Delegation and the Dynamics of Capital Flows

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Abstract

This paper develops a model of financial intermediation that highlights how reputation shapes the allocation of capital, and study the role of tail-risks in reducing the speed of capital flows. An intermediary builds reputation and attracts capital as investors learn from his investment performance. The tension between reputation and performance incentives produces a feedback loop between intermediaries and investors reach-for-yield behavior, which distorts capital allocation decisions and asset prices. Strategic behavior slows down the flow of capital overall, and into low-tail-risk assets in particular. Tail-risk taking opportunities render capital endogenously slow moving.

From the vantage point of standard finance theory, empirical evidence strongly indicates that financial capital flows are too slow and asset prices are too volatile[1]. Market-segmentation based theories[2] have been successful in bridging some of the gap between theory and data. At the same time there is an understanding that in some markets these effects are too persistent to be consistent with reasonable costs of market participation. For example, Mitchell et al. [2007] show that spreads in the convertible bond market take months to recover after a shock. While, the scarce arbitrage capital view provides a useful framework to analyze the slow moving capital

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[1]See Duffie [2010] for a survey of the empirical evidence on slow moving capital, and see Cochrane [2011] for a broader survey of the evidence on discount rates time-variation.

[2]Examples of market-segmentation theories are models based on search frictions [Duffie and Strulovici, 2012], attention frictions [Duffie, 2010], and financial frictions (Xiong [2001] and Gromb and Vayanos [2002]). In each of these models, not all investors are trading in asset markets at any given time, slowing the reallocation of capital.

phenomenon, it raises the question of why arbitrage capital scarcity is so persistent. This paper develops a delegation based explanation as an answer to this question.

The essence of the theory is the idea that an intermediary is more informed about the return profile of his portfolio than the final investor. This informational asymmetry introduces incentives for an intermediary to impact the flow of information according to the quality of his investment opportunities. An intermediary with a bad hand will tilt his portfolio to delay learning. An intermediary with a good opportunity set will try to speed it up. These implicit incentives induce distortions in capital allocation that slows the flow of capital, and produce novel implications relating both the cross-sectional and the time-series of expected returns with the history of past intermediary performance as summarized by his reputation. A novel feature of the analysis is the joint determination of investor's learning behavior and the intermediary's portfolio choice, which leads to amplification of capital allocation distortions.

The model works as follows. The economy consists of local hedgers, outside investors, and a financial intermediary. The financial intermediary enables better risk-sharing of local market endowment risk, and creates value if outside investors have a higher risk-bearing capacity than the local hedgers. While capital might be scarce in the local market, investors entertain the possibility that local hedgers are already well diversified and there is no excess compensation for risk to be earned.

The financial intermediary observe local market financial conditions, but must rely on his fund performance track-record to convince investors that capital is indeed scarce in order to attract capital. The performance of the fund managed by the intermediary reflects both the scarcity of capital in the local market and his portfolio decisions. The fund portfolio consists of multiple assets and it can be summarized by it's weight on a local-hedging portfolio and a tail-risk portfolio. While the local-hedging portfolio has the best performance profile as it is maximally correlated with the local endowment, the tail-risk portfolio is unattractive on a risk-adjusted basis, but produces good returns in the short-term.

Investors are Bayesian and fully understand the different incentives the intermediary face under the two alternative local market conditions. They allocate capital to the intermediary according to their expectation of the fund excess performance, but they cannot evaluate assets' tail-risk exposures and must use the intermediary's equilibrium incentives to form beliefs about the fund's tail-risk. These beliefs are important as they shape how investors respond to performance news and ultimately how fast capital flows. Specifically, investors beliefs about the intermediary's portfolios determine both the informational content of fund performance about the local market conditions and the fund expected profitability. As investors observe the fund performance history, the evolution of their beliefs about the scarcity of capital in the local market is summarized by the intermediary reputation. The intermediary reputation evolve fully endogenously as a function of both intermediary decisions and investors beliefs about these decisions.

As a result of investor behavior, an intermediary portfolio choice trades off static and intertemporal incentives. When capital is scarce, investing in the local-hedging portfolio maximizes both the intermediary current compensation and the fund risk-adjusted performance, but a tilt towards the tail-risk portfolio increases the intermediary reputation growth and reduces the fund liquidation risk. When capital is abundant, assets are correctly priced and as a result, the intermediary focus exclusively on increasing his reputation by investing in the tail-risk portfolio. This difference in portfolios across market conditions becomes more pronounced when reputation is high. Specifically, if capital is scarce, an increase in reputation decreases reputational incentives and induces the intermediary to shift away from the tail-risk portfolio towards the hedging portfolio. Importantly, this shift does not happen when capital is abundant and the hedging portfolio is correctly priced.

Differences in tail-risk taking across market conditions dampen the information content of fund short-term performance and induce endogenous variation in investor's learning behavior. When the difference in tail-risk taking shrinks as a result of decreases in reputation, investors optimally learn more from short-term performance, and less from tail-event performance. This change in learning behavior feeds back into the portfolio choice and amplify the intermediary incentives to boost short-term performance as his reputation deteriorates. The resulting increase in tail-risk taking leads to lower risk-adjusted performance as the intermediary deviates from the local-hedging portfolio, what drives investors to redeem more aggressively from the fund, resulting in a reduction in assets managed and a more sensitive relationship between flow and performance. In equilibrium, the intermediary tilt towards the tail-risk portfolio increases his reputation growth at the expense of lower compensation and higher reputation volatility.

I now discuss the asset pricing implications of these portfolio decisions. As capital flows into (out of) different assets, asset prices and expected returns fluctuate as local hedgers are forced to bear less (more) risk. The cross-sectional of expected returns satisfies a two factor structure where covariance with local endowment risk and tail-risk are both priced. Intermediary reputation impacts factor premia through the total amount of risk-bearing capital invested in the local market, and through the allocation of capital across assets. While covariance-risk premia decreases monotonically as the intermediary builds his reputation and attracts more capital, tail-risk premia exhibits a U-shape pattern with respect to the size of the intermediary balance-sheet. While the increase in reputation decreases the intermediary portfolio tilt towards the tail-risk portfolio, the increase in assets under management amplify the impact of this portfolio tilt on asset prices. As a result, tail-risk premia initially decreases as the intermediary sector expands. High tail risk assets are overpriced during an intermediary sector boom.

Overall, the long lags involved in measuring tail risks properly implies investors learn much slower than they would if they could risk-adjust fund performance using the actual tail-risk of the fund portfolio. As a result, capital is endogenously slow moving.

The endogenous reputation dynamics produces a tension between the mispricing of the two factors. Covariance-risk premia converges most rapidly, exactly when tail risk is more underpriced. The intermediary aggressive portfolio tilt distorts tail-risk premia, but speed up learning and the rate at which capital flows into the local market.

I analyze the model implications for equilibrium prices and the speed of capital flows of several changes in the economic environment. Specifically, I show that: financial innovation slows down the speed of capital flows; loose monetary policy is conducive to reach-for-yield behavior and underpricing of tail risk, as is a demand-driven growth in the intermediary assets; and lastly, I analyze the model implications for the design of optimal investment mandates.

I now turn to a brief review of the literature. The credit market fits the model particularly well as corporate and sovereign bonds alike are heavily exposed to tail risks but are fairly stable during normal times, at least when contrasted with stocks. Credit markets also tend to be segmented and heavily intermediated as in the model. In line with the model predictions, Coval et al. [2009] document that senior claims of collaterized debt obligations were persistently over-priced in the period of rapid intermediary sector growth before the 2007-2008 financial crises. More recently, Kacperczyk and Schnabl [2012] document that money market funds actively increased their exposure to tail risk during the preliminary stages of the financial crises, and they show that tail-risk taking was strongly correlated with the sensitivity of flows to performance. More broadly, the fact that a substantial fraction of investment strategies are negatively skewed (see Agarwal and Naik [2004]) is consistent with the model logic. While these episodes have been interpreted as evidence of neglect of tail risks [Gennaioli et al., 2012, 2013] by investors, my model explains them through endogenous reputational incentives, and implies that investor rationality amplifies tail-risk taking.

The model combines the learning [Berk and Green, 2004, Pastor and Stambaugh, 2010] and the agency view [Chevalier and Ellison, 1997, Basak et al., 2007, Basak and Pavlova, 2013] regarding the role the retail investor behavior plays in the allocation of capital. In our setup, investors' behavior reallocates capital to where it is most scarce, but distorts intermediaries' incentives and asset prices. As a result, prices and the allocation of capital are at the same time more and less efficient. In aggregate, risk-sharing improves as investors chase performance, but in the cross-sectional, high tail-risk assets become artificially expensive.

The model builds on a large body of work that considers the role of market segmentation. In particular, the asset pricing implications of time-variation in the wealth of a limited set of specialists. Examples include Xiong [2001], Gromb and Vayanos [2002], Kondor [2009] and He and Krishnamurthy [2011]. In these papers arbitrage (or intermediary) wealth is special as it enables risk-sharing.

This paper share with several papers the focus on implicit incentives resulting of investor behavior. Basak et al. [2007], Chapman et al. [2009], Makarov and Plantin [2010], and Basak and Makarov [2014] and study a portfolio choice framework, and Shleifer and Vishny [1997], Cuoco and Kaniel [2011], Basak and Pavlova [2013], Vayanos and Woolley [2013], Kaniel and Kondor [2013], and Vayanos [2004], study asset pricing implications of these implicit incentives.

Closest to my work are a few papers that model investor behavior endogenously. Dasgupta and Prat [2008] and Dasgupta et al. [2011] study the asset pricing implications of reputation concerns in a model where prices are determined by a market maker. Guerrieri and Kondor [2012] and Malliaris and Yan [2009] study how reputation concerns impact the will of a fund manager to invest in strategies that pay well with low probability and how these incentives impacts asset prices.

This paper contributes to the literature by building on the signal-jamming framework of Holmstrom [1999] and constructing an economic environment where there is a rich interaction between investors and intermediaries capital allocation decisions. Specifically, the present model highlights the tension between the persistence of aggregate mispricing and mispricing in the cross-sectional of expected returns, and features an incentive based explanation for why capital flows too slowly.

1 Model Setup

Time is continuous $t \in [0, \infty)$ and the economy is populated by three groups of agents: local hedgers (denoted by H), fund investors (denoted by I), and fund managers (denoted by M). Each group has an unit mass of individual agents. Individuals within each group are identical and behave competitively, and for ease of exposition, I will often refer to the representative agent of each group. Each agent takes aggregate allocations as given when considering his own choices.

