager, 1996):

$$w_i = Q\left(\frac{i}{h}\right) - Q\left(\frac{i-1}{h}\right), \quad i = 1, \ldots, h \tag{5}$$

Regarding the group's attitude, it will be gathered at the beginning of the CRP by means of two *attitudinal parameters*, $\vartheta, \varphi \in [0,1]$, used to represent it:

- ϑ represents the group's attitude, which can be optimistic, pessimistic or indifferent; corresponding with a value greater, less or equal than 0.5, respectively. The higher ϑ, the more optimistic the attitude towards consensus. ϑ is also equivalent to the measure of optimism ($orness(W)$) that characterizes OWA operators.
- φ is used to indicate the amount of agreement positions which are given non-null weight in the subsequent aggregation conducted with Attitude-OWA. The higher φ, the more values are considered.

These parameters are the basis to define Attitude-OWA operator:

Definition 2 (Palomares et al (in press)). An *Attitude-OWA operator* of dimension h on a set of values $A = \{a_1, \ldots, a_h\}$ to aggregate, is an OWA operator based on two attitudinal parameters ϑ, φ given by a decision group to indicate their attitude towards consensus,

$$Attitude - OWA_W(A, \vartheta, \varphi) = \sum_{j=1}^{h} w_j b_j \tag{6}$$

where b_j is the jth largest of the a_i values, ϑ, $\varphi \in [0,1]$ are two input attitudinal parameters, and the set of weights, W, is computed by using a RIM quantifier, as shown in Eq. (5).

The attitude $\vartheta \in [0,1]$ of an Attitude-OWA operator can be determined by an associated RIM quantifier with differentiable membership function $Q(r)$, when the number of elements to aggregate h, is sufficiently large, $h \to \infty$ (i.e. when a large number of decision makers participate in the problem), as follows (considering Eqs. (3) and (5)),

$$\vartheta = \lim_{h\to\infty} \frac{1}{h-1} \sum_{i=1}^{h} (h-i) \left[Q\left(\frac{i}{h}\right) - Q\left(\frac{i-1}{h}\right) \right] = \int_0^1 Q(r)dr \tag{7}$$

see Palomares et al. (in press) for further detail. If $Q(r)$ is defined as shown in Eq. (4), then,

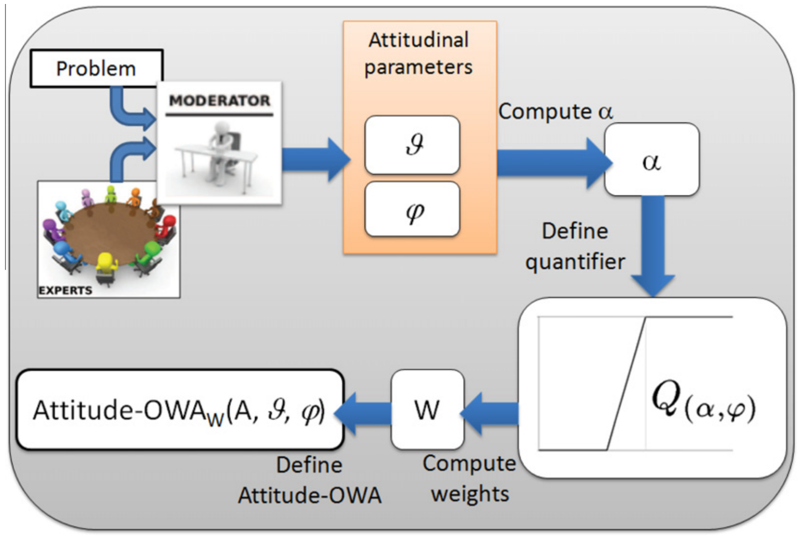

Fig. 2. Process to determine the Attitude-OWA operator based on ϑ and φ.

$$\vartheta = \int_0^1 Q(r)dr = 1 - \alpha - \frac{\varphi}{2} \quad (8)$$

where $\varphi = \beta - \alpha$. Therefore, given attitudinal parameters ϑ, φ, we can determine α, β, necessary to define $Q(r)$, as follows,

(1) $\alpha = 1 - \vartheta - \frac{\varphi}{2}$
(2) $\beta = \alpha + \varphi$

thus showing the capacity of Attitude-OWA to deal with a large number of decision makers (where $h \to \infty$) easily. Once defined the RIM quantifier Q associated to the group's attitude expressed by decision makers, weights w_i, are computed by using Eq. (5). The complete process to define an Attitude-OWA operator upon a group's attitude is shown in Fig. 2.

