NASA CARA

Air Traffic Control *in Spaaaaaaaaace*

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January 20, 2024

Outline

CARA in Theory

Mission Statement:

To take prudent measures, at reasonable cost, to enhance safety of flight, without placing an undue burden on mission operations

CARA in Practice

Inputs:

- Ephemeris data from cooperating missions.
- Catalog of tracked earth-orbiting objects from Combined Space Operations Center (CSpOC).

Outputs:

- Alerts to protected missions on high interest events (HIEs).
- Advisories for protected missions on risk mitigatiions for HIEs.
  - Hopefully avoid more Kosmos-Iridium incidents.

Kepler Orbits

$$\ddot{R} = \ddot{R}_{2\text{B}} = \frac{Gm_{\text{other}}}{||R||^3} R$$

- Solution known since Kepler and Newton.
  - Must be a conic section.
  - If closed, then ellipse.
- A star holds its course and its aim... returns and returns... and is always the same
  - *Mais non*

Perturbation: Third Bodies

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM}$$

- Gravity is a universal force.
- Lots of non-earth mass out there
  - Luna
  - Sol
  - Uncounted others (fortunately negligible)
- Particularly relevant for higher-altitude orbits.

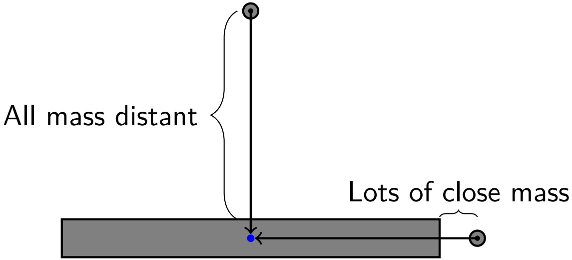
Perturbation: Non-Sphericity

$$\ddot{R} = \ddot{R}_{\text{2B}} + \ddot{R}_{\text{PM}} + \ddot{R}_{\text{NS}}$$

- $\ddot{R}_{\text{2B}}$ uses point-mass equations
  - Works for points
  - Works for spheres (shell theorem)
- Earth is neither
  - Tidal forces (order meters)
  - Centrifugal forces (order kilometers)

Non-Sphericity: Oblation

Gross exaggeration: cylindrical Earth



Equatorial gravity increases. Polar gravity decreases.

Perturbation: Indirect Oblation

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO}$$

- Earth is not an inertial reference frame
  - Has its own orbit around Sol
  - Yanked around by Luna inside that orbit
- "Shaky Camera" effect

Perturbation: Drag

$$\ddot{R} = \ddot{R}_{\mathrm{2B}} + \ddot{R}_{\mathrm{PM}} + \ddot{R}_{\mathrm{NS}} + \ddot{R}_{\mathrm{IO}} + \ddot{R}_{\mathrm{D}}$$

- Drag equation: $F_{\mathrm{D}} = \frac{1}{2}\rho v^2 C_{\mathrm{D}} A$
- Scales by
  - Object shape and orientation (C_{D}, A).
  - Square of object velocity v^2.
  - Atmospheric density *rho*, drops exponentially with altitude.
- LEO objects (altitude ¡ 2000 km) are low and fast
  - experience non-negligible drag
- Bonus: non-periodic and non-conservative.

Perturbation: Solar Radiation Pressure

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_{D} + \ddot{R}_{SRP}$$

- $\gamma := \sqrt{\frac{c^2}{c^2 - v^2}}$
- $p = \gamma m v$
- photons: $m \to 0, \gamma \to \infty$
  - $p \to ?$
  - God's math: $p = \frac{h}{\lambda}$
- Absorbing and emitting light imparts momentum
  - Sunlight never stops: SRP
  - Most impactful on higher altitude orbits
  - Non-periodic and non-conservative

Perturbation: Thrust

$$\ddot{R} = \ddot{R}_{\text{2B}} + \ddot{R}_{\text{PM}} + \ddot{R}_{\text{NS}} + \ddot{R}_{\text{IO}} + \ddot{R}_{\text{D}} + \ddot{R}_{\text{SRP}} + \ddot{R}_{\text{T}}$$

- Orbital payloads commonly come equipped with maneuvering thrusters
  - Chemical burns (fast, short)
  - Electric propulsion (slow, long)
- Good news: allows for doing something about predicted collisions
- Bad news: Non-periodic, non-conservative, AND non-physical(-ish)

