NASA CARA

Air Traffic Control *in Spaaaaaaaaace*

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Outline

CARA in Theory

Mission Statement:

To take prudent measures, at reasonable cost, to enhance safety of flight, without placing an undue burden on mission operations

CARA in Practice

Inputs:

- Ephemeris data from cooperating missions.
- Catalog of tracked earth-orbiting objects from Combined Space Operations Center (CSpOC).

Outputs:

- Alerts to protected missions on high interest events (HIEs).
- Advisories for protected missions on risk mitigations for HIEs.
  - Hopefully avoid more Kosmos-Iridium incidents.

Kepler Orbits

$$\ddot{R} = \ddot{R}_{2\mathrm{B}} = \frac{Gm_{\mathrm{other}}}{||R||^3} R$$

- Solution known since Kepler and Newton.
  - Must be a conic section.
  - If closed, then ellipse.
- A star holds its course and its aim. . . returns and returns. . . and is always the same
  - *Mais non*

Perturbation: Third Bodies

$$\ddot{R} = \ddot{R}_{2\mathrm{B}} + \ddot{R}_{\mathrm{PM}}$$

- Gravity is a universal force.
- Lots of non-earth mass out there
  - Luna
  - Sol
  - Uncounted others (fortunately negligible)
- Particularly relevant for higher-altitude orbits.
- Additional bodies also accelerate Earth.
  - Must subtract out "shaky camera" effect on our reference body.

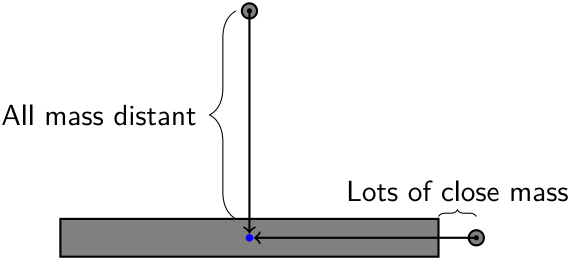
Perturbation: Non-Sphericity

$$\ddot{R} = \ddot{R}_{\text{2B}} + \ddot{R}_{\text{PM}} + \ddot{R}_{\text{NS}}$$

- $\ddot{R}_{\text{2B}}$ uses point-mass equations
    - Works for points
    - Works for spheres (shell theorem)
- Earth is neither
    - Tidal forces (order meters)
    - Centrifugal forces (order kilometers)

Non-Sphericity: Oblation

Gross exaggeration: cylindrical Earth



Equatorial gravity increases. Polar gravity decreases.

Perturbation: Indirect Oblation

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO}$$

- Earth is (still) not an inertial reference frame.
- Point-mass effects already smoothed out in $\ddot{R}_{PM}$ term.
- Non-sphericity effects can further perturb our reference frame.
  - Almost always negligible.
  - Earth-Luna interaction is measurable, however.

Perturbation: Drag

$$\ddot{R} = \ddot{R}_{\mathrm{2B}} + \ddot{R}_{\mathrm{PM}} + \ddot{R}_{\mathrm{NS}} + \ddot{R}_{\mathrm{IO}} + \ddot{R}_{\mathrm{D}}$$

- Drag equation: $F_{\mathrm{D}} = \frac{1}{2}\rho v^2 C_{\mathrm{D}} A$
- Scales by
  - Object shape and orientation (C_{D}, A).
  - Square of object velocity v^2.
  - Atmospheric density ρ, drops exponentially with altitude.
- LEO objects (altitude < 2000 km) are low and fast
  - experience non-negligible drag
- Bonus: non-periodic and non-conservative.

Perturbation: Solar Radiation Pressure

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_{D} + \ddot{R}_{SRP}$$

- $\gamma := \sqrt{\frac{c^2}{c^2 - v^2}}$
- $p = \gamma m v$
- photons: $m \to 0, \gamma \to \infty$
  - $p \to ?$
  - God's math: $p = \frac{h}{\lambda}$
- Absorbing and emitting light imparts momentum
  - Sunlight never stops: SRP
  - Most impactful on higher altitude orbits
  - Non-periodic and non-conservative

Perturbation: Thrust

$$\ddot{R} = \ddot{R}_{2B} + \ddot{R}_{PM} + \ddot{R}_{NS} + \ddot{R}_{IO} + \ddot{R}_{D} + \ddot{R}_{SRP} + \ddot{R}_{T}$$

