Optimization of SocNet Agent-Based Model: Technical Report

Version 1.0

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1 Introduction

This technical report documents a specific optimization used in SocNet ABM (Frey and Šešelja, 2018). This optimization is motivated by the following observation: in most runs of the simulation a large portion of the run is spent in a state where the simulation has reached a 'steady-state' in the sense that all agents have converged on a single theory and there are no further jumps to the other theory. The goal of this optimization is to detect the presence of such steady-states and end the run early to save computational resources.

2 Setup

To understand the reasoning behind this optimization it is instructive to make a case distinction, which will allow us to specify the steady-state of interest. First of all, any state in which the agents have not converged on one theory is obviously not a steady-state and therefore the run should not be aborted.

If all agents have converged on a theory, the question becomes whether this is just temporary or whether the model has reached a steady-state. In SocNet agents make draws from a binomial distribution and update their beliefs (modeled as beta-distributions) on the basis of those draws and the draws of other agents

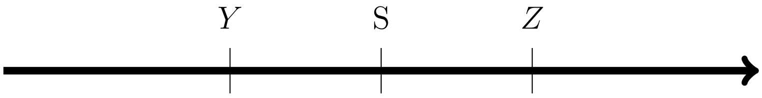

Figure 1: A simple representation of an agent's beliefs throughout a simulation run in our case of interest: the belief about the agent's current theory is denoted with Z, the belief she holds about the rival theory is denoted with Y, and the average of distribution means from which the agent and her neighbors pull is denoted with S.

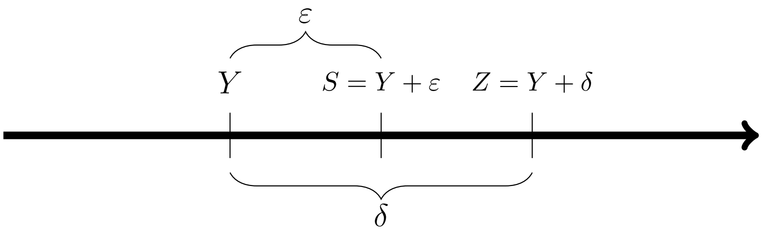

Figure 2: Same as in Figure 1, with further details about each variable.

with whom they're connected in a social network. In order for an agent to switch to another theory she must change the belief about her current theory (denoted as Z) sufficiently in a negative direction such that it becomes worse than the belief she holds regarding the competing theory (denoted as Y) (See Fig. 1).

This leaves us with the case where all agents have converged and there are no further obvious switches (in the sense of the description above). In such cases the question whether an agent will switch becomes a stochastic problem. For an illustration of this setup see Fig. 2.[1]

We can then distinguish between three cases:

1. a run where agents haven't yet converged on a single theory $\rightarrow$ the run hasn't reached a steady-state and will continue;

2. a run where for any of the networked agents S is below Y: the run hasn't reached a steady-state, as given enough time the agents will switch theories $\rightarrow$ the run continues;

3. a run where all agents have converged and for all agents $Y \leq S$: whether

[1]In the code of SocNet variable Y is denoted as 'belief-to-beat'.

agents will switch theories or not is now a stochastic problem. This is the case we focus on in the next section.

3 Calculations

Given that the beliefs are modeled as beta distributions where an agent's belief is simply the quotient of the number of successes (a) divided by the number of pulls (b) the agent observed for a given theory, their current belief Z about that theory can be broken down into $Z = \frac{a}{b}$. The update is equally simple: for any given update add the number of successes in the current try to the numerator, while adding the number of pulls from that try to the denominator. That is, if we have a belief $Z_1 = \frac{a_1}{b_1}$ before the update and $Z_2 = \frac{a_2}{b_2}$ after the update, with $a_2 = a_1 + x_2$ and $b_2 = b_1 + n$ where x_2 denotes the number of successes observed in the current update and n refers to the number of pulls observed in this update, then $Z_2 = \frac{a_2}{b_2} = \frac{a_1+x_2}{b_1+n}$. From now on, we will denote a_1 as α. As mentioned above, for an agent to switch theories their belief regarding their current theory post-update has to be lower than their belief of the rivaling theory (Y). A slightly more conservative way to express this, requests only a less or equal relationship:[2]

$$
\begin{aligned}
Z_2 \leq Y &\implies \frac{a_2}{b_2} \leq Y && (1)\\
&\implies \frac{\alpha + x_2}{b_2} \leq Y && (2)\\
&\implies x_2 \leq b_2 \cdot Y - \alpha && (3)\\
&\implies x_2 \leq (b_1 + n)Y - \alpha && (4)
\end{aligned}
$$

With

$$Z_1 = \frac{\alpha}{b_1} \implies b_1 = \frac{\alpha}{Z_1} = \frac{\alpha}{Y + \delta} \tag{5}$$

[2]When we say we are making the estimate more conservative, this means we increase the probability of the outcome. Recall that this calculation estimates the probability that an agent will switch theories. If this probability is increased, the run will also be likely to continue for longer and therefore we're more cautious when deciding to abort the run.