Perturbation Impacts

- Low-to-medium fidelity diff-eqs can be solved analytically
  - E.g. Brouwer models, SGP4/SDP4
- High-fidelity generally resort to numerical integration
  - E.g. NORAD Special Perturbations (SP)
- Low- and high-fidelity models can diverge significantly, rapidly
  - By kilometers
  - Within a few orbital periods (i.e. hours)

Mission Safety

$$E(\mathrm{Cost}(X)) = P(X) \cdot \mathrm{Cost}(X)$$

- Plausible $\mathrm{Cost}(X)$: 100 million USD
- Plausible $P(X)$: 2e-4
- $E(\mathrm{Cost}(X)) = 2 \cdot 10^{-4} \cdot 10^{8} = 20000$ USD
  - Might be worth mitigating
  - Although, $\sim 85\%$ of likely-lethal conjunctors aren't even tracked ...

Domain Safety I

- Orbit contention is self-reinforcing
  - More objects means more conjunctions
  - More conjunctions means more collisions
  - More collisions means more objects
- Critical density $\rightarrow$ runaway, sustained fragmentation
  - Kessler syndrome
- Sub-critical density increases still increase hazard to ecosystem

Domain Safety II

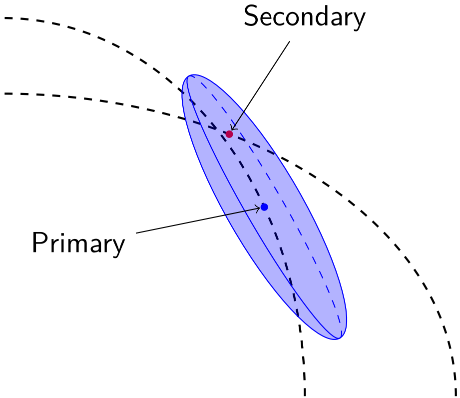
- Vested public interest in controlling flux
- Must avoid collisions, especially between large objects
  - Debris potential strongly linked to object size
  - Largest objects are best tracked
  - Objects follow a power-law distribution: many, many small pieces, comparatively few large

Outline

Conjunction Screening

- Before a conjunction can be analyzed it must be identified
  - Want to know the time of closest approach (TCA)
  - and of course the conjunctors' states at TCA
- Technically not a CARA responsibility
  - Screening computed for CARA by CSpOC using COMBO (Computation of Miss Between Orbits).
  - Operates on a "flying-ellipsoid" volumetric screening paradigm.

Volumetric Screening



Volumetric Screening

```
procedure ScreenConjunctions
    for p, s, t ∈ Primaries × Secondaries × Time Slices do
        V ← ellipse around p(t)
        if s(t).pos ∉ V then
            continue
        d(τ) := ||p(τ).pos − s(τ).pos||
        if sign(d′(t)) = sign(d′(t + Δt)) then
            continue
        t* ← argmin_[t,t+Δt] d
        Emit (p, s, t*)
```

Outline

Standoff Distance

- Most intuitive measure of safety
  - If we are far apart, of course we aren't touching.
  - Implicitly part of volumetric screening regimes.
- Difficult to map distance onto risk.
  - How far apart is far? Meters? Kilometers?
- Cannot capture uncertainty
  - What if our measurements of a satellite's state are known to be imprecise?
- Tendency toward conservatism
  - sometimes desirable, e.g. around human space flight assets

Probability of Collision

- Most commonly used measure of safety.
- Answers the challenges with standoff distance
  - Maps naturally to risk.
  - Captures and describes uncertainty and imprecision.
  - Allows for mindfully tuned risk postures.
- But! Can suffer from probability dilution.
  - Space is big. Really big. Really, really big.
  - Rubbish measurements $\Rightarrow P_C \approx 0$.
  - Probability is "diluted" across space.

Severity Estimation

- For the domain:
  - Different collisions create different amounts of debris.
  - EVOLVE: empirically determined model mapping relative masses and velocities to expected fragmentation count.
  - Basic rules: fast is bad, heavy is bad.
  - Of course, ultimately dependent on O/Os to maneuver/mitigate.
- For O/Os:
  - Everything is moving too damned fast
  - Everything is too damned expensive
  - A hit is a hit is a hit is a bad time

Naive P_C

$$\int \int f(S_1, S_2)\mathbb{I}(\text{collision}|S_1, S_2)dS_1 dS_2$$

- That's a 12-dimensional integral.
- $\mathbb{I}(\text{collision}|S_1, S_2)$ is a nightmare function.
- Let's make some simplifying assumptions

