

BB84

Quantum *Protected* Cryptography

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Outline

Textbook BB84

- Quantum Optics Refresher
- BB84

Textbook to Production

- Error Correction
- Privacy Amplification
- Authentication
- Photon Number Splitting

EM Waves

- ▶ Light is paired waves in electric and magnetic fields

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- ▶ General wave equation

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- ▶ Maxwell's equations

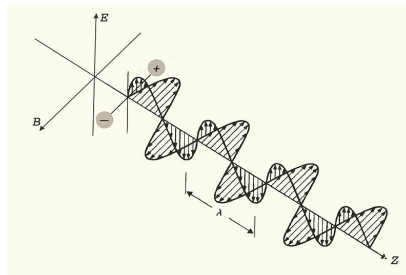
$$\oint_A \mathbf{E} d\mathbf{A} = \oint_A \mathbf{B} d\mathbf{A} = 0$$

$$\oint_C \mathbf{E} d\mathbf{l} = - \int_A \frac{\partial \mathbf{B}}{\partial t} d\mathbf{A}$$

$$\oint_C \mathbf{B} d\mathbf{l} = \mu_0 \epsilon \int_A \frac{\partial \mathbf{E}}{\partial t} d\mathbf{A}$$

Linearly Polarized Light

- ▶ Useful simplified model: assume harmonic, planar waves.
 - ▶ Between Fourier transforms and other superpositions this covers a surprising amount of ground.
 - ▶ Call the orientation of the E -wave the light's *polarization*



Polarizers

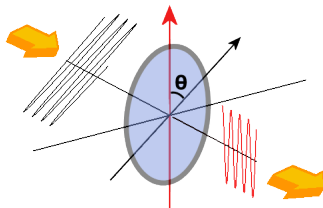
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- ▶ All light passing through a polarizer is dimmed according to the alignment of its polarization with the polarizer.
- ▶ Ideal polarizers follow Malus's law: $I = I_0 \cos^2 \theta$



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- ▶ Still have a polarization due to their wave aspect
 - ▶ or possibly a superposition of different polarizations ...
- ▶ Cannot be dimmed – quanta are indivisible.
- ▶ Instead polarizers absorb with probability $\sin^2 \theta$
 - ▶ or transmit with probability $\cos^2 \theta$

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- ▶ Observing a quantum entity, including photons, collapses it into a single state.
- ▶ Impossible to perfectly capture and reconstruct a photon of unknown polarization
 - ▶ Approximate cloning *is* possible, e.g. via stimulated emission
 - ▶ General upper bound on cloning fidelity of $\frac{5}{6}$
 - ▶ Fidelity bound for the case we'll care about of ≈ 0.8535

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- ▶ Bit flip with probability $\sin^2 \phi$, by Malus' law.

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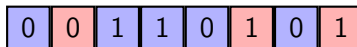
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8. Otherwise, the undropped and unannounced bits are now a shared secret.

Protocol in Pictures

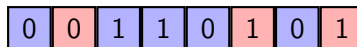
Alice:



Alice selects random bits, bases

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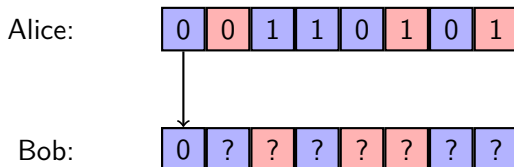


Bob:



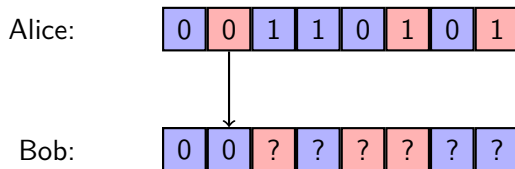
Bob selects random bases

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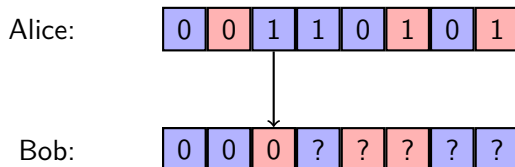
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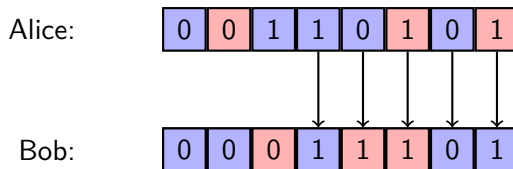
Bases disagree, measurement *may* fail

Protocol in Pictures



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Protocol in Pictures



Etc...

Protocol in Pictures

Alice:



Bob:



Sift out mismatched bases

Protocol in Pictures

Alice:



Bob:



Sample bits to detect Eve

Protocol in Pictures

Alice:



Bob:



Redact sampled bits

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- ▶ Can't stop Eve, but can *detect* her.