3.3. Consensus model

In this subsection, we describe in detail our proposal for a consensus model that extends the one shown in Fig. 1. It consists of six phases as depicted in Fig. 3.

1. *Determining group's attitude towards consensus*: The first phase consists in determining the group's attitude towards the measurement of consensus, gathered by means of attitudinal parameters ϑ, φ as explained in Section 3.2.
2. *Gathering preferences*: Each decision maker e_i provides his/her preferences on alternatives in X, by means of a preference relation P_i, consisting of a $n \times n$ matrix of assessments $p_i^{lk} \in D_i, D_i \in \{numerical, interval-valued, linguistic\}$, on each pair of alternatives (x_l, x_k), $l,k \in \{1,\ldots,n\}$.
3. *Making the information uniform*: Preferences provided by decision makers in different information domains are unified into a single common linguistic domain to facilitate the computations, as previously described in Section 3.1.
4. *Computing consensus degree*: The objective of any CRP is to reach a sufficient level of consensus among decision makers in the group. In this phase, the level of agreement among them is computed and measured as a value in [0, 1]. To do so, the similarity between each pair of decision makers is measured, and these similarities are then aggregated to obtain a consensus degree at different levels. Given that our goal consists in improving the CRP taking into account the group's attitude towards consensus, as well as dealing with large groups effectively, we propose integrating such an attitude in the process to measure consensus by means of Attitude-OWA operator.
The following steps are required to compute the consensus degree:
(a) For each p_i^{lk}, $l \neq k$, a *central value* cv_i^{lk} is computed as follows

$$cv_i^{lk} = \frac{\sum_{j=0}^{g} index(s_j) \cdot \gamma_{ij}^{lk}}{\sum_{j=0}^{g} \cdot \gamma_{ij}^{lk}} \quad (9)$$

where $index(s_j) = j$ and $g + 1$ is the granularity of $S_T = \{s_0, \ldots, s_g\}$.
(b) Based on central values, a *similarity matrix* $SM_{ij} = (sm_{ij}^{lk})^{n\times n}$ is computed for each pair of decision makers e_i, e_j $(i < j)$, where each similarity value $sm_{ij}^{lk} \in [0,1]$ is computed as:

$$sm_{ij}^{lk} = 1 - \left| \frac{cv_i^{lk} - cv_j^{lk}}{g} \right| \quad (10)$$

(c) A *consensus matrix* $CM = (cm^{lk})^{n\times n}$ is obtained by aggregating similarity values at level of pairs of alternatives, using Attitude-OWA to consider the group's desired attitude towards consensus, as follows:

$$cm^{lk} = Attitude - OWA_W(SIM^{lk}, \vartheta, \varphi) \quad (11)$$

where $SIM^{lk} = \{sm_{12}^{lk}, \ldots, sm_{1m}^{lk}, sm_{23}^{lk}, \ldots, sm_{2m}^{lk}, \ldots, sm_{(m-1)m}^{lk}\}$ is the set of all pairs of decision makers' similarities in their opinion on (x_l, x_k). Notice that the more optimistic Attitude-OWA is, the higher similarity values are rather considered in aggregation. $cm^{lk} \in [0,1]$ represents the consensus degree on the pair of alternatives (x_l, x_k).
(d) Consensus degree on each alternative x_l, ca^l, is computed as

$$ca^l = \frac{\sum_{k=1, k\neq l}^{n} cm^{lk}}{n-1} \quad (12)$$

where n is the number of alternatives.
(e) Finally, a global consensus degree, cr, is obtained as follows

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n} \quad (13)$$

5. *Consensus control*
The overall level of agreement, cr, is compared with a consensus threshold $\mu \in [0,1]$ fixed a priori, according to the requirements

