

A Reputation Based Model of Limited Arbitrage

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Abstract

My model shows how limits to arbitrage arises endogenously from a positive self-enforcing feed-back between fund investors' liquidation decisions and manager's portfolio choice. A higher risk of fund liquidation leads managers to favor strategies that pay out quickly. Rational investors anticipate the managers' incentives, learn more from short-term performance and liquidate funds earlier. Investor's decisions feed back into the manager's portfolio through an additional reduction in the manager horizon, further amplifying the initial distortion. Equilibrium pricing reflects this fundamental delegation friction with mispricing becoming more severe as reputational capital becomes scarce.

Investors pull out of funds with poor recent performance. Shleifer and Vishny [1997] famously pointed out that this behavior can be counterproductive as it limits the ability of fund managers to pursue arbitrage opportunities and in equilibrium can lead to mispricing. For example, a fund that is betting that an asset is undervalued will typically exhibit a negative

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relation between past performance and future expected returns: an increase in under-pricing will lead to bad performance, but it will increase the strategy expected returns going forward. Deeper understanding is needed on the economic mechanism that induces investors to pull out when the fund opportunity set improves. Shleifer and Vishny evoke uncertainty about manager skill and higher order uncertainty about the economic environment to motivate investor behavior:

“Both arbitrageurs and their investors are fully rational (...). Implicitly we are assuming that the underlying structural model is sufficiently nonstationary and high dimensional that [fund] investors are unable to infer the underlying structure of the model from past returns data(...)”

While this might be reasonable, the interpretation that the original price deviation is mispricing becomes more challenging if we go down this path. As Brav and Heaton [2002] clearly put it,

“But here the issue comes full circle. The explanation for short horizons [of investors] is a form of rational structural uncertainty (...). This implies that if it is easy for rational arbitrageurs to reject rational explanations for particular patterns of price behavior, then it should be easy to convince rational investors to provide long-term funding for arbitrage.”

This paper endogenizes investor behavior and fund manager horizon. Specifically, the model studies an environment where fund managers reputational capital and trading strategies endogenously determine their trading horizon through fund investors optimal investment behavior, and the endogenous trading horizon feeds-back into manager incentives to invest in different strategies. Even though investors are fully Bayesian, they pull out exactly as strategy expected returns rise and endogenous liquidation decisions happen too early. Conceptually this paper is important, because limited arbitrage in Shleifer and Vishny is a

product of investors being “less” rational than managers, in the sense that they cannot understand the economic environment even if it was fully described to them. In my paper, the blame for limited arbitrage and mispricing rests squarely on the fact that arbitrage is conducted through a financial intermediary. An important advantage of my approach relative to the bounded rationality approach is its predictions about the dynamics of limits to arbitrage effects. Shleifer and Vishny reliance on fund manager trading strategies that are “sufficiently nonstationary” that makes impractical for investors to learn, suggests that mispricing will be a short-lived phenomenon. For example, if one wants to use limited arbitrage as a rational for why individual investor behavioral biases can lead to asset pricing anomalies, one needs a theory that can deliver *persistent* mispricing. If the economic environment changes so fast that fund investors are unable to learn from it, it is reasonable to assume that a financial econometrician would also have a hard time in measuring such mispricing. To be consistent with these asset pricing anomalies, one need a theory of persistent limits to arbitrage.

In addition to having predictions about the persistence of mispricing, my theory has sharp different predictions with respect to where limits to arbitrage are more likely to be severe. While the previous literature emphasizes high strategy idiosyncratic volatility, my model emphasizes the strategy loading on crash risks, such as a financial crisis, a bubble pop, a currency peg break up, the default of a AAA bond, among many other low probability events that are large for a particular asset class. My model also predicts the direction of mispricing, and predicts that crash risks will be under-priced when managers are relatively young and untested, and over-priced when the manager population on the relevant strategy is more seasoned. Only in the limit as fund managers become closer to be fully respected, mispricing disappears. The model also ties asset pricing volatility with the strength of the manager track-record. In addition to several other novel predictions about the nature of arbitrage, the economic environment is fully dynamic, amenable to future quantitative explorations of the magnitude of limited arbitrage effects, the study of the welfare of both investors and manager, and the impact of alternative compensation structures. These questions cannot be

asked when investor behavior is assumed and the environment is static.

I consider a model where investors delegate their portfolio decision to risk-neutral fund managers. A skilled fund manager is endowed with selection skill regarding a particular asset market. The manager selection portfolio is exposed to idiosyncratic risks and to a large low probability event. One can think of a strategy-wide crash. The fund manager chooses how much exposure to have to these crashes, for example, by choosing how many deep out of the money put options on the (particular) market wide portfolio to buy. Managers have a large equity stake in the fund and strongly prefer strategies that maximize expected returns, but they also need to factor in investor behavior. Investors would always prefer to delegate their portfolio to a skilled fund manager, but skill is private information. As time passes, investors update their beliefs about manager type based on the fund's past performance, and decide at each instant whether to liquidate the fund. Because security selection exposes the fund portfolio to idiosyncratic risks, even skilled types face the risk of liquidation. While investors rationally anticipate their manager incentives to trade crash risk, investors' inability to directly observe the fund portfolio tempts managers to use crash exposure to reduce fund liquidation risk. Whether the manager incentives are distorted towards buying or selling out of the money puts depends on the nature of fund liquidation risk. When liquidation risk is high in paths without crashes, the manager will have a bias towards selling put options, as put option sales shift performance from crash events to normal times, reducing current fund liquidation risk at the expense of higher liquidation during crash events. On the other hand, when liquidation risk is relatively high during a crash, skilled fund managers will have a bias towards buying put options as they shift performance from normal times to crash states. The manager chooses her portfolio in a way that trades-off expected return maximization and fund liquidation risk across states.

The key result of the paper is that intertemporal liquidation risk management generates time-variation not only in portfolio choices but in risk premia. An increase in short-term

risk of fund liquidation increases the bias towards selling put options and leads to a decrease in the crash-risk premium. A sequence of good performance periods reduces the risk of fund liquidation in normal times relative to the risk of fund liquidation during a crash, as a result there is a progressive reduction in the bias towards selling put options, and consequently an increase in crash-risk premium towards the efficient level. Interestingly, the convergence towards the efficient level is hump-shaped, with middle-of-the-road managers being so concerned about being liquidated during a crash event that they exhibit a bias towards buying put options, what leads crash-risk to be over-priced. The model also generates endogenous stochastic volatility in put option prices. Because of learning, times of high liquidation risk are also times when reputation is more sensitive to negative performance shocks. This implies that portfolio choices, and consequently crash-risk premium also becomes more volatile. The scarcity of reputational capital increases mispricing and volatility as it reduces fund manager trading horizon.

The model also sheds light in a novel amplification mechanism regarding crash-risk bets. For example, it is well known that highly negative skewed strategies can fool most of standard portfolio metrics [Goetzmann et al., 2007], and recently Stein [2013] emphasized this view

“A fundamental challenge (...) is that many quantitative rules are vulnerable to agents who act to boost measured returns by selling insurance against unlikely events. (...) An example is that if you hire an agent to manage your equity portfolio, and compensate the agent based on performance relative to the S&P 500, the agent can beat the benchmark simply by holding the S&P 500 and stealthily writing puts against it (...)”

The model mechanism does not rely on investors evaluating fund managers against a static benchmark that is subject to manipulation. At the core of the model is the fact that investors endogenously adjust how they benchmark their manager according to the portfolio

the manager chooses in equilibrium. Importantly, the investor optimal reaction *amplifies* the initial distortions in the fund manager incentives. As investors anticipate that the fund manager is selling more crash-risk than she should, they expect a lower expected return but higher short-term performance. The lower expected return leads investors to liquidate managers earlier. Managers that otherwise would have enough reputational capital to operate are liquidated because of the lower expected returns induced by put selling. Higher short-term performance increases performance informativeness, what increases reputation volatility. Both channels increase fund liquidation risk, what feeds back into even stronger incentives to sell put options. While most of the stories of reaching-for-yield implicitly rely on investors failing to properly risk-adjust the fund performance, in this model the proper adjustment amplifies the initial distortions.

The model has several implications for when we should expect mispricing to be larger. My model predicts that lower interest-rates will be associated with stronger under-pricing of tail risk as managers value future earnings more relative to the current arbitrage opportunity. This increases manager aversion to being liquidated, pushing her to distort her portfolio more aggressively. The model also predicts that an increase in competition for investor capital increases mispricing as competition drives out low reputation funds out of business and increases liquidation risk for all surviving funds. The model also shows that mispricing of crash risks are highly sensitive to the frequency/horizon of these crashes, with very long-term risks being more likely to be under-priced.

In the context of hedge funds, a good illustration of behavior consistent with the model is given by Brunnermeier and Nagel [2004]. The authors give a vivid description of the different outcomes for two hedge funds during this period. Both George Soros's Quantum Fund and Julian Robertson's Jaguar Fund took bets against tech stocks during most of 1999, and had outflows during the whole year. Whereas Julian Robertson's fund had a limited exposure to the tech sector and was liquidated just before March 2000, Soros's fund increased its

exposure to the tech sector toward the end of the year and attracted new capital. Although the Quantum Fund performed poorly during the March crash, it did survive. The survival of the Quantum Fund illustrates why deviating from the highest expected return strategy is extremely tempting when fund liquidation is a short-term threat and the probability of the crash is low at any given point in time.

Credit markets also fit the model particularly well as it almost by definition involves a bet on a low probability event. For example, Coval et al. [2009] document how senior claims of collateralized debt obligations traded mostly by financial intermediaries were persistently over-priced in the period before the 2007-2008 financial crises, more recently Kacperczyk and Schnabl [2012] document evidence that some money market funds were over-exposed to crash risk during the financial crises, and Stein [2013] expressed concerns that the narrowing of credit spread in the High Yield debt market was being driven by reaching-for-yield behavior by financial intermediaries. Note that these episodes of mispricing are hard to fit in the original Shleifer and Vishny [1997] story as they are all in asset classes with fairly low volatilities. All these three episodes of under-pricing of crash risk and reaching-for-yield behavior can be understood through the lens of the model. While these episodes have often be interpreted as evidence of neglected risks by investors or managerial myopia, my model explain them in the context of fully rational agents. More broadly, the fact that a substantial fraction of popular investment strategies are negatively skewed (see Jurek [2011], Agarwal and Naik [2004], Duarte et al. [2006]) is consistent with the model logic.

This paper mainly connects with four different lines of work. The first studies the implication of limited arbitrageur capital for asset pricing, such as the works of Gromb and Vayanos [2002] and Xiong [2001]. See Gromb and Vayanos [2010] for a recent survey of this literature. This literature assumes that managers cannot raise new capital and connects the scarcity of intermediary capital with asset prices. The second line of papers studies portfolio choice from the perspective of fund managers. This literature starts from the insight that fund manager

might take into account several implicit incentives when trading, such as the flow sensitivity to performance, the probability of being promoted to another fund, or the probability of being fired. Since Chevalier and Ellison [1997], a large theoretical and empirical literature has flourished that documents and studies how implicit incentives shape the manager choices. This literature is large and being fully inclusive is not feasible, but papers such as Basak et al. [2007], Agarwal et al. [2009], and Brown et al. [1996] represent well the type of questions this literature asks. In this line of work, theory and empirical work complement each other nicely, with empirical work documenting investor and manager behavior and theoretical work rationalizing manager behavior. This literature, however, largely takes investor behavior as given. The third line of work studies investor behavior while assuming away the delegation frictions that are central to the second branch of papers. Papers such as Berk and Green [2004], Dangl et al. [2008], and Berk and Stanton [2007], among others, show theoretically how investor behavior documented in Chevalier and Ellison [1997] and Sirri and Tufano [1998] is optimal if investors are learning about their manager's trading skill. The fourth line of work is related to models with career concerns, such as Zwiebel [1995]. As in my model, in these papers managerial actions are distorted with the aim of convincing outsiders of manager ability. However, these models are based on herding behavior across managers. In my model, the actions of other managers are irrelevant and my mechanism relies on strategic complementarity between the manager and investors' actions. Only recently have researchers made theoretical inroads in studying delegations distortions jointly with investor behavior. Recent work that takes on this challenge includes Dasgupta et al. [2011], Guerrieri and Kondor [2012] and Malliaris and Yan [2009]. Relative to their work, my paper contributes to the literature by showing how asset prices evolve in the absence of any fundamental news and only as a function of the evolution of intermediary reputational capital. My paper also proposes a novel amplification mechanism where investor rational behavior magnifies the impact of fund liquidation risk on portfolio choice and mispricing.

I. Model

I start by describing the features of the model in a setting where crash-risk premium is exogenous. After solving the model in this partial equilibrium case, I introduce equilibrium pricing.