Endowments. The investor receive perishable endowment $\bar{D}dt$ that can be consumed or invested between periods t and $t + dt$. The local hedger receive perishable endowment edD_t at time $t + dt$,

$$dD_t \quad = \quad \bar{D}dt + \sigma dB_t - \nu\sigma Y_t dN_t, \tag{1}$$

where e is a $1 \times n$ row vector, σ is a $n \times n$ matrix of Brownian exposures, B_t is a N-dimensional Brownian motion, N_t is a Poisson process with constant intensity λ, Y_t is a multivariate standard normal $Y_t \sim N(0, I_n)$. The term $\nu\sigma Y_t dN_t$ captures tail risk. Define $\Sigma = \sigma\sigma'$ as the returns covariance matrix during normal times, and $\nu^2\Sigma$ the return covariance matrix during tail events. The scalar ν controls the increase in asset risk during a tail event.

I require jumps to drive a wedge between short-term and long-term performance. This wedge is key for understanding the interaction between investors and intermediaries. The assumption that dividends are perishable and can only be invested for a dt interval keeps the total size of the economy constant and it allows the paper to focus on reputation as the only source of dynamics.[3]

Preferences. Agents discount the future at rate ρ and have preferences over consumption dc_t given by

$$u_i(dc_t) \quad = \quad \mathbb{E}_{i,t}\left[dc_t\left(1+\gamma dN_t\right)\right] - \frac{\alpha_i}{2} var_{i,t}\left(dc_{t-}\right), \tag{2}$$

where all agents have a (common) higher marginal utility γ of consumption during a tail event, but have different degrees of risk-aversion α_i with respect to Brownian risk. This preference structure implies there are potential gains from risk-sharing Brownian risk, but not tail risk. The local hedger's risk aversion can assume two values $\alpha_H \in \{\alpha_{H,b}, \alpha_{H,g}\}$, with $\alpha_{H,g} > \alpha_{H,b} = 0$. If the true risk-aversion is $\alpha_{H,b} = 0$ then transferring risk from local the hedger to the investor decreases welfare, while if $\alpha_{H,g}$ is the true risk aversion, some risk transfer is welfare increasing.

Assets. The local hedger and the manager trade n risky assets in zero net supply and invest in the risk-less technology. These assets have infinitesimal maturity and can be used to transfer risk from the local hedger to the investor, but only the manager and local investors can trade risky assets directly. The assets have prices given by π_t and pay

$$d\tilde{D}_t = dD_t - \eta dN_t,$$

at time $t + dt$, where $ed\tilde{D}_t = edD_t$ and $e\eta = 0$. A portfolio of weights e span the aggregate local endowment risk. The quantity $\eta_+ = \sqrt{\eta^\top \Sigma \eta}$ captures the degree of financial innovation in the local market as it measures the maximum amount of tail-risk per unit of Brownian risk feasible in

[3] For example, Guerrieri and Kondor [2012] take a similar approach.

the opportunity set. I refer to η_+ as financial innovation, because it expands the amount of hidden tail-risk a portfolio can have.[4] The risk assets return vector between t and $t + dt$ is defined as $dS_t \equiv d\tilde{D}_t - \pi_t dt$.

The local hedger problem determines asset prices as a function of the manager capital allocation $X^{M,\theta}$(a row vector $1 \times n$) ,

$$\pi_{\theta,t} = \bar{D} - \rho - \lambda_Q \eta + \alpha_{H,\theta}(X_t^{F,\theta} - e)\Sigma, \tag{3}$$

where $\lambda_Q = (1+\gamma)\lambda$ is the risk-neutral probability of a tail event.

The investor can invest in a fund actively managed by the financial intermediary or the riskless technology, which yields ρdt. The investor problem determines the fund assets as function of perceived fund profitability and risk,[5]

$$a_t = \frac{\mathbb{E}_{I,t}\left[\left(dR_t - \Gamma\left(dR_t\right)\right)\left(1 + \gamma dN_t\right)\right] - \rho}{\alpha_I var_{I,t}\left(dR_{t-} - \Gamma\left(dR_{t-}\right)\right)}, \tag{4}$$

where a is the fraction of wealth allocated to the fund, dR_t is the fund gross return on capital, and $\Gamma\left(dR_t\right)$ is the manager compensation.

Intermediation. The fund manager has no capital of his own, and raises equity capital from investors through an intermediation contract. Importantly, the manager knows the true risk aversion in the local market. I will use $\theta \in \{b, g\}$ to identify the manager type. The Good type $(\theta = g)$ enables risk-sharing as he provides access to a market with risk-averse local hedgers. The Bad type $(\theta = b)$ destroys value as he transfer risk from risk-neutral local hedgers $(\alpha_{H,b} = 0)$ to risk-averse investors. For simplicity we focus on the case of a risk-neutral manager with respect to Brownian risk, $\alpha_M = 0$.

The problem of a manager type θ consists of choosing a portfolio to maximize his lifetime utility

$$V_t^\theta = \max_{X_t} \mathbb{E}_{M,t}\left[\int_0^\tau e^{-\rho s} u_M\left(a_s \Gamma\left(dR_s\right)\right) | \theta\right], \tag{5}$$

where $\Gamma\left(dR_t\right)$ is the manager compensation per unit of capital managed and τ is the first time

[4]Tranching of diversified mortgage portfolios is an useful example of financial innovation that increases η_+.

[5]The problem of the investors is myopic because yields on all technologies are perishable. I omit investment in the risk-less technology because the household is perfectly indifferent between investing at the risk-less rate or consuming.

the fund assets reach zero, which is the time the fund is irreversibly liquidated. Once the fund is liquidated, the manager leaves the money management business forever and earns nothing going forward.

Information and beliefs. Investors do not observe the manager type θ or the fund portfolio X, but observe fund returns in real time. Their information can be represented by $\mathcal{F}_t^I = \{dR_s : s \leq t\}$. Managers know their type θ and their information can be represented as $\mathcal{F}_t^M = \{(dR_s, \theta) : s \leq t\}$. At time $t = 0$ investors believe the manager is Good $(\theta = g)$ with probability P_0 . We refer to P_t as the manager reputation. The uncertainty about the manager true investment opportunities induces investors to learn from past fund performance, and liquidate the fund when the likelihood that the opportunity set is bad is high enough.

Intermediation Contract. We consider an exogenous contract that consists of a symmetric performance fee and a proportional management fee. The symmetric fee assumption buys considerable tractability and allows me to focus on time-variation on reputational incentives.[6] For convenience we assume the management fee scales up with the volatility of the portfolio. Let dR_t be the fund realized return process, then the intermediary payoff flow per amount of dollars under management is given by $\Gamma\left(dR_t\right) = \kappa\left(dR_t - \rho dt\right) + \sqrt{var_{I,t}(dR_{t-})} f dt$.

There are two key frictions on the information environment: The manager has no direct channel to communicate the quality of his opportunities and the tail risk of his portfolio[7]. The first assumption implies that investors will use performance to learn about the investment opportunities the manager has.[8] The second assumption implies the intermediary does not internalize how investors respond to hidden changes in the portfolio composition. This logic implies the following notion of equilibrium,

Definition 1. *A Markov perfect equilibrium consists of functions* $\{X_t^{M,\theta}, X_t^{I,\theta}, a_t, \pi_{\theta,t}, P_t\}_{t \geq 0, \theta \in \{0,1\}}$ *that satisfy*

[6] An extension that either model performance fees using high water marks or an inside equity capital constraint is feasible, but makes the model analysis substantially more complicated due to an additional state variable.

[7] Portfolio opaqueness captures the fact that the intermediary has a unique understanding of the assets he trades, and even though investors are sophisticated and able to fully understand the environment, they are unable to evaluate specific portfolio positions. One could also motivate this assumption as optimal secrecy to protect the profitability of the trading strategy.

[8] For simplicity we assume that returns is the only credible channel of communication, but what is important is that any additional signal is sufficiently imprecise, so investors are left with some uncertainty about the manager quality.

(E1) Investor optimality: a_t is consistent with equation (4) ;

(E2) Manager optimality: $X_t^{M,\theta}$ solves equation (5);

(E3) Investor beliefs: $X_t^{I,\theta} = X_t^{M,\theta}$, and P_t is consistent with Bayes Law;

(E4) Market clearing:$\pi_{\theta,t}$ is consistent with equation (3).

Complementarities between investor's and manager's actions will sometimes lead to multiplicity. Whenever this is the case, I focus on allocations that maximize welfare.

Assumption 1. If Definition 1 admits multiple quantities for a given reputation, the equilibrium allocation is the one that maximizes the total utility , $\sum_{i\in\{H,I,M\}} u_i$.

2 Solution

In this section I characterize the equilibrium behavior of agents and asset prices. I look for a stationary Markov equilibrium in which the only state variables is the manager reputation P_t. The solution of the equilibrium in this economy involve two steps. I first take as given the investor's beliefs about how the manager portfolio policy depends on his reputation, and solve for the evolution of the reputation process (Subsection 2.1) and the equilibrium investment by fund investors (Subsection 2.2). Given the investor behavior, I express the manager problem recursively and solve for the optimal portfolio policy as a function of investors' behavior (Subsection 2.3). I then characterize equilibrium prices as a function of reputational incentives (Subsection 2.4), and close this section by characterizing these reputational incentives in equilibrium. Proofs are in the appendix A.

Because investors do not observe the fund portfolio, but can measure the fund instantaneous volatility[9], it is useful to decompose the portfolio strategy in the observable volatility component, $\sigma_R = var_{i,t}(X^M dS_{t-})$, and the hidden component $x^M \equiv \frac{X^M}{\sigma_R}$. Given the observable volatility choice σ_R, the intermediary can choose any x^M that belongs to the unit-variance space (denoted by Ω). Let $\mu_{E,t}^{M,\theta}$ be the vector of risk-adjusted excess returns the manager type θ faces, and $\mu_{R,t}^{M,\theta} = x_t^{M,\theta}\mu_{E,t}^{M,\theta}$

[9]Since fund returns are observed in continous time.

the fund risk-adjusted excess return. This decomposition implies manager θ fund returns follow

$$dR_t = \rho dt + \sigma_{R,t}\left(\mu_{R,t}^{M,\theta} + \eta_{R,t}^{M,\theta}\lambda_Q\right)dt + \sigma_{R,t}dB_t - \sigma_{R,t}\left(\eta_{R,t}^{M,\theta} + \nu y_t\right)dN_t. \tag{6}$$

The fund tail exposure $\eta_{R,t}^{M,\theta} = x_t^{M,\theta}\eta$ increases the fund performance in any period without a crash. So the uncompensated drift $\mu_{R,t}^{M,\theta} + \eta_{R,t}^{M,\theta}\lambda_Q$ measures the fund expected short-term performance, where the tail exposure effect on short-term performance $\lambda_Q = (1+\gamma)\lambda$, has a risk premia component $\lambda\gamma$, and an expected loss component λ. The shock y_t is a standard normal random variable. The fund volatility is invariant with manager type ($\sigma_{R,t}^{\theta} = \sigma_{R,t}$), because any differences across types would immediately reveal the manager type to the investor, as a result the Bad will always mimic the portfolio volatility of the Good. Importantly, portfolio opaqueness implies the Bad is free to choose any portfolio $x^{M,b}\sigma_R$ (as long $x^{M,b} \in \Omega$).

