CARA Mission
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Conjunction Identification
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Conjunction Analysis
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Risk Remediation
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References

NASA CARA

Air Traffic Control *in Spaaaaaaaaace*

A.C.

January 21, 2024

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Outline

A.C.
NASA CARA

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Purpose

CARA in Theory

Mission Statement:

> *To take prudent measures, at reasonable cost, to enhance safety of flight, without placing an undue burden on mission operations*

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Purpose

CARA in Practice

Inputs:

▶ Ephemeris data from cooperating missions.

▶ Catalog of tracked earth-orbiting objects from Combined Space Operations Center (CSpOC).

Outputs:

▶ Alerts to protected missions on high interest events (HIEs).

▶ Advisories for protected missions on risk mitigations for HIEs.

    ▶ Hopefully avoid more Kosmos-Iridium incidents.

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Complexity

Kepler Orbits

$$\ddot{R} = \ddot{R}_{2B} = \frac{Gm_{\text{other}}}{||R||^3} R$$

▶ Solution known since Kepler and Newton.
  ▶ Must be a conic section.
  ▶ If closed, then ellipse.
▶ A star holds its course and its aim. . . returns and returns. . . and is always the same
  ▶ *Mais non*

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Complexity

Perturbation: Third Bodies

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM}$$

▶ Gravity is a universal force.
▶ Lots of non-earth mass out there
  ▶ Luna
  ▶ Sol
  ▶ Uncounted others (fortunately negligible)
▶ Particularly relevant for higher-altitude orbits.
▶ Additional bodies also accelerate Earth.
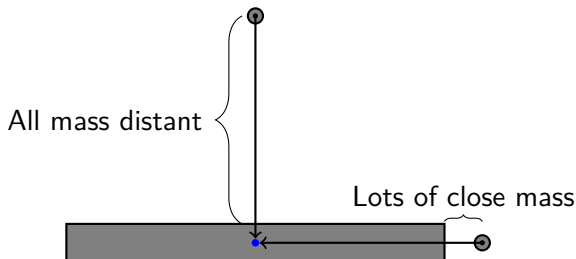 ▶ Must subtract out "shaky camera" effect on our reference body.

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Complexity

Perturbation: Non-Sphericity

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS}$$

- $\ddot{R}_{2B}$ uses point-mass equations
  - Works for points
  - Works for spheres (shell theorem)
- Earth is neither
  - Tidal forces (order meters)
  - Centrifugal forces (order kilometers)

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Complexity

Non-Sphericity: Oblation

Gross exaggeration: cylindrical Earth

All mass distant

Lots of close mass

Equatorial gravity increases. Polar gravity decreases.

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Complexity

Perturbation: Indirect Oblation

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO}$$

▶ Earth is (still) not an inertial reference frame.
▶ Point-mass effects already smoothed out in $\ddot{R}_{PM}$ term.
▶ Non-sphericity effects can further perturb our reference frame.

   ▶ Almost always negligible.
   ▶ Earth-Luna interaction is measurable, however.

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Complexity

Perturbation: Drag

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_{D}$$

▶ Drag equation: $F_D = \frac{1}{2}\rho v^2 C_D A$
▶ Scales by
    ▶ Object shape and orientation (C_D, A).
    ▶ Square of object velocity v^2.
    ▶ Atmospheric density ρ, drops exponentially with altitude.
▶ LEO objects (altitude $<$ 2000 km) are low and fast
    ▶ experience non-negligible drag
▶ Bonus: non-periodic and non-conservative.

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Complexity

Perturbation: Solar Radiation Pressure

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_{D} + \ddot{R}_{SRP}$$

▶ $\gamma := \sqrt{\frac{c^2}{c^2 - v^2}}$

▶ $p = \gamma m v$

▶ photons: $m \to 0, \gamma \to \infty$
  ▶ $p \to ?$
  ▶ God's math: $p = \frac{h}{\lambda}$

▶ Absorbing and emitting light imparts momentum
  ▶ Sunlight never stops: SRP
  ▶ Most impactful on higher altitude orbits
  ▶ Non-periodic and non-conservative

