Predicting Carseats Sales with Decision Trees and Ensemble Methods

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Motivation

Retailers often want to understand what drives **product sales** and how to best predict them. In this project, I use the **Carseats dataset** from the ISLP package to predict Sales as a quantitative response. I apply and compare **tree-based regression methods**:

- Decision Trees (with pruning)
- Bagging
- Random Forests

The goal is to evaluate predictive accuracy, interpretability, and feature importance across methods, building on concepts explored in ISLP.

Data Analysis

Importing Necessary Dependencies

We begin by importing important libraries and the dataset, which is loaded using the `ISLP` libraries. We then split the dataset into 70% training and 30% test observations.

In [1]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import sklearn.model_selection as skm
from ISLP import load_data
from ISLP.models import ModelSpec as MS
from sklearn.tree import DecisionTreeRegressor as DTR
from sklearn.ensemble import RandomForestRegressor as RF

# Load Carseats dataset
Carseats = load_data('Carseats')

# Create design matrix with all predictors except Sales
model = MS(Carseats.columns.drop('Sales'), intercept = False)
D = model.fit_transform(Carseats)

feature_names = list(D.columns)
X = np.asarray(D)

# Split dataset train (70%) and test (30%)
(X_train, X_test, y_train, y_test) = skm.train_test_split(X, Carseats['Sales'], test_size = 0.3, random_state = 0)

Carseats
```

Out[1]:

	Sales	CompPrice	Income	Advertising	Population	Price	ShelveLoc	Age	Education	Urban	US
0	9.50	138	73	11	276	120	Bad	42	17	Yes	Yes
1	11.22	111	48	16	260	83	Good	65	10	Yes	Yes
2	10.06	113	35	10	269	80	Medium	59	12	Yes	Yes
3	7.40	117	100	4	466	97	Medium	55	14	Yes	Yes
4	4.15	141	64	3	340	128	Bad	38	13	Yes	No
...	...	...	...	...	...	...	...	...	...	...	...
395	12.57	138	108	17	203	128	Good	33	14	Yes	Yes
396	6.14	139	23	3	37	120	Medium	55	11	No	Yes
397	7.41	162	26	12	368	159	Medium	40	18	Yes	Yes
398	5.94	100	79	7	284	95	Bad	50	12	Yes	Yes
399	9.71	134	37	0	27	120	Good	49	16	Yes	Yes

400 rows × 11 columns

The response variable is `Sales`, the unit sales (in thousands) at every location. The predictors are:

- `CompPrice` : Price charged by competitor at each location
- `Income` : Community income level (in thousands of dollars)
- `Advertising` : Local advertising budget for company at each location (in thousands of dollars)
- `Population` : Population size in region (in thousands)
- `Price` : Price company charges for car seats at each site
- `ShelveLoc` : A factor with levels Bad, Good and Medium indicating the quality of the shelving location for the car seats at each site
- `Age` : Average age of the local population
- `Education` : Education level at each location
- `Urban` : A factor with levels No and Yes to indicate whether the store is in an urban or rural location
- `US` : A factor with levels No and Yes to indicate whether the store is in the US or not

Decision Tree Regressor

We begin with the simplest model, the decision tree. Decision trees provide an interpretable model for regression problems. However, unpruned trees often overfit, so we compare:

1. A **full tree** (no pruning)
2. A **pruned tree** chosen via cross-validation and cost-complexity pruning

In [2]:
```python
# Create the Decision Tree Regressor model with a fixed random state for reproducibility
reg = DTR(random_state = 0)

# Fit the model to the training dataset
reg.fit(X_train , y_train)

# Predict values of test dataset
preds = reg.predict(X_test)

# Compute and print test MSE (error metric)
full_mse = np.mean((preds - y_test)**2)
print(f"Test MSE (full tree): {full_mse}")
print(f"Number of terminal nodes (full tree): {reg.get_n_leaves()}")
```

```
Test MSE (full tree): 5.083071666666667
Number of terminal nodes (full tree): 277
```

We now train a pruned tree by utilizing k-fold cross validation with $k = 5$ to see if the testing MSE will decrease.

In [3]:
```python
# Cost-complexity pruning path
ccp_path = reg.cost_complexity_pruning_path(X_train, y_train)

