

# NASA CARA

Air Traffic Control *in Spaaaaaaaaace*

A.C.

January 21, 2024

# Outline

## CARA Mission

Purpose

Complexity

Consequence

## Conjunction Identification

## Conjunction Analysis

## Risk Remediation

# CARA in Theory

Mission Statement:

*To take prudent measures, at reasonable cost, to enhance safety of flight, without placing an undue burden on mission operations*

# CARA in Practice

## Inputs:

- ▶ Ephemeris data from cooperating missions.
- ▶ Catalog of tracked earth-orbiting objects from Combined Space Operations Center (CSpOC).

## Outputs:

- ▶ Alerts to protected missions on high interest events (HIEs).
- ▶ Advisories for protected missions on risk mitigations for HIEs.
  - ▶ Hopefully avoid more Kosmos-Iridium incidents.

# Kepler Orbits

$$\ddot{\mathbf{R}} = \ddot{\mathbf{R}}_{2B} = \frac{Gm_{\text{other}}}{||\mathbf{R}||^3} \mathbf{R}$$

- ▶ Solution known since Kepler and Newton.
  - ▶ Must be a conic section.
  - ▶ If closed, then ellipse.
- ▶ A star holds its course and its aim. . . returns and returns. . . and is always the same
  - ▶ *Mais non*

## Perturbation: Third Bodies

$$\ddot{\mathbf{R}} = \ddot{\mathbf{R}}_{2B} + \ddot{\mathbf{R}}_{PM}$$

- ▶ Gravity is a universal force.
- ▶ Lots of non-earth mass out there
  - ▶ Luna
  - ▶ Sol
  - ▶ Uncounted others (fortunately negligible)
- ▶ Particularly relevant for higher-altitude orbits.
- ▶ Additional bodies also accelerate Earth.
  - ▶ Must subtract out “shaky camera” effect on our reference body.

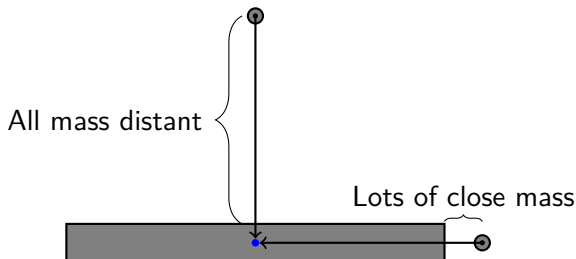
# Perturbation: Non-Sphericity

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS}$$

- ▶  $\ddot{R}_{2B}$  uses point-mass equations
  - ▶ Works for points
  - ▶ Works for spheres (shell theorem)
- ▶ Earth is neither
  - ▶ Tidal forces (order meters)
  - ▶ Centrifugal forces (order kilometers)

## Non-Sphericity: Oblation

Gross exaggeration: cylindrical Earth



Equatorial gravity increases. Polar gravity decreases.



## Perturbation: Indirect Oblation

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{Io}$$

- ▶ Earth is (still) not an inertial reference frame.
- ▶ Point-mass effects already smoothed out in  $\ddot{R}_{PM}$  term.
- ▶ Non-sphericity effects can further perturb our reference frame.
  - ▶ Almost always negligible.
  - ▶ Earth-Luna interaction is measurable, however.

## Perturbation: Drag

$$\ddot{\mathbf{R}} = \ddot{\mathbf{R}}_{2B} + \ddot{\mathbf{R}}_{PM} + \ddot{\mathbf{R}}_{NS} + \ddot{\mathbf{R}}_{IO} + \ddot{\mathbf{R}}_D$$

- ▶ Drag equation:  $F_D = \frac{1}{2}\rho v^2 C_D A$
- ▶ Scales by
  - ▶ Object shape and orientation ( $C_D, A$ ).
  - ▶ Square of object velocity  $v^2$ .
  - ▶ Atmospheric density  $\rho$ , drops exponentially with altitude.
- ▶ LEO objects (altitude  $\leq$  2000 km) are low and fast
  - ▶ experience non-negligible drag
- ▶ Bonus: non-periodic and non-conservative.