Consider an economy in continuous time. Managers are endowed with the ability to identify and invest in under-valued assets for up to one unit of capital in a particular asset market. Security selection exposes the fund to Brownian and crash risks. Portfolio specific idiosyncratic risks cannot be hedged, but the manager can use derivative markets, or choose her portfolio carefully, to control the fund exposure to market-wide crashes. A high crash exposure portfolio performs worse when a crash happens, but performs better during normal times. Specifically, the manager can choose any exposure between fully long and fully short ($q_t \in [-1, 1]$) to the market-wide crash event. Given the manager exposure choice q_t , the realized return process of the actively managed fund can be written as

$$dV_t = (\alpha_s + q_t \pi_t) dt + \sigma dB_t + (q_t \zeta + \epsilon_t) dN_t \quad (1)$$

with $\alpha_s, \pi_t, \sigma > 0$, and $\zeta < 0$. Here α_s captures the manager security selection skill. ϵ and dB are Gaussian shocks that capture the non-diversifiable risk in the fund portfolio. ζ is the strategy crash exposure before any hedging is done ($q = 1$), and π_t is the crash-risk premium. The process N_t is a Poisson jump process with intensity δ_ζ and jump size equal to one. As exemplified in the introduction, the exposure choice captures a trade-off between growth and the timing of returns. When crash risk is under-priced, $\pi_t + \delta_\zeta \zeta < 0$, portfolios that generate higher growth are only expected to over-perform in the long term when the crash event is likely to happen. This tension between expected returns and the timing of when these returns are expected to realize is the center of the model.

A fund is organized as follows. The manager raises outside equity and contributes with

a fraction k of fund equity capital. The manager scale is limited to one unit of capital, excess earnings are distributed when $dV_t > 0$, and new money is raised when $dV_t < 0$ ¹. All agents are risk-neutral and discount cash flows at rate ρ . Liquidated managers and investors have access to an investment that pays ρdt . The investor horizon is determined by an exogenous liquidity shock and an endogenous redemption choice. A liquidity shock arrives with frequency δ_l , and at the time of the liquidity shock τ_l , an investor can either redeem her money at book value or negotiate with the fund manager the ability to sell it's stake directly to other investors. The resale will always be more advantageous, as fund equity trade at a premium for any living fund.² For simplicity, I model this negotiation process by assuming that the manager captures all the surplus of the resale.³

Whereas exogenous liquidity shocks hit a mass δ_l per instant of time, redemption decisions are endogenous and simultaneous because investors are symmetric. When one investors find's redemption optimal, all others do, and the fund fails immediately. At this point, all stakeholders collect their proportional share of the capital, and the manager leaves the arbitrage industry forever. Let $e^I(t)$ be the per dollar premium investors value their equity stake over the book value.⁴ A fund fails as soon as the market value of investors' equity drops below book value, that is, when $e^I(t) < 0$ for the first time. I will refer to this failure time as τ_F . Let $E^M(t)$ be the value of manager inside equity, which captures the value of fund manager capital contribution, but mostly importantly the value of the manager trading skill. We are now ready to write down the investors' and skilled manager's equity valuations

¹The manager also contributes with new money so her equity stake stays constant at k . Note that assumption of contributions after negative performance does not violate limited liability of fund shareholders. As long $dV_t > -1$, investors and the fund manager will always be happy to pony up the extra cash.

²If it was not trading at a premium, investors would cash out immediately what liquidates the fund.

³In the model, the fact that investors capture none of the premium is similar to the assumption in Berk and Green [2004] that fund size adjusts so expected returns are constant for remaining investors.

⁴Investor per dollar equity valuation is $1 + e_t^I$

as

$$E^M(t) = \max_{q(\cdot) \in [-1, 1]} \mathbb{E} \left[\int_t^{\tau_F} e^{-\rho(s-t)} (k dV_s + \delta_l e^I(s) ds) + e^{-\rho(\tau_F-t)} k | \mathcal{F}_t^M \right] \quad (2)$$

$$e^I(t) = \max_{\tau_F} \mathbb{E} \left[\int_t^{\tau_F \wedge \tau_l} e^{-\rho(s-t)} dV_s | \mathcal{F}_t^I \right]. \quad (3)$$

Both equity stakes are valued almost identically, because the only differences are the size of the stake, the proceedings of secondary-market equity transactions, and the information set under which the expectations are formed. Note that a flow of $\delta_l e^I(s)$ accrues to the manager as old investors leave the fund and new investors enter, where $\delta_l dt$ is the mass of investors that need to sell. Total enterprise value is given by the sum of total equity in the fund, $(1 - k)(1 + e^I(t)) + E^M(t)$. Tables I and II summarize the notation I used throughout the paper.

A. Reputation

In this economy, there is a second type of fund managers, which do not have ability to deliver superior performance ($\alpha_u = 0$) and are not strategic with respect to their crash exposure choices. These unskilled managers believe the crash premium, π , is their alpha, and are always long crash-risk and have exposure $q_t^u = 1$. Before deciding whether to invest their money with a fund manager, investors study the manager track-record and act as Bayesians. They use their initial beliefs regarding the manager type and fund performance to update their belief about manager skill and fund expected returns.

The log-likelihood ratio is a convenient way to express investors beliefs. If P_0 is the time-zero probability that the manager is skilled, I express investors' initial beliefs as $x_0 = \ln(P_0/(1 - P_0))$. I will refer to x_t as the manager reputation at time t . The manager reputation

can be shown to follow a jump diffusion process in which drift and shock elasticities are endogenous to the manager portfolio q_t and investors beliefs about the manager portfolio q_t^I . I will refer to these beliefs as $q^I(x_t, \phi)$, which states investors beliefs about a manager of type ϕ with reputation x_t . Given the behavior of the unskilled manager, it follows that $q^I(x_t, u) = 1$ in equilibrium. Because beliefs about the unskilled-manager portfolio are constant, it is convenient to define $q_t^{I,s} \equiv q^I(x_t, s)$ as a shortcut for investors' beliefs about the skilled-manager portfolio. I will state the reputation dynamics for the case of a constant the crash-risk premium as $\pi_t = \pi$, but everything follows when risk premium varies. In this case, Bayes law implies⁵

$$dx_t = \Delta_B(q_t^{I,s}) \left(\frac{dV_t - \mu_B(q_t^{I,s})dt}{\sigma_B} \right) + \Theta(dV_{t+}, q_t^{I,s}) dN_t \quad (4)$$

where,

$$\Delta_B(q) = \frac{\alpha_s + (q-1)\pi}{\sigma_B}, \quad \mu_B(q) = \frac{\alpha_s + (q+1)\pi}{2}$$

$$\Theta(dV, q) = \frac{(q-1)\zeta}{\sigma_N^2} \left(dV - \frac{(q+1)\zeta}{2} \right).$$

An important aspect of the model mechanism is that while investors cannot observe the manager portfolio allocations directly, they adjust their interpretation of fund performance according to their perception of manager incentives. This portfolio opaqueness friction is important, because it leads the manager to have a temptation to boost short-term performance by selling crash protection even when value maximization calls for the opposite choice. This temptation is a direct consequence of the fund-portfolio opaqueness. If the fund holdings were transparent, a manager would take into account how her choice impacts investors' beliefs and her own reputation dynamics. I refer to the reputation dynamics coefficient $\{\Delta_B, \mu_B, \Theta\}$ as investors' rational benchmarking policy. Whenever convenient, I will use $\Delta_B(x_t) \equiv \Delta_B(q_t^{I,s})$ as a short cut.⁶

⁵See Lipster and Shiryaev [2002] for a standard treatment of optimal filtering.

⁶And analogously for $\mu_B \equiv \mu_B(q^I(x))$ and $\Theta(dV, x) \equiv \Theta(dV, q^I(x))$.

Assumption 1. $\alpha_s > 2\pi$

An important property of reputation dynamics that I need to preserve throughout the paper is that reputation should always be increasing in fund performance. In any equilibrium in which reputation does not increase with fund performance, managers will have an incentive to waste money with transaction costs or to divert cash. To keep the model parsimony, I will restrict the model analysis to a set of parameters such that good performance is always good news about manager skill. The most practical way to restrict reputation to be increasing in performance is to choose parameters for which $\Delta_B(x) > 0$ always holds regardless of investors beliefs, which can be easily accomplished by focusing on a parameter set for which $\alpha_s > 2\pi$. This assumption says that even when the manager goes all in betting on the crash ($q = q^I = -1$), the manager is expected to perform better than the unskilled manager even in the short term.

Given this positive relationship between performance and reputation, the intuition of equation (4) is as follows. In periods without a crash, investors compare the fund realized performance with the average expected performance across manager types in periods without crashes, $\mu_B(x)$, and update their manager reputation upwards (downwards) if the fund realized performance is above (below) this average. How much they update depend on the perceived spread in expected performance (in periods without crashes) across types and the amount of risk⁷ in the manager portfolio. When investors expect a skilled manager to bet on the crash, $q^{I,s} = -1$, the normal times benchmark μ_B and the spread in expected performance, Δ_B are low. As a result, for a given return distribution the manager is more likely to experience an increase in her reputation and the reputation is less volatile during these periods when performance has little information about the manager skill. However, as investors expect the manager to bet on the crash less aggressively, $q^{I,s} \rightarrow 1$, the reputation dynamics moves against the fund manager. For a given return distribution the

⁷More generally, the learning depends on the residual risk in the portfolio after investors filtered out all measurable sources of variation, such as market volatility, and other factors not specific to the manager.

manager becomes less likely to experience and increase in her reputation, and reputation becomes more volatile. In a period with a crash, the learning mechanics is similar, but performance informativeness increases as investors expect the manager to bet on the crash more aggressively.

Investors beliefs about the fund manager strategy shift how they interpret performance. More precisely, beliefs about the portfolio crash exposure shift the timing of the reputation news and the shift introduce complementarity between investors' beliefs and manager choices as we will study in section (C.).

B. Valuation

With the evolution of reputation characterized, it follows from the Ito's formula that investors and managerial equity solve a system of coupled integro-differential equations in which the manager's reputation is the only state variable. Since unskilled manager always chooses $q(x, u) = 1$, I use $q(x)$ as notation for the optimal policy of the skilled manager, thereby allowing us to write each agent Hamilton-Jacobi-Belman equation as⁸⁹

$$\begin{aligned} \rho E^{M,s}(x) = & \max_{q(x) \in [-1,1]} k(\alpha_s + q(x)(\pi + \delta_\zeta \zeta)) + \delta_l e^I(x) + E_x^{M,s}(x) \Delta_B(x) \left(\frac{\alpha_s + q(x)\pi - \mu_B(x_t)}{\sigma_B} \right) + \\ & + \frac{1}{2} E_{xx}^{M,s}(x) \Delta_B(x)^2 + \delta_\zeta \left(\mathbb{E}_\epsilon \left[E^{M,s}(x + \Theta(dV_+, x)) \right] - E^{M,s}(x) \right) \end{aligned} \quad (5)$$

$$\begin{aligned} \rho e^I(x) = & \mathbb{E}_\phi \left[\alpha_\phi + q^I(x, \phi)(\pi + \delta_\zeta \zeta) \right] - \delta_l e^I(x) \\ & + \frac{1}{2} e_{xx}^I(x) \Delta_B(x)^2 + \delta_\zeta \left(\mathbb{E}_{\epsilon, \phi} \left[e^I(x + \Theta(dV_+, x)) \right] - e^I(x) \right). \end{aligned} \quad (6)$$

Where $E^{M,\phi}$ denotes the equity valuation of a manager of type ϕ ¹⁰. Manager equity valuation depends on the investor equity valuation because investors' equity value determines when the

⁸The HJB is obtained by applying Ito's lemma to equations (2). For standard treatment of the topic see Øksendal and Sulem [2005]

⁹Where I use notation $\mathbb{E}_z[\cdot]$ makes explicit that the expectation is over random variable z .

¹⁰Equity valuations of unskilled type are almost identical. The only difference is that unskilled managers always choose exposure $q^u = 1$ and have no selection skill, $\alpha_u = 0$.

fund fails and how much the manager captures from resales from investors hit with a liquidity shock. A fund fails when investors are strictly better off converting their investment back into cash. Mathematically this implies that the optimal fail threshold has to satisfy smooth pasting $e_x^I(x_F) = 0$ and value matching $e^I(x_F) = 0$. The second condition specifies that the value of investor equity has to be equal to its cash value at liquidation, which will always hold for any chosen redemption threshold, whereas the first condition is only satisfied by the optimal redemption behavior. This smooth-pasting says that investors gain exactly nothing of delaying their decision for a bit longer. At liquidation, the manager gets his capital back but loses his intermediation ability forever, $E^{M,s}(x_F) = k$. The manager equity valuation impacts investors indirectly. The sensitivity of manager equity to her reputation shapes her equilibrium portfolio, which feeds back into investors' valuations through their assessment of fund expected returns and different reputation dynamics. This three-way feedback between portfolio choice, beliefs, and reputation dynamics is a key model insight.

