Real Estate Investment Optimization

Using Mixed-Integer Linear Programming

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Agenda

- Introduction
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- Data Source
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- Optimization Methodology
- Objective function and constraints
- Result
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Introduction

We are consulting a fictitious Real Estate Firm based out of Chicago

Our objective is to optimize a Real Estate Investment portfolio by allocating the appropriate number of 1, 2 & 3 bedroom properties in three distinct Chicago area zip codes that maximize returns.

Return is defined as the difference between the forecasted value of the property at month 12 and the list cost for that property.

Our profit maximisation solution relies on current and expected future price data.

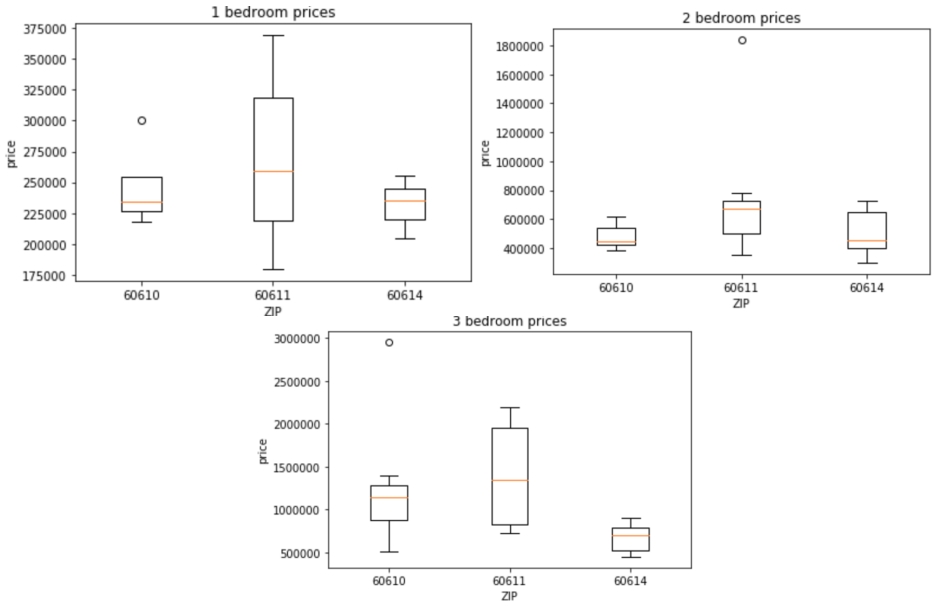
Data Sources:

- Price trend forecasting: Historic Median Home Values dataset (**Zillow Research** Bedroom & Zipcode series)
- For Optimization: Current MRED/MLS Property Listings in Chicago (**RedFin**)

Variable considerations & other insights:

- Notable variables include age of listing, property type, sq.ft, no. of bedrooms and baths, price, and days on market.
- Cannot normalise price by sq.ft, as most of the listings don't have this information.

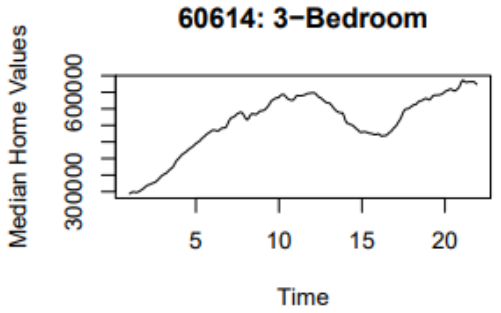
Assumption: Forecasted value of each property within a zip-code is contingent on the property type (1,2 or 3 bdr) and its' location (zip-code).



"This analysis uses price trends forecasted with historic median home values to arrive at expected future price points of property listings"

Utilizing Time-Series Forecasting to Project Prices

- We utilized the Zillow Research MHV data by Zip Code & Bedroom Type to build ARIMA models to forecast out Median Home Values to 12 months (Monthly TS data, 1996-2017 was smoothed and seasonally adjusted).
- Indexes were calculated for each Zip Code and Bedroom combination by dividing the point-forecast at month 12 by the last MHV for that Zip Code & Bedroom combination.
- The list cost of each 1,2 & 3 bedroom properties were multiplied by their respective index (Zip Code & Bedroom combination) in order to get an estimated home value at month 12..