2D P_C: Assumptions

1. State position vectors $R_1 \sim \mathcal{N}(\bar{R}_1, C_1), R_2 \sim \mathcal{N}(\bar{R}_2, C_2)$
2. $R_1 \perp R_2$
3. Velocity uncertainty is negligible
4. Position uncertainty is stable throughout the encounter
5. Relative motion is linear throughout the encounter
6. Both objects are spheres

2D P_C

Core idea: Don't think about two objects, think about the distance separating them. More precisely, let

$$R_{\text{miss}} := R_2 - R_1$$

We wish to integrate

$$\int f_{\text{miss}}(R)\mathbb{I}(\text{collision}|R)dR$$

2D P_C

$$\int f_{\text{miss}}(R)\mathbb{I}(\text{collision}|R)dR$$

Recall: $R_1 \perp R_2$, both Gaussian. So

$$R_{\text{miss}} \sim \mathcal{N}(\bar{R}_2 - \bar{R}_1, C_2 - C_1)$$

f_{miss} is just ϕ!

2D P_C

$$\int f_{\text{miss}}(R)\mathbb{I}(\text{collision}|R)dR$$

Linear motion, spherical objects. Collision iff $\exists t \in \mathbb{R}$ s.t.

$$||R_{\text{miss}} + v_{\text{miss}} \cdot t|| < r$$

That's a cylinder!

2D P_C

Observe: cylinder is infinite. Rotate our z-axis to align with it, and it will marginalize to 1.
A little final massaging and we arrive at a numerical-quadrature-friendly integral.

$$P_C = \frac{1}{\sqrt{\det(2\pi C)}} \int \int_A \exp\left(-\frac{r^T C^{-1} r}{2}\right) dxdy$$

Hyperkinetic Assumptions

- 2D P_C treated conjunctors like a pair of bullets
- Pretty good model when relative velocity is high, encounter is short
- Not all close approaches occur at high (relative) velocity, though
- Can we do better?

3D P_C

What happens if we trace our miss vector through the encounter?

$$\int P(\text{conjunctors touching at } t)dt$$

That's actually N_C, the expected number of collisions, but close enough.

3D P_C

$$P(\text{conjunctors touching at } t) = P(||R_{\text{miss}}(t)|| = r)$$

- The integrand is a surface integral over a sphere!
  - Total integration dimension: 3
  - Curse of dimensionality remains weak
- Can we compute and integrate $R_{\text{miss}}(t)$?
  - Yes, with two-body equations of motion. . . but the math is kind of involved.
  - Please read the full pub if interested.

Motivation

- All this integration, it's making my head hurt.
- Even the fancy 3D P_C variant had to make simplifying assumptions.
- This is a stochastic process, let's try modeling it stochastically.

From-Epoch MC

procedure FROMEPOCHMC(*primOD*, *secOD*, *trials*)
    $hits \leftarrow 0$
    for $i \in [0, \ldots, trials)$ **do**
        $p \leftarrow$ draw from *primOD*
        $s \leftarrow$ draw from *secOD*
        if *p*.propagate() collides with *s*.propagate() **then**
            $hits \leftarrow hits + 1$
    return $\frac{hits}{trials}$

From-Epoch MC

- No simplifying assumptions! Estimate is as good as
  - Our orbit propagation algorithm
  - The number of trials we run
- Heinously expensive, though
  - Millions and millions of trials
  - Each trial doing high-fidelity numerical ODE solving from epoch up through TCA
- Most useful to prove out the accuracy of other algorithms.

From-TCA MC

```
procedure FromTCAMC(primTCA, secTCA, trials)
    hits ← 0
    for i ∈ [0, ..., trials) do
        p ← draw from primTCA
        s ← draw from secTCA
        if p.propForward() collides with s.propForward() then
            hits ← hits + 1
        else if p.propBack() collides with s.propBack() then
            hits ← hits + 1
    return hits/trials
```

From-TCA MC

- Faster than From-Epoch
  - Only need to propagate motion around the encounter
  - Shorter motion-prop window means lower fidelity propagation schemes can be used.
- Depends on assumptions about the shape of propagated uncertainty
  - Not really a practical issue if we're careful to sample in curvilinear coordinates.
- Still much slower than analytic methods.
  - Empirically not much better results
  - Not frequently used in CARA in practice

References

[allowframebreaks]

Self Link

The source for this presentation is hosted at `https://github.com/alan-christopher/cara-edu`.