

NASA CARA

Air Traffic Control *in Spaaaaaaaaace*

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Outline

CARA Mission

Purpose

Complexity

Consequence

Conjunction Identification

HIE Identification

Risk Remediation

CARA in Theory

Mission Statement:

To take prudent measures, at reasonable cost, to enhance safety of flight, without placing an undue burden on mission operations

CARA in Practice

Inputs:

- ▶ Ephemeris data from cooperating missions.
- ▶ Catalog of tracked earth-orbiting objects from Combined Space Operations Center (CSpOC).

Outputs:

- ▶ Alerts to protected missions on high interest events (HIEs).
- ▶ Advisories for protected missions on risk mitigations for HIEs.
 - ▶ Hopefully avoid more Kosmos-Iridium incidents.

Kepler Orbits

$$\ddot{R} = \ddot{R}_{2B} = \frac{Gm_{\text{other}}}{||R||^3} R$$

- ▶ Solution known since Kepler and Newton.
 - ▶ Must be a conic section.
 - ▶ If closed, then ellipse.
- ▶ A star holds its course and its aim. . . returns and returns. . . and is always the same
 - ▶ *Mais non*

Perturbation: Third Bodies

$$\ddot{\mathbf{R}} = \ddot{\mathbf{R}}_{2B} + \ddot{\mathbf{R}}_{PM}$$

- ▶ Gravity is a universal force.
- ▶ Lots of non-earth mass out there
 - ▶ Luna
 - ▶ Sol
 - ▶ Uncounted others (fortunately negligible)
- ▶ Particularly relevant for higher-altitude orbits.
- ▶ Additional bodies also accelerate Earth.
 - ▶ Must subtract out “shaky camera” effect on our reference body.

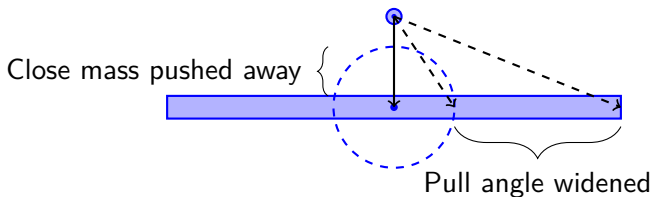
Perturbation: Non-Sphericity

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS}$$

- ▶ \ddot{R}_{2B} uses point-mass equations
 - ▶ Works for points
 - ▶ Works for spheres (shell theorem)
- ▶ Earth is neither
 - ▶ Tidal forces (order meters)
 - ▶ Centrifugal forces (order kilometers)

Non-Sphericity: Oblation

Gross exaggeration: cylindrical Earth



Polar gravity ↓↓↓.

Perturbation: Indirect Oblation

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO}$$

- ▶ Earth is (still) not an inertial reference frame.
- ▶ Point-mass effects already smoothed out in \ddot{R}_{PM} term.
- ▶ Non-sphericity effects can further perturb our reference frame.
 - ▶ Almost always negligible.
 - ▶ Earth-Luna interaction is measurable, however.

Perturbation: Drag

$$\ddot{\mathbf{R}} = \ddot{\mathbf{R}}_{2B} + \ddot{\mathbf{R}}_{PM} + \ddot{\mathbf{R}}_{NS} + \ddot{\mathbf{R}}_{IO} + \ddot{\mathbf{R}}_D$$

- ▶ Drag equation: $F_D = \frac{1}{2}\rho v^2 C_D A$
- ▶ Scales by
 - ▶ Object shape and orientation (C_D, A).
 - ▶ Square of object velocity v^2 .
 - ▶ Atmospheric density ρ , drops exponentially with altitude.
- ▶ LEO objects (altitude < 2000 km) are low and fast
 - ▶ experience non-negligible drag
- ▶ Bonus: non-periodic and non-conservative.

Perturbation: Solar Radiation Pressure

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_D + \ddot{R}_{SRP}$$

- ▶ $\gamma := \sqrt{\frac{c^2}{c^2 - v^2}}$
- ▶ $p = \gamma mv$
- ▶ photons: $m \rightarrow 0, \gamma \rightarrow \infty$
 - ▶ $p \rightarrow ?$
 - ▶ God's math: $p = \frac{h}{\lambda}$
- ▶ Absorbing and emitting light imparts momentum
 - ▶ Sunlight never stops: SRP
 - ▶ Most impactful on higher altitude orbits
 - ▶ Non-periodic and non-conservative

Perturbation: Thrust

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_D + \ddot{R}_{SRP} + \ddot{R}_T$$

- ▶ Orbital payloads commonly come equipped with maneuvering thrusters
 - ▶ Chemical burns (fast, short)
 - ▶ Electric propulsion (slow, long)
- ▶ Good news: allows for doing something about predicted collisions
- ▶ Bad news: Non-periodic, non-conservative, AND non-physical(-ish)

Perturbation Impacts

- ▶ Low-to-medium fidelity diff-eqs can be solved analytically
 - ▶ E.g. Brouwer models, SGP4/SDP4
- ▶ High-fidelity generally resort to numerical integration
 - ▶ E.g. NORAD Special Perturbations (SP)
- ▶ Low- and high-fidelity models can diverge significantly, rapidly
 - ▶ By kilometers
 - ▶ Within a few orbital periods (i.e. hours)
- ▶ None of the models correspond to beautiful, eternal shapes.

