ناو (کوردی/عربي):

بەروار: ٢٠٢٣/١٠/١٢

1. Test the below series for convergence or divergence, and give the reason:

$$\sum_{n=0}^{\infty} e^{-2n}$$

29. convergent geometric series with sum $\frac{1}{1-\left(\frac{1}{e^2}\right)} = \frac{e^2}{e^2-1}$

2. Express the below number as a ratio of two integers:

$0.\overline{23} = 0.23\ 23\ 23\ldots$

51. $0.\overline{23} = \sum_{n=0}^{\infty} \frac{23}{100}\left(\frac{1}{10^2}\right)^n = \frac{\left(\frac{23}{100}\right)}{1-\left(\frac{1}{100}\right)} = \frac{23}{99}$

3. Use integral test to determine if the below series converges or diverges?

$$\sum_{n=2}^{\infty} \frac{\ln n}{n}$$

9. diverges by the Integral Test: $\int_2^n \frac{\ln x}{x}\,dx = \frac{1}{2}\left(\ln^2 n - \ln 2\right) \Rightarrow \int_2^{\infty} \frac{\ln x}{x}\,dx \to \infty$

4. Test convergence or divergence by using the limit comparison test ($1/n^2$)

$$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^3}$$

11. converges by the Limit Comparison Test (part 2) when compared with $\sum_{n=1}^{\infty} \frac{1}{n^2}$, a convergent p-series:

$$\lim_{n\to\infty} \frac{\left[\frac{(\ln n)^2}{n^3}\right]}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{(\ln n)^2}{n} = \lim_{n\to\infty} \frac{2(\ln n)\left(\frac{1}{n}\right)}{1} = 2\lim_{n\to\infty} \frac{\ln n}{n} = 0$$