2.1 Reputation

Taking as given the manager strategy $x^{M,\theta}$, and investors' beliefs about these strategies, $x^{I,\theta} \equiv \mathbb{E}_{I,t}\left[x^{M,\theta}|\theta\right]$, I solve for the dynamic evolution of investor posterior about the manager type. Optimal learning in this two type environment consists of comparing the likelihood that a given return realization was generated by the two candidate statistical models, $\theta \in \{g, b\}$. I divide the learning process into normal periods and periods with a tail event. In both situations the statistical learning problem investors face is equivalent to that of an econometrician who tries to differentiate between two normal random variables of identical variance but different means.

The log-likelihood ratio is a convenient way to express the manager reputation. The function $\Psi(\underline{P}, p) = \frac{\underline{P}e^p}{1-\underline{P}+\underline{P}e^p}$ maps p, the log-likelihood distance into P, the perceived probability. I refer to both P and p as reputation, and I will abuse notation and refer to functions that were defined in probability space as they were defined in log-likelihood space (and vice versa).

Proposition 1. *Given $x^{i,\theta}(p)\ \ for\ \ i \in \{M, I\}$, the reputation $p_t = \ln\left(\frac{Prob(\theta=1|dR_s, s\leq t)}{Prob(\theta=0|dR_s, s\leq t)}\frac{1-\underline{P}}{\underline{P}}\right)$ satisfies,*

$$dp_t = \sigma_{p,t}\left(\mu_{p,t}dt + dZ_{t-}\right) - \sigma_{p,t}^N\left(\mu_{p,t}^N - y_t\right)dN_t \tag{7}$$

for $p_t > 0$ and $dp_t = 0$ if exists $s \leq t$ such that $p_s \leq 0$, with coefficients

$$
\begin{aligned}
\sigma_{p,t} &= \mu_{R,t}^{I,\Delta\theta} + \eta_{R,t}^{I,\Delta\theta}\lambda_Q \quad &, \; \sigma_{p,t}^N = -\frac{\eta_{R,t}^{I,\Delta\theta}}{\nu} \\
\mu_{p,t} &= \mu_{R,t}^{M,\theta} + \eta_{R,t}^{M,\theta}\lambda_Q - \left(\overline{\mu_{R,t}^{I,\theta}} + \overline{\eta_{R,t}^{I,\theta}}\lambda_Q\right) &, \; \mu_{p,t}^N = \frac{1}{\nu}\left(\eta_{R,t}^{M,\theta} - \overline{\eta_{R,t}^{I,\theta}}\right)
\end{aligned}
$$

where $Z_{R,t}^{I,\Delta\theta} \equiv Z_{R,t}^{I,g} - Z_{R,t}^{I,b}$ and $\overline{Z_{R,t}^{I,\theta}} \equiv \frac{Z_{R,t}^{I,g} - Z_{R,t}^{I,b}}{2}$. Note that $\mathbb{E}_{I,t}\left[\mu_{p,t}\right] = \mathbb{E}_{I,t}\left[\mu_{p,t}^N\right] = 0$ and the reputation process is a martingale from the investor's vantage point.

As soon as the fund is liquidated, the reputation ceases to evolve and the manager stays liquidated forever. This implies that a sufficient low reputation today reduces the manager earnings for the entire future. This non-linearity at low reputation values makes the Good type risk-averse with respect to reputational shocks.

The filter described in Proposition 1 works as follows: if investors expect the Good type to perform better than the Bad type in normal times, $\mu_{R,t}^{I,\Delta\theta} + \eta_{R,t}^{I,\Delta\theta}\lambda_Q > 0$, then positive return surprises, $dR_t - \left(\overline{\mu_{R,t}^{I,\theta}} + \overline{\eta_{R,t}^{I,\theta}}\lambda_Q\right) > 0$, will signal good news about manager quality, where performance news is measured relative what is expected from a p_t reputation manager. The endogenous informational content of normal times performance is measured by $\sigma_{p,t}$, which consists of the difference in Sharpe ratios across types. During tail events learning is lumpy, but works in a similar fashion. In contrast to the literature on herding behavior (For example, Scharfstein and Stein [1990] and Zwiebel [1995]), the reputation evolution does not depend on the action of other managers. The reputation dynamics depends exclusively on investors' beliefs and the manager's own actions.

A novel feature of this framework is to make this standard filter endogenous to the manager trading behavior. Previous work has mostly abstracted from the interaction between portfolio choice and learning by either simplifying the investment opportunity set [Berk and Green, 2004], or arguing that the investment opportunity set was "sufficient non stationary" that made this type of endogenous response by fund investors infeasible [Shleifer and Vishny, 1997].[10]

[10] For important exceptions please see the discussion in the literature review.

2.2 Investor

The investor's portfolio problem is static and pins down the total normal-times risk of the intermediary portfolio, $A_t = a_t \sigma_{R,t}$, which I refer to as the fund portfolio size. The portfolio size is increasing in the local hedgers risk aversion, intermediary reputation, and Good type excess returns, but decreasing in the investor's risk aversion. Because the Bad type trades at fair prices $(\alpha_{H,b} = 0)$, his portfolio does not impact directly the investor's capital allocation decision.

Proposition 2. *Let $A_t = a_t \sigma_{R,t}$ be the fund portfolio size and $\underline{P}$ the liquidation threshold. Given $x^{I,\theta}(p)$,* investor maximization implies,

$$\underline{P} = \frac{f}{(1-\kappa)\alpha_{H,g} x^{I,g}(0)\Sigma e^{\top}} \tag{8}$$

$$A(p) = \frac{\Psi(p,\underline{P})\alpha_{H,g} x^{I,g}(p)\Sigma e^{\top} - f(1-\kappa)^{-1}}{\alpha_I(1-\kappa) + \alpha_{H,g}\Psi(p,\underline{P})}, \ \forall p \geq 0 \tag{9}$$

The portfolio problem of the local hedger pins down equilibrium prices as a function of the amount of capital allocated to the different assets. Manipulating expression (3) I can write the vector of risk-adjusted excess returns (Equation (10)) as,

$$\mu_E^{i,\theta}(p) = \alpha_{H,\theta}\Sigma\left(e - x^{i,\theta}(p)A(p)\right)^{\top}, \tag{10}$$

where the index i denotes the fact that beliefs about equilibrium excess returns depends on each agent beliefs about the manager portfolio policy.

2.3 Manager

Given investor learning (Equation (7)) and capital allocation decisions (Equations (8) and (9)), the manager valuation in equation (5) must satisfy an integro-differential equation in the manager reputation. Loosely speaking, Equation (11) can be derived by applying Ito's formula to (5).

Proposition 3. *Given $x^{I,\theta}$ and A, the manager lifetime utility (equation (5)) satisfies $V^{\theta}(0) = 0$ and the* *following integro-differential equation,*

$$\rho V^{\theta} = \max_{x\in\Omega} A\left(\kappa x\mu_E^{M,\theta}+f\right)+V_p^{\theta}\mu_p + \frac{1}{2}V_{pp}^{\theta}\sigma_P^2+\lambda\mathbb{E}_{M,t}\left[V^{\theta}\left(p-\sigma_p^N\left(\mu_p^N-y\right)\right)-V^{\theta}|\theta\right]. \tag{11}$$

The first term in the right is the instantaneous compensation flow, which includes both performance and management fees. The following three terms capture the valuation consequences of the reputation dynamics. The first two two capture growth and risk during normal times. The last term captures that after a tail event, the manager reputation jumps to a new (and uncertain) value because of lumpy amount of information revealed during a crash. In the LHS we have discounting.

From the first order condition of the manager problem in Equation (11), we obtain that the optimal portfolio can be represented as the tangent portfolio with an endogenous tail-risk tilt ω, which I define next.

$$x(\mu_E,\omega) = \frac{\Sigma^{-1}\left(\mu_E+\omega\lambda\eta\right)}{\sqrt{\left(\mu_E+\omega\lambda\eta\right)^{\top}\Sigma^{-1}\left(\mu_E+\omega\lambda\eta\right)}}. \tag{12}$$

Where μ_E is the vector of tail-risk-adjusted expected excess returns and the scalar ω is the manager reputational incentive, which summarizes the incentives to over-weight high-tail-risk assets. The reputational incentive implied by Equation (11) is

$$\omega^{\theta} = \frac{(1+\gamma)\sigma_p V_p^{\theta}-\frac{\sigma_p^N}{v}\mathbb{E}_M\left[V_p^{\theta}\left(p-\sigma_p^N\left(\mu_p^N-y\right)\right)|\theta,p\right]}{\kappa A+\sigma_p V_p^{\theta}}. \tag{13}$$

Where A, σ_p and σ_p^N are a function of investor's beliefs $x^{I,\theta}$, and μ_p^N is a function of investor's beliefs and the manager actual portfolio choice (see Propositions 7 and 2). To get intuition on Equation (13), it is useful to have in mind two properties that hold in equilibrium (See Proposition 7). First, more reputation is always good, $V_p \geq 0$, and reputation is always increasing in performance, $(\sigma_p, \sigma_p^N \geq 0)$. Given these properties, short-term reputational incentives, $\sigma_p V_p^{\theta}$, push the manager to over-weight high-tail-risk assets, and long-term reputational incentives, $-\frac{\sigma_p^N}{\nu}\mathbb{E}_M\left[V_p^\theta\left(p-\sigma_p^N\left(\mu_p^N-y\right)\right)|\theta\right]$, pushes in the opposite direction. Performance based compensation reduce the importance of reputational forces as a whole. Importantly, the strength of the two different reputational forces depends critically on investor's beliefs through both the reputation-performance elasticities and portfolio size.

2.4 Incentives and capital allocation

I now show how reputational incentives impact equilibrium capital allocation decisions. I start with the fund manager portfolio policy.

Using Equations (10) and (12) and imposing that the portfolio must belong to the unit-variance space and local markets must clear we obtain in closed form equilibrium allocations as function of manager type and reputational incentive ω^θ.

Proposition 4. *Given θ-type reputational incentive $\omega^\theta=\omega$, equilibrium in local asset markets implies*

$$x^{M,\theta}(\omega) \quad = \quad \frac{\psi^\theta(\omega)}{\sigma_e\alpha_{H,\theta}}\left(\alpha_{H,\theta}e+\omega\lambda\eta^\top\Sigma^{-1}\right) \tag{14}$$

where $\psi^\theta(\omega)=\sqrt{\frac{\alpha_{H,\theta}^2\sigma_e^2}{\alpha_{H,\theta}^2\sigma_e^2+(\omega\lambda\eta_+)^2}}$ and $\sigma_e^2=e\Sigma e^\top$.