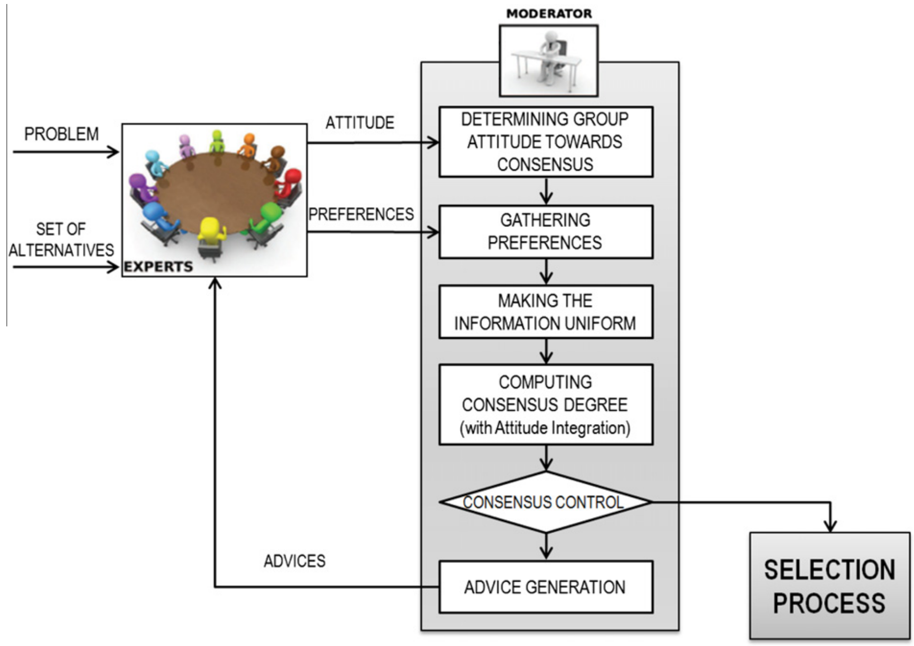

Fig. 3. Proposed consensus model scheme.

of the particular GDM problem. If $cr \geqslant \mu$, the consensus process ends and the group moves onto the selection process; otherwise, the process requires further discussion. A parameter *Maxrounds* controls the maximum number of discussion rounds allowed.

6. *Advice generation*
When $cr < \mu$, another discussion round is required, therefore decision makers are advised to modify their preferences to make them closer to each other and increase the consensus degree in the following round. Such pieces of advice could be computed with different methods (Herrera-Viedma et al., 2005; Mata et al., 2009). In our approach, we use the method proposed in Mata et al. (2009).

4. Web-based CSS integrating group attitude towards consensus

This section presents a Web-based CSS that implements the consensus model presented in Section 3, and describes the communication and work flow that summarizes the functions of such a system. The main advantage of this CSS is the automation of the human moderator's tasks, thus eliminating any controversy caused by his/her possible subjectivity. The system also allows ubiquitous CRPs, so that no physical meetings are required anymore.

The most widely used architecture for web applications is the *client/server* architecture (see Fig. 4), in which the client is a computer. When a client sends a request to the server, it processes the request and sends a response back to the client. An advantage of using a client/server architecture is that the client users (decision makers) do not have to install the Web-based CSS application in their computer.

Regarding web technologies and programming languages considered, the application has been implemented using *Java* and *Java Server Pages* (JSP), which allow to generate dynamic web pages; *Servlets*, that control the system and carry out any necessary operation; *Javascript* and *Cascade Style Sheets*, to develop the web interface; and *MySQL*, to manage the database.

Another important feature of the Web-based CSS is its ubiquity, i.e. it can be used anytime and anywhere, which facilitates the elicitation of preferences and the overall CRP.