- Orbital payloads commonly come equipped with maneuvering thrusters
  - Chemical burns (fast, short)
  - Electric propulsion (slow, long)
- Good news: allows for doing something about predicted collisions
- Bad news: Non-periodic, non-conservative, AND non-physical(-ish)

Perturbation Impacts

- Low-to-medium fidelity diff-eqs can be solved analytically
  - E.g. Brouwer models, SGP4/SDP4
- High-fidelity generally resort to numerical integration
  - E.g. NORAD Special Perturbations (SP)
- Low- and high-fidelity models can diverge significantly, rapidly
  - By kilometers
  - Within a few orbital periods (i.e. hours)

Mission Safety

$$E(\text{Cost}(X)) = P(X) \cdot \text{Cost}(X)$$

- Plausible Cost(X): 100 million USD
- Plausible $P(X)$: 2e-4
- $E(\text{Cost}(X)) = 2 \cdot 10^{-4} \cdot 10^{8} = 20000$ USD
  - Might be worth mitigating
  - Although, $\sim$ 85% of likely-lethal conjunctors aren't even tracked ...

Domain Safety I

- Orbit contention is self-reinforcing
  - More objects means more conjunctions
  - More conjunctions means more collisions
  - More collisions means more objects
- Critical density $\rightarrow$ runaway, sustained fragmentation
  - Kessler syndrome
- Sub-critical density increases still increase hazard to ecosystem

Domain Safety II

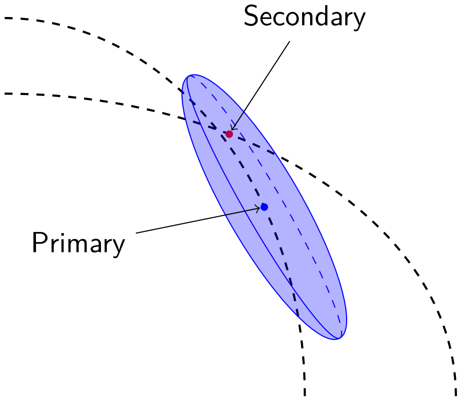
- Vested public interest in controlling flux
- Must avoid collisions, especially between large objects
  - Debris potential strongly linked to object size
  - Largest objects are best tracked
  - Objects follow a power-law distribution: many, many small pieces, comparatively few large

Outline

Conjunction Screening

- Before a conjunction can be analyzed it must be identified
  - Want to know the time of closest approach (TCA)
  - and of course the conjunctors' states at TCA
- Technically not a CARA responsibility
  - Screening computed for CARA by CSpOC using COMBO (Computation of Miss Between Orbits).
  - Operates on a "flying-ellipsoid" volumetric screening paradigm.

Volumetric Screening



Volumetric Screening

procedure $\textsc{ScreenConjunctions}$
  for $p, s, t \in \mathit{Primaries} \times \mathit{Secondaries} \times \mathit{Time\ Slices}$ **do**
    $V \leftarrow$ ellipse around $p(t)$
    if $s(t).\mathrm{pos} \notin V$ **then**
      continue
    $d(\tau) := ||p(\tau).\mathrm{pos} - s(\tau).\mathrm{pos}||$
    if $\mathrm{sign}(d'(t)) = \mathrm{sign}(d'(t + \Delta t))$ **then**
      continue
    $t^* \leftarrow \mathrm{argmin}_{[t, t+\Delta t]}\, d$
    Emit (p, s, t^*)

Outline

Standoff Distance

- Most intuitive measure of safety
  - If we are far apart, of course we aren't touching.
  - Implicitly part of volumetric screening regimes.
- Difficult to map distance onto risk.
  - How far apart is far? Meters? Kilometers?
- Cannot capture uncertainty
  - What if our measurements of a satellite's state are known to be imprecise?
- Tendency toward conservatism
  - sometimes desirable, e.g. around human space flight assets

Probability of Collision

- Most commonly used measure of safety.
- Answers the challenges with standoff distance
  - Maps naturally to risk.
  - Captures and describes uncertainty and imprecision.
  - Allows for mindfully tuned risk postures.
- But! Can suffer from probability dilution.
  - Space is big. Really big. Really, really big.
  - Rubbish measurements $\Rightarrow P_C \approx 0$.
  - Probability is “diluted” across space.