# Cross-validation to select optimal alpha
kfold = skm.KFold(5, shuffle = True, random_state = 10)
grid = skm.GridSearchCV(
    reg, {'ccp_alpha': ccp_path.ccp_alphas},
    refit = True, cv = kfold, scoring='neg_mean_squared_error'
)

# Fit the model to the training dataset
G = grid.fit(X_train, y_train)
best_ = grid.best_estimator_

# Compute and print test MSE for pruned tree
cv_mse = np.mean((y_test - best_.predict(X_test))**2)
print(f"Test MSE (pruned tree): {cv_mse}")
print(f"Number of terminal nodes (pruned tree): {best_.get_n_leaves()}")
```

```
Test MSE (pruned tree): 4.431070971603666
Number of terminal nodes (pruned tree): 55
```

Despite dropping to 55 terminal nodes in the pruned tree as opposed to the 277 terminal nodes in the full tree, we improve the test MSE by over 10%, showcasing the power of pruned trees in capturing data patterns and delivering accurate predictions while reducing complexity by preventing overfitting.

Bagging

Bagging (Bootstrap Aggregating) reduces variance by training many trees on bootstrap samples and averaging their predictions. This typically improves predictive accuracy compared to a single tree by preventing overfitting and introducing randomization (through bootstrapping). We also examine **feature importance** to see which predictors matter most for Sales.

In [4]:
```python
# Create the bagging model (random forest with all predictors considered)
bag_carseats = RF(max_features = X_train.shape[1], random_state = 0)

# Fit the model to the training dataset
bag_carseats.fit(X_train, y_train)

# Predict values of the test dataset
bag_preds = bag_carseats.predict(X_test)

# Compute and print test MSE
bag_mse = np.mean((bag_preds - y_test)**2)
print(f"Test MSE (bagging): {bag_mse}")

# Compute feature importance
importances = bag_carseats.feature_importances_
feature_importance_df = pd.DataFrame({
    'Feature': feature_names,
    'Importance': importances
})

# Sort feature importances in descending order and print
feature_importance_df = feature_importance_df.sort_values(by='Importance', ascending=False)
print("Feature importance for bagging model:")
print(feature_importance_df)
```

```
Test MSE (bagging): 2.0077445197500015
Feature importance for bagging model:
              Feature  Importance
4               Price    0.278666
5      ShelveLoc[Good]    0.222431
7                 Age    0.108637
0           CompPrice    0.097085
6   ShelveLoc[Medium]    0.082965
2         Advertising    0.074627
1              Income    0.051799
3          Population    0.040945
8           Education    0.030836
10            US[Yes]    0.006815
9          Urban[Yes]    0.005195
```

We see that bagging places the most emphasis on price and good shelf location. These are both logical indicators of sales performances, as the value of a good carseat and its selling position in a marketplace would heavily influence buyer traffic and encourage potential customers to consider and purchase the carseat.

Random Forests

Random forests improve upon bagging by decorrelating trees. At each split, only a random subset of features is considered, which reduces the dominance of strong features and leads to more diverse trees.

We evaluate random forests in two ways:

1. **Varying** m (the number of features considered at each split) and plotting test MSE.
2. Using the default recommendation of $m = \sqrt{p}$ (rounded) and examining feature importance.

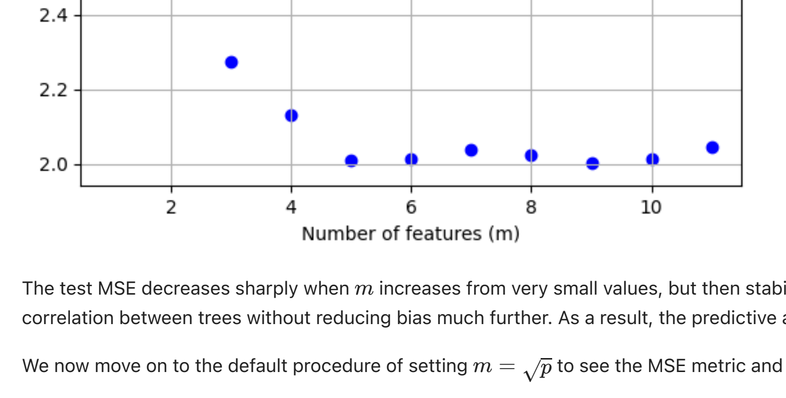
Our random forests will use 1,000 trees and vary in number of features considered.

In [5]:
```python
# Set up number of features
p = len(feature_names)