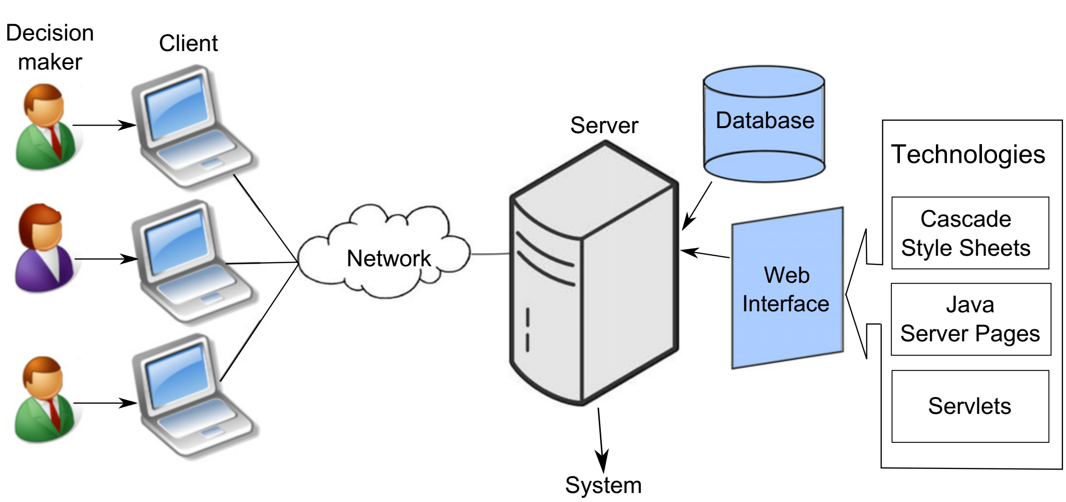

Fig. 4. Client/Server architecture.

The performance of the Web-based CSS has been divided into two categories: client and server performances. They are briefly described below.

4.1. Client

The Web-based CSS shows the following four interfaces to the decision makers involved in a GDM problem.

- *Authentication*: The web application requests decision maker his/her username and password to log in the CSS (see Fig. 5).
- *Assigned problems*: When a decision maker logs in the system, the CSS shows him/her some information about the problems where he/she has been invited to participate (see Fig. 6).
- *Elicitation of preferences*: This interface implements the *Gathering Preferences* phase of the proposed consensus model. Decision makers use such an interface to elicit their preferences, indicating the type of information (numerical, interval-valued, linguistic) and domain (range in case of numerical or interval-valued information, and syntax of the linguistic terms in case of linguistic information) used to provide their preferences (see Fig. 7).
- *Checking current problem status*: The application shows decision makers the preferences provided in the last round. If the system has generated any recommendation for decision makers (as a result of the *Advice generation* phase), they must submit new preference values in order to increase the consensus degree. Recommendations are highlighted in the interface by means of a colored font (red color to increase and green color to decrease), as shown in Fig. 8.

4.2. Server

The server implements three main modules and manages the database that stores all the information about the defined problems, decision makers involved in each problem and the information generated during the decision process.

The communication with the client to send/receive information from/to decision makers is carried out by the Internet (see Fig. 9). The implemented modules in the server side are as follows:

- *Computing consensus degree*: Once all decision makers involved in the GDM problem have introduced their preferences, the server carries out the phases of the consensus model, *Making the information uniform* and *Computing consensus degree*. The latter one computes the consensus and similarity measures to determine the degree of agreement in the group, taking into account the decision makers' attitude towards consensus.
- *Consensus control*: This module implements the *consensus control* phase of the proposed model, checking whether the consensus level has achieved the minimum consensus level desired, in which case the CRP ends. Otherwise, more discussions rounds are required.

Fig. 5. User authentication screen.

Fig. 6. Assigned problems to a decision maker.