Naive P_C

$$\int \int \rho(S_1, S_2)\mathbb{I}(\text{collision}|S_1, S_2)dS_1 dS_2$$

- That's a 12-dimensional integral.
- $\mathbb{I}(\text{collision}|S_1, S_2)$ is a nightmare function.
- Let's make some simplifying assumptions

2D P_C: Assumptions

1. State position vectors $R_1 \sim \mathcal{N}(\bar{R}_1, C_1), R_2 \sim \mathcal{N}(\bar{R}_2, C_2)$
2. $R_1 \perp R_2$
3. Velocity uncertainty is negligible
4. Position uncertainty is stable throughout the encounter
5. Relative motion is linear throughout the encounter
6. Both objects are spheres

2D P_C

Core idea: Don't think about two objects, think about the distance separating them. More precisely, let

$$R_{\text{miss}} := R_2 - R_1$$

We wish to integrate

$$\int \rho_{\text{miss}}(R)\mathbb{I}(\text{collision}|R)dR$$

2D P_C

$$\int \rho_{\text{miss}}(R)\mathbb{I}(\text{collision}|R)dR$$

Recall: $R_1 \perp R_2$, both Gaussian. So

$$R_{\text{miss}} \sim \mathcal{N}(\bar{R}_2 - \bar{R}_1, C_2 - C_1)$$

ρ_{miss} is just ϕ!

2D P_C

$$\int f_{\text{miss}}(R)\mathbb{I}(\text{collision}|R)dR$$

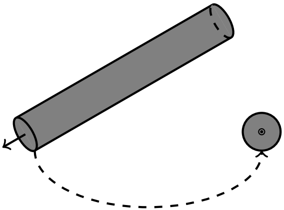
Linear motion, spherical objects. Collision iff $\exists t \in \mathbb{R}$ s.t.

$$||R_{\text{miss}} + v_{\text{miss}} \cdot t|| < \text{HBR}_1 + \text{HBR}_2$$

That's a cylinder!

2D P_C

Rotate coordinates to align cylinder with z-axis.



Cylinder is infinite: z-axis marginalizes to 1.

2D P_C

Integral form after massaging:

$$P_C = \frac{1}{\sqrt{\det(2\pi C)}} \int \int_A \exp\left(-\frac{r^T C^{-1} r}{2}\right) dxdy$$

Not fully analytic, but quite amenable to numerical quadrature.

Hyperkinetic Assumptions

- 2D P_C treated conjunctors like a pair of bullets
- Pretty good model when relative velocity is high, encounter is short
- Not all close approaches occur at high (relative) velocity, though
- Can we do better?

3D P_C

What happens if we try to count collisions throughout the encounter?

$$\bar{N}_C = \int_{t_0}^{t_{\text{end}}} E(\dot{N}_C)dt$$

That's not actually P_C, but for single-encounter conjunctions, the distinction is academic.

3D P_C

$$E(\dot{N}_C) = \int \int \rho(S_1, S_2)\dot{N}_C(S_1, S_2, t)dS1dS2$$

To find $\dot{N}_C(S_1, S_2, t)$, let's start with N_{overlap}. It's a step function on the ball around our miss vector, propagated to time t!

$$N_{\text{overlap}} = \mathbb{I}(||R_{\text{miss}}|| < r) = \begin{cases} 0 & r^2 - R^T R \leq 0 \\ 0 & r^2 - R^T R > 0 \end{cases}$$

Chain rule it out to get

$$\dot{N}_{\text{overlap}} = \delta(r^2 - R^T R)(-2R^T \dot{R})$$

3D P_C

$$E(\dot{N}_C) = \int \int \rho(S_1, S_2) \dot{N}_C(S_1, S_2, t) dS1dS2$$

$\dot{N}_{\text{overlap}}$ is not $\dot{N}_{\text{C}}$ – it adds ingress (yay!) and subtracts egress (boo!). Thresholding at zero solves our problem

$$\dot{N}_C = max(0, \dot{N}_{\text{overlap}}) = \delta(r^2 - R^T R)[-2R^T \dot{R})]_+$$

3D P_C

$$E(\dot{N}_C) = \int\int \rho(S_1, S_2)\dot{N}_C(S_1, S_2, t)dS1dS2$$

- Hello ~~darkness~~ 12-D integral, my old friend
- With an assumption of elliptical movement, this *can* actually be reduced to a manageable dimension.
  - Requires a heroic amount of massaging algebra and calculus.
  - Interested parties are referred to the full pub.
- Final integral dimensions post-massage:
  - 1-D time integral over the encounter interval
  - 2-D surface integral over the miss ball

Motivation

- All this integration, it's making my head hurt.
- Even the fancy 3D P_C variant had to make simplifying assumptions.
- This is a stochastic process, let's try modeling it stochastically.

