Coordinate Direction Angles

Angles between vector and positive x,y,z.
α, β, γ ϵ [0 deg, 180 deg]
$\cos\alpha = \frac{A_x}{A}$ $\cos\beta = \frac{A_y}{A}$ $\cos\gamma = \frac{A_z}{A}$
$\vec{A} = A\cos\alpha\,\hat{i} + A\cos\beta\,\hat{j} + A\cos\gamma\,\hat{k}$
$\hat{u}_A = \frac{\vec{A}}{A} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Dot Product

$\vec{A}\cdot\vec{B} = AB\cos\theta$
$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Projection of a Vector

$||\vec{F_{1A}}|| = \vec{F_1}\cdot\hat{u_A}$
$\text{proj}_{\vec{A}}(\vec{B}) = \frac{\vec{B}\cdot\vec{A}}{\vec{A}\cdot\vec{A}}\vec{A}$

Transverse & Azimuthal Angles

θ - Transverse angle - angle from xy projection to positive x-axis
ϕ - Azimuthal angle - angle from vector to z-axis
$A_z = A\cos\phi$

$A_{xy} = A\sin\phi$
$A_x = A_{xy}\cos\theta$
$A_y = A_{xy}\sin\theta$

Cross Product

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Triple Scalar Product

$\vec{C}\cdot(\vec{A}\times\vec{B})$

1 Spring

$F = ks$

Chandelier

1. draw diagram 2. write $\vec{r}$ for all cables
3.find direction vector $\hat{u}$ for all $\vec{r}$
4. $\sum\vec{F}$ in x,y,z
5. $\vec{F_i} = F_iu_{ix}\hat{i} + F_iu_{iy}\hat{j} + F_iu_{iz}\hat{k}$
6. from equilibrium equations determine relationships between forces
7. sub max tension into forces to check which one breaks first
8. sub max tension into force that breaks first to find max weight

Moments

$\vec{M_O} = \vec{r_O}\times\vec{F}$
projection along an axis A:
$\vec{M_a} = \hat{u_A}\cdot\vec{M_O} = \hat{u_A}\cdot(\vec{r}\times\vec{F})$

Reduction to a wrench

1. Draw diagram
2. Write out $\hat{i}\,\hat{j}\,\hat{k}$ for all $\vec{F}$ and $\vec{M}$
3. $\vec{F_R} = \sum\vec{F} \mid \vec{M_{R_O}} = \sum\vec{M_o} + \sum\vec{M_c}$
4. $\vec{M_{||}} = (\vec{M_{R_O}}\cdot\hat{u_{F_R}})\hat{u_{F_R}}$
5. $\vec{M_\perp} = \vec{M_{R_O}} - \vec{M_{||}}$
6. Find $\vec{r_p}$ such that $\vec{r_p}\times\vec{F_R} = \vec{M_\perp}$ *7. Move $\vec{M_{||}}$ to $\vec{r_p}$
Solution:

"The wrench is $\vec{F_R} = (Fx\hat{i} + Fy\hat{j} + Fz\hat{k})$ acting at $\vec{r_p} = (rx\hat{i} + ry\hat{j} + rz\hat{k})$ with $\vec{M_{||}} = (M_{||}x\hat{i} + M_{||}y\hat{j} + M_{||}\hat{k})$"