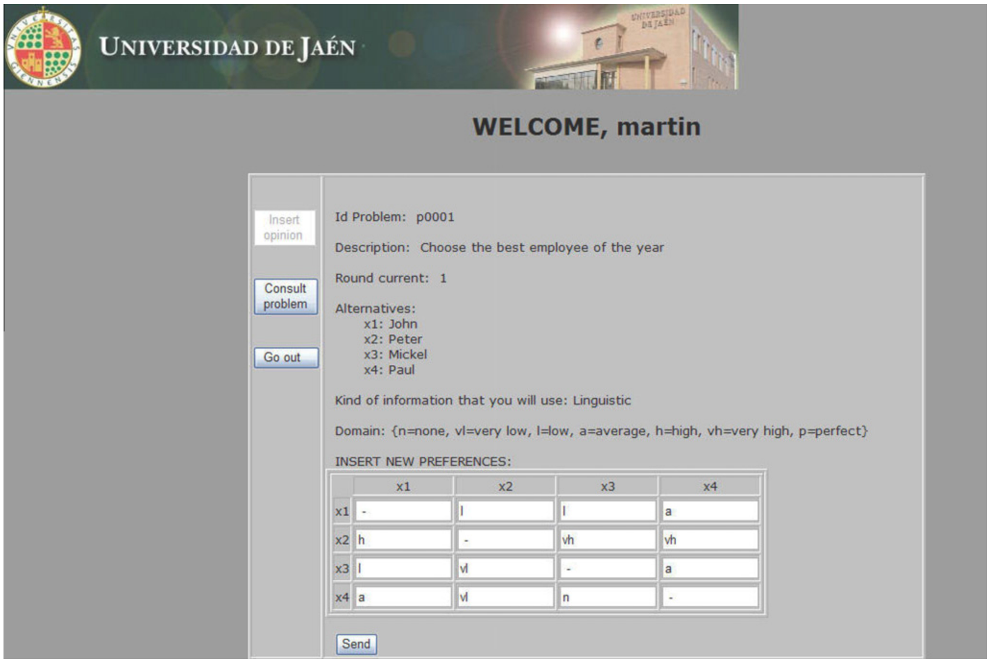

Fig. 7. A decision maker introduces his/her preferences by using linguistic information.

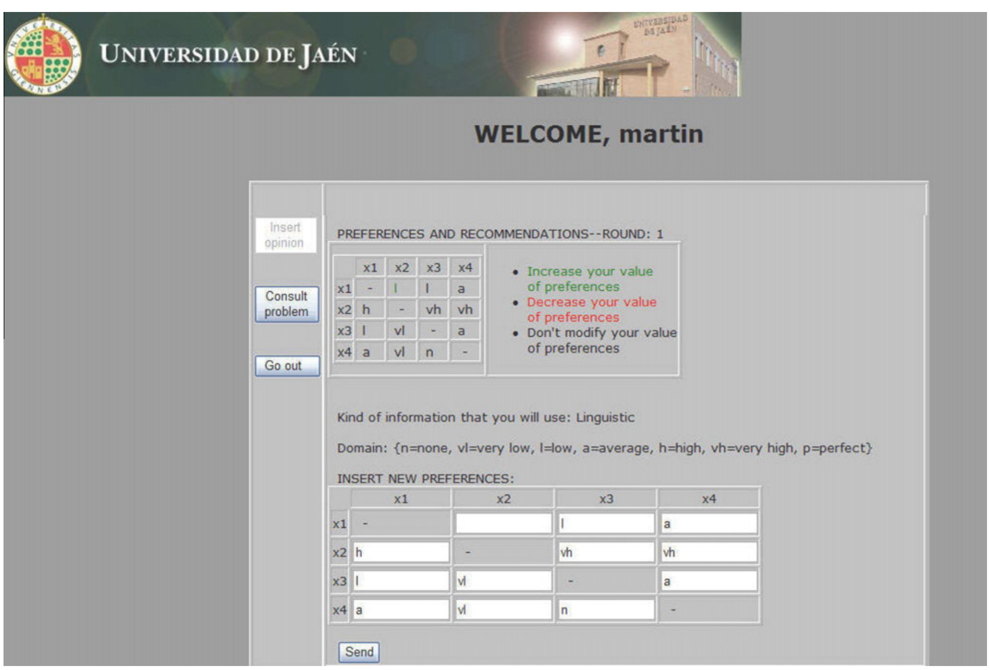

Fig. 8. A decision maker who provided his/her assessments by using linguistic terms, receives some recommendations.