Examine equations (5) and (6) more closely is instructive. Both have the same overall structure, with manager equity having one extra term, five in total. First note the terms they have in common: (i) The first term captures expected cash distributions; whereas managers are only uncertain about shocks $\{dB, dN, \epsilon\}$, investors are also uncertain about manager quality $\{\phi\}$ and form expectations accordingly. (ii) The second term captures the expected cash flow due to resales, due to secondary-market resales flow of $\delta_l e^I(x)$ accrues to the fund manager, which leads her to capture the surplus value her skill generates; investors, on the other hand, expect to have a capital loss of $-e^I(x) < 0$ as soon they are hit with a liquidity shock. (iii) The third term captures the valuation effects of expected reputational growth and is absent from the investor equation because manager reputation is a martingale under their information set. (iv) The quadratic term captures valuation effects of reputation volatility. (v) The fifth term captures the reputational consequences of crashes, which I refer to as long-term reputational concerns.

The interpretation of terms (iii-v) is particularly important to the portfolio problem being studied. Contrasting terms (iii) and (iv), we see the manager's actual choice shows up only in the growth term. The volatility term is pinned down in equilibrium by investors' beliefs. The fact that managers do not internalize the impact of their choices on the volatility of their reputation implies that the usual mean-variance trade-off that arises in a standard portfolio choice will not play a role in my analysis. As in a standard portfolio problem, a measure such as $-E_{xx}^{M,s}(x)/E_x^{M,s}(x)$ still captures the manager's "reputation risk aversion", it captures how the manager values growth versus risk. However, the manager does not impact her reputational volatility directly so is unable to do this trade-off properly. As a result, she will end up with too much reputational risk in equilibrium. Even though the nature of crash bets makes this local risk-return trade-off nonoperational in this setting, long-term reputation considerations (term v) will play a key role in improving manager choices.

C. Portfolio Choice

I describe here the trade-offs the manager faces when choosing her crash exposure $q(x)$. First I will examine the interesting case in which crash risk is under-priced, $\pi < -\delta_\zeta \zeta$. A marginal increase in the manager's crash exposure reduces the expected flow of fund dividend distributions, increases the manager's reputational growth in paths without crashes, and decreases the manager's reputation growth when a crash hits. I refer to these different forces as "payoff incentives," "short-term reputational incentives," and "long-term reputational incentives"

$$\max_{q(x) \in Q} \underbrace{q(x)k(\pi + \delta_\zeta \zeta)}_{\text{Performance}} + \underbrace{q(x) \frac{\pi}{\sigma_B} \Delta_B(x) E_x^{M,s}}_{\text{Short-Term}} + \underbrace{\delta_\zeta \mathbb{E}_\epsilon \left[E^{M,s}(x + \Theta(dV_+, x)) \right]}_{\text{Long-Term}}. \quad (7)$$

In (7), we see that although both payoff and short-term reputational incentives are linear in the manager portfolio, long-term incentives depend on the curvature of the manager

valuation. Local linear incentives imply the manager will be pushed to the boundaries of her portfolio leverage constraints. Due to portfolio opaqueness, the manager cannot directly affect how investors learn from fund performance, which leads the manager to not internalize the impact of her portfolio choice on reputation volatility in equilibrium.

I first analyze the case where long-term incentives are weak, $\frac{\zeta}{\sigma_N} \rightarrow 0$. In this case, investors do not learn from crash performance ($\Theta_{dV} \rightarrow 0$), leverage constraints always bind, and characterizing the manager portfolio choice is easy, $q(x) = 1 \times \text{Sign}\{k(\pi + \delta_\zeta \zeta) + \frac{\pi}{\sigma_B} \Delta_B(x) E_x^{M,s}\}$. When crash risk is under priced, payoff and short-term reputational incentives are always in conflict as long as manager equity is increasing in reputation, $E_x^{M,s} > 0$ ¹¹. While the dividend stake pushes the manager to “bet-on-the-crash” ($q(x) = -1$), short-term reputational incentives push the manager to engage in “put-selling” ($q(x) = 1$). If the manager is averse to reputational risk ($E_{xx}^{M,s} < 0$), short-term reputation incentives will be decreasing in reputation. Holding investors beliefs fixed, $\Delta_x(x) = 0$, it is easy to see the manager portfolio will be decreasing in reputation.

In equilibrium, investors adjust their beliefs so that $q^{I,s}(x) = q(x)$ holds. Response that further amplifies manager incentives to sell put options. This complementarity leads to multiplicity, if investors are optimistic, $q^{I,s}(x) = -1$, managers with lower reputation will find it optimal to “bet on the crash.” Because $E_x^{M,s}$ is monotonic, for any investors’ beliefs $q^{I,s}$, we can solve for the reputation threshold, $x_i(q^{I,s})$, at which the manager is perfectly indifferent across portfolios¹²

$$x_i(q^{I,s}) = [E_x^{M,s}]^{-1} \left(\frac{-k(\pi + \delta_\zeta \zeta) \sigma_B^2}{\pi(\alpha_s + (q^{I,s} - 1)\pi)} \right). \quad (8)$$

We can easily show this threshold is increasing in investors’ beliefs¹³. It therefore follows

¹¹ $\Delta_B(x)$ is always positive in the range of parameters considered in this paper. As discussed in Section 2.B this guarantees that reputation is always increasing in fund performance.

¹²In this case the manager chooses $q = -1$ for $x > x_i(q^I)$, and $q = 1$ for $x < x_i(q^I)$.

¹³This result follows if $E_x^{M,s} > 0$ $E_{XX}^{M,s} < 0$.

that the state space can be split into three regions: An upper-dominance region $x > x_i(1)$, where the manager “bet on the crash” regardless of investors beliefs. A lower-dominance region $x < x_i(-1)$, where the manager “put sells” even if investors were optimistic. A multiplicity region $x \in [x_i(-1), x_i(1)]$, where the manager choice can go either way depending on investors’ beliefs. Although multiplicity is important in some settings, managers and investors have many communication channels available to coordinate in a good equilibrium. I formally introduce this coordination assumption in my equilibrium definition by assuming the manager chooses the equilibrium with the highest expected returns. In the case of weak long-term reputation incentives, this assumption leads to a simple cutoff-equilibrium portfolio strategy, $q(x) = 1 - 2(x \geq x_{bc})$, where $x_{bc} = x_i(-1)$ is the lowest reputation of a manager that finds it optimal to bet on the crash when investors believe that she is doing so.

Definition 1. (Equilibrium) The equilibrium in this economy consists of mutual consistency between three objects: (i) the skilled-manager portfolio choice $q(x)$, (ii) investors’ beliefs about the skilled-manager portfolio choice $q^{I,s}(x)$, and (iii) the fund failure threshold x_F . It can be summarized in three conditions: (a) given investors beliefs $q^{I,s}(x)$, reputation evolution in equation (4) and the failure threshold x_F , portfolio choice $q(x)$ maximizes the value of the manager stake (equation (5)), (b) given skilled-manager portfolio policy $q(x)$, investors have correct beliefs about her choices $q^{I,s}(x) = q(x)$, manager reputation evolution is consistent with Bayes Law, and the fund failure threshold maximizes the value of investor’s equity (equation (6)), (c) $q(x)$ maximizes expected returns given conditions (a) and (b) hold.

When crash performance is potentially informative about manager skill, it becomes an additional force pushing the manager to bet on the crash instead of selling puts¹⁴, partially or completely offsetting the short-term incentives the threat of fund liquidation induces. From the previous analysis, we know that for reputations higher than x_{bc} , the manager bets on

¹⁴This follows directly from the fact that $q_{\Delta\phi}^I(x) \leq 0$, what implies $\Theta_{dV}(dV, x) = \Theta_{dV}(q^I(x)) = q_{\Delta\phi}^I(x) \frac{\zeta^2}{\sigma_N^2} \leq 0$.

the crash even without long-term reputational incentives. Long-term reputation incentives will push the bet-on-the-crash threshold x_{bc} lower as long-term reputational incentives are negative, $\mathbb{E}_\epsilon \left[E_x^{M,s} (x + \Theta(dV_+, x)) \Theta_{dV} | q = q^{I,s} = -1 \right] \leq 0$, when investors have optimistic beliefs. However this effect is typically small as a result of the concavity of manager's equity valuation and the fact that the manager expects a large reputational boost when the crash hits (If she is indeed betting on the crash). A simple calculation says that for a manager that bets on the crash the probability that her reputation goes down during a crash is $\Phi(\frac{\zeta}{\sigma_N})$. For example, if idiosyncratic volatility is half of the expected crash size, the probability of a reputation reduction is less than 3%, and the probability of a large increase in reputation is large. This high expected reputational boost implies that *marginal* long-term reputational incentives ($\mathbb{E}_\epsilon \left[E_x^{M,s} (x + \Theta(dV_+, x)) \right]$) will be fairly small when compared with $E_x^{M,s}(x)$. However, long-term reputational incentives play a critical role once the first-best portfolio is no longer an equilibrium. For managers with reputations below x_{bc} , an equilibrium will always exist in which $q = q^{I,s} = 1$ as in this case, $\Theta_{dV} = 0$, and long-term reputational incentives play no role, as in the first case of “no crash learning.” Typically, however, an alternative equilibrium will exist in which *marginal* long-term reputational incentives are large enough to balance short-term incentives and expected returns are higher. I use $\Omega(q, x | q^{I,s})$ to denote the manager's first-order condition with respect to the crash exposure q given investors beliefs $q^{I,s}$,

$$\Omega(q, x | q_s^I) = k(\pi + \delta_\zeta \zeta) + E_x^{M,s}(x) \Delta(q^{I,s}) \frac{\pi}{\sigma_B} + \delta_\zeta \zeta \Theta_{dV}(q_s^I) \mathbb{E}_\epsilon \left[E_x^{M,s} (x + \Theta(q\zeta + \epsilon, q^{I,s})) \right], \quad (9)$$

where I specify these marginal incentives depend on investor beliefs through $\Delta(\cdot)$ and $\Theta(\cdot)$. For an interior choice $q \in (-1, 1)$ to be optimal, we need $\Omega(q, x | q^{I,s}) = 0$, and for this choice to be an equilibrium, $\Omega(q, x | q) = 0$. Obviously the size of this interior region, and the magnitude of the impact on the equilibrium portfolio—how far from 1 the equilibrium choice q is—will strongly depend on both how informative the crash performance is, as measured by Θ_{dV} , and the horizon of these crashes, δ_ζ^{-1} . In proposition 1, I prove that the equilibrium portfolio is decreasing in manager reputation, as more respected managers bet more heavily on the crash event. The following proposition provides a formal characterization of the manager equilibrium trading behavior.:

Proposition 1. (Equilibrium portfolio) Let $\alpha_s > 2\pi$, $\delta_\zeta > 0$, $\zeta < 0$, $\frac{\partial^n E^{M,s}(x)}{\partial x^n}(-1)^{n-1} > 0$ and $\lim_{x \rightarrow \infty} \frac{\partial^n E^{M,s}(x)}{\partial x^n} = 0$, for $3 \geq n \geq 1$, than if $\pi + \delta_\zeta \zeta \geq 0$, $q^*(x) = 1$ for any $x \geq x_F$. If $\pi + \delta_\zeta \zeta < 0$, than portfolio policy can be split in at most three regions: (i) if $x \in [x_F, x_{ps})$, $q^*(x) = 1$; (ii) if $x \in [x_{ps}, x_{bc})$, $q^*(x) = \min_{q \in [-1,1]} q$ s.t. $\Omega(q, x|q) = 0$, and if $(\frac{\zeta}{\sigma_N})^2 > \underline{c}$ we have that $\frac{\partial q^*(x)}{\partial x} < 0$; and (iii) if $x \in [x_{bc}, \infty)$, $q^*(x) = -1$. Where $x_{ps} = \min_{x \in [x_F, x_{bc})} x$ s.t. $\exists q \in (-1, 1] \mid \Omega(q, x|q) = 0$ and $x_{bc} = \min_{x \in [x_F, x_{bc})} x \mid \Omega(-1, x|-1) \leq 0$.

The proposition starts by telling us the manager will always maximize returns when put-selling is profitable. In this case, no trade-off exists between short-term and long-term incentives. The most profitable strategy also pays out quickly. In the more interesting case when betting on the crash is profitable, the proposition tells us the manager will maximize expected returns as long she has a high-enough reputation. Managers in the region closer to fund failure will engage in put-selling even though this strategy minimizes expected returns. For these managers, short-term reputation concerns are too strong, because the managers expect to fail in the near future and therefore long-term reputational gains are not very valuable. Managers in the middle of the pack follow a reputational-hedging strategy. They choose portfolios that exactly balance marginal long-term reputational gains against short-term incentives. The regions are pinned down by equation $\Omega(q, x|q) = 0$, which I refer to as the manager's equilibrium-first-order condition. It is called an *equilibrium*-first-order condition because it imposes investors' belief consistency, $q^{I,s}(x) = q(x)$. The proof of the proposition explores the fact that incentives to bet on the crash are increasing in reputation, $\Omega_x(q, x|q^{I,s}) < 0$, which follows directly from concavity of manager's equity. In general $\Omega(q(x), x|q(x)) = 0$ has multiple solutions as Ω is non-monotonic in q , but I focus on the smallest of the roots because my equilibrium definition selects the solution with the highest expected returns. This equilibrium is the natural one on which to focus, because it maximizes the total value of the equity in the fund and communication between investors and the fund manager will always coordinate on this equilibrium.