I start with the Good type. When there is zero reputational incentive, $\omega^g=0$, the Good-type's portfolio is proportional to the local-market hedging portfolio, $x^{M,g}(0)\propto e$. In general it will be a combination of the maximum tail-risk portfolio $\Sigma^{-1}\eta$ and the hedging portfolio. For a positive incentive ω^g, the Good-type's portfolio exhibits a tail-risk tilt. The function $\psi(\cdot)$ keeps the allocation in the unit-variance space for different values of ω^g and measures the efficiency of how the the risk-bearing capital A is allocated. Capital is allocated most efficiently when there is exactly zero reputational incentive and the manager invests only in the hedging portfolio. For the Bad type, the optimal portfolio is of the Bang-Bang type as $x^{M,b}(\omega^b)=\frac{\omega^b}{|\omega^b|}\frac{\Sigma^{-1}\eta}{\eta_+}$. In the knife-edge case $\omega^B=0$, the portfolio is undetermined as the Bad type is exactly indifferent across all feasible portfolios.

I turn back to the investor problem, where reputational incentives impact investor's decisions through the impact it has on investor's beliefs about the Good manager profitability. Substituting

Equation (14) in Equation (10), I obtain the liquidation threshold and the portfolio size as function of Good-type incentives,

$$\underline{P}(\omega^g) = \frac{f}{(1-\kappa)\alpha_{H,g}\psi^g(\omega^g)\sigma_e} \tag{15}$$

$$A(p,\omega^g) = \frac{\Psi(p,\underline{P})\alpha_{H,g}\psi^g(\omega^g)\sigma_e - f(1-\kappa)^{-1}}{\alpha_I(1-\kappa)+\alpha_{H,g}\Psi(p,\underline{P})}. \tag{16}$$

These expressions show the intensive and extensive margin effects of incentive distortions on the quantity of intermediation services. First, Equation (15) shows that an increase in the incentive distortion ($\omega^2 \uparrow$) leads to earlier termination ($\underline{P}'(\cdot) > 0$). Second, holding the distance to liquidation fixed, Equation (16) shows the overall size of the intermediary portfolio shrinks as capital is less efficiently allocated.

Plugging Equations (14) and (9) into the fund return dynamics (Equation (6)) one can write it as a function of equilibrium incentives and reputation,

$$\mu_R^{i,g}(p,\omega^g) = \alpha_{H,g}\left(\frac{(1-\kappa)\psi^g(\omega^g)\sigma_e\alpha_I + f(1-\kappa)^{-1}}{\alpha_I(1-\kappa)+\alpha_{H,g}\Psi(p,\underline{P})}\right) \tag{17}$$

$$\eta_R^{i,g}(\omega^g) = \frac{\omega^g}{|\omega^g|}\sqrt{1-\psi^g(\omega^g)^2}\eta_+ \tag{18}$$

In equilibrium, the Good-type alpha is maximized when there are no reputational distortions $(\omega^g = 0)$, but short-term performance, $\mu_R^{i,g} + \eta_R^{i,g}\lambda_Q$, is maximized at $\omega^g = 1+\gamma$. This tension is the ultimate source of intermediation inefficiency and amplification. A positive incentive ω^g increases short-term performance, which feeds back into an increase in the reputational incentive (Equation (13)). For the Bad type, the risk-adjusted performance $\mu_R^{i,b}(p,\omega^b)$ is zero and the tail exposure is $\eta_R^{i,b}(\omega^b) = \frac{w^b}{|w^b|}\eta_+$, since $\mu_E^{i,b} = 0$.

2.5 Equilibrium incentives

I now turn to the equilibrium determination of the reputational incentive ω^θ. I start by characterizing the Bad-type equilibrium portfolio as a function of the Good-type reputational incentive

ω^g.

Equation (19) characterizes the spread in tail exposure across types $\delta_\eta = \eta_R^{I,b} - \eta_R^{I,g}$ consistent with indifference by the Bad-type, $\omega^b = 0$. When such portfolio is not feasible, $\delta_\eta > \left(1 - \sqrt{1 - \psi^g(\omega^g)^2}\right)\eta_+$, it follows that the Bad type max out tail-risk taking $\eta_R^{M,b} = \eta_+$ and $\omega^b > 0$. Whenever the Bad-type portfolio policy is not constrained, he responds to decreases in Good-type risk-adjusted performance by reducing the tail-exposure differential $\eta_R^{M,\Delta\theta}$.

Proposition 5. *(Bad type)Let $\delta_\eta(p, \omega^g)$ be a fixed point of*

$$\delta_\eta(p, \omega^g) = \mu_R^{I,g}(p, \omega^g)\left(\frac{1}{(1+\gamma)\nu^2}\frac{\int V_p^b\left(p - \frac{\delta_\eta(p,\omega^g)}{\nu}\left(\frac{\delta_\eta(p,\omega^g)}{2\nu} - y\right)\right)\mathcal{N}(dy)}{V_p^b(p)} + \lambda_Q\right)^{-1}, \tag{19}$$

then the Bad-type portfolio satisfies

$$\eta_R^{M,b}(p, \omega^g) = \min\left\{\eta_+, \delta_\eta(p, \omega^g) + \frac{\omega^g}{|\omega^g|}\sqrt{1 - \psi^g(\omega^g)^2}\eta_+\right\}. \tag{20}$$

Because the Good type must generate positive risk-adjusted performance in any equilibrium with an operational fund, Equation (19) implies the Bad type always chooses a higher tail exposure than the Good type. Using these results I can express the reputation dynamics (Equation (7)) as a function of reputation and incentives.

The information content of short-term (tail-event) performance is increasing (decreasing) in reputational incentives ω^g in the constrained region and in the unconstrained region the opposite holds. This implies that in some states (the constrained region) there is strategic complementarity between portfolio choice and incentives, and in other states (the unconstrained region) there is strategic substitutability.

I substitute everything back into the reputational-incentive equation (Equation (13)) to obtain a fixed point equation that any equilibrium incentive must satisfy,

$$\omega^g = \frac{(1+\gamma)\sigma_p(p,\omega^g)V_p^g - \frac{\sigma_p^N(p,\omega^g)}{\nu}\int V_p^g\left(p - \sigma_p^N(p,\omega^g)\left(\frac{1}{2}\sigma_p^N(p,\omega^g) - y\right)\right)\mathcal{N}(dy)}{\kappa A(p,\omega^g) + \sigma_p(p,\omega^g)V_p^g}. \tag{21}$$

An incentive ω^g is consistent with equilibrium if it is self-generated: It must induce manager's portfolio policies $\{x^{M,\theta}\}$, which in equilibrium must induce investor's capital allocation and learning policies $\left\{\sigma_p, \sigma_p^N, A\right\}$ that generate incentive ω^g. Whenever there are multiple solutions to this fixed point problem, the equilibrium definition selects the one that maximizes welfare.

Proposition 6. *(Good type)Let $\omega_{fp}(p)$ the set of solutions to equation (21). For a reputation p, (ii) if $\omega_{fp}(p)$ is a singleton, than $\omega(p) = \omega_{fp}(p)$ characterizes equilibrium allocations and prices;(ii) if $\omega_{fp}(p)$ has multiple elements, then the reputational incentives that characterize equilibrium price and allocations solves*

$$\omega(p) = \arg \min_{\omega \in \omega_{fp}(p)} \quad \omega^2 \tag{22}$$

3 Analysis

I start with three set of results that describe some general features of the equilibrium and provide an useful benchmark for the model results.

I then solve the model by characterizing the solution $V^\theta(p_t)$ of the integro-differential equation in (11), which I solve numerically using finite-difference methods. Appendix B provides details. Given equilibrium value functions, I characterize equilibrium incentives ω^θ following Proposition (6). Given incentives the model equilibrium is solved in closed form as described in Section 2 . Our baseline parameter values are in Table I. We discuss the parameter choice in the Appendix C.

Table I about here.

3.1 Preliminary results

In this section I characterize some qualitative features of the equilibrium. Proposition 7 shows that as long reputation is valuable, reputation is increasing in fund returns.

Proposition 7. *(Returns surprises are always good news)In any equilibrium where reputation is (weakly) valuable ($V_p \geq 0$), reputation growth is (weakly) increasing in performance ($\sigma_p, \sigma_p^N \geq 0$). Furthermore, if*

$\nu < \infty$, *the inequalities are strict.*

This result is a consequence of the optimal Bad-type behavior. If there was a negative relationship between reputation and performance, the Bad type would improve his reputation growth by taking on less tail-risk. Propositions 7 together with Equations (13) ,(14) , and (10) imply that tail incentives ω^g of the Good type will be different from zero.[11]

I now provide a sharper characterization of the equilibrium in a special case where tail-event performance has no information about the manager type because tail-event returns are assumed to be very volatile. This special case is simpler because it transforms the problem from solving an integro-differential equation into solving a differential equation, consequently simplifying the characterization of equilibrium incentives. It is useful because it allows me to prove some features of the equilibrium that hold more generally as long tail-event volatility ν is not too low.

Proposition 8. *(No learning during tail events) Let* $\nu \to \infty$, $V_p \geq 0$, $V_{pp} \leq 0$, and $\lim_{P\to 1} V_p^g = 0$ *then there is an unique equilibrium and there is a threshold* $\bar{\eta}$ *such that* [12]*: (i) if* $\eta_+ < \bar{\eta}$*, reputation is increasing in short-term performance,* $\omega^g(P)$ *is decreasing, with* $\lim_{P\to 1}\omega^g(P) = 0$*, and the Bad-type tail exposure is* $\eta_R^{M,b} = \eta_+$ *; (ii) otherwise, reputation is constant,* $\eta_R^{M,b} = \bar{\eta}$ *and* $\omega^g = 0$*.*

When η_+ is not too high, the tail exposure that completely counteract the short-term performance advantage of the Good type is not in the opportunity set. As a result, the expected short-term performance of the Bad type is always lower, and a positive return surprise is good news about the manager type. Because reputation is less valuable when it is high, Good-type reputational incentive and tail-risk taking decreases with reputation. Whenever η_+ is high enough, performance becomes uninformative as the Bad type has enough flexibility to reproduce the short-term performance of the Good type.

As a benchmark I characterize allocations in a setting where investors can directly invest in the local market, but where they are faced with the same uncertainty about locals' hedging needs. The "direct-investing" allocation can be implemented with an intermediary if his portfolio is *trans-*

[11]The only exception is if there is a tail-exposure difference across managers, $\eta_{R,t}^{I,G} - \eta_{R,t}^{I,B}$, that induces identical net reputational concerns for both types. This typically does not happen because the Good type values reputation differently from a Bad type.