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Complexity

Perturbation: Thrust

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_{D} + \ddot{R}_{SRP} + \ddot{R}_{T}$$

▶ Orbital payloads commonly come equipped with maneuvering thrusters
  ▶ Chemical burns (fast, short)
  ▶ Electric propulsion (slow, long)
▶ Good news: allows for doing something about predicted collisions
▶ Bad news: Non-periodic, non-conservative, AND non-physical(-ish)

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Complexity

Perturbation Impacts

▶ Low-to-medium fidelity diff-eqs can be solved analytically
  ▶ E.g. Brouwer models, SGP4/SDP4
▶ High-fidelity generally resort to numerical integration
  ▶ E.g. NORAD Special Perturbations (SP)
▶ Low- and high-fidelity models can diverge significantly, rapidly
  ▶ By kilometers
  ▶ Within a few orbital periods (i.e. hours)

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Consequence

Mission Safety

$$E(\text{Cost}(X)) = P(X) \cdot \text{Cost}(X)$$

▶ Plausible $\text{Cost}(X)$: 100 million USD

▶ Plausible $P(X)$: 2e-4

▶ $E(\text{Cost}(X)) = 2 \cdot 10^{-4} \cdot 10^8 = 20000$ USD

    ▶ Might be worth mitigating

    ▶ Although, $\sim 85\%$ of likely-lethal conjunctors aren't even tracked . . .

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Consequence

Domain Safety I

▶ Orbit contention is self-reinforcing
  ▶ More objects means more conjunctions
  ▶ More conjunctions means more collisions
  ▶ More collisions means more objects
▶ Critical density $\rightarrow$ runaway, sustained fragmentation
  ▶ Kessler syndrome
▶ Sub-critical density increases still increase hazard to ecosystem

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Consequence

Domain Safety II

▶ Vested public interest in controlling flux
▶ Must avoid collisions, especially between large objects
  ▶ Debris potential strongly linked to object size
  ▶ Largest objects are best tracked
  ▶ Objects follow a power-law distribution: many, many small pieces, comparatively few large

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Volumetric Screening

Conjunction Screening

▶ Before a conjunction can be analyzed it must be identified
  ▶ Want to know the time of closest approach (TCA)
  ▶ and of course the conjunctors' states at TCA
▶ Technically not a CARA responsibility
  ▶ Screening computed for CARA by CSpOC using COMBO
    (Computation of Miss Between Orbits).
  ▶ Operates on a "flying-ellipsoid" volumetric screening paradigm.

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Volumetric Screening

Volumetric Screening

Secondary

Primary

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References

Volumetric Screening
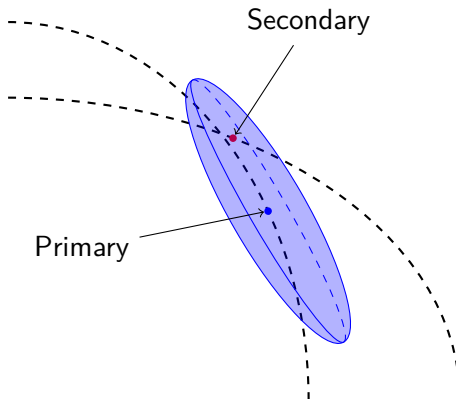
Volumetric Screening

> **procedure** SCREENCONJUNCTIONS
>   **for** $p, s, t \in$ *Primaries* $\times$ *Secondaries* $\times$ *Time Slices* **do**
> $V \leftarrow$ ellipse around $p(t)$
>   **if** $s(t).\text{pos} \notin V$ **then**
>   **continue**
> $d(\tau) := ||p(\tau).\text{pos} - s(\tau).\text{pos}||$
>   **if** $\text{sign}(d'(t)) = \text{sign}(d'(t + \Delta t))$ **then**
>   **continue**
> $t^* \leftarrow \text{argmin}_{[t, t+\Delta t]} \, d$
>   Emit (p, s, t^*)

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Risk Measures

Standoff Distance

▶ Most intuitive measure of safety
  ▶ If we are far apart, of course we aren't touching.
  ▶ Implicitly part of volumetric screening regimes.
▶ Difficult to map distance onto risk.
  ▶ How far apart is far? Meters? Kilometers?
▶ Cannot capture uncertainty
  ▶ What if our measurements of a satellite's state are known to be imprecise?
▶ Tendency toward conservatism
  ▶ sometimes desirable, e.g. around human space flight assets