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- ▶ Sending and receiving individual quanta is *hard*
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- ▶ Secrets which are mostly the same are not nearly as useful as secrets which are exactly the same.
- ▶ We need to somehow rectify the differences between what Alice sent and Bob received.

Single Error Detection

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- ▶ Can write as a check matrix:

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 - ▶ Each parity bit will only cover some bits in the code word.
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- ▶ Hamming solved this for us in 1950 using powers of two
 - ▶ Code word: $(p_1, p_2, d_1, p_3, d_2, d_3, d_4)^T$
 - ▶ Check matrix:

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Full SECDED

- ▶ Glue a total parity bit onto a Hamming SEC scheme
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- ▶ Commonly known as Hamming(8,4).
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- ▶ We can reconcile errors by exchanging Hamming syndromes!

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7. Go back to (1) until convinced that the probability of surviving errors is negligible.

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- ▶ *Eve is allowed to have partial information on our secret!*
 - ▶ Or we can just never succeed in negotiating a secret
- ▶ Need to do two things
 - ▶ Estimate how many bits of information Eve has
 - ▶ Somehow scrub those bits out of our secret

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 - ▶ Too easy to have $\hat{p} < p$
- ▶ Bayes rule to pick \hat{p} s.t. $P(p > \hat{p} | \text{QBER}) < \epsilon$?
 - ▶ No sensible prior to put on p

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- ▶ Calculating CIs for a binomial distribution on the fly can be cumbersome
 - ▶ Common to pick m , p_{crit} in advance
 - ▶ Always trim off m bits, bail if $QBER > p_{\text{crit}}$

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 - ▶ choose our hash function randomly, although it *can* be public
- ▶ This is a direct result of the leftover hash lemma
- ▶ Approach is obvious:
 1. Alice publicly announces a random seed
 2. Alice and Bob both use the seed to hash their secrets down

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 - ▶ But we can still use the scheme to stretch the bootstrap secret, e.g. to allow for one-time pads in the data plane.
- ▶ Our goal is unconditional security, so a standard HMAC won't cut it.

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3. Encrypt the tag with a one-time pad.
4. Discard the m bits used for the OTP.

One Time MAC Intuitions

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 - ▶ Otherwise, she could backward engineer the pad used to encrypt legitimate MACs
- ▶ The hash value is encrypted, so she can't manufacture a hash collision.
 - ▶ Universal hash function means she can't even make intelligent guesses about the hash value.
- ▶ Best she can do is guess the tag
 - ▶ Succeeds with probability $\frac{1}{2^m}$
 - ▶ Places a tradeoff between key expenditure and probability that Eve gets lucky.

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- ▶ Can also block single-photon pulses to stop Alice/Bob from using them in the secret.
 - ▶ Bob expects a lot of transmissions to get lost, this won't necessarily set off alarm bells

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 - ▶ Possible for a pulse to include multiple photons.
- ▶ Eve could sniff multi-photon pulses without corrupting them
- ▶ Can also block single-photon pulses to stop Alice/Bob from using them in the secret.
 - ▶ Bob expects a lot of transmissions to get lost, this won't necessarily set off alarm bells
- ▶ Attack is commonly known as photon number splitting

Decoy States

- ▶ Alice uses two (or more) signal sources S and S'
 - ▶ S is the real photon source, averages μ photons per pulse
 - ▶ S' is a decoy, with $\mu' > \mu$
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- ▶ Impossible for Eve to tell whether a given pulse came from S or S'
- ▶ By blocking single photon pulses, Eve will affect the transmission rate for S more than for S' .
- ▶ Alice and Bob can compare empirical transmission loss for S and S' at the end.
 - ▶ If they differ, they conclude Eve did something naughty.

TL;DR

1. Send/receive a stream of quanta in random bases.
2. Throw away the bits where bases disagreed.
3. Reconcile keys using (adaptations of) standard error correction schemes.
4. Hash your keys down according to pessimistic estimates of how much information Eve could have.
5. Make sure to authenticate all your messages throughout.
6. Profit!

References and Additional Materials

- ▶ Presentation source: <https://github.com/alan-christopher/bb84/tree/master/edu>
- ▶ Quantum Cryptography Intro: <https://arxiv.org/abs/1002.1237>
- ▶ Quantum Cloning: <https://arxiv.org/abs/quant-ph/0511088>
- ▶ Privacy Amplification: <https://link.springer.com/article/10.1007/BF00191318>
- ▶ Winnow: <https://arxiv.org/abs/quant-ph/0203096>
- ▶ Decoy states: <https://arxiv.org/abs/quant-ph/0211153>
- ▶ A QKD implementation: <https://arxiv.org/abs/1603.08387>

Included Works

- ▶ Transverse Wave Image
- ▶ Polarizer Image

Questions?