- *Advice generation*: When a consensus round is conducted without having achieved the consensus threshold, the server carries out the *Advice generation* phase, which generates some recommendations to help decision makers to change their preferences on some alternatives in order to reach the consensus in the following rounds.

Once described the main functionalities of the system from the viewpoints of the client and the server, in the following we briefly show a general scheme of the work flow between them, as well as the interaction between the modules and the system's database.

1. *Initialization*: An initial step is carried out to insert in the database all the information about the GDM problem and decision makers involved in such a problem.
2. *Authentication*: When a decision maker wants to access the web application, he/she has to log in. The server checks the username and password in the database and if they are right, the

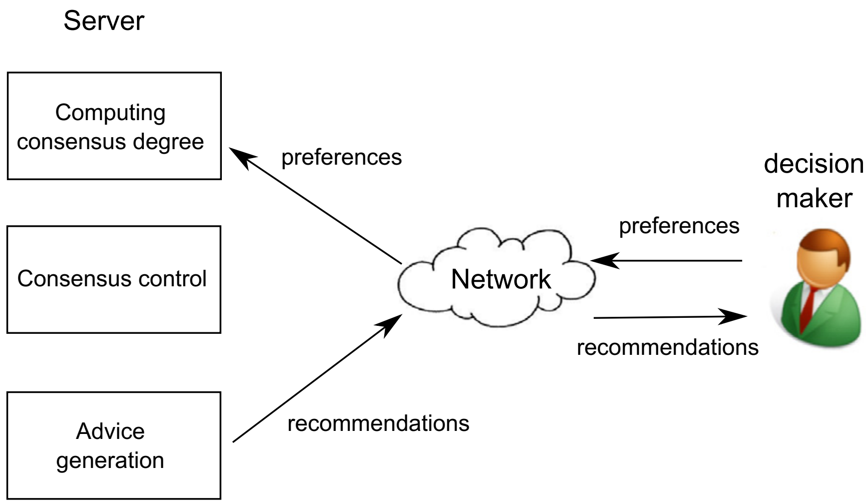

Fig. 9. Server modules.

decision maker accesses and sees the GDM problems in which he/she is involved. The decision maker can then carry out two tasks:

(a) Elicitation of preferences: decision makers can provide their preferences by using numerical, interval-value or linguistic information.

(b) Checking current problems status: decision makers can see their preferences provided in each round and their recommendations, if any.

3. *Computing consensus degree*: If all decision makers involved in the GDM problem have provided their preferences, the server starts the consensus process, makes the information uniform and computes the consensus and similarity measures.
4. *Consensus control*: The server checks if the required agreement degree has been achieved, in such a case the consensus process must finish. Otherwise, the server proceeds to step 5 before beginning a new consensus round.
5. *Advice generation*: The server generates some recommendations for decision makers to modify their preferences. In order to prevent the CRP from taking too long without having achieved an agreement, the system fixes a parameter *Maxrounds* to control the maximum number of discussion rounds allowed.

4.3. Web-based CSS performance

Once presented the main characteristics of the Web-based CSS, an example of a GDM problem is introduced and solved by using such a CSS.

Let us suppose that in a company there are 50 employees, $E = \{e_1, \ldots, e_{50}\}$ and they must choose the best employee of the year. The director of the company has selected 4 candidates $X = \{x_1 = John,\ x_2 = Peter,\ x_3 = Mickel,\ x_4 = Paul\}$. In the company there are 3 different departments whose employees have different backgrounds, so the type of information used for employees might