Mission Safety

$$E(\text{Cost}(X)) = P(X) \cdot \text{Cost}(X)$$

- ▶ Plausible $\text{Cost}(X)$: 100 million USD
- ▶ Plausible $P(X)$: $2e-4$
- ▶ $E(\text{Cost}(X)) = 2 \cdot 10^{-4} \cdot 10^8 = 20000$ USD
 - ▶ Might be worth mitigating
 - ▶ Although, $\sim 85\%$ of likely-lethal conjunctors aren't even tracked ...

Domain Safety

- ▶ Any collision increases hazard to ecosystem
 - ▶ Debris potential strongly linked to object size
 - ▶ Largest objects are best tracked
 - ▶ Objects follow a power-law distribution: many, many small pieces, comparatively few large

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- ▶ Critical density → runaway, sustained fragmentation
 - ▶ Kessler syndrome

Outline

CARA Mission

Conjunction Identification
Volumetric Screening

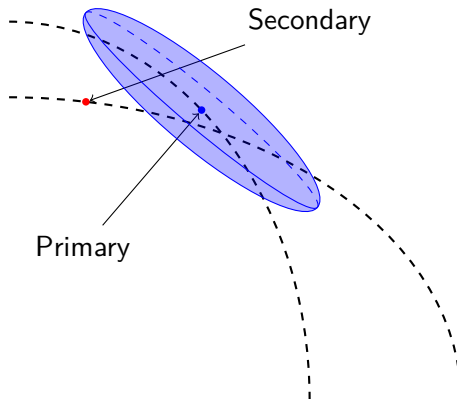
HIE Identification

Risk Remediation

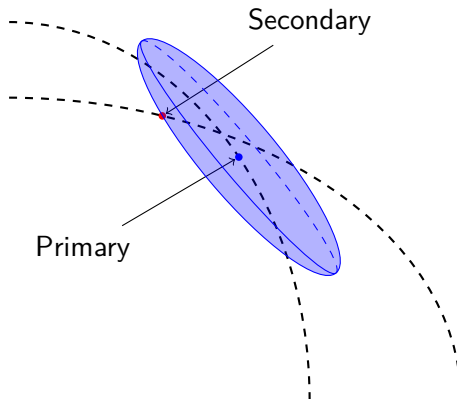
Conjunction Screening

- ▶ Before a conjunction can be analyzed it must be identified
 - ▶ Want to know the time of closest approach (TCA)
 - ▶ and of course the conjunctors' states at TCA
- ▶ Technically not a CARA responsibility
 - ▶ Screening computed for CARA by CSpOC using COMBO (Computation of Miss Between Orbits).
 - ▶ Operates on a “flying-ellipsoid” volumetric screening paradigm.

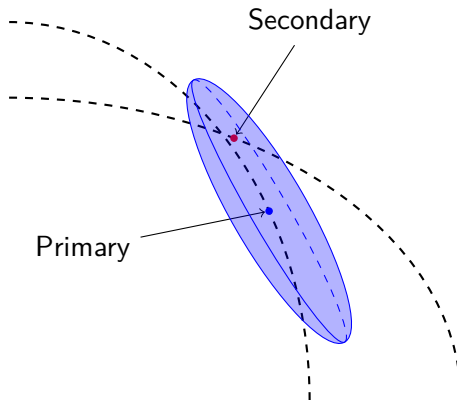
Volumetric Screening



Volumetric Screening



Volumetric Screening



Volumetric Screening

procedure SCREENCONJUNCTIONS

for $p, s, t \in \text{Primitives} \times \text{Secondaries} \times \text{Time Slices}$ **do**

$V \leftarrow$ ellipse around $p(t)$

if $s(t).\text{pos} \notin V$ **then**

continue

$d(\tau) := ||p(\tau).\text{pos} - s(\tau).\text{pos}||$

if $\text{sign}(d'(t)) = \text{sign}(d'(t + \Delta t))$ **then**

continue

$t^* \leftarrow \text{argmin}_{[t, t+\Delta t]} d$

Emit (p, s, t^*)