## Perturbation: Solar Radiation Pressure

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_D + \ddot{R}_{SRP}$$

- ▶  $\gamma := \sqrt{\frac{c^2}{c^2 - v^2}}$
- ▶  $p = \gamma mv$
- ▶ photons:  $m \rightarrow 0, \gamma \rightarrow \infty$ 
  - ▶  $p \rightarrow ?$
  - ▶ God's math:  $p = \frac{h}{\lambda}$
- ▶ Absorbing and emitting light imparts momentum
  - ▶ Sunlight never stops: SRP
  - ▶ Most impactful on higher altitude orbits
  - ▶ Non-periodic and non-conservative

## Perturbation: Thrust

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_D + \ddot{R}_{SRP} + \ddot{R}_T$$

- ▶ Orbital payloads commonly come equipped with maneuvering thrusters
  - ▶ Chemical burns (fast, short)
  - ▶ Electric propulsion (slow, long)
- ▶ Good news: allows for doing something about predicted collisions
- ▶ Bad news: Non-periodic, non-conservative, AND non-physical(-ish)

## Perturbation Impacts

- ▶ Low-to-medium fidelity diff-eqs can be solved analytically
  - ▶ E.g. Brouwer models, SGP4/SDP4
- ▶ High-fidelity generally resort to numerical integration
  - ▶ E.g. NORAD Special Perturbations (SP)
- ▶ Low- and high-fidelity models can diverge significantly, rapidly
  - ▶ By kilometers
  - ▶ Within a few orbital periods (i.e. hours)

# Mission Safety

$$E(\text{Cost}(X)) = P(X) \cdot \text{Cost}(X)$$

- ▶ Plausible  $\text{Cost}(X)$ : 100 million USD
- ▶ Plausible  $P(X)$ :  $2\text{e-}4$
- ▶  $E(\text{Cost}(X)) = 2 \cdot 10^{-4} \cdot 10^8 = 20000 \text{ USD}$ 
  - ▶ Might be worth mitigating
  - ▶ Although,  $\sim 85\%$  of likely-lethal conjunctors aren't even tracked ...

# Domain Safety I

- ▶ Orbit contention is self-reinforcing
  - ▶ More objects means more conjunctions
  - ▶ More conjunctions means more collisions
  - ▶ More collisions means more objects
- ▶ Critical density → runaway, sustained fragmentation
  - ▶ Kessler syndrome
- ▶ Sub-critical density increases still increase hazard to ecosystem

## Domain Safety II

- ▶ Vested public interest in controlling flux
- ▶ Must avoid collisions, especially between large objects
  - ▶ Debris potential strongly linked to object size
  - ▶ Largest objects are best tracked
  - ▶ Objects follow a power-law distribution: many, many small pieces, comparatively few large



# Outline

CARA Mission

Conjunction Identification  
Volumetric Screening

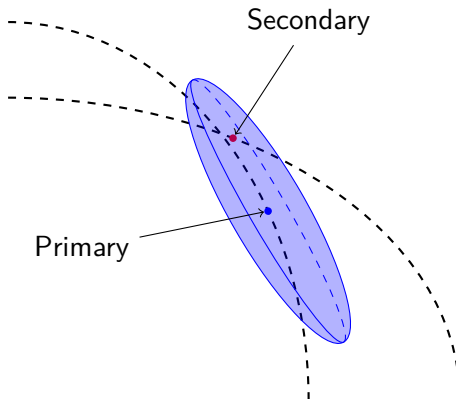
Conjunction Analysis

Risk Remediation

# Conjunction Screening

- ▶ Before a conjunction can be analyzed it must be identified
  - ▶ Want to know the time of closest approach (TCA)
  - ▶ and of course the conjunctors' states at TCA
- ▶ Technically not a CARA responsibility
  - ▶ Screening computed for CARA by CSpOC using COMBO (Computation of Miss Between Orbits).
  - ▶ Operates on a “flying-ellipsoid” volumetric screening paradigm.

# Volumetric Screening



# Volumetric Screening

**procedure** SCREENCONJUNCTIONS

**for**  $p, s, t \in \text{Primitives} \times \text{Secondaries} \times \text{Time Slices}$  **do**

$V \leftarrow$  ellipse around  $p(t)$

**if**  $s(t).\text{pos} \notin V$  **then**

**continue**

$d(\tau) := ||p(\tau).\text{pos} - s(\tau).\text{pos}||$

**if**  $\text{sign}(d'(t)) = \text{sign}(d'(t + \Delta t))$  **then**

**continue**

$t^* \leftarrow \text{argmin}_{[t, t+\Delta t]} d$

Emit  $(p, s, t^*)$

# Outline

CARA Mission

Conjunction Identification

Conjunction Analysis

Risk Measures

2D  $P_C$

3D  $P_C$

Monte Carlo

Risk Remediation

## Standoff Distance

- ▶ Most intuitive measure of safety
  - ▶ If we are far apart, of course we aren't touching.
  - ▶ Implicitly part of volumetric screening regimes.
- ▶ Difficult to map distance onto risk.
  - ▶ How far apart is far? Meters? Kilometers?
- ▶ Cannot capture uncertainty
  - ▶ What if our measurements of a satellite's state are known to be imprecise?
- ▶ Tendency toward conservatism
  - ▶ sometimes desirable, e.g. around human space flight assets