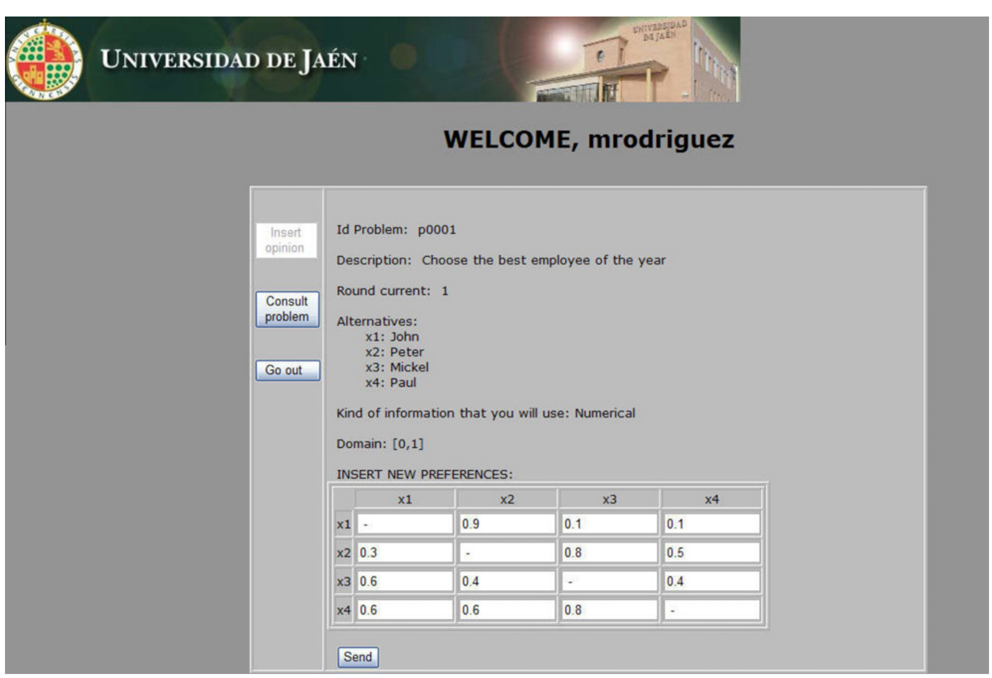

Fig. 10. The employee e_1, introduces his/her preferences by using numerical information.

be different. The employees have to reach a minimum level of agreement of $\mu = 0.85$, taking into account that employees follow an pessimistic attitude of $\vartheta = 0.25$ and the amount of agreement positions to be considered is $\varphi = 0.1$. The maximum number of discussion rounds allowed is $Maxrounds = 10$.

The employees provide their preferences by using different types of information: numerical, interval-valued and linguistic. The domains used for each type of information are:

- Numerical: $[0,1]$
- Interval-valued: $I([0,1])$
- Linguistic: $\{nothing(n), very\_low(vl), low(l), average(a), high(h), very\_high(vh), perfect(p)\}$

Fig. 10 shows the preferences provided by an employee e_1, who has used numerical information.

Once all employees have introduced their preferences, the consensus process begins with the first round, following the phases of the proposed consensus model in Fig. 3.

1. *Determining group's attitude towards consensus*
2. *Gathering preferences*
3. *Making the information uniform*
4. *Computing consensus degree*: The global consensus degree obtained in the first round is

$$cr = 0.5$$

5. *Consensus control*: As the global consensus degree, $cr = 0.5 < 0.85 = \mu$, it is then concluded that there is not enough consensus among the employees of the company, and consequently, the Web-based CSS should continue with another round.
6. *Advice generation*: Once the system verifies that the minimum level of agreement has not been reached, the system generates some recommendations for employees to modify their preferences in order to increase the level of agreement, and then the second round of discussion begins. Fig. 11 shows the recommendations generated for the employee e_1.

Table 1
Attitudinal parameters and RIM quantifiers used.

Attitude	ϑ	φ	α	$Q_{(\alpha,\varphi)}$
Pessimistic	0.25	0.1	0.7	$Q_{(0.7,0.1)}$
Indifferent	0.5	0.6	0.2	$Q_{(0.2,0.6)}$
Optimistic	0.75	0.3	0.1	$Q_{(0.1,0.3)}$

Table 2
Global consensus degree for each round.

Round	Pessimistic	Indifferent	Optimistic
1	0.5	0.7	0.87
2	0.59	0.76	
3	0.68	0.81	
4	0.77	0.86	
5	0.84		
6	0.88		

In this GDM problem, due to the choice of a pessimistic attitude, it is necessary to carry out six rounds of discussion to reach the consensus threshold $\mu = 0.85$. The proposed consensus model in Section 3 integrates the group's attitude towards consensus to deal with large groups and improve the CRP. In order to illustrate the effect of integrating different attitudes in the CRP, we will solve the GDM problem three times by using for each resolution a different attitude ϑ, including an optimistic, indifferent and pessimistic attitude, and different values for the amount of information (agreement positions between decision makers) considered in aggregation, φ.