# Create lists for MSE values and number of features
rf_mse_vals = []
m_vals = np.arange(1, p+1)

# Fit a random forest for varying number of features
for i in m_vals:
    RF_carseats = RF(max_features = i, n_estimators = 1000, random_state = 0) # set random state for reproducibility
    RF_carseats.fit(X_train, y_train)
    rf_preds = RF_carseats.predict(X_test)
    rf_mse_vals.append(np.mean((rf_preds - y_test)**2))

# Plot MSE vs. m (number of features)
plt.scatter(m_vals, rf_mse_vals, color = "b")
plt.title("Test MSE of Random Forest with m features")
plt.xlabel("Number of features (m)")
plt.ylabel("Test MSE")
plt.grid()
```



The test MSE decreases sharply when m increases from very small values, but then stabilizes. This is expected: once the strongest predictors are available to the trees, adding more predictors only increases correlation between trees without reducing bias much further. As a result, the predictive accuracy levels off, with small fluctuations due to randomness.

We now move on to the default procedure of setting $m = \sqrt{p}$ to see the MSE metric and feature importances.

In [6]:
```python
# Print number of features and m = sqrt(p) value
print("Number of features (p):", p)
m = round(np.sqrt(p))
print("m = sqrt(p):", m)

# Create random forest with m = sqrt(p) features and random state for reproducibility
RF_carseats_opt = RF(max_features = m, n_estimators = 1000, random_state = 0)

# Fit the model to the training dataset
RF_carseats_opt.fit(X_train, y_train)

# Predict values of the test dataset
rf_preds_opt = RF_carseats_opt.predict(X_test)

# Compute and print test MSE
rf_opt_mse = np.mean((rf_preds_opt - y_test)**2)
print(f"Test MSE (random forest with sqrt(p) features): {rf_opt_mse}")

# Compute feature importance
importances = RF_carseats_opt.feature_importances_
feature_importance_df = pd.DataFrame({
     'Feature': feature_names,
     'Importance': importances
})

# Sort feature importances in descending order and print
feature_importance_df = feature_importance_df.sort_values(by='Importance', ascending=False)
print("Feature importance for model with m = sqrt(p) features:")
print(feature_importance_df)
```

```
Number of features (p): 11
m = sqrt(p): 3
Test MSE (random forest with sqrt(p) features): 2.2756299329358276
Feature importance for model with m = sqrt(p) features:
              Feature  Importance
4               Price    0.243342
5      ShelveLoc[Good]    0.160471
7                 Age    0.122006
0           CompPrice    0.097027
2         Advertising    0.087855
1              Income    0.085006
3          Population    0.076153
8           Education    0.050686
6   ShelveLoc[Medium]    0.048134
10            US[Yes]    0.016977
9          Urban[Yes]    0.012344
```

The rule-of-thumb $m = \sqrt{p}$ is a reasonable default, but it does not always minimize test error. In this dataset, the optimal m could be slightly higher or lower depending on which features are most predictive and how correlated they are, reflecting the practical challenges of balancing bias and variance in real-world machine learning.

Overall Takeaways

Decision Trees:

- Easy to interpret and visualize.
- Full trees tend to overfit, leading to higher test MSE.
- Pruning improves generalization by reducing the number of terminal nodes, balancing bias and variance.

Bagging:

- Aggregating multiple trees reduces variance and stabilizes predictions.
- Test MSE is lower than a single decision tree, showing the benefit of ensemble averaging.
- Feature importance analysis highlights the predictors most strongly influencing Sales, providing actionable insights.

Random Forests:

- Further reduces variance by decorrelating trees through random feature selection at splits.
- Provides the best predictive performance in this dataset.
- The choice of m (number of features per split) affects performance, and the default heuristic $m = \sqrt{p}$ is reasonable but not guaranteed optimal.

Practical insights:

- Certain predictors (e.g., Price, ShelveLoc, Age) consistently emerge as important, reflecting key drivers of product sales.
- Ensemble methods balance interpretability and accuracy, making them useful for both predictive modeling and business decision-making.

Overall learning:

- Tree-based methods offer flexible modeling for tabular data.
- Combining multiple trees via bagging or random forests improves predictive reliability.
- Understanding hyperparameters, like pruning complexity or m in random forests, is critical to achieving strong generalization.