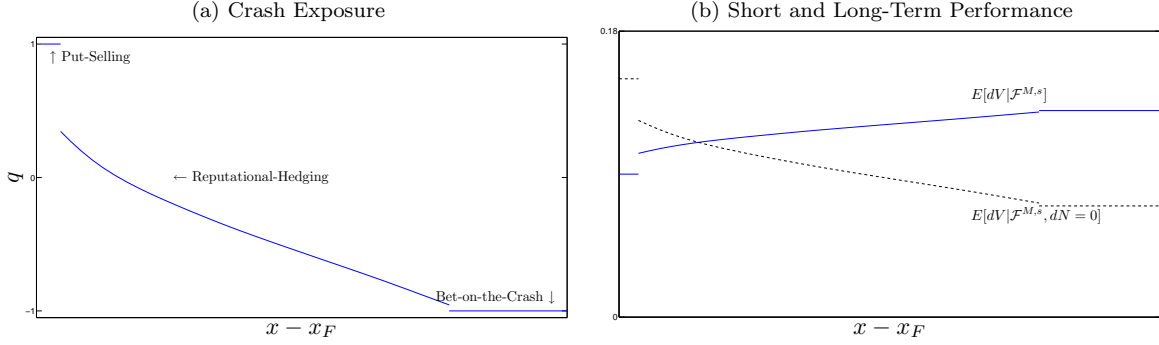


Figure 1: Equilibrium Portfolio Choice, Short and Long-term Fund Performance. These two figures describe the equilibrium portfolio and resulting fund performance as a function of manager reputation. The horizontal axis shows the difference between the current manager reputation and the reputation at which the manager is liquidated, x_F . In the left panel, we have the equilibrium crash exposure q . In the right panel, the equilibrium fund performance. The continuous line shows the fund's expected returns from a skilled manager's vantage point. The dashed line shows the fund expected performance in periods without crashes, what I refer as short-term performance.

Figure 1 illustrate proposition (1) in the interesting case in which crash risk is under priced. We can see the three regions depicted in Panel (a). On the left we see that low-reputation managers engage in as much put-selling as possible. These managers are constrained, and if they could sell even more puts, they would. Middle-of-the-pack managers with reputation $x \in [x_{ps}, x_{bc}]$ engage in reputational hedging, choosing a crash exposure that optimally balances performance and short- and long-term reputational incentives. These managers are happy with their portfolios. They could sell more or fewer puts but choose not to. In the third region, we have the highly respected managers that buy as much protection as possible. Respected managers, like low-reputation managers, are constrained. If they could bet on the crash even more aggressively, they would

From a specific manager's vantage point, these cross-sectional statements have time-series implications. As a manager loses reputational capital, her horizon gets shorter, which pushes her to give more weight to short-term than long-term reputational incentives, leading to an increase in the incentive reach for yield, $\frac{\partial \Omega(q, x|q)}{\partial x} > 0$. Equilibrium is "restored" as the manager increases her exposure to crash risk, which increases the long-term reputational risk the manager bears when a crash hits. The equilibrium portfolio has expected returns that

are increasing in manager reputation (even from the manager’s perspective) . Eventually, managers with sufficient reputational capital go all in on the bet-on-the-crash strategy. High-reputation managers can afford the low expected short-term performance, because they bear low liquidation risk.

Figure 2 describes how equity stakes change in value as investors learn about their manager and as manager reputation evolves. Two important features stand out. (i) Investors’ valuation has a slope of zero at liquidation, and (ii) manager valuation has a slope that is maximum at liquidation, approaching zero when the manager becomes infinitely respected ($x \rightarrow \infty$) . (i) follows directly from investors optimally exercising their option to cash out. At the optimal x_F threshold, the investor does not gain/lose anything from delaying his redemption decision. Whereas convexity of investor valuation follows from optimality of x_F from the investors’ point of view, (ii) follows from the fact that x_F is suboptimal from a skilled-manager perspective. Because skilled managers expect to have a high expected abnormal performance going forward, they expect any delay in fund liquidation to disproportionately increase the probability they survive when her reputation is close to x_F . Their private information about future superior performance together with the high rents of being a fund manager induces the concavity in the skilled-manager equity valuation. Recall that if the manager is liquidated, she can never raise capital again. Because of this irreversibility, manager capital becomes valued just like investors capital.

D. Endogenous Reputational Risk

Reputation volatility moves perversely against the manager. The more concerned she is about her reputation, the more volatile her reputation becomes. The feed back loop between endogenous risk aversion and volatility drives portfolio distortions. A manager concerned

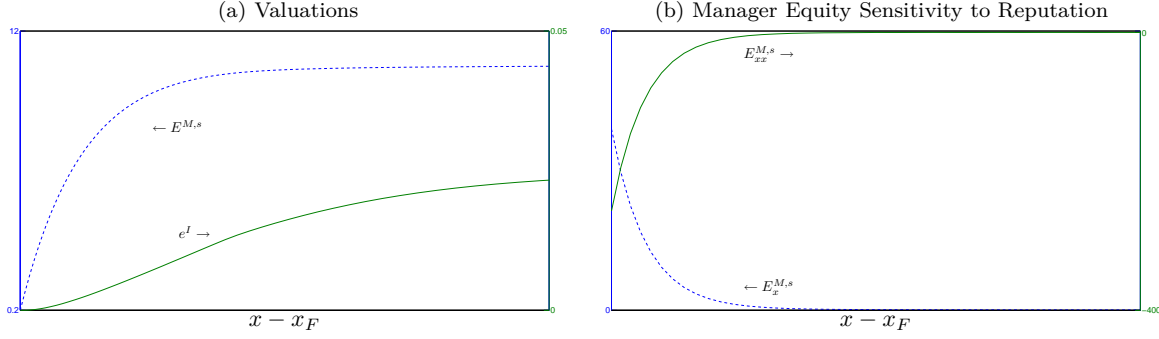


Figure 2: Valuations. Panel (a) describes the value of fund investment to managers and investors as a function of manager reputation. Panel (b) depicts the first and second derivatives of manager valuation. The horizontal axis represents the distance to liquidation in reputation units.

about her reputation $E^{M,s}_x \uparrow$ tilt her portfolio more toward the short term because gains in reputation are particularly valuable in paths where the crash bet does not payout. The bias toward short-term bets leads to an increase in both performance informativeness and in expected reputational growth, but also to an increase in reputational risk as investors react more to shocks. The fact that the portfolio is opaque leads the investor to only consider the change in expected reputational growth due to her actions. Reputation volatility changes as an equilibrium response of investors that understand manager incentives. Portfolio opacity leads managers to choose portfolios that lead to a reputation process with unfavorable mean-variance trade-offs. If the manager could credibly communicate her position, the short-term component of the manager problem in equation (7) would inherit an additional term proportional to the manager's endogenous risk aversion ($E^{M,s}_{xx}(x) < 0$). This term would dampen the impact of short-term concerns on portfolio choice,

$$k(\pi + \delta_\zeta \zeta) + \left(\underbrace{E^{M,s}_x(x)}_{>0} + \underbrace{E^{M,s}_{xx}(x)}_{<0} \right) \Delta_B(x) \frac{\pi}{\sigma_B} < st(q, x). \quad (10)$$

Figure 3 (b) shows how reputation volatility decreases with the manager's reputation. As managers become more respected, they focus less on short-term performance. Investors anticipate this response and learn less from short-term performance, leading to a less volatile

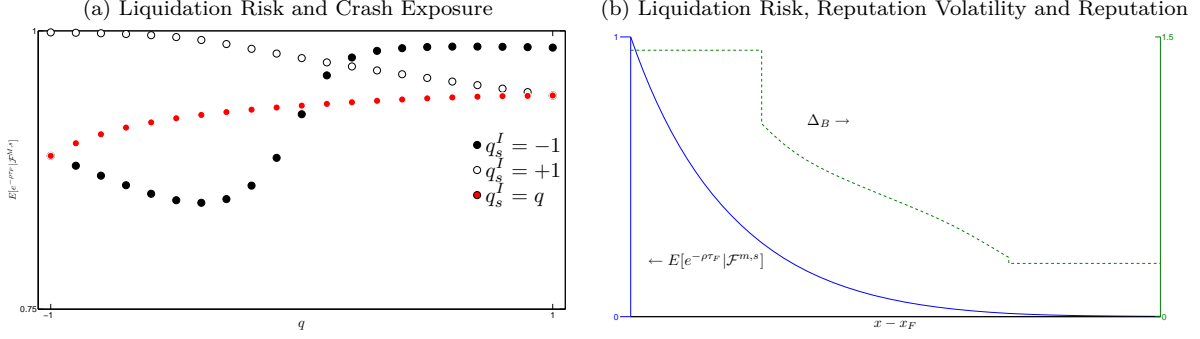


Figure 3: Endogenous Liquidation Risk. These figures illustrate how liquidation risk is amplified by mutually reinforcing responses between manager trading policy and investors learning and liquidation decisions. In Panel (a) the black dots show that when investors expect managers be bet on the crash ($q^{I,s} = -1$), a portfolio that bets on the crash moderately minimizes fund liquidation risk, and a portfolio that sells put options maximizes liquidation risk. The white dots show that if investors believe the manager is selling puts ($q^{I,s} = 1$), selling puts minimizes fund-liquidation risk. The red dots show that when portfolio choices and beliefs are mutually consistent ($q^{I,s} = q$), higher crash exposure will lead to higher liquidation risk. Panel (b) plots reputation volatility Δ_B and fund-liquidation risk as a function of manager reputation.

reputation evolution.¹⁵ Figure 3 (a) illustrates how the amplification mechanism works. Holding fixed the distance to liquidation and investor beliefs q_s^I , the figure shows how the probability of liquidation changes with changes in the crash exposure q . When investors are optimistic ($q^{I,s} = -1$), the manager can reduce the likelihood of liquidation by being slightly less aggressive in the bet in the crash strategy. Because investors understand this incentive, they will expect higher crash exposures. These beliefs further increase manager incentives to increase fund exposure to crash risk. In the limit when investors are pessimistic, liquidation risk is minimized by validating investors' pessimistic beliefs. We see that even though the manager tilts her portfolio toward put-selling in an attempt to reduce fund-liquidation risk, in any equilibrium ($q^{I,s} = q$), liquidation risk will increase with the amount of distortion taking place. The manager's attempt to survive results in a higher likelihood of failure.

¹⁵Note that here I only show the endogenous effect as I am reporting the volatility in log-likelihood space as I have done through out the paper. In probability space reputation volatility also feature a mechanical effect due to the binomial nature of the type distribution.

II. Equilibrium Crash Premium

In this section, I close the model and derive the impact of reputational capital on asset prices. I follow Kondor [2009] and assume demand for crash-risk protection by other participants in the market is given by $D(\pi) = \bar{D} - \gamma\pi$. I shall refer to these other participants as local hedgers. Local hedgers demand less protection as the premium increases, $\gamma > 0$. This structure is convenient and used as a reduced-form description of local risk-averse behavior. Given this structure, equilibrium pricing follows directly from the aggregate manager positions, q^a , which is the the sum of crash exposures across all funds active in the market. In this case equilibrium pricing is as follows,

$$D(\pi) = q^a \Rightarrow \pi(q^a) = \frac{\bar{D} - q^a}{\gamma}. \quad (11)$$

The more put-selling, $q^a \uparrow$, takes place in aggregate, the lower the crash premium. I emphasize the role of *reputational capital* by considering local crash-exposure demand levels, \bar{D} , so that as the aggregate arbitrageur position moves from betting on the crash to put-selling, the equilibrium crash premium goes from over- to under priced, $|\delta_\zeta \zeta| \in (\frac{\bar{D}-1}{\gamma}, \frac{\bar{D}+1}{\gamma})$, so that enough arbitrageur capital exists to correct prices.

Assumption 2. $|\delta_\zeta \zeta| \in (\frac{\bar{D}-1}{\gamma}, \frac{\bar{D}+1}{\gamma})$

As in the partial equilibrium setting I restrict parameters so that better than expected short-term performance is always good news about manager type. In the case of partial equilibrium this restriction was $\alpha_s > 2\pi$. When prices are determined in equilibrium this restriction is slightly changed to take into account that equilibrium prices are different if the managers are skilled versus if they are not.

