Coordinate Direction Angles

Angles between vector and positive x,y,z.
$\alpha, \beta, \gamma \, \epsilon \, [0 \deg, 180 \deg]$
$\cos\alpha = \frac{A_x}{A}$ $\cos\beta = \frac{A_y}{A}$ $\cos\gamma = \frac{A_z}{A}$
$\vec{A} = A\cos\alpha\,\hat{i} + A\cos\beta\,\hat{j} + A\cos\gamma\,\hat{k}$
$\hat{u}_A = \frac{\vec{A}}{A} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Dot Product

$\vec{A} \cdot \vec{B} = AB\cos\theta$
$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

Projection of a Vector

$||\vec{F_{1A}}|| = \vec{F_1} \cdot \hat{u_A}$
$\text{proj}_{\vec{A}}(\vec{B}) = \frac{\vec{B}\cdot\vec{A}}{\vec{A}\cdot\vec{A}}\vec{A}$

Transverse & Azimuthal Angles

θ - Transverse angle - angle from xy projection to positive x-axis
ϕ - Azimuthal angle - angle from vector to z-axis
$A_z = A\cos\phi$

$A_{xy} = A\sin\phi$
$A_x = A_{xy}\cos\theta$
$A_y = A_{xy}\sin\theta$

Cross Product

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Triple Scalar Product

$\vec{C} \cdot (\vec{A} \times \vec{B})$

Chandelier

1. draw diagram 2. write $\vec{r}$ for all cables
3.find direction vector $\hat{u}$ for all $\vec{r}$
4. $\sum \vec{F}$ in x,y,z
5. $\vec{F_i} = F_iu_{ix}\hat{i} + F_iu_{iy}\hat{j} + F_iu_{iz}\hat{k}$
6. from equilibrium equations determine relationships between forces
7. sub max tension into forces to check which one breaks first
8. sub max tension into force that breaks first to find max weight

Moments

$\vec{M_O} = \vec{r_O} \times \vec{F}$
projection along an axis A:
$\vec{M_a} = \hat{u_A} \cdot \vec{M_O} = \hat{u_A} \cdot (\vec{r} \times \vec{F})$

Reduction to a wrench

1. Draw diagram
2. Write out $\hat{i}\,\hat{j}\,\hat{k}$ for all $\vec{F}$ and $\vec{M}$
3. $\vec{F_R} = \sum \vec{F} \,|\, \vec{M_{R_O}} = \sum \vec{M_o} + \sum \vec{M_c}$
4. $\vec{M_{||}} = (\vec{M_{R_O}} \cdot \hat{u_{F_R}})\hat{u_{F_R}}$
5. $\vec{M_\perp} = \vec{M_{R_O}} - \vec{M_{||}}$
6. Find $\vec{r_p}$ such that $\vec{r_p} \times \vec{F_R} = \vec{M_\perp}$ *7. Move $\vec{M_{||}}$ to $\vec{r_p}$
Solution:
"The wrench is $\vec{F_R} = (Fx\hat{i} + Fy\hat{j} + Fz\hat{k})$ acting at $\vec{r_p} = (rx\hat{i} + ry\hat{j} + rz\hat{k})$ with $\vec{M_{||}} = (M_{||}x\hat{i} + M_{||}y\hat{j} + M_{||}\hat{k})$"

Rigid Body Equilibrium (2D)

$\sum F_x = 0$ $\sum F_y = 0$ $\sum M_O = 0$

Rigid Body Equilibrium (3D)

$\sum F_x = 0$ $\sum F_y = 0$ $\sum F_z = 0$
$\sum M_x = 0$ $\sum M_y = 0$ $\sum M_z = 0$
$\vec{M_O} = \vec{r} \times \vec{F}$
$\vec{F_R} = \sum \vec{F}$ $\vec{M_{R_O}} = \sum(\vec{r_i} \times \vec{F_i}) + \sum \vec{M_{c_i}}$

Trusses

Method of Joints

$\sum F_x = 0$ $\sum F_y = 0$
Zero-force rules:
1. Two non-colinear members at unloaded joint $\Rightarrow$ both zero-force
2. Three members, two colinear, joint unloaded $\Rightarrow$ non-colinear member zero-force

Method of Sections

Cut ≤ 3 unknowns:
$\sum F_x = 0$ $\sum F_y = 0$ $\sum M = 0$

Friction

Static: $F_s \leq \mu_s N$ Impending: $F_s = \mu_s N$
Kinetic: $F_k = \mu_k N$

Incline: $N = W\cos\theta$ $W_{||} = W\sin\theta$

Tipping vs sliding:
$\theta_{sliding}:\ \mu_s = \tan\theta$
$\theta_{tipping}:\ \tan\theta = \dfrac{b}{2h}$