# Probability of Collision

- ▶ Most commonly used measure of safety.
- ▶ Answers the challenges with standoff distance
  - ▶ Maps naturally to risk.
  - ▶ Captures and describes uncertainty and imprecision.
  - ▶ Allows for mindfully tuned risk postures.
- ▶ But! Can suffer from probability dilution.
  - ▶ Space is big. Really big. Really, really big.
  - ▶ Rubbish measurements  $\Rightarrow P_C \approx 0$ .
  - ▶ Probability is “diluted” across space.

# Naive $P_C$

$$\int \int f(S_1, S_2) \mathbb{I}(\text{collision} | S_1, S_2) dS_1 dS_2$$

- ▶ That's a 12-dimensional integral.
- ▶  $\mathbb{I}(\text{collision} | S_1, S_2)$  is a nightmare function.
- ▶ Let's make some simplifying assumptions



## 2D $P_C$ : Assumptions

1. State position vectors  $R_1 \sim \mathcal{N}(\bar{R}_1, C_1)$ ,  $R_2 \sim \mathcal{N}(\bar{R}_2, C_2)$
2.  $R_1 \perp R_2$
3. Velocity uncertainty is negligible
4. Position uncertainty is stable throughout the encounter
5. Relative motion is linear throughout the encounter
6. Both objects are spheres

2D  $P_C$ 2D  $P_C$ 

Core idea: Don't think about two objects, think about the distance separating them. More precisely, let

$$R_{\text{miss}} := R_2 - R_1$$

We wish to integrate

$$\int f_{\text{miss}}(R) \mathbb{I}(\text{collision}|R) dR$$

2D  $P_C$ 

$$\int f_{\text{miss}}(R) \mathbb{I}(\text{collision}|R) dR$$

Recall:  $R_1 \perp R_2$ , both Gaussian. So

$$R_{\text{miss}} \sim \mathcal{N}(\bar{R}_2 - \bar{R}_1, C_2 - C_1)$$

$f_{\text{miss}}$  is just  $\phi$ !

2D  $P_C$ 2D  $P_C$ 

$$\int f_{\text{miss}}(R) \mathbb{I}(\text{collision}|R) dR$$

Linear motion, spherical objects. Collision iff  $\exists t \in \mathbb{R}$  s.t.

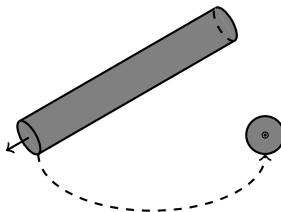
$$||R_{\text{miss}} + v_{\text{miss}} \cdot t|| < r$$

That's a cylinder!



2D  $P_C$ 2D  $P_C$ 

Rotate coordinates to align cylinder with z-axis.



Cylinder is infinite: z-axis marginalizes to 1.

2D  $P_C$ 2D  $P_C$ 

Integral form after massaging:

$$P_C = \frac{1}{\sqrt{\det(2\pi C)}} \int \int_A \exp\left(-\frac{r^T C^{-1} r}{2}\right) dx dy$$

Not fully analytic, but quite amenable to numerical quadrature.

## Hyperkinetic Assumptions

- ▶ 2D  $P_C$  treated conjunctors like a pair of bullets
- ▶ Pretty good model when relative velocity is high, encounter is short
- ▶ Not all close approaches occur at high (relative) velocity, though
- ▶ Can we do better?

# 3D $P_C$

What happens if we trace our miss vector through the encounter?

$$\int P(\text{conjunctors touching at } t) dt$$

That's actually  $N_C$ , the expected number of collisions, but close enough.



3D  $P_C$ 

$$P(\text{conjunctors touching at } t) = P(\|R_{\text{miss}}(t)\| = r)$$

- ▶ The integrand is a surface integral over a sphere!
  - ▶ Total integration dimension: 3
  - ▶ Curse of dimensionality remains weak
- ▶ Can we compute and integrate  $R_{\text{miss}}(t)$ ?
  - ▶ Yes, with two-body equations of motion... but the math is kind of involved.
  - ▶ Please read the full pub if interested.

# Motivation

- ▶ All this integration, it's making my head hurt.
- ▶ Even the fancy 3D  $P_C$  variant had to make simplifying assumptions.
- ▶ This is a stochastic process, let's try modeling it stochastically.