Monthly Index Example
Zip Code: 60614 - 3 Bedrooms

Month 1	0.9987
Month 2	0.9991
Month 3	1.0012
Month 4	1.0031

Methodology

Mixed Integer Linear Programming

- A mixed-integer linear programming problem is one where some of the decision variables are constrained to be integer values at the optimal solution.

$$\begin{aligned} \text{maximize} \quad & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b}, \\ & \mathbf{x} \geq \mathbf{0}, \\ \text{and} \quad & \mathbf{x} \in \mathbb{Z}^n, \end{aligned}$$

- Objective Function must be linear expression
- Constraints must be linear expression

Julia - JuMP + GLPKMathProgInterface

- x[1:m1], y[1:m2], and z[1:m3], (variable vectors), are declared as **binary** types as decisions to buy or not buy.
- Assume the listing prices of properties be the cost of unit thus the total investment is represented by multiplication of listing price vector and decision vector.
- To simplify the problem, use the average forecasts of 12 month return by zone and bed-type, to compute the expected return.
- Due to the potential errors of forecasted return in a year, we choose to diversify the investment portfolio .

Definition Of Optimization Model

Variables

x : properties in zip 60610
y : properties in zip 60611
z : properties in zip 60614
rv_ : expected revenue of _
c_ : total investment of _
_1 : one-bed unit out of _
_2 : two-bed unit out of _
_3 : three-bed unit out of _

```
@variables md1 begin
    x1[1:m1], Bin
    x2[1:m2], Bin
    x3[1:m3], Bin
    y1[1:m4], Bin
    y2[1:m5], Bin
    y3[1:m6], Bin
    z1[1:m7], Bin
    z2[1:m8], Bin
    z3[1:m9], Bin
    rvx >=0
    rvy >=0
    rvz >=0
    cx >=0
    cy >=0
    cz >=0
end
```

Objective Function

Maximize the return of investment

```
#expected profit = revenue - cost
@objective(md1, Max, rvx+rvy+rvz-cx-cy-cz)
```

