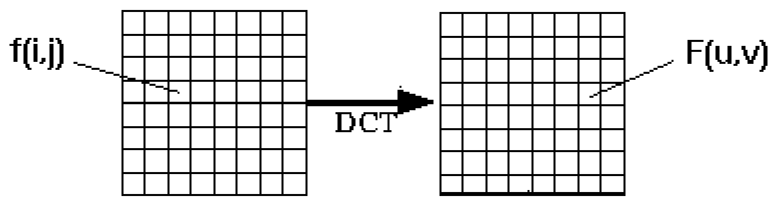
# The Discrete Cosine Transform (DCT)

The discrete cosine transform (DCT) helps separate the image into parts (or spectral sub-bands) of differing importance (with respect to the image's visual quality). The DCT is similar to the discrete Fourier transform: it transforms a signal or image from the spatial domain to the frequency domain (Fig 7.8).




**DCT Encoding**

The general equation for a 1D ($N$ data items) DCT is defined by the following equation:

$$F(u) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \Lambda(i).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] f(i)$$

and the corresponding ***inverse*** 1D DCT transform is simple $F^{-1}(u)$, i.e.:

where

$$\Lambda(i) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

The general equation for a 2D ($N$ by $M$ image) DCT is defined by the following equation:

$$F(u,v) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \left(\frac{2}{M}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(i).\Lambda(j).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] cos\left[\frac{\pi.v}{2.M}(2j+1)\right] .f(i,j)$$

and the corresponding ***inverse*** 2D DCT transform is simple $F^{-1}(u,v)$, i.e.:

where

$$\Lambda(\xi) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

The basic operation of the DCT is as follows:

- The input image is N by M;
- f(i,j) is the intensity of the pixel in row i and column j;
- F(u,v) is the DCT coefficient in row k1 and column k2 of the DCT matrix.
- For most images, much of the signal energy lies at low frequencies; these appear in the upper left corner of the DCT.
- Compression is achieved since the lower right values represent higher frequencies, and are often small - small enough to be neglected with little visible distortion.
- The DCT input is an 8 by 8 array of integers. This array contains each pixel's gray scale level;
- 8 bit pixels have levels from 0 to 255.
- Therefore an 8 point DCT would be:

where

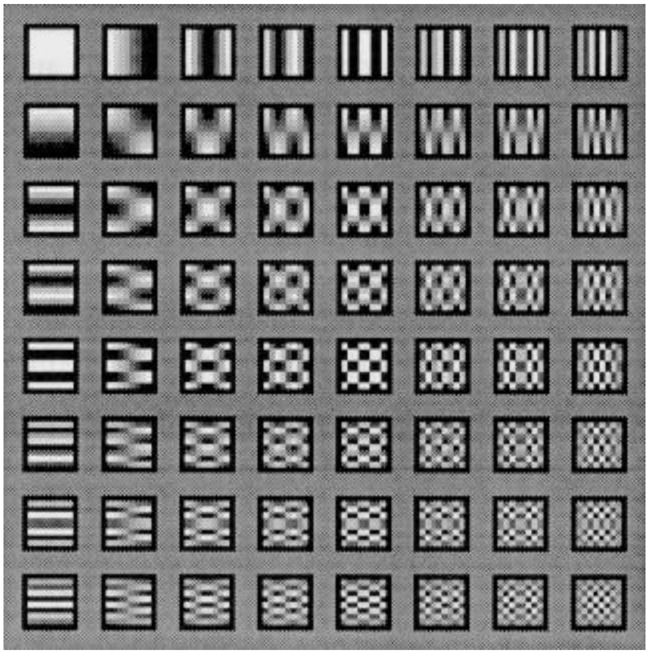
$$\Lambda(\xi) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

**Question**: What is F[0,0]?

***answer:*** They define DC and AC components.

- The output array of DCT coefficients contains integers; these can range from -1024 to 1023.
- It is computationally easier to implement and more efficient to regard the DCT as a set of **basis functions** which given a known input array size (8 x 8) can be precomputed and stored. This involves simply computing values for a convolution mask (8 x8 window) that get applied (summ values x pixelthe window overlap with image apply window accros all rows/columns of image). The values as simply calculated from the DCT formula.

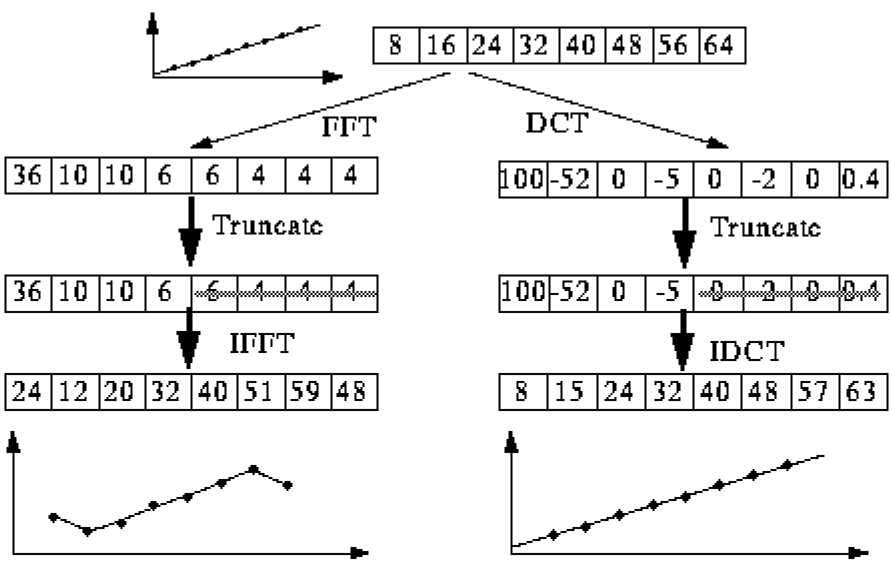
The 64 (8 x 8) DCT basis functions are illustrated in Fig 7.9.



**DCT basis functions**

- Why DCT not FFT?

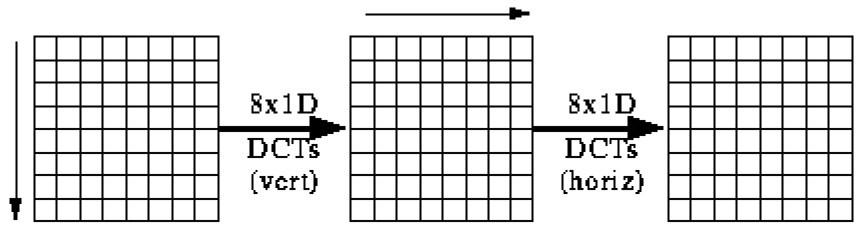
DCT is similar to the Fast Fourier Transform (FFT), but can approximate lines well with fewer coefficients (Fig 7.10)



**DCT/FFT Comparison**

- Computing the 2D DCT
  - Factoring reduces problem to a series of 1D DCTs (Fig 7.11):
    - apply 1D DCT (Vertically) to Columns
    - apply 1D DCT (Horizontally) to resultant Vertical DCT above.
    - or alternatively Horizontal to Vertical.

The equations are given by:




  - Most software implementations use fixed point arithmetic. Some fast implementations approximate coefficients so all multiplies are shifts and adds.