Assumption 3. $\alpha_s \geq \frac{2\bar{D}}{\gamma}$

First, I consider the case of a manager that has exclusive market access and is a monopolist trading against these local hedgers. Then I introduce competition between managers. The key difference relative to the partial equilibrium analysis is that fund managers can also be over-hedged when pricing is endogenous. This doesn't happen in partial equilibrium because the portfolio that minimizes fund liquidation risk during a crash is also the one that maximizes expected returns. So long-term reputation concerns always push the manager towards return maximization. When prices are determined in equilibrium and there is enough arbitrageur capital to close the mispricing this is no longer true. Because the portfolio that maximizes returns is interior, return maximization will not minimize liquidation risk during crashes.

A. Exclusive Market Access

With exclusive market access, three things change relative to the portfolio choice we characterized in proposition 1: the crash premium will become time-varying and is directly impacted by the manager choice, $\pi_t = \pi(q(x_t))$, and investors will adjust their benchmarking policies to take into account how equilibrium prices change with the skill of their manager. Given that unskilled types always sell put options, benchmarking policies become

$$\Delta_B(x) = \frac{\alpha_s + q^{I,s}(x)\pi(q^{I,s}(x)) - 1 \times \pi(1)}{\sigma_B}, \quad \mu_B(x) = \frac{\alpha_s + q^{I,s}(x)\pi(q^{I,s}(x)) + 1 \times \pi(1)}{2}. \quad (12)$$

Equilibrium leads to an additional amplification mechanism. Short-term performance might be low because a skilled manager is aggressively betting on the crash and the premium is high, or because an unskilled manager is aggressively selling puts and the premium is low. Incorporating these changes in equation (7) and taking the derivative with respect to q , we obtain the monopolist optimal portfolio. Contrasting with expression (7), the key difference is that the manager now takes into account how the premium moves against her as she

trades:

$$\begin{aligned}\Omega^m(q, x|q_s^I) &= k\delta_\zeta\zeta + (\pi(q) + q\pi'(q)) \left(k + E_x^{M,s} \frac{\alpha_s + q^{I,s}\pi(q^{I,s}) - \pi(1)}{\sigma_B^2} \right) + \\ &\quad + \delta_\zeta\zeta\Theta_{dV}(q^{I,s}) \int E_x^{M,s} \left(x + \Theta(q\zeta + \sigma_N z, q^{I,s}) \right) \phi(z) dz.\end{aligned}\quad (13)$$

If for a given reputation, bet-on-the-crash is an equilibrium, it should be the case that $\Omega^m(-1, x|-1) < 0$. Analogously, $\Omega^m(1, x|1) > 0$ in the put-selling equilibrium, and $\Omega^m(q^*, x|q^*) = 0$ in the reputation-hedging equilibrium. A useful benchmark is the choice of an unconstrained self-financed monopolist (*sfm*). She would choose q^{sfm} such that $\pi(q^{sfm}) + q^{sfm}\pi'(q^{sfm}) + \delta_\zeta\zeta = 0$, which maximizes arbitrage revenue. It can be seen from Ω^m that an arbitrageur that cares about her reputation will not usually make this choice. Substituting the condition that pins down q^{sfm} in equation (13), we see that performance incentives drop out. q^{sfm} will be an equilibrium only if short-term reputational concerns exactly off set long-term reputational concerns or when the manager's reputation is so high that both sources of reputational concerns are mute. For low-reputation managers, short-term concerns dominate, which leads them to sell too many put options, leading to underpricing of crash risk. As the manager's reputation increases, long-term concerns become more salient and the manager buys more protection than a self-financed arbitrageur would. The rise of long-term concerns leads the equilibrium crash premium to be "too high" in the sense that it is higher than it would be if no delegation frictions were present. But of course because a monopolist restricts insurance provision, from an efficient stand point higher prices are efficient as they push premiums closer to be fairly priced. In this loose sense, intermediary reputation is a public good because it helps the crash premium become more informationally efficient, i.e. $\pi \rightarrow -\delta_\zeta\zeta$. Eventually both long-term and short-term concerns fade as the manager builds enough of reputational capital. As performance incentives dominate, the crash premium converges to the equilibrium premium without delegation frictions ($\pi(q^{sfm})$). Figure (4) shows the behavior of the equilibrium premium as a function of intermediary reputation:

as reputational capital is depleted, concerns over liquidation during a crash event initially and crash risk becomes over-priced (from the perspective of a monopolist), but as reputational capital keeps going down short-term liquidation risks become more important, what pushes the fund manager to under-hedge and crash risk to be increasingly under priced. Figure (4) shows the premium volatility spikes as the manager reputation decreases. Stochastic volatility is generated endogenously from the rationale discussed in section D.: liquidation risk induces put-selling, which leads investors to learn from short-term performance, which increases liquidation risk.

The two bottom panels in Figure (4) contrast the fund equilibrium expected return given equilibrium exposure and equilibrium premium (left) with the strategy expected return (right). The strategy expected return is the expected return as measured by an econometrician. It is the expected return a skilled manager that maximizes fund expected returns would capture if equilibrium prices were determined by the reputation concerned fund managers of our model. This contrast captures the essence of limits to arbitrage. Even though the fund opportunity set is improving, the high liquidation risk limits the ability of the fund manager to explore this mispricing.

B. A Competitive Arbitrage Industry

In this section, I show that competition makes delegation distortions worse and accentuate the impact of intermediary reputation volatility in equilibrium prices. For this analysis to be tractable, I restrict dissemination of performance information to investors of other funds. I build on the premise that a lot of arbitrage activity is highly opaque, and assume that for each manager of mass di , a mass of investors Wdi exists, with $W > 1$, that tracks her performance. These investors do not track the performance of any other manager and do not observe any additional information about the crash-risk market. As before, investors only

observe fund performance. To aggregate easily, I assume all shocks are perfectly correlated across managers, $\mathbb{E}[\sigma_B dB^i \times \sigma_B dB^j] = \sigma_B^2$, $\mathbb{E}[\epsilon^i \times \epsilon^j] = \sigma_N^2$, and $\mathbb{E}[\phi^i(1 - \phi^j)] = 0$, which means manager reputations move perfectly together. This set of assumptions leads to a lot of tractability because I only need to track one reputation to pin down equilibrium prices.¹⁶

One should think about this setting as follows: A new strategy is discovered by several managers, these managers pitch the new asset class to their clients and ask for equity financing. These new fund managers either have identified a truly profitable arbitrage opportunity and are all skilled, or all these managers are just “asset gatherers” and don’t know what they are doing. In this case, they are all unskilled. These assumption make the problem tractable because I do not need to track the evolution of the entire distribution of manager reputations.

Given this convenient set of assumptions, the problem is essentially the same as the one solved for the monopolist. The only difference is that now individual managers do not factor in the direct consequences their trades have on equilibrium pricing:

$$\begin{aligned} \Omega^c(q, x|q_s^I) &= k(\pi(q) + \delta_\zeta \zeta) + \pi(q) E_x^{M,s} \frac{\alpha_s + q^{I,s} \pi(q^{I,s}) - \pi(1)}{\sigma_B^2} + \\ &\quad + \delta_\zeta \zeta \Theta_{dV}(q^{I,s}) \int E_x^{M,s} (x + \Theta(q\zeta + \sigma_N z, q^{I,s})) \phi(z) dz. \end{aligned} \quad (14)$$

As in the monopolist case, it is convenient to use the equilibrium with self-financed arbitrageurs as a benchmark. Because agents are risk neutral and enough capital exists to correct mispricing, it is clear that in the self-finance case, we will have $\pi(q^{sfc}) = \delta_\zeta \zeta$. As in the monopolist case, at portfolio $q(x) = q^{sfc}$, it is easy to see that payoff incentives disappear from equation (14). Exactly as in the case of the monopolist, the balance between short- and long-term concerns will push the manager away from this efficient choice. When

¹⁶Not surprisingly tracking the cross-sectional distribution of different managers reputations is very challenging. Tracking the evolution of this cross-sectional distribution is necessary if one wants to compute the aggregate arbitrageur position, which pin down equilibrium premium. Krussel and Smith types of techniques are not useful in my setting as fund liquidation introduces sharp non-linear dynamics.

managers have a low reputation, their portfolios will be tilted toward put-selling, leading to low and volatile crash premiums. Volatility in the crash-risk market is a direct by-product of the high volatility of managers' reputations as they approach liquidation. As low-reputation managers build reputation, long-term reputation concerns get progressively more weight. Initially reputation building leads to higher expected returns *and* lower mispricing, because fund managers get bolder in betting on the crash, but eventually long-term concerns are so strong that they push managers to over insure. They buy protection even though in equilibrium, the crash premium is over priced. When managers have middle-of-the-road reputations, the chance that they are liquidated during a crash is much larger than the probability of receiving a long enough stream of bad Brownian shocks. As in the case of the monopolist, as reputation keeps increasing, both short- and long-term concerns fade, and equilibrium choices and prices converge to the outcomes of a frictionless arbitrage industry.

Figure (4) compares the equilibrium outcomes as a function of the degree of competition in the arbitrage industry. Relative to the monopolistic case, prices become more efficient when reputation is high, but less efficient when reputation is low. Higher efficiency in the right tail of the reputation distribution is a direct consequence of competition in the crash-insurance market. Competitive arbitrageurs do not inefficiently restrict insurance supply, which obviously leads to more efficient prices when delegation frictions become small. In the left tail of the reputation distribution, the opposite occurs. Because competitive arbitrageurs do not take into account that their reach for yield behavior reduces yields, they bid crash premiums lower than a monopolist as their reputation goes down. In equilibrium, mispricing is more severe and pricing more volatile as managers approach liquidation when the industry is competitive.

In the bottom left panel of Figure (4) we see what is another important insight of this analysis. Equilibrium funds expected returns are a non-monotonic function of the amount of reputational capital in the intermediary sector. When reputation is high, competition

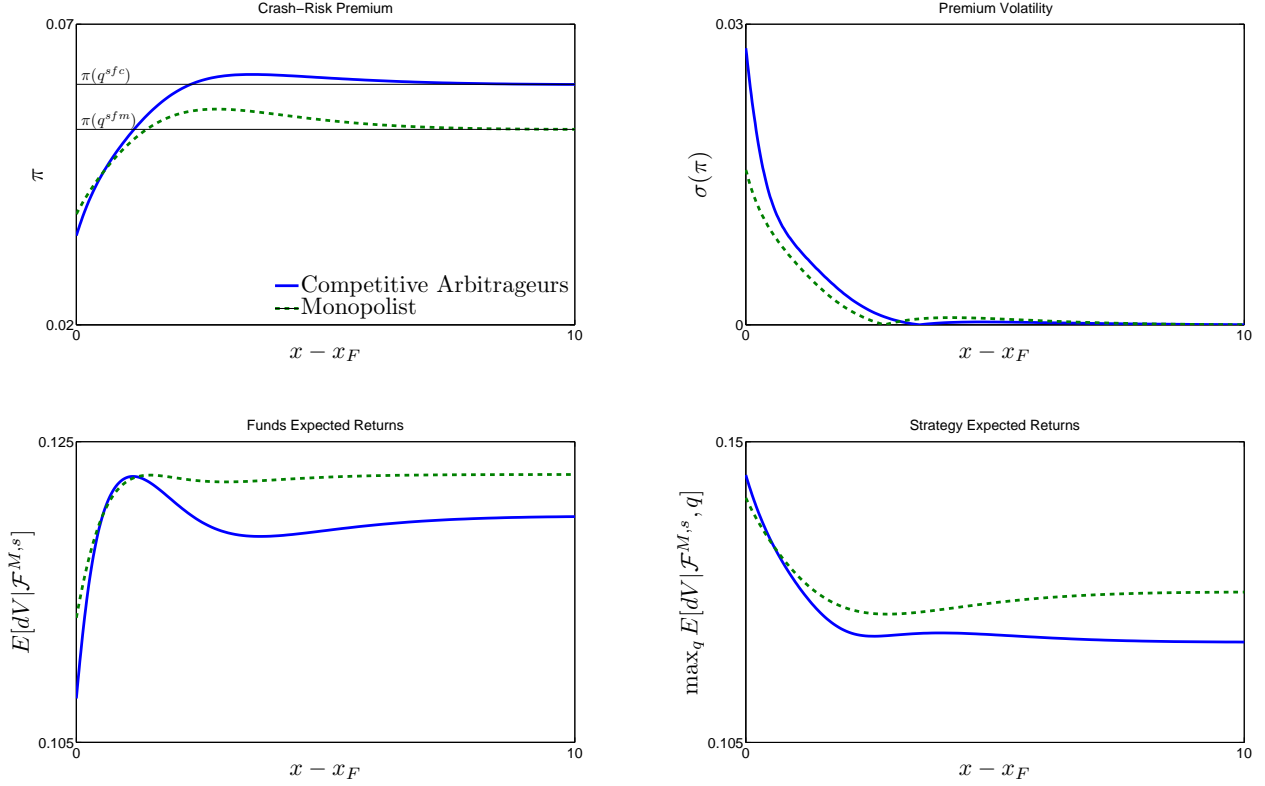


Figure 4: Equilibrium Pricing. The four figures describe equilibrium pricing as a function of manager reputation distance to liquidation. The continuous line shows the results when there is competition among arbitrageurs and the dashed line shows the results for an arbitrageur that has exclusive local market access. The top panels depict equilibrium crash-risk premium and its volatility. The bottom panels show fund expected returns *given* the manager equilibrium portfolio and the strategy expected return, the expected return of the portfolio that maximizes expected returns given equilibrium prices. A note on parameters: I set \bar{D} and γ so that crash risk is correctly priced when aggregate manager position is $q = -0.75$ and that crash premium varies from 0.025 to 0.065 as managers move from put selling to bet on the crash. Remaining parameters are from Table I.

induces managers to bid premiums so high that they make no money on their bet. In fact, middle-of-the-road managers are so afraid of being liquidated during a crash, that premiums overshoot. Their expected performance is actually reduced because they insure too much. Profitability peaks in the middle when reputation is high enough for managers to bet on the crash, but not enough to push premiums too high. The scarcity of reputational capital generates mispricing, this increases the value of accumulating reputation, what feeds back into higher reputational concerns and mispricing. As is the case for the monopolist, when the market is competitive strategy expected returns are maximum exactly when investors pull out.

