ناو (كوردى/عربي):

بەروار: ٢٠٢٣/١٠/١٩

Q/ Test the convergence and the divergence of the below series:

1. **(Use ratio test):**

$$\sum_{n=1}^{\infty} \frac{n^{\sqrt{2}}}{2^n}$$

1. converges by the Ratio Test: $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{\left[\frac{(n+1)^{\sqrt{2}}}{2^{n+1}}\right]}{\left[\frac{n^{\sqrt{2}}}{2^n}\right]} = \lim_{n \to \infty} \frac{(n+1)^{\sqrt{2}}}{2^{n+1}} \cdot \frac{2^n}{n^{\sqrt{2}}}$ $= \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{2}} \left(\frac{1}{2}\right) = \frac{1}{2} < 1$

2. (Use root test):

$$\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$$

23. converges by the Root Test: $\lim_{n \to \infty} \sqrt[n]{a_n} = \lim_{n \to \infty} \sqrt[n]{\frac{n}{(\ln n)^n}} = \lim_{n \to \infty} \frac{\sqrt[n]{n}}{\ln n} = \lim_{n \to \infty} \frac{1}{\ln n} = 0 < 1$

3. Determine if the below series diverges, converges, or converges absolutely or conditionally? Give the reason

$$\sum_{n=1}^{\infty} (-1)^{n+1}(0.1)^n$$

11. converges absolutely since $\sum_{n-1}^{\infty} |a_n| = \sum_{n-1}^{\infty} \left(\frac{1}{10}\right)^n$ a convergent geometric series

4.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$

13. converges conditionally since $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}} > 0$ and $\lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0 \Rightarrow$ convergence; but $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ is a divergent p-series