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Risk Measures

2D P_C

3D P_C

Monte Carlo

Risk Remediation

Standoff Distance

- ▶ Most intuitive measure of safety
 - ▶ If we are far apart, of course we aren't touching.
 - ▶ Implicitly part of volumetric screening regimes.
- ▶ Difficult to map distance onto risk.
 - ▶ How far apart is far? Meters? Kilometers?
- ▶ Cannot capture uncertainty
 - ▶ What if our measurements of a satellite's state are known to be imprecise?
- ▶ Tendency toward conservatism
 - ▶ sometimes desirable, e.g. around human space flight assets

Probability of Collision

- ▶ Most commonly used measure of safety.
- ▶ Answers the challenges with standoff distance
 - ▶ Maps naturally to risk.
 - ▶ Captures and describes uncertainty and imprecision.
 - ▶ Allows for mindfully tuned risk postures.
- ▶ But! Can suffer from probability dilution.
 - ▶ Space is big. Really big. Really, really big.
 - ▶ Rubbish measurements $\Rightarrow P_C \approx 0$.
 - ▶ Probability is “diluted” across space.

Naive P_C

$$\int \int \rho(S_1, S_2) \mathbb{I}(\text{collision} | S_1, S_2) dS_1 dS_2$$

- ▶ That's a 12-dimensional integral.
- ▶ $\mathbb{I}(\text{collision} | S_1, S_2)$ is a nightmare function.
- ▶ Let's make some simplifying assumptions

2D P_C : Assumptions

1. State position vectors $R_1 \sim \mathcal{N}(\bar{R}_1, C_1)$, $R_2 \sim \mathcal{N}(\bar{R}_2, C_2)$
2. $R_1 \perp R_2$
3. Velocity uncertainty is negligible
4. Position uncertainty is stable throughout the encounter
5. Relative motion is linear throughout the encounter
6. Both objects are spheres

2D P_C

Core idea: Don't think about two objects, think about the distance separating them. More precisely, let

$$R_{\text{miss}} := R_2 - R_1$$

We wish to integrate

$$\int \rho_{\text{miss}}(R) \mathbb{I}(\text{collision}|R) dR$$

2D P_C

$$\int \rho_{\text{miss}}(R) \mathbb{I}(\text{collision}|R) dR$$

Recall: $R_1 \perp R_2$, both Gaussian. So

$$R_{\text{miss}} \sim \mathcal{N}(\bar{R}_2 - \bar{R}_1, C_2 - C_1)$$

ρ_{miss} is just ϕ !

2D P_C 2D P_C

$$\int \rho_{\text{miss}}(R) \mathbb{I}(\text{collision}|R) dR$$

Linear motion, spherical objects. Collision iff $\exists t \in \mathbb{R}$ s.t.

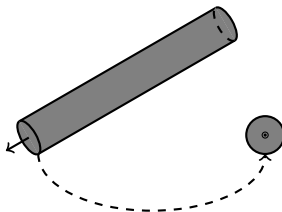
$$||R_{\text{miss}} + v_{\text{miss}} \cdot t|| < \text{HBR}_1 + \text{HBR}_2$$

That's a cylinder!



2D P_C 2D P_C

Rotate coordinates to align cylinder with z-axis.



Cylinder is infinite: z-axis marginalizes to 1.

2D P_C 2D P_C

Integral form after massaging:

$$P_C = \frac{1}{\sqrt{\det(2\pi C)}} \int \int_A \exp\left(-\frac{r^T C^{-1} r}{2}\right) dx dy$$

Not fully analytic, but quite amenable to numerical quadrature.

Hyperkinetic Assumptions

- ▶ 2D P_C treated conjunctors like a pair of bullets
- ▶ Pretty good model when relative velocity is high, encounter is short
- ▶ Not all close approaches occur at high (relative) velocity, though
- ▶ Can we do better?

3D P_C 3D P_C

What happens if we try to count collisions throughout the encounter?

$$\bar{N}_C = \int_{t_0}^{t_{\text{end}}} E(\dot{N}_C) dt$$

That's not actually P_C , but for single-encounter conjunctions, the distinction is academic.