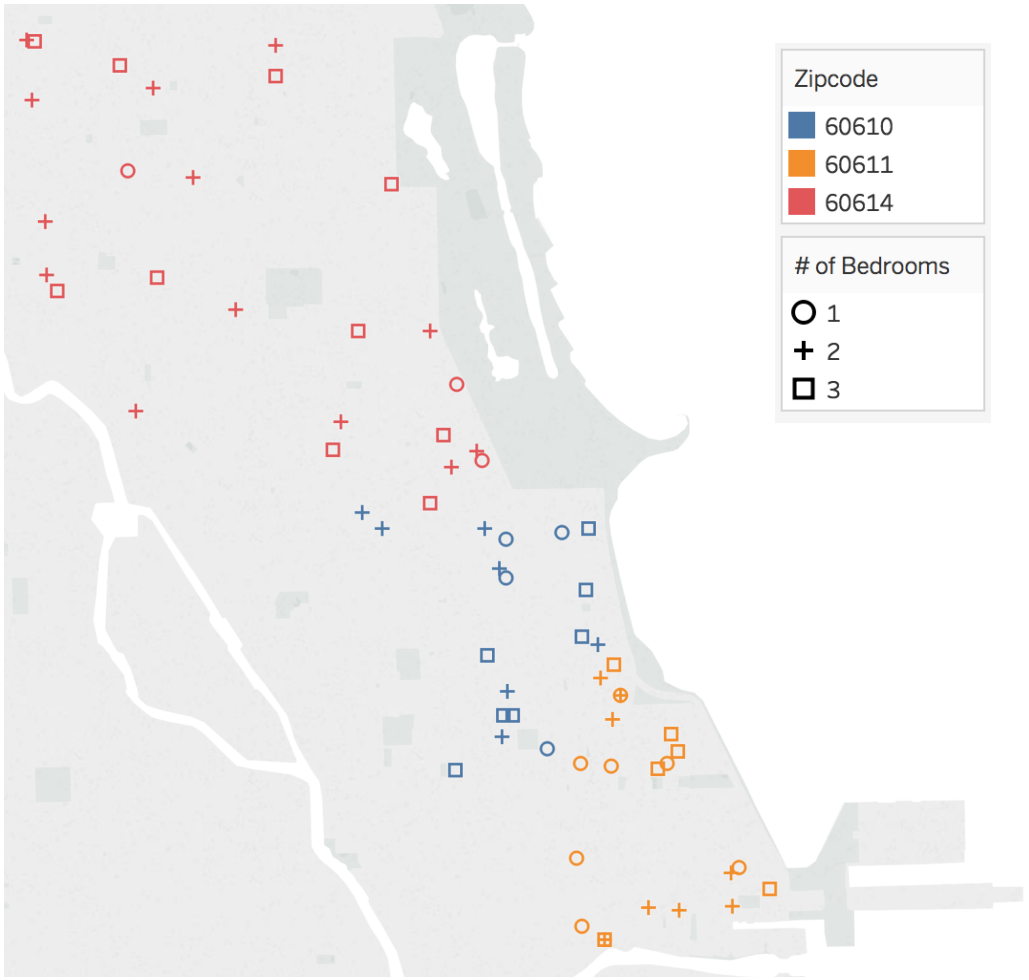
Constraints

Rule 1: Total inversement has an upper boundary which is the budget.
Rule 2: Diversity of Investment Portfolio.
Rule 3: Either Costs or Expected Return must be positive.

```
@constraints(md1, begin
    cx - ( AffExpr(x1,cx1,0) + AffExpr(x2,cx2,0) + AffExpr(x3,cx3,0) )== 0 #cost of zip 1
    cy - ( AffExpr(y1,cy1,0) + AffExpr(y2,cy2,0) + AffExpr(y3,cy3,0) )== 0 #cost of zip 2
    cz - ( AffExpr(z1,cz1,0) + AffExpr(z2,cz2,0) + AffExpr(z3,cz3,0) )== 0 #cost of zip 3
    cx + cy + cz <= tot # upper boundary for total invs
    cx <= 0.5 * tot # diversify the inversement
    cy <= 0.5 * tot
    cz <= 0.5 * tot
    sum(x1)+sum(y1)+sum(z1)>=1
    sum(x2)+sum(y2)+sum(z2)>=1
    sum(x3)+sum(y3)+sum(z3)>=1
    rvx - ( AffExpr(x1,cx1,0)*rrx1 + AffExpr(x2,cx2,0)*rrx2 + AffExpr(x3,cx3,0)*rrx3 ) == 0 #Rev of zip1
    rvy - ( AffExpr(y1,cy1,0)*rry1 + AffExpr(y2,cy2,0)*rry2 + AffExpr(y3,cy3,0)*rry3 ) == 0 #Rev of zip2
    rvz - ( AffExpr(z1,cz1,0)*rrz1 + AffExpr(z2,cz2,0)*rrz2 + AffExpr(z3,cz3,0)*rrz3 ) == 0 #Rev of zip3
end)
```