[12]Where $\bar{\eta} = \frac{1}{\lambda(1+\gamma)}\frac{\alpha_{H,g}\sigma_e + f}{\alpha_I + \alpha_{H,G}(1-\kappa)}$.

parent. In this case, investors can observe the manager portfolio and properly measure its tail-risk, and as a result the Bad type must mimic the Good type portfolio choice to stay in business. As a consequence of these portfolio choices, tail-risk taking does not impact learning and the tail-risk taking incentives disappear ($\omega^g = \omega^b = 0$).

Proposition 9. *(Transparent intermediaries) If the portfolio is transparent, both Good and Bad managers invest in the hedging portfolio $x^{M,\theta} = \sigma_e^{-1} e$ and investors learn at rate $\sigma_p^2 + \lambda \left(\sigma_p^N\right)^2 = \alpha_{H,g}^2 \left(\sigma_e - A\right)^2$, with $\sigma_p^N = 0$.*

Transparent intermediation implies that allocations are efficient and the scarcity of capital in the local market fully determines the speed of learning and capital flows.

3.2 Intermediary incentives, portfolio choice and performance

Figure 1(a) shows the managers' valuations V^θ. The Good type has a higher valuation as a result of higher performance based pay and higher reputational growth. The higher valuation also makes the Good type more risk-averse (value function more concave) as liquidation is more costly in terms of future foregone earnings. The increasing slope of the value function as reputation approaches the liquidation threshold on the left, reflects the high (and volatile) reputation growth at low reputations. While the Good type expects to drift up increasingly faster as his reputation shrinks, the Bad type expects to drift down, making his valuation convex and risk-loving. The absence of allocative distortions in the transparent portfolio case leads to a lower liquidation risk, what increases the valuation of both Good and Bad types as they approach liquidation. Far from the liquidation threshold, portfolio transparency hurts the Bad type and helps the Good type. The next section shows this result is driven by the faster rate of learning in the transparent case.

In Figure 1(b) we see the equilibrium incentive $\omega^\theta(p)$ for each manager type. For the baseline case, the increasing slope of the Good-type valuation strengthens the manager reputational incentives. As the manager approaches liquidation, the short-term reputational incentive dominates and the reputational incentive approaches the short-term performance advantage of a high tail-risk asset, $\omega^g(\underline{P}) \to 1 + \gamma$. In the opposite extreme, as the reputation grows, the reputational incentive approaches zero, $\omega^g(1) \to 0$. A similar pattern emerges for the Bad-type incentives, but the Bad type incentives decay at a lower rate and remain positive through out.

Rearranging Equation (14), I can express the portfolio weight on the tail-risk portfolio as a function of equilibrium incentives ω^θ

$$x_{tail}^{\theta} = \frac{\omega^\theta \lambda \eta_+}{\omega^\theta \lambda \eta_+ + \alpha_{H,g}\sigma_e}.$$

Figure 1(c) shows the portfolio composition of both the hedging and the tail-risk portfolio. while the local hedging needs have equal weights on all five assets, the tail-risk portfolio is a long-short portfolio that buys high-tail-risk assets and sells low tail-risk assets. These two portfolios are an exogenous feature of the environment. The hedging portfolio shaped by the supply of local undiversifiable risks present in the local market, the second shaped by financial innovation η_+.

Figure 1(d) shows how the weight on the tail-risk portfolio fades only slowly as the Good manager builds his reputation. This contrasts with the transparent portfolio case, where the Good type always holds the hedging portfolio. The positive incentives together with the Bang-Bang property of the Bad manager policy implies he will invest everything in the tail-risk portfolio as long his reputational incentives are positive. Once his reputational incentive hits zero his choice is no longer constrained by η_+, and he invests just enough in the tail-risk portfolio to balance out short-and long-term reputational incentives. This situation arises in Section 4.1 where I consider higher values of η_+.

The Good-type portfolio tail exposure decreases with reputation (Figure 1(e)), reaching zero once his reputation is very high. In the transparent case, investors can measure the portfolio tail-risk and there is no tail-risk taking in equilibrium.

Figure 1 about here.

Because covariance-risk is not priced in the "Bad" market, the Bad manager short-term performance is exclusively a function of the tail-risk in his portfolio, $\mu_R^{M,b} + \lambda_Q \eta_R^{M,b} = \lambda_Q \eta_R^{M,b}$. When the portfolio is transparent both short-term and risk-adjusted performance coincide at 0 because $\eta_R^{M,b} = 0$.

Figure 1(f) depicts the Good manager performance profile. The Good-type performance is decreasing in reputation, as a respected fund manager attracts more capital, it reduces the risk the hedgers must bear, pushing downward risk premia. For the transparent portfolio case, both short-term and risk-adjusted performance coincide since portfolios have no tail risk. When the portfolio is opaque, short-term and risk-adjusted performance diverge sharply as the manager rumps up the tail risk of his portfolio. Portfolio opaqueness induces the Good manager to increase his reputation growth in the short-term at the expense of higher risk-adjusted returns.

The dynamics of risk-adjusted performance, which goes down because the manager is concerned about the risk of liquidation has similarities with the mechanism in Shleifer and Vishny [1997], where investor behavior induces the manager to choose low expected return strategies. In their setting, more volatile trading strategies lead to higher liquidation risk because investors cannot adjust their learning appropriately. Here, it is about the timing of different strategies. A manager tilts his portfolio towards strategies that are are likely to pay well in the short term even if they are very risky. The inability to adjust learning with respect to (hidden) changes in tail-risk taking has similarities to Shleifer and Vishny [1997] assumption that investors do not adjust their learning for variation in fund volatility. The formal distinction between short-term observable volatility (the Brownian risk in my framework) and long-term risk (the tail events) enables me to analyze how persistent these portfolio distortions are.

I show next, that endogenous adjustment in investor's learning behavior is an important source of amplification.

3.3 Investor behavior

Investors decide how much to invest in the fund for a given manager reputation, and how much to learn from fund returns. Figures 2(a) and (b) illustrate how variation in reputational incentives impacts investor behavior. As incentives deviate from zero, the fund portfolio shrinks and the fund is liquidated at a higher reputation threshold. The reduction in the fund portfolio weakens performance incentives, and the earlier liquidation threshold strengthens short-term incentives. Both forces are a response to the lower profitability resulting from the Good manager increasing tilt towards the tail-risk portfolio.

Learning behavior is also sensitive to the manager incentives. Increases in reputational incentives induce investors to learn less from tail-event performance and more from short-term performance, as they shift their expectation of performance differences across manager types.

These endogenous responses introduce strategic complementarity between investor's beliefs and the manager portfolio allocation. For example, a reduction in the distance to liquidation increases the marginal value of reputation $V_p^g \uparrow$ and incentives ω^g (see (13)). Understanding the change in incentives, investors adjust their beliefs about the informativeness of short-term performance upwards, $\sigma_p(p) \uparrow$, and long-term performance downwards, $\sigma_p^N(p) \downarrow$, causing investors to reduce their investment in the fund, $A(p) \downarrow$, and to liquidate the fund earlier $\underline{P} \uparrow$. All four responses further increase the reputational incentive ω^g, which feeds back in more tail-risk taking.

Figure 2 about here.

Figures 2(c) and (d) show equilibrium policies as function of reputation. Changes in reputation drive fund assets through changes in manager's incentives and changes in investors' beliefs. The belief channel operates both in the transparent and the opaque cases: as reputation goes up, investor's perceived fund profitability increases since the Good type invests in assets with higher expected returns. The incentive channel is exclusive to the opaque case: as reputation goes up, reputational incentive ω^g weakens. The incentive channel also induces earlier liquidation, which is reflected in the shorter line in Figure 2(c).

The learning channel is represented by the reputation-performance sensitivities σ_p and σ_p^N (Figure 2(d)). Figure 1(c) shows that the short-term performance of the Good type increases as his reputation goes down, while the short-term performance of the Bad type stays flat throughout.[13] As a result of the large spread across types, short-term performance becomes more informative as reputation deteriorates.

Note that the negative slope of σ_p in Figure 2(b) is a feature of both transparent and opaque cases, reproducing the behavior of Good-type short-term performance seen in Figure 1(b), which is a result of the reduction in asset's expected returns as the amount of risk-bearing capital increases.

[13] In general, the the Bad type short-term performance might decrease once his reputation is high enough as he chooses to reduce the tail-risk of his portfolio. This is the case when tail-risk taking opportunities are very rich (η_+ high). I analyze this case in section 4.1.

When the portfolio is opaque, the reduction in the Good-type portfolio tail-risk compounds this efficient reduction in expected returns, leading to a steeper reduction in short-term performance informativeness.

The transparent portfolio case always features more learning. When the portfolio is opaque, the Bad type loads on tail risk more aggressively, and short-term performance understate the true performance differences across types. While allocation distortions increase as reputation shrinks (higher tail-risk taking), learning distortions go down. Portfolio opaqueness introduces a trade off between the static and inter-temporal efficiency in the allocation of capital.

In Figure 2(d), we see that that a decrease in short-term learning (as reputation goes up) is compensated by an increase in learning during tail events. As the Good-type loads less on tail-risk, his performance during a tail event becomes easier to distinguish from the Bad type, what increases the informativeness of tail-event performance as reflected in the positive slope of σ_p^N in Figure 2(b). In the transparent there is no information in tail-event performance since investors know the fund tail risk.

The model provides a different interpretation of Kacperczyk and Schnabl [2012] analysis of money market funds during the 2007-2009 financial crisis. They document an expansion of tail-risk taking opportunities for money market funds, and show how investors' flows during this period were extremely sensitive to the fund yield, and that high-flow-sensitivity funds allocated a larger fraction of capital to assets that turn out to be highly exposed to the financial crisis.

The academic literature and policy makers have interpreted the evidence in Kacperczyk and Schnabl [2012] as a result of lack of market discipline. Driven either by investors neglect of the link between higher fund yields and tail risk, or because investors had confidence on a government bailout of these funds. In my model market discipline breaks down because investors are not able to direct measure the tail exposure of their fund investments, but their full understanding of fund manager incentives amplifies the reach-for-yield behavior of investors and managers.

3.4 Asset pricing

The model asset pricing implications work trough the amount of risk local hedgers have to bear in equilibrium, which is determined by the amount of capital allocated by outside investors, and how this capital is invested across assets. By construction the framework has only something to say about pricing in the "Good" market with real hedging needs. Expected returns are constant and equal to $\mathbb{E}_{M,t}\left[dS_t - \rho|\theta = b\right] = \lambda\eta\gamma$ in the "Bad" market, since local hedgers have unlimited risk-bearing capacity of variance risk, $\alpha_{H,b} = 0$. So in what follows I focus in the interesting $\theta = g$ case.