C. Mispricing Dynamics

Compensation for bearing crash risk is too low when managers have a low reputation and reach for yield to avoid liquidation, but is too high when managers over hedge against crash risk. But how persistent is mispricing? Models that rely on mechanical investor behavior are silent on this question. It is unlikely investors never change their behavior after years of repeating the same mistakes. How investors change these policies is key to understanding how fast mispricing will fade. The Bayesian investor is a natural benchmark for exploring this question, because it tells us how investors *should* change their policies in light of data. The Bayesian benchmark connects the size of the distortion with its persistence—a connection that is absent in the more behavioral rational for limited arbitrage. In the model, as the manager distorts her portfolio to boost short-term performance, she reduces expected returns but boosts investors’ learning. The larger the distortions are, the more informative short-term performance. This connection leads to a positive relation between the size of the distortion and the speed of convergence in prices. We see this pattern in Figure (5) (a), where I plot the response of the equilibrium premium after a year, with fund performance one standard deviation above (below) the mean. This shock leads to an increase in mispricing of 100 basis points as the equilibrium premium dips from 4.5% to 3.5%. This increase in mispricing recovers after 26 months on average. For mispricing to reduce by another 100 basis points takes additional 42 months. We see the equilibrium premium overshoots the fair price because respected managers over hedge due to long-term reputation concerns.

On one level the link between persistence and the magnitude of limits to arbitrage distortions is intuitive. Skilled managers who sell more put options will over perform in the short term, boosting their reputations. The more the manager distorts, the faster they build up their reputations. This dynamic mechanism attenuates limits to arbitrage distortions, because reputation concerns are weaker when the option to build reputation faster in the future exists. The option value of distorting the portfolio in the future, reduces incentives

to distort today. While the dynamic nature of reputation concerns reduces how large are the distortions, and makes limits to arbitrage less severe, investor rationality amplifies it. Because rational investor behavior link the growth and volatility of reputation together¹⁷, periods of high reputational build up are also periods of high reputational risk. This link makes a manager averse to going down paths in which large distortions are needed, because these distortions will also expose her to higher liquidation risk. Higher future liquidation risk increases the incentives to distort the portfolio in the present to avoid these risky regions of the state space. Inter-temporal liquidation risk management leads managers to distort their portfolios when their reputation volatility is counter cyclical.

Whereas panel (a) characterizes short-run mispricing dynamics in paths without a crash, panel (b) describes how the distribution of mispricing evolves over time across economies across all paths. Not surprisingly, mispricing fades out eventually as asymmetric information between managers and investors is resolved as managers build a strong track record. In more realistic cases in which skill depreciates, delegation can induce long-term mispricing as even managers known to be skilled will eventually be doubted, if they do not perform. This type of extension is important if one wants to think more seriously about the model calibration¹⁸.

D. Comparative Statics

Volatility

Because investors are rational and all agents are risk neutral, any Brownian aggregate risk has no impact on the problem/valuations of investors or managers. As long as investors know the exposure to this aggregate risk factor, they can perfectly filter out this aggregate

¹⁷You can see this in the manager HJB equation (5), where in equilibrium $E_x^{M,s}$ and $E_{xx}^{M,s}$ coefficients are equal to $\frac{\Delta_B(x)^2}{2}$.

¹⁸See Moreira [2012] for an extension in this direction.

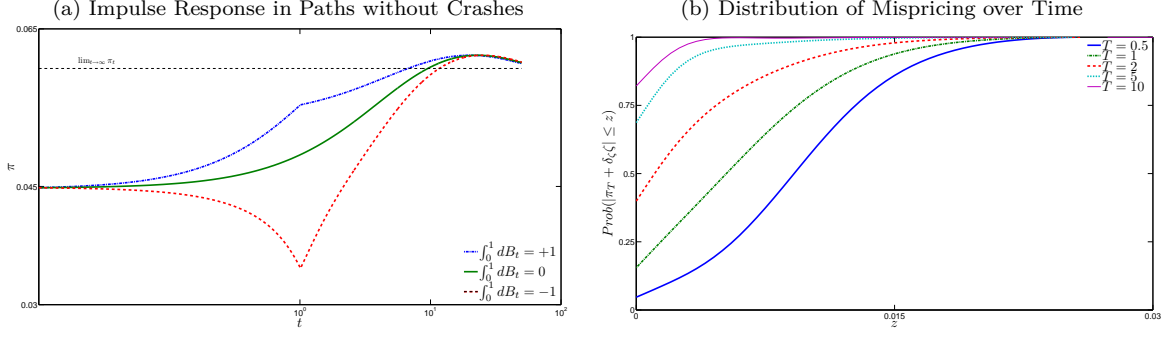


Figure 5: Mispricing Dynamics. These figures illustrate the time-series behavior of crash-risk premium. Panel (a) shows how pricing responds to a one standard deviation to the manager returns in the following year in path without crashes. In the x-axis, we have time in the long scale. In the y-axis, the equilibrium premium. The continuous line shows the expected price path. The lower dashed line, the price path when the manager receives a negative one-standard deviation shock. The upper dashed line represents a positive shock. Panel (b) shows how the distribution of mispricing changes over time. In the x-axis, we have different degrees of mispricing, and in the y-axis, the mispricing cumulative distribution. Initial reputation is set at $x_0 - x_F = 0.5$. A note on parameters: I set \bar{D} and γ so that crash risk is correctly priced when aggregate manager position is $q = -0.75$ and that crash premium varies from 0.025 to 0.065 as managers move from put selling to bet on the crash. Remaining parameters are from Table I.

component from fund performance. If investors don't know this aggregate exposure, they can quickly learn differences in exposures from the amount of co-variation between fund returns and the aggregate risk factor. The fund's idiosyncratic volatility, however, cannot be filtered out by investors. The more idiosyncratic volatility the manager has to take to generate her selection alpha, α^s , the less informative fund short-term performance is. Higher volatility decreases the signal-noise ratio of short-term performance, which everything else constant, makes fund liquidation less likely. The reduction in liquidation risk induces the manager to be bolder and less myopic. Additionally an increase in volatility also reduces the gains of boosting short-term performance.

As a result of this logic, we see in Figure 6 that under-pricing of crash-risk decreases with strategy idiosyncratic volatility. This property of the model is fundamentally different from more traditional models of limits to arbitrage in which investors follow mechanical policies, such as the fund investors in Shleifer and Vishny [1997]. In Shleifer and Vishny, an increase in volatility increases the risk of fund liquidation and hence limits to arbitrage. When investors are rational, they anticipate that low-return months become more likely as volatility

increases, thereby leading them to rationally learn less from short-term performance, which has the side benefit of tilting the manager toward the more profitable long-term strategy. This prediction tells us the “reach for yield” behavior will be stronger in periods of low idiosyncratic volatility or in asset classes where idiosyncratic volatility is low. A reduction in idiosyncratic volatility makes investors more sensitive to small changes in fund performance, which leads to higher liquidation risk and ultimately to fund managers’ stronger reach for yield behavior. As a result of the impact on the intensive margin of limits to arbitrage, we see in Figure 6 that a reduction in idiosyncratic volatility leads to a higher fund-failure threshold when investors have a short-enough horizon.

Crash horizon

The strategy horizon δ_ζ^{-1} controls how different the timing of resolution of uncertainty is between the put-selling and the bet-on-the-crash strategy. When $\delta_\zeta \rightarrow \infty$, the heterogeneity vanishes and a useful distinction between the long-term and the short-term is no longer present. The model therefore has something interesting to say when δ_ζ is low enough that the manager faces a difficult trade-off between surviving and delivering high expected returns. Figure 6 shows that the longer the horizon on the crash δ_ζ^{-1} , the more crash-risk is underpriced. This comparative statics implies that managers reach for yield behavior will be stronger when risks are large ($|\zeta| \uparrow$), but very long term $\delta_\zeta^{-1} \uparrow$. Note these comparative statics are driven solely by the horizon effect, because I hold expected return and learning constant.¹⁹ This result indicates that the intermediaries in our model will have a particularly hard time to arbitrage very long-term risks like the 2000 tech sector bust or the 2007-2008 financial crises. In both case we had academics, practitioners and policy makers predicting a crash as soon as 4 years in advance²⁰. I interpret this result as placing endogenous bounds

¹⁹Increases in δ_ζ^{-1} are balanced out with increases in ζ so $\delta_\zeta \zeta$ is constant, and increases in ζ are balanced out with increases in σ_N so the amount of learning during a crash is constant $\frac{\zeta}{\sigma_N}$.

²⁰for example, Greenspan famous “irrational exuberance” speech was given in the end of 1996.

on maturity transformation by the intermediary sector. In an environment with asymmetric information, intermediaries cannot be trusted to maturity transform effectively because their fear of being liquidated will push them to shorten the duration of expected profits.

Interest rates

The impact of overall level of interest rates can be studied by changing ρ of all agents. Low discount rate environments induce intermediaries to put more weight into future cash-flow streams relative to current performance. This implies that reputation concerns become more important relative to pay-off incentives. So the model predicts that mispricing is higher when interest rates are low. This behavior is consistent with policy makers concerns (see Stein [2013]) that a low interest rate environment induce reach-for-yield behavior. While stories linking reach-for-yield behavior with interest rate are typically related to risk-shifting of institutions that have fixed value liabilities, in my model reach-for-yield behavior increases with reductions in interest rates because fund liquidation becomes more costly relative to the foregone expected returns when the future earnings represent a larger fraction of fund valuation.

Crash volatility

Crash volatility, σ_N , captures agents uncertainty regarding how the crash will impact the cross-sectional of assets. When σ_N is low the impact is expected to be fairly uniform, when σ_N is high agents are fairly uncertain how the crash will impact their portfolio. The uncertainty in a crash σ_N determines how much learning happens during crashes. If σ_N is low, investors can accurately infer ex-post the manager position q because they know that most of innovation in fund returns can be traced out to ex-ante exposure choices. Clean inference allow them to distinguish easily skilled and unskilled managers as long as investors

expect skilled managers to have crash exposures minimally different from unskilled managers. In other words, when the crash is “systematic” $\frac{\zeta}{\sigma_N} \uparrow$, performing well during a crash is a powerful signaling mechanism for a skilled manager. The power of the mechanism can cut both ways, however. Because returns to reputation are decreasing ($E_{xx}^{M,s} < 0$), the benefit of choosing a portfolio sufficiently different from unskilled managers dies out quickly, as small portfolio differences will result in large reputation spikes when a crash hits. As a result, long-term reputation concerns are effective in nudging the manager out of distorting her portfolio too closely to the unskilled type, but is toothless in avoiding small deviations from the efficient choice. The result is an intricate relationship between how systematic the crash is and how severe limits to arbitrage are. In Figure 6 we see that more uncertain agents are about the cross-sectional consequences of the crash, the smaller the mispricing of crash-risk. Managers unsure about how their specific portfolio will perform during the crash avoid distorting their portfolios too much, reducing under-pricing of crash risk in equilibrium.