Therefore

$$x_2 \leq (b_1 + n)Y - \alpha \implies x_2 \leq \left(\frac{\alpha}{Y+\delta} + n\right) Y - \alpha \tag{6}$$

Recall that we draw from a binomial distribution. We can approximate the mean of this distribution by averaging all the means of distributions from which the agent and her neighbors pull. We call this averaged mean $S = Y + \varepsilon$. Therefore we make draws from a Binomial distribution with $\mu = nS = n(Y + \varepsilon)$ and $\sigma = \sqrt{nS(1-S)} = \sqrt{n(Y+\varepsilon)(1-Y-\varepsilon)}$.

Therefore the probability of pulling less than x_2 successes i.e. the probability of x_2 fulfilling above condition (6) is given by

$$P(x_1 \leq T_n \leq x_2) = \sum_{k=x_1}^{x_2} \binom{n}{k} \cdot t^k \cdot (1-t)^{n-k} \tag{7}$$

with $x_1 = 0$. This can be approximated by the normal distribution:

$$P(x_1 \leq T_n \leq x_2) \approx \Phi\left(\frac{x_2 + 0.5 - \mu}{\sigma}\right) - \underbrace{\Phi\left(\frac{x_1 - 0.5 - \mu}{\sigma}\right)}_{\approx 0} \approx \Phi\left(\frac{x_2 + 0.5 - \mu}{\sigma}\right) \tag{8}$$

Where Φ is the CDF of the standard-normal distribution. The $\Phi\left(\frac{x_1 - 0.5 - \mu}{\sigma}\right)$ part of the equation is approximately equal to 0 for values which occur in the model and can therefore be dropped which has the added benefit of making our estimate more conservative.

With $x_2 = \left(\frac{\alpha}{Y+\delta} + n\right) Y - \alpha$ and $\mu = nS = n(Y+\varepsilon)$ and $\sigma = \sqrt{nS(1-S)} = \sqrt{n(Y+\varepsilon)(1-Y-\varepsilon)}$ we obtain

$$\Phi\left(\frac{x_2 + 0.5 - \mu}{\sigma}\right) \tag{9}$$

$$= \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{((2\,n\varepsilon + 2\,\alpha - 1)\,\delta + y\,(2\,n\varepsilon - 1))\sqrt{2}}{\sqrt{-n\,(y+\varepsilon)\,(-1+y+\varepsilon)}\,(4\,y + 4\,\delta)}\right) \tag{10}$$

In this equation all the parameters except for n are known. n is unknown as we don't know how long the run will continue and therefore how many more updates will happen. We will therefore choose the most conservative n i.e. the n which maximizes the probability of above function and therefore the probability that a switch occurs. We find this optimal n by setting the first partial derivative equal to 0.

$$\frac{\partial}{\partial n}\left(\frac{1}{2}-\frac{1}{2}\,\mathrm{erf}\left(\frac{\left(\left(2\,n\varepsilon+2\,\alpha-1\right)\delta+y\left(2\,n\varepsilon-1\right)\right)\sqrt{2}}{\sqrt{-n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(4\,y+4\,\delta\right)}}\right)\right)=0 \quad (11)$$

$$\implies -\frac{1}{4}\,\frac{\left(\left(n\varepsilon-\alpha+1/2\right)\delta+y\left(n\varepsilon+1/2\right)\right)\sqrt{2}}{\sqrt{-n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)}\sqrt{\pi}n\left(y+\delta\right)}\mathrm{e}^{1/8\,\frac{\left(2\,\delta\,n\varepsilon+2\,ny\varepsilon+2\,\alpha\,\delta-\delta-y\right)^2}{n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(y+\delta\right)^2}}=0 \quad (12)$$

$$\implies n=\frac{1}{2}\,\frac{\left(2\,\alpha-1\right)\delta-y}{\varepsilon\,\left(y+\delta\right)} \quad (13)$$

Replacing n in above equation yields

$$\Phi\left(\frac{x_2+0.5-\mu}{\sigma}\right)=\frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(\frac{\left(-2\,\alpha+1\right)\delta+y}{y+\delta}\,\frac{1}{\sqrt{\frac{\left(-2\,\alpha\,\delta+\delta+y\right)\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)}{\varepsilon\,\left(y+\delta\right)}}}\right) \quad (14)$$

Equation (14) is what is used in SocNet to determine the probability of a switch occurring, by running those calculations for each agent.