The Good manager tilt towards the tail-risk portfolio implies local hedgers' portfolios have too little of high-tail-risk assets and too much of low-tail-risk assets relative to their hedging needs. As a result, the equilibrium compensation to hold high-tail-risk assets goes down (and goes up for low-tail-risk assets). This can be seen in Figure 3(a), which plots assets' expected returns as a function of their tail exposure for different levels of reputation. In the transparent case, the expected return line shifts down in parallel as invested capital grows with the increase in reputation. An increase in capital reduces the total risk in the local-hedgers' portfolios, but it does not change it's composition, reducing all their asset holdings equally. With opaque intermediaries this parallel shift is combined with a rotation of the expected return line, which is the result of the changes in the composition of the local-hedgers' portfolio as the new capital that flows in is allocated with a tilt towards high-tail-risk assets.

In fact it is easy to show that the cross-sectional of expected returns satisfies a two-factor structure, where normal-times betas and tail betas are determined by the cash-flow process and risk-premia are a function of reputational incentives ω^θ and the amount of capital allocated by investors.

Proposition 10. *Given the Good-type reputational incentive $\omega^g = \omega$, equilibrium in local asset markets implies*

$$\mathbb{E}_{M,t}\left[dS_t - \rho|\theta = g, \omega\right] = \Sigma e \Pi^{dB,g}(p,\omega) + \lambda\eta\Pi^{dN,g}(p,\omega) \tag{23}$$

where covariance-risk premium is $\Pi^{dB,g}(p,\omega) = \alpha_{H,g}\left(1-\psi^g(\omega)\frac{A(p,\omega)}{\sigma_e}\right)$ *and the tail-risk premium* $\Pi^{dN,g}(p,\omega) = \gamma - \psi^g(\omega)\frac{A(p,\omega)}{\sigma_e}\omega$.

The variation in the level of expected returns in Figure 3(a) measures variation the covariance-risk premia $\Pi^{dB,g}$, and the slope of the expected return line measures variation in the tail-risk premia $\Pi^{dN,g}$.

Figure 3(b) shows how the covariance-risk premia decreases with reputation. While in the transparent portfolio case the reduction is the result of the increase in total quantity of outside capital in the local market, in the opaque case the reduction in covariance-risk premia is further amplified by an improvement on how the capital is allocated across assets,

$$\Pi_p^{dB,g} = -\frac{\alpha_{H,g}}{\sigma_e}\left(\psi^g \underbrace{\left(A_p + A_\omega \omega_p^g\right)}_{\text{Quantity}(+)} + A \underbrace{\psi_\omega^g \omega_p^g}_{\text{Allocation}(+)}\right), \tag{24}$$

which impacts risk-premia through the endogenous investors' response A_ω and the direct improvement in the allocation of capital.

Figure 3 about here.

By design all agents (including the fund manager) have identical distaste γ for suffering losses during a tail event. As a result, when there are no incentive distortions, as in the transparent case, tail-risk premia reflects agents risk-preferences, $\Pi^{dN,g}(p,\omega) = \gamma$. In the opaque case, Figure 3(c) shows the tail-risk premia is state-dependent and U-shaped in the manager reputation. Now changes in quantity and allocation counteract each other,

$$\Pi_p^{dN,g} = -\sigma_e^{-1}\left(\psi^g \omega^g \underbrace{\left(A_p + A_\omega \omega_p^g\right)}_{\text{Quantity}(+)} + A \underbrace{\left(\psi^g + \psi_\omega^g \omega^g\right) \omega_p^g}_{\text{Allocation (+/-)}}\right). \tag{25}$$

When reputation is very low, reputational incentives are high and the total amount of capital is low. As result, the expansion in capital initially dominates , new capital is deployed with a severe tail-risk tilt, pushing tail-risk premia downwards. Simultaneously, as intermediary assets expand, reputational incentives fade, improving how new and old capital are allocated. Tail-risk premia is determined by a race between asset growth and endogenous improvement in investment incentives. Tail-risk premia is particularly low when outside capital is very responsive to small changes in the perceived fundamentals. For example, when the local market hedging needs are small relative to the aggregate risk-bearing capacity. Figures 3(a) and 3(c) show how time-variation in reputation concerns has different asset pricing implications from time-varying risk aversion. The slope of the expected returns line not only gets flatter, but can flip. Tail-risk premia can be negative .

Overall these results suggest a novel explanation of why financial markets provide few warning signs about financial crises[14]. The model actually produces a *negative* correlation between the expected size of aggregate losses in the intermediary sector and tail-risk premia. Figure 3(d) shows the negative relationship between intermediary losses during a tail event $A \times \eta_R^{M,g}$ and the equilibrium tail-risk premia $\Pi^{dN,g}$. In the model tail-risk premia is low exactly because intermediaries exposure to tail risk is high.

3.5 Slow moving capital

The model links the speed of capital flows with the evolution of investor's beliefs about the profitability of investment in the local market and the incentives the manager faces when allocating investor's capital. The speed at which capital flows into each of the local market assets can be found by totally differentiating the aggregate capital allocation,

$$\mathbb{E}\left[\frac{dX^{M,g}}{dt}\right] = \left(Ax_\omega^{M,g}\omega_p^g + \left(A_p + A_\omega\omega_p^g\right)x^{M,g}\right)\left(\sigma_p^2 + \lambda\left(\sigma_p^N\right)^2\right). \tag{26}$$

Note how these forces feedback into each other: Change in incentives impact reputation growth and outside capital thorough its effect on allocations, reputation growth and assets shape investment incentives as they shift the relative importance of performance based pay relative to reputation considerations. All these feedback effects are reflected in the equilibrium evolution of ω^g.

Figure 4(a) depicts the speed of capital flows as a function of the amount of capital allocated to

[14]See for example, Manela and Moreira [2013] and Baron and Xiong [2014].

the different assets. To evaluate whether capital flows are "too" slow due to intermediation I again use the transparent portfolio case as benchmark. In the transparent case, capital flows into different assets proportionally to the local hedging needs, and at the speed determined by how scarce is capital as measured by the local-market Sharpe ratio, $\mathbb{E}\left[\frac{dX^{M,g}}{dt}\right] = \sigma_e^{-1} e A_p \left(\alpha_{H,g}\left(\sigma_e - A\right)\right)^2$.

In Figure 4(b) I show the aggregate average of these cross-sectional quantities. Aggregate capital flows are about 50% slower when the portfolio is opaque. The speed-scarcity curve has a slope that implies new capital flows are expected to reduce 60% of the gap per year in the transparent case, while only 33% in the opaque case– a very large slowdown. This slowdown in capital flows is driven by η_+, the degree of financial innovation in the local market. The higher η_+ is, the higher the flexibility a Bad type has in "manufacturing" a desirable performance profile to preserve his reputation.

Figure 4 about here.

While the the Bad-type tail-risk taking distorts the allocation of capital inter-temporally, as it slows down learning by investors and the flow of capital into (out of) the local market, the Good-type portfolio distorts the allocation of capital across assets. Figure 4(a) shows how the slowdown in capital flows is substantially more pronounced for assets that are good hedges for tail risk. Initially, when reputation is low and capital scarcity is high, capital flows in the wrong direction. Capital becomes even more scarce and expected returns go up as the manager attracts more capital. Eventually capital starts flowing into these assets, but at very low speeds. This contrast with the high-tail-risk assets, which converge much faster. These cross-sectional differences are driven by the strong tilt in the manager portfolio when his reputation is low, which causes new capital to be deployed with a tilt towards high tail-risk assets , but recedes as the intermediary builds reputation.

The sluggishness of capital flows have implications for the dynamics of risk premia. Figures 4(c) and (d) show convergence rates for the both the tail- and the covariance-risk premia. The slow flow of capital implies covariance-risk premia persists too high for a longer period due to intermediation. Not surprisingly, the effect is qualitatively and quantitatively very similar to the behavior of capital flows.

The model generates large heterogeneity in the speed of capital flows across assets traded by the same agents (Figure 4(a)). This heterogeneity implies the rich dynamics for tail-risk premia we see in Figure 4(d). Initially when reputation is low and capital is low, tail-risk mispricing increases at a very high speed as the amount of outside capital in the local market increases. Convergence back to fundamentals is extremely slow as it takes place only as the manager incentives start improving faster than new capital comes in (See Equation (25)). Figure 4(d) suggests that tail-risk can stay under-priced for quite a long period.

4 Applications

4.1 Financial innovation

In this section, I show how the degree of financial innovation η_+ impacts portfolios and asset pricing. Figure 5(a) shows equilibrium tail exposure $\eta_R^{M,\theta}$ as a function of the maximum tail exposure η_+. Both the Good- and the Bad-type initially increase their tail-risk taking as the opportunity set expands. However, the Bad type respond more aggressively. As a result, short-term performance informativeness decreases as η_+ increases, making tail-risk taking progressively less valuable as a reputation building tool. This results in a downward slopping demand for financial innovation and implies equilibrium tail-risk premia increases in tail-risk taking opportunities (Figure 5(b)).

Figure 5 about here.

Once tail-risk taking opportunities are rich enough, the Bad-type can respond one-for-one to increases in the informativeness of short-term performance (see Proposition 5), breaking the strategic complementarity between Good-type tail-risk taking and investor's learning behavior. This flexibility dampens amplification and leads to the hump-shaped pattern we see in Figure 5(a). The Bad-type tail exposure eventually flattens out, and the Good-type tail exposure goes down back to zero.

Covariance-risk premia follows the hump-shaped pattern of the Good-type tail exposure (Figure 5(c)). It initially increases as the Good manager allocates capital less efficiently, but goes down once short-term performance informativeness drops. Dynamically, financial innovation slows down the flow of capital (Figure 5(d)). Covariance-risk premia goes down at a much slower rate due to the higher flexibility the Bad type has to boost his short-term performance. Financial innovation leads to capital immobility.

The model suggests delegation induced incentive distortions can be an important driver of financial innovation. The very high value managers attach to assets with high tail-risk exposures (low tail-risk premia) when tail-risk taking opportunities are very poor implies a financial innovator can profit by tranching assets and enabling intermediaries to increase tail-risk taking. Figure 5(b) implies the financial innovator would be able to charge a positive (but decreasing) fee for enabling a clean (with less Brownian risk) trade of the tail-risk factor.

4.2 Rationality and amplification

I formally study the implication of investors' sophistication by analyzing the equilibrium consequences of investors having naive beliefs. Under the interpretation of Definition 2, a naive investor is an investor that does not adjust his investment and learning behavior to the changing incentives the manager faces. Comparing quantities across the "sophisticated" and this "naive" equilibrium illustrates the role endogenous investor behavior in amplifying delegation distortions.

Definition 2. *(Naive investor) Let investors beliefs be given by $\omega_\phi(\omega) = \phi\omega$, where ϕ measures the degree of investor sophistication.*

Proposition 11 in the appendix A.6 incorporates Definition 2 into the fixed point equation (Equation (21)) that defines the equilibrium incentive. Figure 6 compares equilibrium objects across fully sophisticated ($\phi = 1$) and fully naive ($\phi = 0$). Investor's naivety impacts equilibrium quantities directly by changing how the investor allocates capital, and indirectly by changing the equilibrium reputational incentives the manager faces.