Competition

Competition between fund manager can increase or decrease limits to arbitrage. The impact of competition depends on how it impacts the value of a marginal improvements in the manager track-record. In other words, what matters is how competition changes the value of a marginally better past performance history. When investors capital becomes scarce, for example because the pool of investors have to other better investment opportunities, the hurdle rate investors demand increases, what increases the reputation needed for a fund manager to survive. This mechanism implies that for manager close to liquidation, the value of a marginal improvement in their tracking history increases substantially. While the value of being a respected manager decreases, because a higher share of her skill is shared with fund investors, the marginal value of reputation increases for manager close to being liquidated. This liquidation risk effect dominates when the strategy is populated by young managers

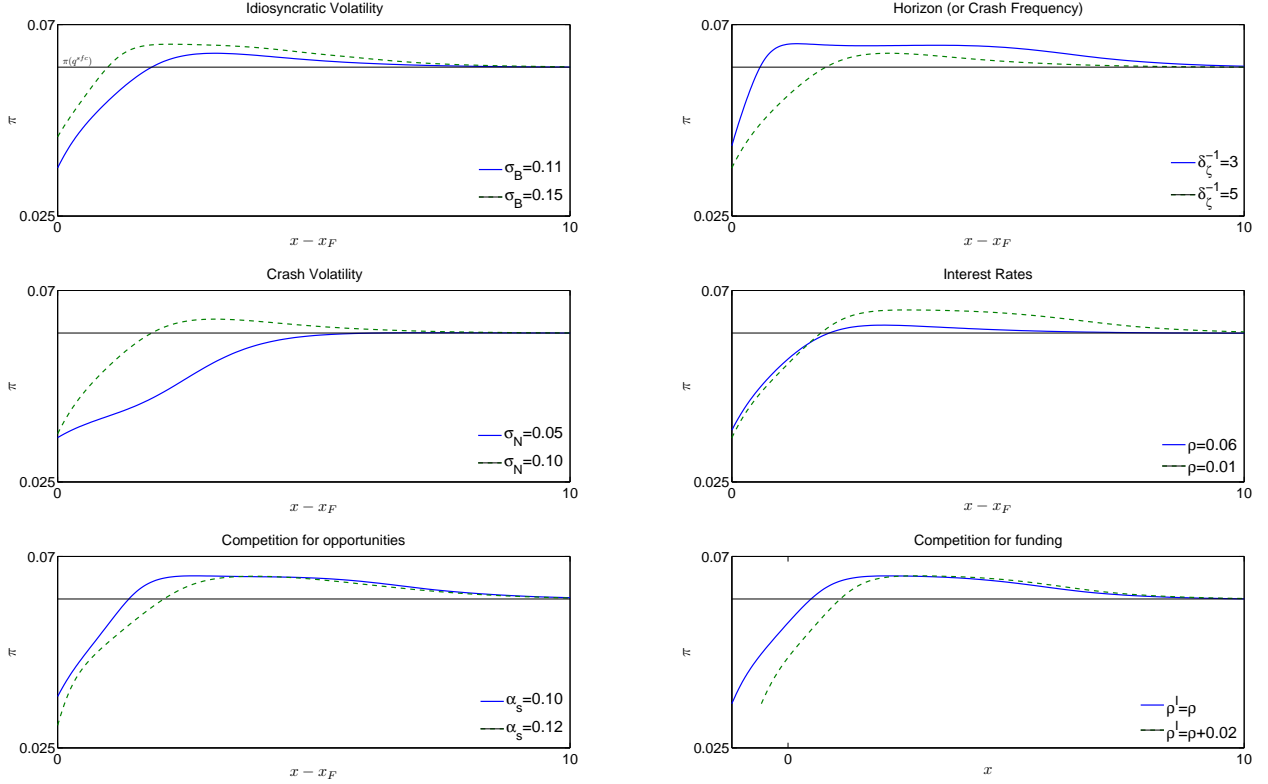


Figure 6: Comparative Statics These figures illustrate how limits to arbitrage are related to different characteristics of the economic environment. In the x-axis we have the manager reputation and in the y-axis the equilibrium crash-risk premium. A note on parameters: I set \bar{D} and γ so that crash risk is correctly priced when aggregate manager position is $q = -0.75$ and that crash premium varies from 0.025 to 0.065 as managers move from put selling to bet on the crash. Remaining parameters are from Table I.

(low reputation), what leads to an increase in under-pricing of crash risk. Figure 6 (bottom right panel) illustrates this mechanism at work.

When investor capital is abundant, but there are too many managers chasing too few investment opportunities we expect the selection alpha to go down ($\alpha_s \downarrow$). As in the case of a higher hurdle rate, this increases the minimum reputation needed to attract capital. But in contrast to the first mechanism, a reduction in selection alpha reduces learning from short-term performance, what reduces the benefits of marginal improvements in the manager track-record. Figure 6 (bottom left panel) illustrates this second mechanism by showing how crash-risk premium changes for different selection alphas while holding the distance to liquidation constant.

III. Discussion

A. Mapping to Data

The model asset pricing predictions relates the reputation of a representative manager with the degree of mispricing. In reality managers in the same strategy are likely to have different track-records. I think there are two reasonable approaches to map the model to data. The first approach is to use the age of an asset class as a proxy for the reputation of the typical manager trading these assets. The idea is that because the asset class is new, managers are on average low reputation, and investors are learning a lot from short-term performance. We can interpret Coval et al. [2009] results in this spirit. They show that there was substantial under-pricing of crash risk in the CDO market (new asset class) relative to the option market (old asset class).

An alternative approach is to use manager compensation as a proxy for reputation. A possible way to measure the reputation of the representative manager is to use the median manager compensation. Periods where this median compensation are low, are periods where the typical manager has a low compensation and likely faces high liquidation risk, while the opposite is true when this median compensation is high. This predicts that periods of low median manager compensation are periods that crash risk is under priced.

This same idea of using within strategy compensation variation to measure variation in reputations can be used to test the model cross-sectional prediction that managers with less reputation will be more exposed to crash-risk, performing better during normal times but worse during crashes. In particular the model predicts that manager reputation predicts crash performance only in crash events where several funds are liquidated. If investors do not expect skilled managers to be differentially exposed to a particular shock, than reputation should neither predict crash-performance and crash performance should not trigger

redemptions.

B. Assumptions

My aim here is to discuss the role different assumptions play in my analysis. First and foremost the model presumes that contracts and other types of screening tools cannot perfectly tell apart skilled from unskilled managers . This residual uncertainty is essential for me to be able to talk about the role of reputational capital. A second key assumption that this paper relies on is that fund portfolios are opaque, at least with respect to bets on low-probability events. This assumption was clearly valid for several asset-backed securities that before the 2008 financial crisis were seen as super safe, and after the crisis became known as economic catastrophe bonds (see Coval et al. [2009]), but the assumption might not hold when investors are sophisticated enough to interpret complex portfolio disclosures. Of course one can find many reasons for why fund managers might not want to be fully transparent with their portfolios, such as predatory trading, investors mimicking these portfolios for their own benefit (and without paying the fee), or competitors eroding their profit margins when chasing the same assets. These two assumptions, residual uncertainty about manager skill and portfolio opaqueness, are at the core of the model mechanism , but I made several additional assumptions for tractability.

For example, in this paper, I keep the scale of the fund and the assets under management constant and assume investors and the manager cash in and out of the fund as needed to keep the fund operating at its maximum capacity. This assumption is not realistic as the relationship between flow and performance is one of the most studied empirical facts in financial economics. However, by assuming of that managers take a cut of the secondary-market transactions, I capture the main consequence of fund size, that is, the transfer of surplus to skilled managers due to competition in the market for trading skill. This point,

originally made by Berk and Green [2004], is at the center of the “extensive margin” of limits to arbitrage. Because managers cannot pledge their future human capital, they end up being liquidated too soon. One could easily add time-varying fund size but not without complicating the model. One assumption that is more relevant is the fact that managers can always maintain their equity stake at 20%. Important papers, such as He and Krishnamurthy [2008] have studied what happens when intermediaries cannot keep their share of investment and are forced to lever up or downsize. One could add an additional leverage choice to my model, but my mechanism which relies on intermediary *reputational* capital is a complement to rather than a substitute for their seminal work. A third assumption that I make is that unskilled managers are non-strategic and manage money as long people are willing to give them money to manage. One could think of ways to make this behavior optimal by introducing private benefits of controlling investors assets, through for example soft dollars from brokerages, participation in the Greenwich high society, or outright diversion of cash. None of those rational would be particularly easy to introduce in the current setup, but they all share key aspects of my model choice: being a manager is profitable even if you are not good at, it and selling crash insurance is a easy way to avoid being caught too soon.

In the interest of parsimony I assume the background drift differential and fund idiosyncratic diffusion volatility are exogenous, and not a subject of the manager choice. This assumption allows the paper to focus on the components of the manager portfolio that investors will take the longest to learn about. Heterogeneity in strategy risky are manifested in returns relative quickly (in the limit case of continuous time immediately), and a fully rational manager will take into account how her current portfolio choices will impact investors beliefs in the future. This implies that any limits to arbitrage effects depend on how fast investors can learn from performance about the manager positions. Modeling this learning dynamics is challenging and my focus on crash risk allows me to study the component of the manager portfolio choice that takes a particularly long time to show up in performance. This logic tells that limits of arbitrage induced by investor learning is the most severe for bets on crash events.

The last aspect of my model that I want to discuss is the parameter restriction I imposed on the manager selection skill. I impose that $\alpha_s > 2\pi$. This restriction ensures investors always interpret good performance as good news about the manager's skill. If this were not satisfied, positive abnormal performance would be associated with reductions in manager reputation if the manager is expected to be betting on the crash. Note that an equilibrium with negative reputation-performance sensitivity would be sensitive to strategies such as, over-trading, cash diversion, among others. For example, if managers had a cash diversion technology every time the reputation-performance sensitivity dropped below zero, they could increase their reputation and their consumption by diverting cash. If the cost of diversion is lower for the unskilled manager, we will never have an equilibrium with negative reputation-performance sensitivity. These potential model perturbations makes an equilibrium where good performance is bad news implausible. However, modeling this mechanism would complicate my analysis substantially. A somewhat related concern is that I am assuming that fund managers keep a constant allocation to the selection portfolio. Obviously any manager should have the freedom to adjust her allocation. In my setting changes in allocations that are reflected in portfolio diffusion variance are immediately recognized by investors. It follows that in contrast to choices related to the crash portfolio, the manager properly take into account how investors actions respond to her choices. As before, I would need to make the portfolio choices of the unskilled manager strategic to avoid the skilled manager from mechanically revealing her type by choosing a zero volatility portfolio with negative drift. Return profile that an unskilled manager can easily replicate. As I discussed before making the unskilled manager strategic would be realistic, but it would substantially complicate the analysis.

C. Optimal Contracts

I model contracts to mimic the main feature of money-management contracts among sophisticated intermediaries. Some inside equity and a substantial chunk of financing in the form of demandable equity. Demandable equity capital share loses but can be converted at the net asset value at any time. The zero maturity nature of these contract leads to the frictions discussed in this paper. In other work [Moreira, 2012], I use this as motivation to study the role of lockup contracts in mitigating the limits to arbitrage effects studied in this paper. Some recent work has also asked what optimal contracting looks like in environments with some similarities to the one I study. For example, Biais et al. [2010] study an environment in which managers face a private cost of taking an action that avoids a low probability risk and they show a relationship a optimal relation between scale and past performance. Makarov and Plantin [2010] study a problem similar to mine, in which managers are tempted to boost their reputation by taking on a low-probability risk and they derive what optimal contract looks like in this environment. Importantly, they suggest making the maturity of claw-back provisions endogenous to fund past performance. A challenging but important way forward is to study the optimality of the contractual arrangements that we do observe in the industry and use those choices as a way to elicit market perception of the importance of different distortions. For example, in Moreira [2012], I use managers' choice of lockup maturities to study how important limits to arbitrage are relative to the cost of managerial entrenchment introduced by long-term contracts. Another example of recent work that studies the relationship between financing policies and limits to arbitrage is Hombert and Thesmar [2011]

.

IV. Conclusion

In this paper, I present a novel theory of limits to arbitrage. My theory emphasizes the role of reputational capital in empowering intermediaries to sustain long-term bets. When reputational capital is scarce, fear of liquidation leads intermediaries to focus on short-term strategies, resulting in mispricing, low expected returns, and higher liquidation risk. At the core of my theory is an amplification mechanism that is a direct consequence of investors' and managers' rational responses to each other behavior. After a sequence of low-performance months, managers become increasingly likely to be liquidated as investors correctly update their opinion about their manager, and the resulting reduction in the investment horizon leads the manager to tilt her portfolio toward bets that pay out quickly but are less profitable. The reduction in expected returns leads investors to respond by liquidating the fund earlier than they otherwise would, and the tilt toward short-term bets lead investors to learn more from short-term performance. Investor response to the initial change in the manager portfolio compounds the initial reduction in the manager horizon, further exacerbating the manager incentives to focus on short-term performance at the expense of the more profitable long-term opportunities.