Table 1 shows the different attitudes, given by ϑ, φ, and the different values of α and RIM quantifiers obtained, denoted as $Q_{(\alpha,\varphi)}$.

The global consensus degree obtained for each attitude and the number of necessary rounds to reach the minimum consensus level are shown in Table 2. As can be seen, the problem resolution with an optimistic attitude is the only one where the consensus has been achieved in the first round, whereas more rounds are necessary with the other two attitudes.

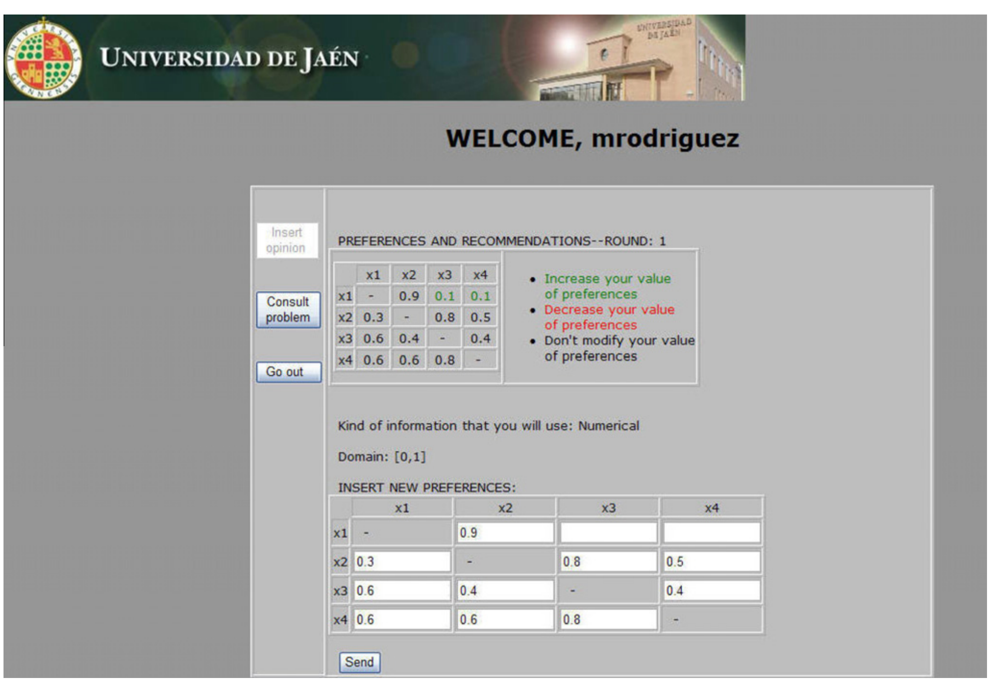

Fig. 11. The employee e_1, receives some recommendations.

The results affirm that the use of Attitude-OWA operator based on an optimistic attitude favors a greater convergence towards consensus, whereas Attitude-OWA operator based on a pessimistic attitude favors a lower convergence and a further discussion process, regardless of the proportion of values considered, φ. Therefore, depending on decision makers' priority to reach the minimum level of consensus, they can use an optimistic attitude if their priority is to achieve a consensus quickly, or a pessimistic attitude for a problem that requires further discussion.

5. Concluding remarks

The evolution of group decision making problems with increasingly larger scales of decision makers who may have different backgrounds, makes necessary to modify the present vision on current existing models. In this paper, we have presented a consensus model which deals with heterogeneous information and manages the attitude of decision makers. In addition, we have implemented a Web-based consensus support system upon such a model, that automates real consensus reaching processes. Due to its capacity to deal with large groups of decision makers, we aim to apply the system to real-life problems involving entire societies of individuals, such as e-democracy processes and social networks.

Acknowledgements

This work is partially supported by the Research Project TIN-2009–08286 and FEDER funds.

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