The investor's capital allocation decision is less responsive to changes in reputation as the naive investor does not take into account how reputation impacts the portfolio decisions of the Good manager. As a result the fund is larger and survives for longer than in the baseline case. [15]

Figure 6 about here.

[15] These results follow immediately from evaluating Equations (15) and (16) at $\omega_\phi = 0$.

Reputational incentives are weaker when investors are naive as naive beliefs breaks the positive reinforcement between investor's and manager's actions, ultimately resulting on a reduction in the portfolio tilt towards the tail-risk portfolio.

The size and the incentive effects push tail-risk premia in opposite directions. While the size effect increases the impact of delegation on tail-risk premia, the incentive effect reduces it. Figure 6(b) shows that the incentive effect dominates for most reputations, with the size effect dominating only for very low levels of reputation.

Typically one thinks that investor sophistication acts as a disciplining force. For example, Gennaioli et al. [2013] shows how investors' neglect for tail risks can induce a financial sector boom and induce welfare decreasing financial innovation. Contrary to standard intuition, in our setting investor's sophisticated understanding of incentives and the investment opportunity set amplify the distortions introduced by delegation.

It is worthwhile to contrast the present mechanism with other theories of amplification in delegated asset management. This paper mechanism is unique as amplification arises from the strategic interaction between the fund manager and the fund investors. In Chen et al. [2010] amplification arises from complementarity between investors' behavior, due to the illiquidity of the fund assets. In Dasgupta et al. [2011] amplification arises from complementarity across different managers, due to reputational concerns.

4.3 Intermediation booms

The model tightly links asset growth with the growth in reputation. In this section I break this link and exploit the effect of demand driven intermediation booms. Recent episodes include the growth in private equity investing by endowments during the 2000's, hedge fund investing by pension funds during the late 90's, and the globalization of the market for U.S. mortgages during the early 2000's.

During these episodes demand drives an expansion of intermediaries balance-sheets, without the the reduction in reputational incentives that takes place during a reputation-driven boom. As result, incentive distortions have a larger impact on asset prices.

Consider the size of the investor base as a source of exogenous variation in the size of the intermediary sector. Let m be the mass of investors, then the size of the portfolio is

$$A(p|m) = \frac{\Psi(p, \underline{P})\alpha_{H,g}\sqrt{\psi^{g}(\omega^{g}(p|m))}\sigma_e - f(1-\kappa)^{-1}}{\frac{\alpha_I}{m} + \alpha_{H,g}(1-\kappa)\Psi(p, \underline{P})}. \; \forall p \geq 0$$

Figure 7(a) shows that increases in m lead to a faster convergence of the covariance-risk premia. Capital moves faster into/out of the fund as investors' capital supply is more elastic due to the higher risk-tolerance of outside investors . Figure 7(b) shows that an expansion in the investor base has the opposite effect in the cross-sectional of expected returns. Tail-risk premia becomes more under priced as a consequence of the faster asset growth.

Figure 7 about here.

This tension between an improvement in absolute valuation, and a deterioration in relative valuation, helps to reconcile conflicting perspectives on the role intermediaries play when they enter a new market. It shows that the optimistic and pessimistic view of their role can both be right at the same time. Intermediaries do increase welfare by reducing expected returns in local markets and providing access to good investment opportunities to outside investors, but their activity has the side effect of making relative prices less reflective of fundamentals. The negative consequences of intermediation are more pronounced the flatter the investor's capital supply curve; for example, when local markets are relatively small relative to the potential capital pool.

4.4 Monetary policy

The very low interest rates in the years following the 2002 recession and more recently following the 2007-2008 financial crises has sparked debate among policy makers whether accommodative monetary policy can induce reach-for-yield behavior by financial intermediaries. Existing theories for the connection between risk-taking and interest rates rely on intermediaries having a fixed nominal liability. The reduction in interest rate increases the risk that intermediaries cannot pay this liability inducing risk-shifting incentives. Typical examples of intermediaries cited by policy makers include pension funds, when they must pay a defined benefit, and money market funds, which need to pay for it's operational expenses.

Figure 8 about here.

In the model, a low interest rate also induces reach-for-yield behavior (Figure 8), but through an entirely novel mechanism. A low interest rate pushes fund managers to place more weight on their future compensation relative to their current compensation, increasing their reputational incentives and, consequently, their loading on tail risk. As a result, a persistent reduction in interest rate reduces tail-risk premia and increases reach-for-yield behavior by investors. In contrast to existing theories and conventional wisdom, in the model the fund manager invests excessively on high-tail-risk assets exactly because he cares about the future. A low interest rate makes performance incentives less powerful.

4.5 Investment mandates

The results on financial innovation suggest why investors (and Good managers) might benefit from restrictive investment mandates as observed in practice. The model leads to a trade-off between the increase in profitability and the increase in static and dynamic distortions associated with a richer investment opportunity set.

Consider the following extension of the model where both the amount of risk in the local market σ_e and the local tail-risk opportunities η_+ are increasing in the flexibility of the investment mandate χ. Let's first abstract from the dynamic implications, and focus on the investment mandate that maximizes the Good manager static compensation flow $\mathbb{E}[\Gamma(dR_t)]$, which is an increasing function of the local hedging needs (σ_e) and how well the manager invests (ω^g),

$$\sqrt{\psi^g(\omega^g)}\sigma_e = \sqrt{\frac{\alpha_{H,\theta}^2\sigma_e^2(\chi)}{\alpha_{H,\theta}^2\sigma_e^2(\chi) + (\omega^g(\chi)\lambda\eta_+(\chi))^2}}\sigma_e(\chi).$$

The investment mandate impacts the manager payoff through three different channels. The profitability channel $\frac{\partial\Gamma}{\partial\sigma_e}\frac{\partial\sigma_e}{\partial\chi} > 0$, the tail-risk taking channel $\frac{\partial\Gamma}{\partial\eta_+}\frac{\partial\eta_+}{\partial\chi} < 0$, and the incentive channel $\frac{\partial\Gamma}{\partial\omega^g}\frac{\partial\omega^g}{\partial\chi} \lesseqqgtr 0$. The analysis in Section 4.1 implies the incentive channel is likely to have a negative impact on compensation when the mandate set is very limited to begin with (very low η_+), but to have a positive effect when the mandate is very broad. This u-shaped effect will push optimal mandates towards the extremes. The initial reputation also plays an important role, as respected managers have very low reputational incentives, the increase in η_+ has little impact on their portfolios. This suggests that more respected managers will have broader investment mandates, and these mandates will change very sharply through the life-cycle of a manager, going from very tight to very loose.

The Good manager cares not only about how to maximize his current compensation, but also about how quickly his reputation and compensation will grow under different mandates. The Bad type on the other hand benefits from a slower rate of learning. Section 4.1 results predict increases in η_+ lead to a slower rate of learning. A higher profitability ($\sigma_e \uparrow$) on the other hand increase the rate of learning. Incentives can again go both ways depending on the manager reputation and how rich the opportunity set is to begin with. Note again the tension between static and dynamics. Whenever the change in incentives push the Good manager to take on less tail-risk, this incentive change will reduce the speed of learning. However, signaling considerations suggests that the equilibrium mandate will have a tilt towards mandates that prioritize learning speeds at the expense of profitability. For example, if a broad investment mandate reduces the speed of learning, one expect managers to offer mandates that are "too restrictive" as a signaling tool, leading to more capital immobility across assets.

I have provided a preliminary analysis of how endogenous reputational incentives will shape the design of an optimal investment mandate. This analysis shows how an optimal investment mandate should trade-off static and dynamic consequences of changes in the investment opportunity set. I left for future work a full analysis of investment mandates.

4.6 Conclusion

I have developed a fully consistent narrative of how intermediaries and investors interact. I have shown how this interaction leads to time-variation in tail-risk taking and has repercussions for the pricing of risk and the speed at which capital flows. Tail-risk premia is low and tail risk builds up in periods of rapid expansion of the intermediary sector. It implies capital flows only slowly to profitable opportunities and that the reduction in flows is particularly severe for strategies that are good hedges against tail risks. The endogenous learning dynamics makes flow volatility counter-cyclical producing a feedback look between liquidation risk and excessive tail-risk taking. In contrast with previous literature, the model results are driven by investors sophisticated understanding of the environment. Information and capital flows shape and are shaped by the incentives intermediaries face.

The importance of tail-risk opportunities in slowing the flow of capital is the fundamental new insight of my framework. It complements the slow-moving-capital literature by developing a mechanism that can account for substantially more persistent asset pricing distortions.

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Table I: Parameter Values

This table contains the values for the model parameters used for producing the baseline results to the paper.

Variable	Symbol	Parameter value
Risk aversion	$\{\alpha_{H,g}, \alpha_{H,b}, \alpha_I, \alpha_M\}$	{1,0,1,0}
Maximum risk-adjusted Sharpe ratio	$\sigma_e \alpha_{H,g}$	1
Maximum tail-risk	η_+	1.5
Ratio of tail-event to normal-times volatility	ν	2.5
Tail-event frequency	λ	0.25
Risk-free rate	ρ	0.05
Tail-event risk-aversion	γ	0.1
Performance fee	κ	0.2
Management fee	f	0.4
Number of assets	n	5
Covariance matrix	$\Sigma = \sigma_f^2 \beta' \beta + I_n \sigma_i^2$	
	$\{\sigma_f, \sigma_i, \beta\}$	{0.15,0.2,1_n}

Figure 1: Incentives, portfolio allocation and performance

Equilibrium value functions (a), incentives(b), Sharpe-ratios(c), and tail-risk exposure (d) for managers with opaque and transparent portfolios. Panel (e) shows asset weights of the strategy that is long the hedging portfolio and for the strategy that maximizes portfolio tail-risk. Panel (f) shows the manager weight on the tail-risk portfolio.