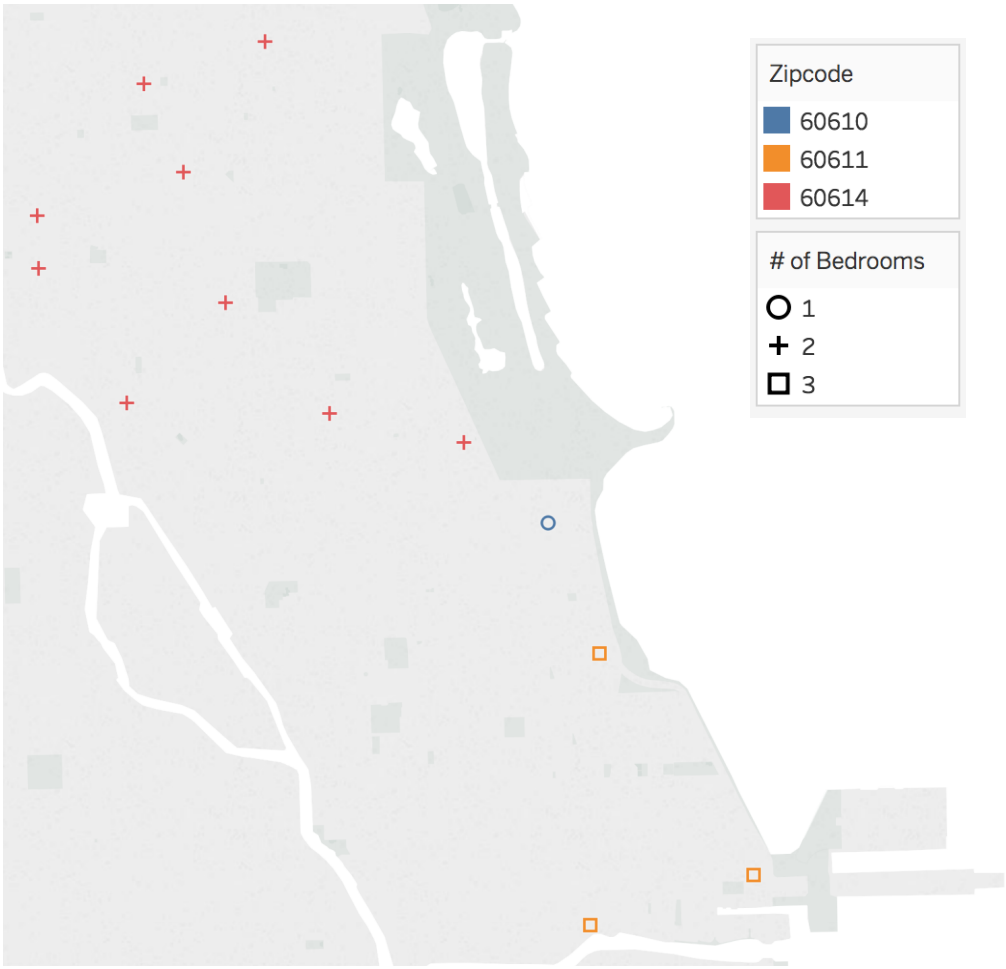
Optimization Model Inputs

- Pre-selected properties by zip codes
  - 22 from Streeterville
  - 18 from Gold Coast
  - 26 from Lincoln Park
- Identify properties by features
  - 14 1-bedroom Condominiums
  - 28 2-bedroom Condominiums/Townhouses
  - 24 3-bedroom Condominiums/Townhouses
- Forecast indexes by features and zip codes obtained using time series model



Optimization Model Result

- Objective value or Maximum Profit $125,211
- 13 properties were selected
  - 1-bedroom: 1 (60610)
  - 2-bedroom: 9 (60614)
  - 3-bedroom: 3 (60611)
- Constraints checklist
  - Total cost = $9,969,700 (less than $10mm)
  - ✓ Properties diversified (each zipcode is less than ½ of total investment by value)
  - At least 1 property for each feature
  - ✓ Each property has a net positive profit
  - ✓



With additional refinements, our prototype can mimic sophisticated real world real estate investment strategies.

Conclusion

- Our solution is extendable to other geographic locations/ additional zip codes.
- Additional criteria, such as per-zip code exposure weights can be implemented as additional constraints.
- Possible additional factors such as rental cash-flows, operational & transaction costs and risk-free interest rates, can also be incorporated to the objective function.

Possible extensions

- Portfolio diversification with risk under consideration. Consider a problem specification with N_t properties, W_t the weight of that investment which is a function of its risk components, cashflow and sunk costs.
- Stochastic dynamic programming based market timing strategy, to make periodic 'hold', 'buy' and 'sell' weight decisions in the investment horizon, using state vectors defining cash available, properties owned, price movements, rent, managing costs, etc.,.
- Dynamic extension to the popular ski rental problem, with projected price points at different times.

"Portfolio diversification, market timing, 'rent or buy?': with tweaks to the objectives and constraints, real estate portfolio optimization opportunities galore!"