3.1 Proof of optimal n

We can prove that the n found above (13) maximizes the probability by checking the 2nd derivative:

$$\frac{\partial^2}{\partial n^2}\left(\frac{1}{2}-\frac{1}{2}\operatorname{erf}\left(\frac{\left(\left(2\,n\varepsilon+2\,\alpha-1\right)\delta+y\left(2\,n\varepsilon-1\right)\right)\sqrt{2}}{\sqrt{-n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)}\left(4\,y+4\,\delta\right)}\right)\right)=$$

$$-1/4\,\frac{\left(\delta\,\varepsilon+y\varepsilon\right)\sqrt{2}}{\sqrt{-n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)}\sqrt{\pi}n\left(y+\delta\right)}\mathrm{e}^{1/8\,\frac{\left(2\,\delta\,n\varepsilon+2\,ny\varepsilon+2\,\alpha\,\delta-\delta-y\right)^2}{n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(y+\delta\right)^2}}$$

$$-1/4\,\frac{\left(\left(n\varepsilon-\alpha+1/2\right)\delta+y\left(n\varepsilon+1/2\right)\right)\sqrt{2}}{\sqrt{-n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)}\sqrt{\pi}n\left(y+\delta\right)}$$

$$\cdot\left(1/4\,\frac{\left(2\,\delta\,n\varepsilon+2\,ny\varepsilon+2\,\alpha\,\delta-\delta-y\right)\left(2\,\delta\,\varepsilon+2\,y\varepsilon\right)}{n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(y+\delta\right)^2}\right.$$

$$\left.-1/8\,\frac{\left(2\,\delta\,n\varepsilon+2\,ny\varepsilon+2\,\alpha\,\delta-\delta-y\right)^2}{n^2\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(y+\delta\right)^2}\right)\mathrm{e}^{1/8\,\frac{\left(2\,\delta\,n\varepsilon+2\,ny\varepsilon+2\,\alpha\,\delta-\delta-y\right)^2}{n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(y+\delta\right)^2}}$$

$$-1/8\,\frac{\left(\left(n\varepsilon-\alpha+1/2\right)\delta+y\left(n\varepsilon+1/2\right)\right)\sqrt{2}\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)}{\left(-n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\right)^{3/2}\sqrt{\pi}n\left(y+\delta\right)}\mathrm{e}^{1/8\,\frac{\left(2\,\delta\,n\varepsilon+2\,ny\varepsilon+2\,\alpha\,\delta-\delta-y\right)^2}{n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(y+\delta\right)^2}}$$

$$+1/4\,\frac{\left(\left(n\varepsilon-\alpha+1/2\right)\delta+y\left(n\varepsilon+1/2\right)\right)\sqrt{2}}{\sqrt{-n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)}\sqrt{\pi}n^2\left(y+\delta\right)}\mathrm{e}^{1/8\,\frac{\left(2\,\delta\,n\varepsilon+2\,ny\varepsilon+2\,\alpha\,\delta-\delta-y\right)^2}{n\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(y+\delta\right)^2}}\tag{15}$$

Replacing n with the optimal n found in (13) greatly simplifies this expression to

$$\frac{\left(y+\delta\right)\varepsilon^2}{\sqrt{\pi}\left(\left(-2\,\alpha+1\right)\delta+y\right)}\mathrm{e}^{-\frac{\left(\left(-2\,\alpha+1\right)\delta+y\right)\varepsilon}{\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)\left(y+\delta\right)}}\frac{1}{\sqrt{\frac{\left(-2\,\alpha\,\delta+\delta+y\right)\left(y+\varepsilon\right)\left(-1+y+\varepsilon\right)}{\varepsilon\left(y+\delta\right)}}}\tag{16}$$

- which will always be smaller than 0 if $y<\left(2\,\alpha-1\right)\delta$,[3] thereby proving that the n found is actually a local maximum for the probability of a switch occurring. To check whether this is a global maximum we can check the extreme values n can take on the interval $[0,\infty)$. For $n=0$ this obviously can't be a maximum as this refers to a situation where no pulls are made and the belief of the agent doesn't change so the probability of a switch occurring is 0. For $n\to\infty$ the belief will converge to the means of the distributions the agent updates on i.e. $S=Y+\varepsilon$. Because we made sure that $S>Y$[4] the probability of a switch occurring is similarly 0 in this case. Therefore we can reasonably conclude that the n found in eq. 13 is actually a global maximum.

[3] SocNet checks for this and lets the run continue otherwise.

[4] SocNet checks for this and lets the run continue otherwise.

References

Frey, Daniel and Dunja Šešelja (2018). “Robustness and Idealization in Agent-Based Models of Scientific Interaction”. In: *British Journal for the Philosophy of Science*. DOI: 10.1093/bjps/axy039.