Curr. & Res. & Pow.

$n = N/V = e^{-}$density
$V_{\text{drift}} = \frac{I}{nAe}$ where $e = 1.6 \times 10^{-19}C$
Resistivity $[\rho] = \Omega m$
$R = \frac{\rho L}{A}$
$P = IV = I^2R = \frac{V^2}{R}$ Node: sum of currents into a node = sum out.
Junction: algebraic sum of potential differences around a loop = 0.

Electrostatics

$\vec{F} = \frac{|q_1 q_2|}{4\pi\varepsilon_0 r^2}$
$\varepsilon_0 = 8.85 \times 10^{-}12 \frac{C^2}{Nm^2}$
$e = 1.6 \times 10^{-19}C$
$\vec{E} \equiv \frac{\vec{F}}{Q} \mid [\vec{E}] = \frac{N}{C} = \frac{V}{m}$
$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$
$\vec{E} = \sum \vec{E}_i = \frac{q}{4\pi\varepsilon_0 r_i^2}\hat{r}_i$
Energy in a field $U_E = \frac{1}{2}\varepsilon_0 E^2$
$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r}$
$\Phi = \vec{E} \cdot \vec{A}$

Steps for Continuous Charge Distributions

1. Divide into small charges dQ
2. Find var for pos of dQ w/ bounds
3. Determine field due to dQ
4. Break field into components if needed
5. Express all in terms of const and pos. var
6. Integrate wrt pos var

Gauss's Law

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$
Always true but only useful to find E when symmetrical (typically spherical, cylindrical, planar)

Conductors arrange charge to have zero internal electric field.
Charge will always move to the surface of a conductor.
$|\vec{E}| = \frac{\sigma}{\varepsilon}$

Electric Potential

$U = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$ for point charges
$\Delta U = -W_E = -\int F dr$
Electric potential: $\Delta V = \frac{\Delta U}{Q} = -\int E dr$
For a uniform field: $\Delta V = -\vec{E} \cdot \Delta\vec{r}$
Potential increases moving against the electric field
Conductors are equipotential surfaces.
$\vec{E} = \vec{0}$ inside conductor
V in conductor = V surface and is constant
$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$
$\Delta V = -\int_{\vec{r_0}}^{\vec{r}} \vec{E} \cdot d\vec{l}$

Electric Field from Potential

$\vec{E} = -\vec{\nabla} V$
$\vec{E}_x = -\frac{\partial V}{\partial x}\hat{\imath}$
$\vec{E}_y = -\frac{\partial V}{\partial y}\hat{\jmath}$
$\vec{E}_z = -\frac{\partial V}{\partial z}\hat{k}$

Capacitors

$C = \frac{Q}{V}$
Capacitance depends only on geometry of conductor
Parallel plate: $C = \frac{\epsilon_0 A}{d}$
Dielectrics weaken field by factor κ (on ε_0), increase cap. by same factor.
In circuits, potential change across cap. $V = \frac{Q}{C}$

Energy in a capactior

$U_C = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q}{C}^2$

Dielectric Effects

Battery Connected (constant V):
$C' = \kappa C_0$
$Q' = C'V = \kappa Q_0$
$U' = \frac{1}{2}C'V^2 = \kappa U_0$
Battery Disconnected (constant Q):
$V' = \frac{Q}{C'} = \frac{V_0}{\kappa}$
$U' = \frac{Q^2}{2C'} = \frac{U_0}{\kappa}$

Fields & Potentials

Charge Densities

$\lambda = \frac{Q}{L} \mid \sigma = \frac{Q}{A} \mid \rho = \frac{Q}{V}$

Point Charge

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$
$V = \frac{Q}{4\pi\varepsilon_0 r}$

Infinite Plane of Charge

$\vec{E} = \frac{\sigma}{2\varepsilon_0}$, directed $\perp$ to plane
$V = -\frac{\sigma}{2\varepsilon_0}x$ (for distance x from plane)

Infinite Line of Charge

$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}$, directed $\perp$ to line
$V = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r}{r_0}\right)$

Conducting Hollow Sphere

Same as point charge for $r > R$ of sphere:
$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$,
$V = \frac{Q}{4\pi\varepsilon_0 r}$
Inside ($r < R$): $\vec{E} = 0$,
$V = \frac{Q}{4\pi\varepsilon_0 R}$

Uniformly Charged Solid Sphere

For $r > R$: same as point charge
For $r < R$: $\vec{E} = \frac{Qr}{4\pi\varepsilon_0 R^3}\hat{r}$,
$V = \frac{Q}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)$

Infinitely Long Cylindrical Volume (uniform ρ)

$$\vec{E} = \begin{cases} \dfrac{\rho r}{2\varepsilon_0}, & r < R \\ \dfrac{\rho R^2}{2\varepsilon_0 r}, & r > R \end{cases}$$

Finite Cylinder (no charge on ends)

$\vec{E} = \frac{Q}{4\pi h\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+h^2}}\right]$ along axis

Charged Disc (on axis, total charge Q)

$\vec{E} = \frac{Q}{2\pi\varepsilon_0 R^2}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$ along axis