3D P_C

$$E(\dot{N}_C) = \int \int \rho(S_1, S_2) \dot{N}_C(S_1, S_2, t) dS_1 dS_2$$

To find $\dot{N}_C(S_1, S_2, t)$, let's start with N_{overlap} . It's a step function on the ball around our miss vector, propagated to time t !

$$N_{\text{overlap}} = \mathbb{I}(\|R_{\text{miss}}\| < r) = H(r^2 - R^T R)$$

Chain rule it out to get

$$\dot{N}_{\text{overlap}} = \delta(r^2 - R^T R)(-2R^T \dot{R})$$

3D P_C

$$E(\dot{N}_C) = \int \int \rho(S_1, S_2) \dot{N}_C(S_1, S_2, t) dS_1 dS_2$$

\dot{N}_{overlap} is not \dot{N}_C – it adds ingress (yay!) and subtracts egress (boo!). Thresholding at zero solves our problem

$$\dot{N}_C = \max(0, \dot{N}_{\text{overlap}}) = \delta(r^2 - R^T R)[-2R^T \dot{R}]_+$$

3D P_C

$$E(\dot{N}_C) = \int \int \rho(S_1, S_2) \dot{N}_C(S_1, S_2, t) dS_1 dS_2$$

- ▶ Hello darkness 12-D integral, my old friend
- ▶ With an assumption of elliptical movement, this *can* actually be reduced to a manageable dimension.
 - ▶ Requires a heroic amount of massaging algebra and calculus.
 - ▶ Interested parties are referred to the full pub.
- ▶ Final integral dimensions post-massage:
 - ▶ 1-D time integral over the encounter interval
 - ▶ 2-D surface integral over the miss ball

Motivation

- ▶ All this integration, it's making my head hurt.
- ▶ Even the fancy 3D P_C variant had to make simplifying assumptions.
- ▶ This is a stochastic process, let's try modeling it stochastically.

From-Epoch MC

```
procedure FROMEPOCHMC(primOD, secOD, trials)  
  hits  $\leftarrow$  0  
  for  $i \in [0, \dots, \textit{trials})$  do  
     $p \leftarrow$  draw from primOD  
     $s \leftarrow$  draw from secOD  
    if  $p.\text{propagate}()$  collides with  $s.\text{propagate}()$  then  
      hits  $\leftarrow$  hits + 1  
  return  $\frac{\textit{hits}}{\textit{trials}}$ 
```


From-Epoch MC

- ▶ No simplifying assumptions! Estimate is as good as
 - ▶ Our orbit propagation algorithm
 - ▶ The number of trials we run
- ▶ Heinously expensive, though
 - ▶ Millions and millions of trials
 - ▶ Each trial doing high-fidelity numerical ODE solving from epoch up through TCA
- ▶ Most useful to prove out the accuracy of other algorithms.

From-TCA MC

```

procedure FROMTCAMC(primTCA, secTCA, trials)
  hits  $\leftarrow$  0
  for  $i \in [0, \dots, trials)$  do
     $p \leftarrow$  draw from primTCA
     $s \leftarrow$  draw from secTCA
    if  $p.\text{propForward}()$  collides with  $s.\text{propForward}()$  then
      hits  $\leftarrow$  hits + 1
    else if  $p.\text{propBack}()$  collides with  $s.\text{propBack}()$  then
      hits  $\leftarrow$  hits + 1
  return  $\frac{hits}{trials}$ 

```

From-TCA MC

- ▶ Faster than From-Epoch
 - ▶ Only need to propagate motion around the encounter
 - ▶ Shorter motion-prop window means lower fidelity propagation schemes can be used.
- ▶ Depends on assumptions about the shape of propagated uncertainty
 - ▶ Not really a practical issue if we're careful to sample in curvilinear coordinates.
- ▶ Still much slower than analytic methods.
 - ▶ Empirically not much better results
 - ▶ Not frequently used in CARA in practice

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Risk Remediation

Maneuver Planning

HIE Response

Two choices for dealing with an HIE.

1. Change course

- ▶ Costs fuel/reaction mass.
- ▶ Generally cheaper the earlier an orbit is modified.

2. Wait and see

- ▶ Most HIEs come to nothing
- ▶ Can self-resolve “early enough” as conjunction nears and uncertainty drops off.

Common pattern: plan maneuver immediately in response to HIE, then abort if HIE resolves before the commit point.

Maneuver Trade Space

- ▶ Several knobs to turn when modifying trajectory
 - ▶ Burn intensity
 - ▶ Burn timing
 - ▶ Satellite orientation (differential drag)
- ▶ CARA maintains analysis tools for mapping possible maneuvers to expected P_C post-maneuver.
 - ▶ Not very well documented in the public domain. . .
- ▶ Allows us to mitigate the risk to the primary from this particular secondary.
- ▶ What about all the other possible conjunctors?

Maneuver Validation

- ▶ Space is big.
- ▶ The lane we plan to swerve into is *probably* open.
- ▶ Approach: Plan the remediation in isolation, then pass the planned orbit back into the next round of conjunction screening.
 - ▶ If no new HIEs crop up, we're good.
 - ▶ Else, try planning a different maneuver.

References and Further Reading



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Self Link

The source for this presentation is hosted at
<https://github.com/alan-christopher/cara-edu>.