Prisoner's Dilemma

Theme and Variations

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Outline

Nash Equilibrium

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- Players can communicate. Their strategies are assumed transparent to each other (e.g., by declaration).
- Players needn't cooperate. It's still a Nash Equilibrium if both players changing strategy *together* yields improvement.

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Defect	5, 0	1, 1

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- Nash equilibrium on D-D
- Optimal move is D, independent of opponent choice

Homo Economicus

Iterated Prisoner's Dilemma

What happens if we play several rounds of prisoner's dilemma? A tension between developing good will (cooperating) and spending it (defecting)?

Simple rules:

- N rounds of play.
- Same reward-space in every round.
- Final score is sum of individual rounds.
- Every player has perfect recall.

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- Iterated PD still yields universal defection under optimal, rational play.
- Irrational partners could cooperate.
  - We might want to (sometimes) cooperate as well, depending on partner.

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What happens if we play forever?

- ∞ rounds of play.
- Same reward-space in every round.
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- Final score is the *discounted* sum of individual rounds.
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  - Also, it makes the sum converge.

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Defect-always is still *an* equilibrium.

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Grim Trigger is an always-cooperating equilibrium.

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How realistic is that, anyways?

Credibility II: Roko's Basilisk

Credibility III: Declaratory Strategy

Outline

Tournament Format

Practical experiment run in 1980 by Robert Axelrod.

- Field of 14 prisoner's dilemma actors.
- Opponent strategy was *not* announced.
  - A few actors tried to infer opposing strategies from observed play.
- Each actor plays every other actor, plus a `Random` round and a mirror round.
  - 200 round iterated PD per matchup
- Actor with the highest average score wins.

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- Rank 10 with an average score of 391.

Grim Trigger

```
procedure FRIEDMAN(history)
    if D ∈ history then
        return D
    return C
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- Co-operates well with other "Nice" strategies.
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- Rank 7 with an average score of 473.
- Scores poorly without the benefit of a declaratory strategy.

Tit-for-Tat

procedure TitForTat(history)
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- Co-operates well with other "Nice" strategies.
- Effectively defends itself against extortion strategies.
- Won Axelrod's tournament with an average score of 504.

Tournament Results

The key features of good performance in Axelrod's tournament were

1. Niceness: early cooperation paid enormous dividends.
2. Forgiveness: reverting to a cooperative posture allowed "resets" after a limited defection.

Obvious question: why is this so out of line with our theoretical, defection-happy discussions of PD?

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Nice, forgiving population produces a **founder effect**.

The Red Queen

The goal of Axelrod's tournament is not to maximize score, it's to maximize probability of winning (or surviving).

Mixed Signals

Problem variant: Cooperate/Defect decision is randomly flipped with some probability p.

What happens?

TFT

C-C

TFT

C-C, C-C

TFT

C-C, C-C, C-C

TFT

C-C, C-C, C-C, D-C

TFT

C-C, C-C, C-C, D-C, C-D

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C-C, C-C, C-C, D-C, C-D,D-C,. . .

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C-C, C-C, C-C, D-C, C-D,D-C,. . .

- Cooperate-then-defect chain continues until an error or error(s) puts us into C-C again.
- Harder to fall out of the non-cooperation loop than into it.

TF2T

Simple robustification strategy: retaliate only on two defects:

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- Very difficult to fall into accidental conflict.

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- No harder to fall out of it.

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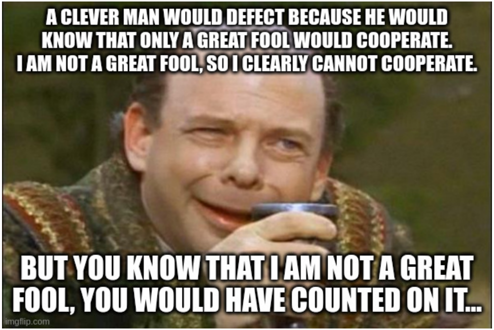
- Very difficult to fall into accidental conflict.
- No harder to fall out of it.
- Can be exploited by “predatory” strategies.

Outline

Super-Rationality



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$$R(\text{player}) \to R(\text{player}) + \alpha R(\text{other})$$

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- *Common interest is common*.

  See, e.g., kin selection.

Fin

The source for this presentation is hosted at
`https://github.com/alan-christopher/edu`.

Questions?