## From-Epoch MC

```
procedure FROMEPOCHMC(primOD, secOD, trials)  
  hits  $\leftarrow$  0  
  for  $i \in [0, \dots, \textit{trials})$  do  
     $p \leftarrow$  draw from primOD  
     $s \leftarrow$  draw from secOD  
    if  $p.\text{propagate}()$  collides with  $s.\text{propagate}()$  then  
      hits  $\leftarrow$  hits + 1  
  return  $\frac{\textit{hits}}{\textit{trials}}$ 
```

# From-Epoch MC

- ▶ No simplifying assumptions! Estimate is as good as
  - ▶ Our orbit propagation algorithm
  - ▶ The number of trials we run
- ▶ Heinously expensive, though
  - ▶ Millions and millions of trials
  - ▶ Each trial doing high-fidelity numerical ODE solving from epoch up through TCA
- ▶ Most useful to prove out the accuracy of other algorithms.

## From-TCA MC

```
procedure FROMTCAMC(primTCA, secTCA, trials)  
  hits  $\leftarrow$  0  
  for  $i \in [0, \dots, \textit{trials}]$  do  
     $p \leftarrow$  draw from primTCA  
     $s \leftarrow$  draw from secTCA  
    if  $p.\text{propForward}()$  collides with  $s.\text{propForward}()$  then  
      hits  $\leftarrow$  hits + 1  
    else if  $p.\text{propBack}()$  collides with  $s.\text{propBack}()$  then  
      hits  $\leftarrow$  hits + 1  
  return  $\frac{\textit{hits}}{\textit{trials}}$ 
```

## From-TCA MC

- ▶ Faster than From-EPOCH
  - ▶ Only need to propagate motion around the encounter
  - ▶ Shorter motion-prop window means lower fidelity propagation schemes can be used.
- ▶ Depends on assumptions about the shape of propagated uncertainty
  - ▶ Not really a practical issue if we're careful to sample in curvilinear coordinates.
- ▶ Still much slower than analytic methods.
  - ▶ Empirically not much better results
  - ▶ Not frequently used in CARA in practice

# Outline

CARA Mission

Conjunction Identification

Conjunction Analysis

Risk Remediation

Maneuver Planning

# HIE Response

Two choices for dealing with an HIE.

1. Change course
  - ▶ Costs fuel/reaction mass.
  - ▶ Generally cheaper the earlier an orbit is modified.
2. Wait and see
  - ▶ Most HIEs come to nothing
  - ▶ Can self-resolve “early enough” as conjunction nears and uncertainty drops off.

Common pattern: plan maneuver immediately in response to HIE, then abort if HIE resolves before the commit point.



# Maneuver Trade Space

- ▶ Several knobs to turn when modifying trajectory
  - ▶ Burn intensity
  - ▶ Burn timing
  - ▶ Satellite orientation (differential drag)
- ▶ CARA maintains analysis tools for mapping possible maneuvers to expected  $P_C$  post-maneuver.
  - ▶ Not very well documented in the public domain. . .
- ▶ Allows us to mitigate the risk to the primary from this particular secondary.
- ▶ What about all the other possible conjunctors?

# Maneuver Validation

- ▶ Space is big.
- ▶ The lane we plan to swerve into is *probably* open.
- ▶ Approach: Plan the remediation in isolation, then pass the planned orbit back into the next round of conjunction screening.
  - ▶ If no new HIEs crop up, we're good.
  - ▶ Else, try planning a different maneuver.

# References and Further Reading



JO Cappellari, CE Vélez, and Arthur J Fuchs. *Mathematical theory of the Goddard trajectory determination system*. Vol. 71106. Goddard Space Flight Center, 1976.



Doyle T Hall et al. "High fidelity collision probabilities estimated using brute force monte carlo simulations". In: *AAS/AIAA Astrodynamics Specialist Conference*. GSFC-E-DAA-TN58551. 2018.



Doyle T Hall. "Expected collision rates for tracked satellites". In: *Journal of Spacecraft and Rockets* 58.3 (2021), pp. 715–728.



Frederic J Krage. *Nasa spacecraft conjunction assessment and collision avoidance best practices handbook*. Tech. rep. 2023.



SPACEFLIGHT SAFETY HANDBOOK FOR SATELLITE OPERATORS. Available at [https://www.space-track.org/documents/SFS\\_Handbook\\_For\\_Operators\\_V1.7.pdf](https://www.space-track.org/documents/SFS_Handbook_For_Operators_V1.7.pdf). 18th and 19th Space Defense Squadron. 2023.

## Self Link

The source for this presentation is hosted at  
<https://github.com/alan-christopher/cara-edu>.