P346/475 Computer Lab

End-Semester examination, 2025

NISER, Bhubaneswar

Full marks: 30 Time: 3 hours

Marks are given in bold along with the questions. Attempt all.

1. Consider a box divided into two equal halves separated by a wall. At $t = 0$, there are $N = 5000$ particles on the left side. A hole is punched in the wall and one particle can pass through it at a time. All particles have equal probabilities of going to either sides. Graphically determine the equilibrium state (number of particles in left and right sides) when sufficient time ($\gg N$) has passed. **[4]**

2. Solve the following linear equation by Gauss-Seidel method to a precision of 10^{-6}. **[4]**

$$\begin{pmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 2 \\ 1 \\ 2 \end{pmatrix}$$

3. A spring is attached to a rigid wall at one end and a force of $F = 2.5$ Newton is applied at the other. For an arbitrary displacement x, the force acting on the system is $F(x) = F - x\exp(x)$. Find, using Newton-Raphson method, how far the spring can be stretched. **[4]**

4. A 2 meter long beam has a linear mass density $\lambda(x) = x^2$, where x is measured from one its ends. Find the center of mass of the beam numerically using method of your choice. Report answer correct up to 4 decimal places. **[4]**

5. An object is thrown vertically upward with velocity $v_0 = 10$ from the ground. It experiences air resistances proportional to its velocity at any instant. The differential equation describing the motion is

$$\ddot{y} = -\gamma\,\dot{y} - g \quad \text{or, alternatively} \quad v\frac{dv}{dy} = -\gamma\,v - g$$

where $g = 10$ and $\gamma = 0.02$ in appropriate unit. Determine the maximum height reached by the object. Plot the variation of the velocity with height. Use RK4. `[Hint : first determine maximum height reached without air resistance and then set it as upper limit for` y`.]` **[5]**

6. Solve the 1-dimension heat equation $u_{xx} = u_t$ over a metal rod of length 2 units, with the initial conditions,

$$\begin{aligned} u(0,t) = 0^o\text{C} = u(2,t) \qquad & \text{for} \quad 0 \le t \le 4 \\ u(x,0) = 20\,|\sin(\pi x)\,|^o\text{C} \qquad & \text{for} \quad 0 \le x \le 2 \end{aligned}$$

Take number of position grid $nx = 20$ and time grid $nt = 5000$. Show the temperature profile across the length of the rod at time steps 0, 10, 20, 50, 100, 200, 500 and 1000 in a plot. **[4]**

7. Fit the data given in the datafile `esem4fit.txt` with a quartic polynomial of the form $a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. **[5]**