More broadly, this paper contributes to the literature by pushing us away from thinking exclusively about balance-sheet effects and flashing out a fully dynamic theory of how reputational capital impacts intermediaries' portfolios and ultimately asset prices. This theory shows that even well-capitalized intermediaries can be exposed to severe delegation frictions with respect to bets that are likely to take a long time to pay out. From a public policy or investment-advice perspective, the model's central message is that the focus of scrutiny should be on portfolio exposures with respect to risks that take time to realize. These risks are the ones a competitive market will take a long time to sort out. With respect to these risks, distortions are likely to arise even when intermediaries are profitable and have substantial skin in the game.

V. Appendix

A. Equilibrium Portfolio

Proposition 1. (*Equilibrium portfolio*) Let $\alpha_s > 2\pi$, $\delta_\zeta > 0$, $\zeta < 0$, $\frac{\partial^n E^{M,s}(x)}{\partial x^n}(-1)^{n-1} > 0$ and $\lim_{x \rightarrow \infty} \frac{\partial^n E^{M,s}(x)}{\partial x^n} = 0$, for $4 > n > 0$, than if $\pi + \delta_\zeta \zeta \geq 0$, $q^*(x) = 1$ for any $x \geq x_F$. If $\pi + \delta_\zeta \zeta < 0$, than portfolio policy can be split in at most three regions; (i) if $x \in [x_F, x_{ps})$, $q^*(x) = 1$, (ii) if $x \in [x_{ps}, x_{bc})$, than $q^*(x) = \min_{q \in [-1, 1]} q$ s.t. $\Omega(q, x|q) = 0$, and if $(\frac{\zeta}{\sigma_N})^2 > \underline{c}$ we have that $\frac{\partial q^*(x)}{\partial x} < 0$ (iii) if $x \in [x_{bc}, \infty)$, than $q^*(x) = -1$. Where $x_{ps} = \min_{x \in [x_F, x_{bc})} x$ s.t. $\exists q \in (-1, 1] \mid \Omega(q, x|q) = 0$ and $x_{bc} = \min_{x \in [x_F, x_{bc})} x \mid \Omega(-1, x| -1) \leq 0$.

Proof. From the equilibrium definition in (1) we have that a portfolio choice is an equilibrium if it maximizes returns *and* solves the manager problem when the reputation evolves in a way that is consistent with the manager choice in equilibrium. From the manager HJB in equation (5) we have that the manager FOC for her portfolio q , given reputation x , and beliefs $q^{I,s}$ is given by $\Omega(q, x|q^I)$ in equation (9). If for a given q we have $\Omega(q, x|q^{I,s}) > 0$ ($\Omega(q, x|q^{I,s}) < 0$), it follows that the manager would like to increase (decrease) q further. If $\pi + \delta_\zeta \zeta \geq 0$ than $q = 1$ maximizes expected returns. To check if the maximum expected return strategy is an equilibrium it is enough to check if $\Omega(1, x|1) \geq 0$. Indeed when $q^{I,s} = 0$ long-term reputational concerns disappear, performance incentives are positive since $\pi + \delta_\zeta \zeta \geq 0$ and because $E_x^{M,s}(x) > 0$ and $\alpha_s > 2\pi > 0$ it follows that $E_x^{M,s}(x) \frac{\alpha_s + (q^{I,s} - 1)\pi}{\sigma_B} \frac{\pi}{\sigma_B} > 0$ for $q^{I,s} = 1$. This proves the case where crash risk is over-priced. If $\pi + \delta_\zeta \zeta < 0$, than $q = -1$ maximizes expected returns. To check if the maximum expected return strategy is an equilibrium it is enough to check if $\Omega(-1, x| -1) \leq 0$. If for a given x this is the condition holds than $q(x) = -1$ is an equilibrium. Note that because risk is under-priced performance incentives are negative, (If $q_s^I < 1$) long-term reputational incentives are negative for any reputation, but short-term reputation concerns are always positive (from $\alpha_s > 2\pi > 0$ and $E_x^{M,s}(x) > 0$ follows $E_x^{M,s}(x) \frac{\alpha_s + (q^{I,s} - 1)\pi}{\sigma_B} \frac{\pi}{\sigma_B} > 0$ for any $q^{I,s} \in [-1, 1]$). Because $E_{xx}^{M,s}(x) < 0$ it follows that if $\Omega(-1, x| -1) \leq 0$ for some reputation x than $\Omega(-1, x'| -1) \leq 0$ also holds for any $x' \geq x$. The simplest way to see that this is true is to abstract from long-term reputational concerns. If $k(\pi + \delta_\zeta \zeta) + E_x^{M,s}(x) \Delta(-1) \frac{\pi}{\sigma_B} < 0$ and given that $E_{xx}^{M,s}(x) < 0$ it follows that $k(\pi + \delta_\zeta \zeta) + E_x^{M,s}(x') \Delta(-1) \frac{\pi}{\sigma_B} < 0$ for any $x' > x$. This proves that for any $x > x_{bc}$ it follows that $q(x) = -1$. Now if $\Omega(-1, x| -1) > 0$ we know that the maximum expected return strategy is not feasible. If a $q \in [-1, 1]$ satisfies $\Omega(q, x|q) = 0$ than it solves the manager problem. When there is only one solution to this equation, this is our equilibrium choice, if there is more than one $\{q^a, q^b, \dots\}$, the smallest one is the equilibrium choice as it

maximizes expected returns. In order to prove that portfolios are decreasing in reputation let implicit differentiate $\Omega(q, x|q) = 0$. We have that $\frac{\partial q}{\partial x} = -\frac{\frac{\partial \Omega(q, x|q)}{\partial x}}{\frac{\partial \Omega(q, x|q)}{\partial q}}$ so it is sufficient to prove that $\text{Sign}\{\frac{\partial \Omega(q, x|q)}{\partial x}\} = \text{Sign}\{\frac{\partial \Omega(q, x|q)}{\partial q}\} = -1$. Note that since $\Omega(-1, x| -1) > 0$ for $x < x_{bc}$, and q is the minimum root of $\Omega(q, x|q)$, by continuity of $\Omega(\cdot)$ we have that $\Omega(q, x|q)$ has to cross zero from above, otherwise there would be another root q that is smaller than the minimum root, what cannot be by construction. It follows that $\frac{\partial \Omega(q, x|q)}{\partial q} < 0$. Now to prove that $\frac{\partial \Omega(q, x|q)}{\partial x} < 0$ rearrange $\frac{\partial \Omega(q, x|q)}{\partial x}$ to get,

$$\frac{\partial \Omega(q, x|q)}{\partial x} = \frac{\zeta^2}{\sigma_N^2} \left(\left(\frac{\alpha_s + (q-1)\pi}{\sigma_B^2 \frac{\zeta^2}{\sigma_N^2}} \right) \pi E_{xx}^{M,s}(x) + \delta_\zeta (q-1) \mathbb{E}^x [E_{xx}^{M,s}(x + \Theta(q\zeta + z, q^{I,s}))] \right) \quad (15)$$

Taking the limit as crash informativeness grows large

$$\lim_{\frac{\zeta^2}{\sigma_N^2} \rightarrow \infty} \frac{\partial \Omega(q, x|q)}{\partial x} = \left(\frac{\alpha_s + (q-1)\pi}{\sigma_B^2} \right) \pi E_{xx}^{M,s}(x) < 0 \quad (16)$$

since $\lim_{\frac{\zeta^2}{\sigma_N^2} \rightarrow \infty} \mathbb{E}^x [E_{xx}^{M,s}(x + \Theta(q\zeta + z, q^{I,s}))] = 0$. By continuity there is a \bar{c} such that if $\frac{\zeta^2}{\sigma_N^2} \geq \bar{c}$ we also have $\frac{\partial \Omega(q, x|q)}{\partial x} < 0$. Hence $\frac{\partial q}{\partial x} < 0$ in the region where $\Omega(q, x|q) = 0$. To prove the proposition remains to characterize the put selling region. Let $\Omega(q_{ps}, x_{ps}|q_{ps}) = 0$ and by construction for any $x < x_{ps}$ there is no $q \in [-1, 1]$ such that $\Omega(q, x|q) = 0$. In this case it is trivial that $q = 1$ is the equilibrium. Note that $\Omega(-1, x| -1) > 0$ since $x < x_{bc}$ so it follows that $\Omega(1, x|1) > 0$. So $q = 1$ is clearly consistent with the manager problem. Since there is no other $q \in [-1, 1)$ consistent with the manager problem $q = 1$ is the equilibrium. This completes the proof. □

B. Numerical Solution

I apply the finite-difference method to solve the system of integro partial differential equations. To solve for optimal policies I sequentially iterate until the value functions converges. Two value functions ($e^I(x)$, $E^{M,s}(x)$), two choice variables ($q(x)$, x_L) are determined jointly. The state space consists of manager reputation in log-likelihood space ($x \in \mathbb{R}$).

I first discretize the state space as follows. I construct a grid with limits $\{-\bar{x}, \bar{x}\}$ and N grid points. Let $z = (\bar{x} + 1)^{2/N}$, I populate the grid by setting $x(i) = z^i - 1$ for $i \geq N/2$ and $x(i) = 1 - z^{N/2-i}$ for $i < N/2$. Using this grid I discretize equations (6) and (5) using central differences as described in Candler [1998].

First I hold constant the skilled manager's portfolio choice at the efficient choice $q(x) = -1$ and iterate to find what I denote the efficient solution $\{e^I(x), E^{M,s}(x), x_F\}^{ef}$. The efficient liquidation threshold x_F^{ef} is a lower bound to the equilibrium liquidation threshold x_F . Starting from the efficient liquidation policy and value function, I iterate on the HJB, and each time I solve for the optimal portfolio $q(x)$ policy and liquidation policy x_F at each step.

More specifically, the iteration procedure can be divided in steps.

1. Given choices $q^{i-1}(x)$ and a candidate x_F^j solve for $\{e^I(x)^i\}$ using the discretization of equation (6). Find x_F^i the such that $e^I(x_F^i)^i = 0, e_x^I(x_F^i)^i = 0$.
2. Given $\{x_F^i, e^I(x)^i\}$ and $q^{i-1}(x)$ solve for $E^{M,s}(x)^i$ using the discretization of equation (5) and the boundary condition $E^{M,s}(x_F^i)^i = k$.
3. Given $E^{M,s}(x)^i$ solve for $q^i(x)$ using proposition (1).²¹
4. If $|e^I(x)^i - e^I(x)^{i-1}| < \epsilon$, $|E^{M,s}(x)^i - E^{M,s}(x)^{i-1}| < \epsilon$, $|x_F^i - x_F^{i-1}| < \epsilon$, and $|q^i(x) - q^{i-1}(x)| < \epsilon$ and stop. If not satisfied, repeat.

²¹When solving for the efficient solution, I skip step 3 as the portfolio choice is always held at $q(x) = -1$.

The procedure converges extremely fast and takes no more than 10 iterations for a typical solution. The typical solution time is 30 seconds. Code is available upon request.

Table I: Parameter Values. Otherwise specifically noted this table summarizes the choices of parameters utilized through out the figures shown in this paper. Please refer to Moreira [2012] for a discussion of why such numbers are plausible for the hedge fund industry.

Variable	Symbol	Value	Variable	Symbol	Value
Risk-free rate	ρ	4%	Crash premium	π	4%
Brownian volatility	σ_B	11%	Crash Size	ζ	-30%
Selection skill	α_s	11%	Liquidity Shocks Frequency	δ_l	4
Crash Volatility	σ_N	10%	Manager Equity Stake	κ	20%
Crash Horizon	δ_ζ^{-1}	5			

Table II: Notation

Description	Symbol
Reputation: Log-Likelihood Ratio between Skilled vs Unskilled Manager Events	x
Reputation at Which Investors pull out and the Fund Fails	x_F
Minimum Reputation at which managers <i>Maximize</i> Expected Returns (Bet-on-the-crash)	x_{bc}
Maximum Reputation at which Managers <i>Minimize</i> Expected Returns (Put-selling)	x_{ps}
Investors Beliefs about the Crash Exposure of a Manager Type ϕ with Reputation x	$q^I(x, \phi)$
Investors Beliefs about the Crash Exposure of a Skilled Manager with Reputation x	$q^{I,s}(x)$
Crash Exposure of a Skilled Manager with Reputation x	$q(x)$
Short-term Performance Signal-to-Noise Ratio for a Manager with Reputation x	$\Delta_B(x)$
Expected Short-term Performance for a Manager with Reputation x	$\mu_B(x)$
Long-term Performance Learning Function	$\Theta(dV_+, x)$
Fund Returns	dV
Crash Event	dN
Everyday Shocks to Fund Performance	dB
Shocks to Fund Performance in Crash Periods	ϵ
Investor equity per Dollar Premium Over Book	e^I
Value of Skilled Manager Equity Stake	$E^{M,s}$
Investor Information set	\mathcal{F}^I
Skilled Manager Information set	$\mathcal{F}^{M,s}$
Time of Fund liquidation/Failure	τ_F
Time of Liquidity Shock	τ_l

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