From-Epoch MC

procedure $\textsc{FromEpochMC}(\mathit{primOD}, \mathit{secOD}, \mathit{trials})$
  $\mathit{hits} \leftarrow 0$
  for $i \in [0, \ldots, \mathit{trials})$ **do**
    $p \leftarrow$ draw from primOD
    $s \leftarrow$ draw from secOD
    if p.propagate() collides with s.propagate() **then**
      $\mathit{hits} \leftarrow \mathit{hits} + 1$
  return $\frac{\mathit{hits}}{\mathit{trials}}$

From-Epoch MC

- No simplifying assumptions! Estimate is as good as
  - Our orbit propagation algorithm
  - The number of trials we run
- Heinously expensive, though
  - Millions and millions of trials
  - Each trial doing high-fidelity numerical ODE solving from epoch up through TCA
- Most useful to prove out the accuracy of other algorithms.

From-TCA MC

procedure $\textsc{FromTCAMC}(\mathit{primTCA}, \mathit{secTCA}, \mathit{trials})$
    $\mathit{hits} \leftarrow 0$
    for $i \in [0, \ldots, \mathit{trials})$ **do**
        $p \leftarrow$ draw from $\mathit{primTCA}$
        $s \leftarrow$ draw from secTCA
        if p.propForward() collides with s.propForward() **then**
            $\mathit{hits} \leftarrow \mathit{hits} + 1$
        else if p.propBack() collides with s.propBack() **then**
            $\mathit{hits} \leftarrow \mathit{hits} + 1$
    return $\frac{\mathit{hits}}{\mathit{trials}}$

From-TCA MC

- Faster than From-Epoch
  - Only need to propagate motion around the encounter
  - Shorter motion-prop window means lower fidelity propagation schemes can be used.
- Depends on assumptions about the shape of propagated uncertainty
  - Not really a practical issue if we're careful to sample in curvilinear coordinates.
- Still much slower than analytic methods.
  - Empirically not much better results
  - Not frequently used in CARA in practice

Outline

HIE Response

Two choices for dealing with an HIE.

1. Change course
   - Costs fuel/reaction mass.
   - Generally cheaper the earlier an orbit is modified.
2. Wait and see
   - Most HIEs come to nothing
   - Can self-resolve "early enough" as conjunction nears and uncertainty drops off.

Common pattern: plan maneuver immediately in response to HIE, then abort if HIE resolves before the commit point.

Maneuver Trade Space

- Several knobs to turn when modifying trajectory
  - Burn intensity
  - Burn timing
  - Satellite orientation (differential drag)
- CARA maintains analysis tools for mapping possible maneuvers to expected P_C post-maneuver.
  - Not very well documented in the public domain...
- Allows us to mitigate the risk to the primary from this particular secondary.
- What about all the other possible conjunctors?

Maneuver Validation

- Space is big.
- The lane we plan to swerve into is *probably* open.
- Approach: Plan the remediation in isolation, then pass the planned orbit back into the next round of conjunction screening.
  - If no new HIEs crop up, we're good.
  - Else, try planning a different maneuver.

References and Further Reading

- JO Cappellari, CE Vélez, and Arthur J Fuchs. *Mathematical theory of the Goddard trajectory determination system*. Vol. 71106. Goddard Space Flight Center, 1976.
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- Doyle T Hall. "Expected collision rates for tracked satellites". In: *Journal of Spacecraft and Rockets* 58.3 (2021), pp. 715–728.
- Frederic J Krage. *Nasa spacecraft conjunction assessment and collision avoidance best practices handbook*. Tech. rep. 2023.
- *SPACEFLIGHT SAFETY HANDBOOK FOR SATELLITE OPERATORS*. Available at `https://www.space-track.org/documents/SFS_Handbook_For_Operators_V1.7.pdf`. 18th and 19th Space Defense Squadron. 2023.

Self Link

The source for this presentation is hosted at
https://github.com/alan-christopher/cara-edu.