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Risk Measures

Probability of Collision

▶ Most commonly used measure of safety.
▶ Answers the challenges with standoff distance
  ▶ Maps naturally to risk.
  ▶ Captures and describes uncertainty and imprecision.
  ▶ Allows for mindfully tuned risk postures.
▶ But! Can suffer from probability dilution.
  ▶ Space is big. Really big. Really, really big.
  ▶ Rubbish measurements $\Rightarrow P_C \approx 0$.
  ▶ Probability is "diluted" across space.

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2D P_C

Naive P_C

$$\int \int f(S_1, S_2)\mathbb{I}(\text{collision}|S_1, S_2)dS_1 dS_2$$

▶ That's a 12-dimensional integral.

▶ $\mathbb{I}(\text{collision}|S_1, S_2)$ is a nightmare function.

▶ Let's make some simplifying assumptions

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2D P_C

2D P_C: Assumptions

1. State position vectors $R_1 \sim \mathcal{N}(\bar{R_1}, C_1), R_2 \sim \mathcal{N}(\bar{R_2}, C_2)$
2. $R_1 \perp R_2$
3. Velocity uncertainty is negligible
4. Position uncertainty is stable throughout the encounter
5. Relative motion is linear throughout the encounter
6. Both objects are spheres

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2D P_C

2D P_C

Core idea: Don't think about two objects, think about the distance separating them. More precisely, let

$$R_{\text{miss}} := R_2 - R_1$$

We wish to integrate

$$\int f_{\text{miss}}(R)\mathbb{I}(\text{collision}|R)dR$$

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2D P_C

2D P_C

$$\int f_{\mathsf{miss}}(R)\mathbb{I}(\mathsf{collision}|R)dR$$

Recall: $R_1 \perp R_2$, both Gaussian. So

$$R_{\mathsf{miss}} \sim \mathcal{N}(\bar{R}_2 - \bar{R}_1, C_2 - C_1)$$

f_{miss} is just ϕ!

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2D P_C

2D P_C

$$\int f_{\mathsf{miss}}(R)\mathbb{I}(\mathsf{collision}|R)dR$$

Linear motion, spherical objects. Collision iff $\exists t \in \mathbb{R}$ s.t.

$$||R_{\mathsf{miss}} + v_{\mathsf{miss}} \cdot t|| < \mathsf{HBR}_1 + \mathsf{HBR}_2$$

That's a cylinder!

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2D P_C

2D P_C

Rotate coordinates to align cylinder with z-axis.
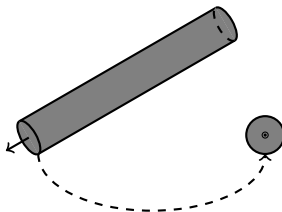
Cylinder is infinite: z-axis marginalizes to 1.

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2D P_C

2D P_C

Integral form after massaging:

$$P_C = \frac{1}{\sqrt{\det(2\pi C)}} \int \int_A \exp\left(-\frac{r^T C^{-1} r}{2}\right) dxdy$$

Not fully analytic, but quite amenable to numerical quadrature.

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3D P_C

Hyperkinetic Assumptions

▶ 2D P_C treated conjunctors like a pair of bullets
▶ Pretty good model when relative velocity is high, encounter is short
▶ Not all close approaches occur at high (relative) velocity, though
▶ Can we do better?

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3D P_C

3D P_C

What happens if we trace our miss vector through the encounter?

$$\int P(\text{conjunctors touching at } t)dt$$

That's actually N_C, the expected number of collisions, but close enough.

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3D P_C

3D P_C

$$P(\text{conjunctors touching at } t) = P(\|R_{\text{miss}}(t)\| = r)$$

▶ The integrand is a surface integral over a sphere!
  ▶ Total integration dimension: 3
  ▶ Curse of dimensionality remains weak
▶ Can we compute and integrate $R_{\text{miss}}(t)$?
  ▶ Yes, with two-body equations of motion... but the math is kind of involved.
  ▶ Please read the full pub if interested.

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Monte Carlo

Motivation

- ▶ All this integration, it's making my head hurt.
- ▶ Even the fancy 3D P_C variant had to make simplifying assumptions.
- ▶ This is a stochastic process, let's try modeling it stochastically.

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Monte Carlo

From-Epoch MC

procedure FROMEPOCHMC(*primOD*, *secOD*, *trials*)
  hits ← 0
  for $i \in [0, \ldots, trials)$ **do**
    p ← draw from *primOD*
    s ← draw from *secOD*
    if p.propagate() collides with s.propagate() **then**
      hits ← *hits* + 1
  return $\frac{hits}{trials}$

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Monte Carlo

From-Epoch MC

▶ No simplifying assumptions! Estimate is as good as
  ▶ Our orbit propagation algorithm
  ▶ The number of trials we run
▶ Heinously expensive, though
  ▶ Millions and millions of trials
  ▶ Each trial doing high-fidelity numerical ODE solving from epoch up through TCA
▶ Most useful to prove out the accuracy of other algorithms.

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Monte Carlo

From-TCA MC

> **procedure** FROMTCAMC(*primTCA*, *secTCA*, *trials*)
> $hits \leftarrow 0$
>   **for** $i \in [0, \ldots, trials)$ **do**
> $p \leftarrow$ draw from *primTCA*
> $s \leftarrow$ draw from *secTCA*
>   **if** *p*.propForward() collides with *s*.propForward() **then**
> $hits \leftarrow hits + 1$
>   **else if** *p*.propBack() collides with *s*.propBack() **then**
> $hits \leftarrow hits + 1$
>   **return** $\frac{hits}{trials}$

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Monte Carlo

From-TCA MC

▶ Faster than From-Epoch
  ▶ Only need to propagate motion around the encounter
  ▶ Shorter motion-prop window means lower fidelity propagation schemes can be used.

▶ Depends on assumptions about the shape of propagated uncertainty
  ▶ Not really a practical issue if we're careful to sample in curvilinear coordinates.

▶ Still much slower than analytic methods.
  ▶ Empirically not much better results
  ▶ Not frequently used in CARA in practice

Outline

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Risk Remediation
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References

Maneuver Planning

HIE Response

Two choices for dealing with an HIE.

1. Change course
   ▶ Costs fuel/reaction mass.
   ▶ Generally cheaper the earlier an orbit is modified.
2. Wait and see
   ▶ Most HIEs come to nothing
   ▶ Can self-resolve "early enough" as conjunction nears and uncertainty drops off.

Common pattern: plan maneuver immediately in response to HIE, then abort if HIE resolves before the commit point.

CARA Mission
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Conjunction Identification
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Conjunction Analysis
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Risk Remediation
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References

Maneuver Planning

Maneuver Trade Space

▶ Several knobs to turn when modifying trajectory
  ▶ Burn intensity
  ▶ Burn timing
  ▶ Satellite orientation (differential drag)
▶ CARA maintains analysis tools for mapping possible maneuvers to expected P_C post-maneuver.
  ▶ Not very well documented in the public domain. . .
▶ Allows us to mitigate the risk to the primary from this particular secondary.
▶ What about all the other possible conjunctors?

CARA Mission Conjunction Identification Conjunction Analysis Risk Remediation References
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Maneuver Planning

Maneuver Validation

▶ Space is big.

▶ The lane we plan to swerve into is *probably* open.

▶ Approach: Plan the remediation in isolation, then pass the planned orbit back into the next round of conjunction screening.

  ▶ If no new HIEs crop up, we're good.
  ▶ Else, try planning a different maneuver.

CARA Mission
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Conjunction Identification
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Conjunction Analysis
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Risk Remediation
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References

References and Further Reading

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📄 *SPACEFLIGHT SAFETY HANDBOOK FOR SATELLITE OPERATORS*. Available at https://www.space-track.org/documents/SFS_Handbook_For_Operators_V1.7.pdf. 18th and 19th Space Defense Squadron. 2023.

Self Link

The source for this presentation is hosted at
https://github.com/alan-christopher/cara-edu.