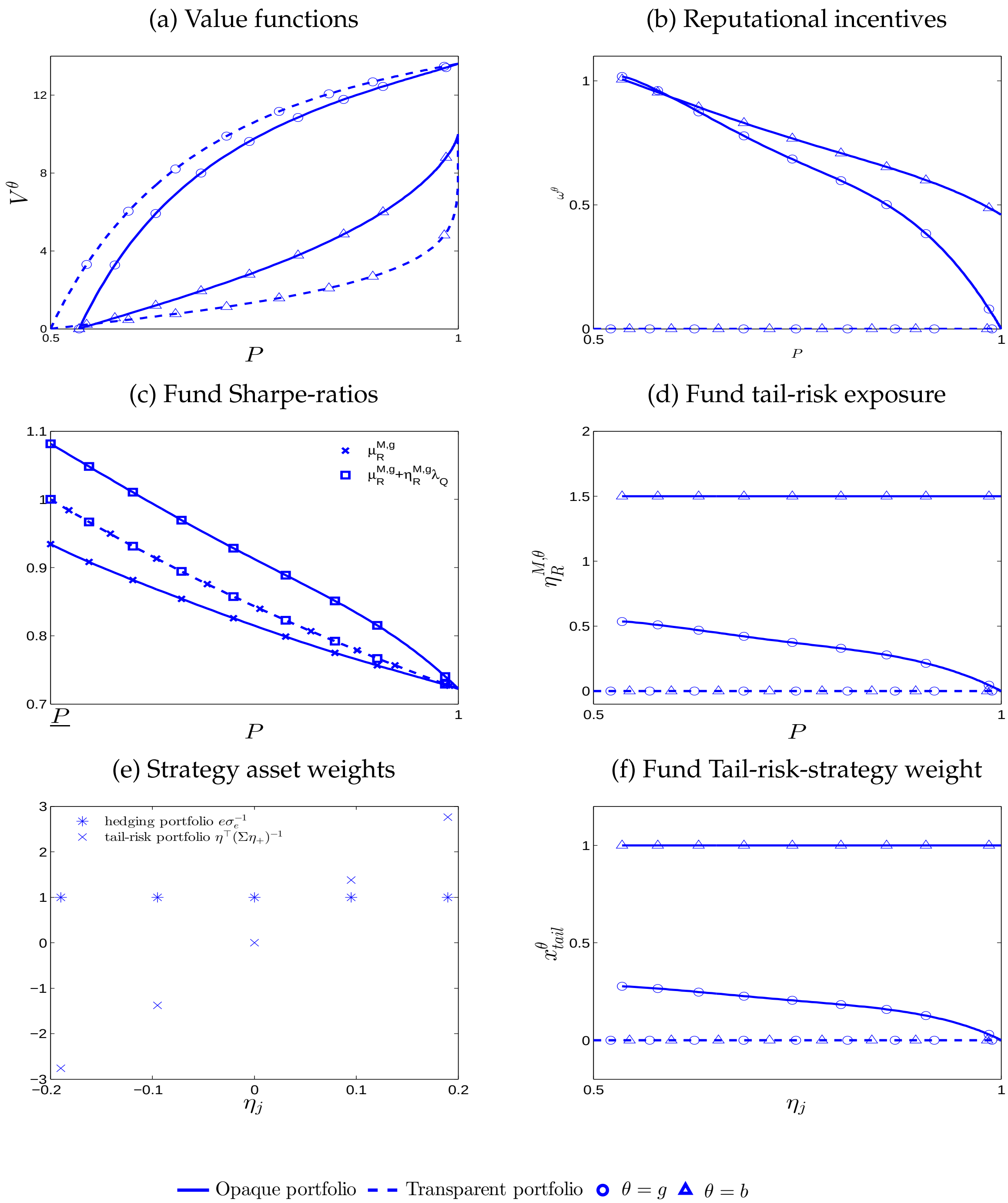


Figure 2: Investor behavior

Panels (a) and (b) show investor's capital allocation and learning policies as a function of the Good manager reputational incentive. Values reported for reputation P=0.7. Panels (c) and (d) show theses policies in equilibrium as a function of the manager reputation.

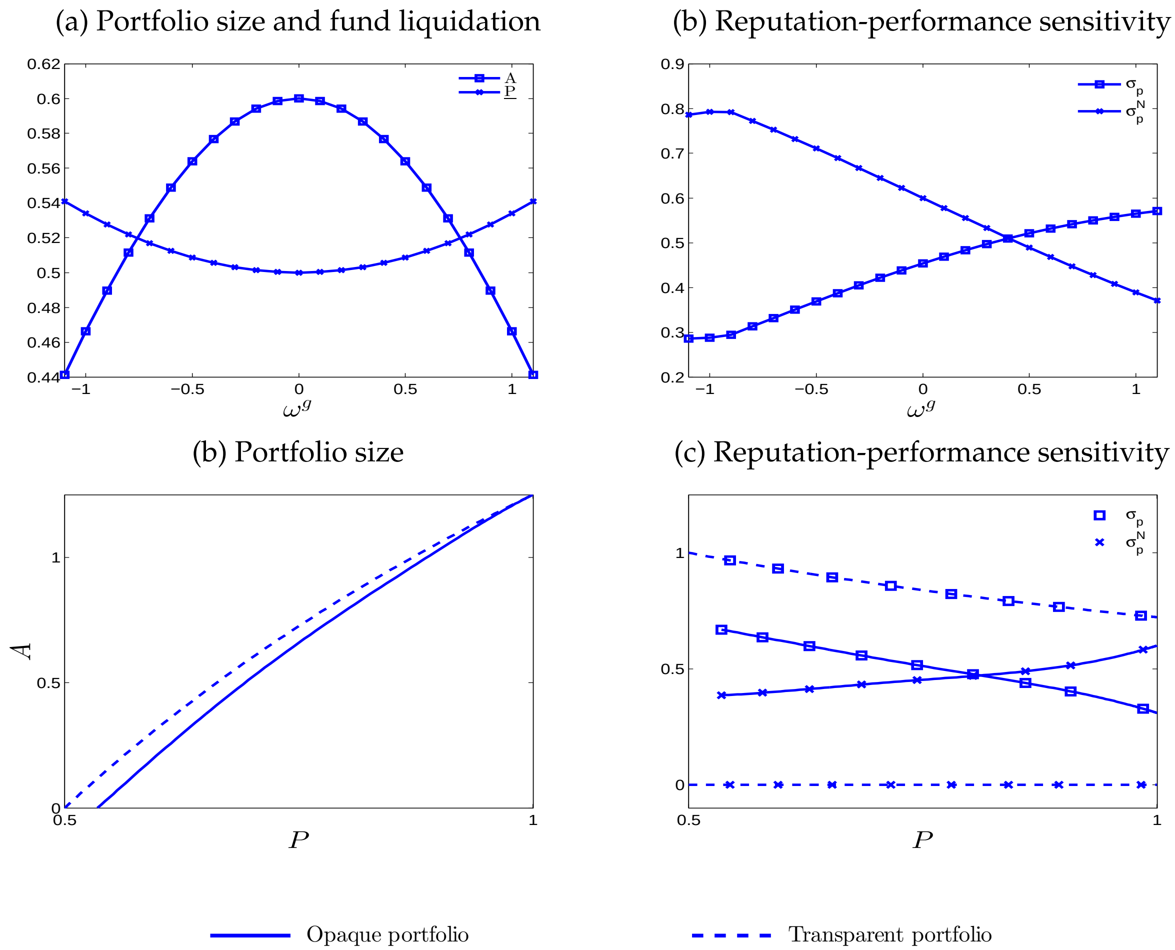


Figure 3: Asset pricing

Expected returns, covariance-risk premia, tail-risk premia, and aggregate intermediary tail-risk exposure as a function of reputation.

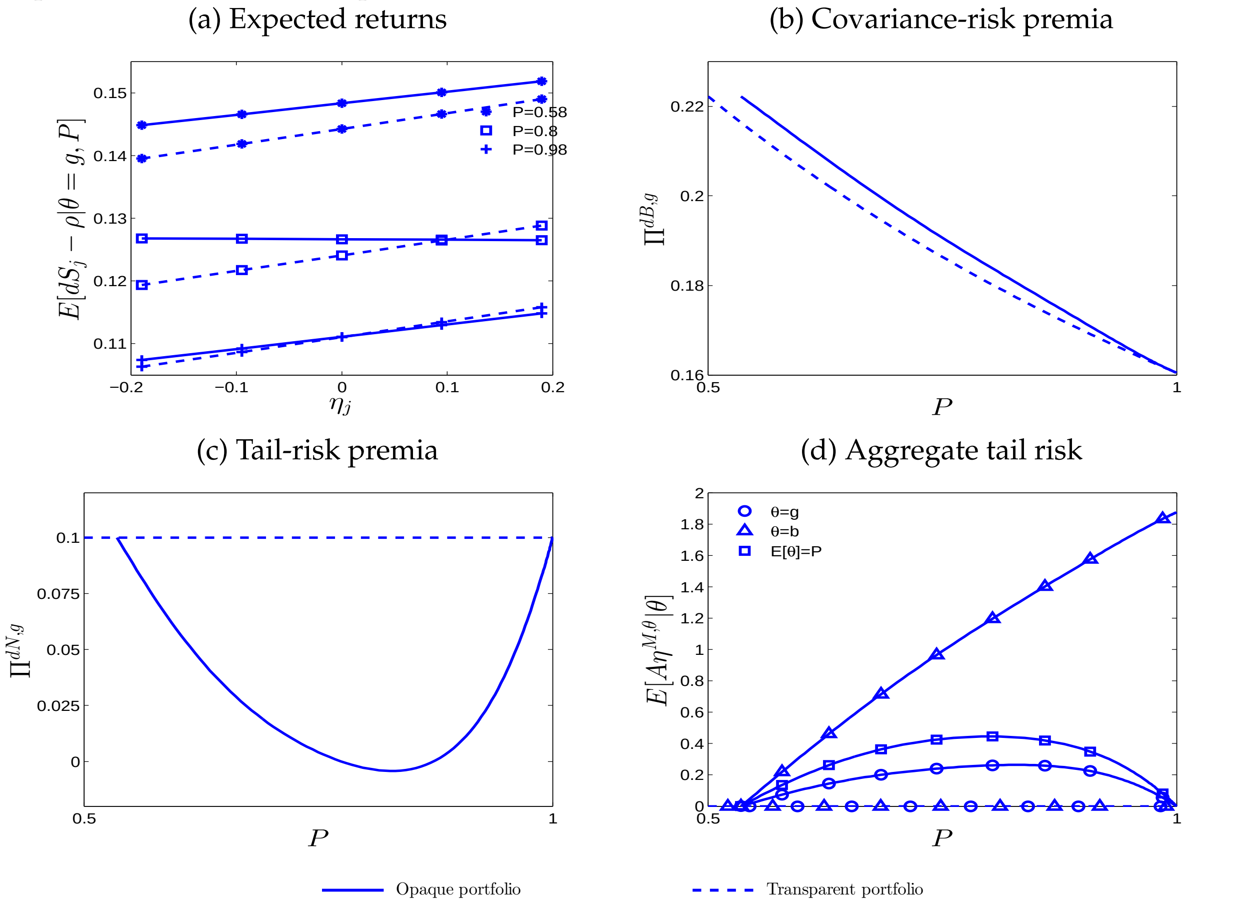


Figure 4: Slow-moving capital

Convergence rates for capital and risk premia as a function of the current state of the economy.The red circle denotes the economy equilibrium at $P = 1$.

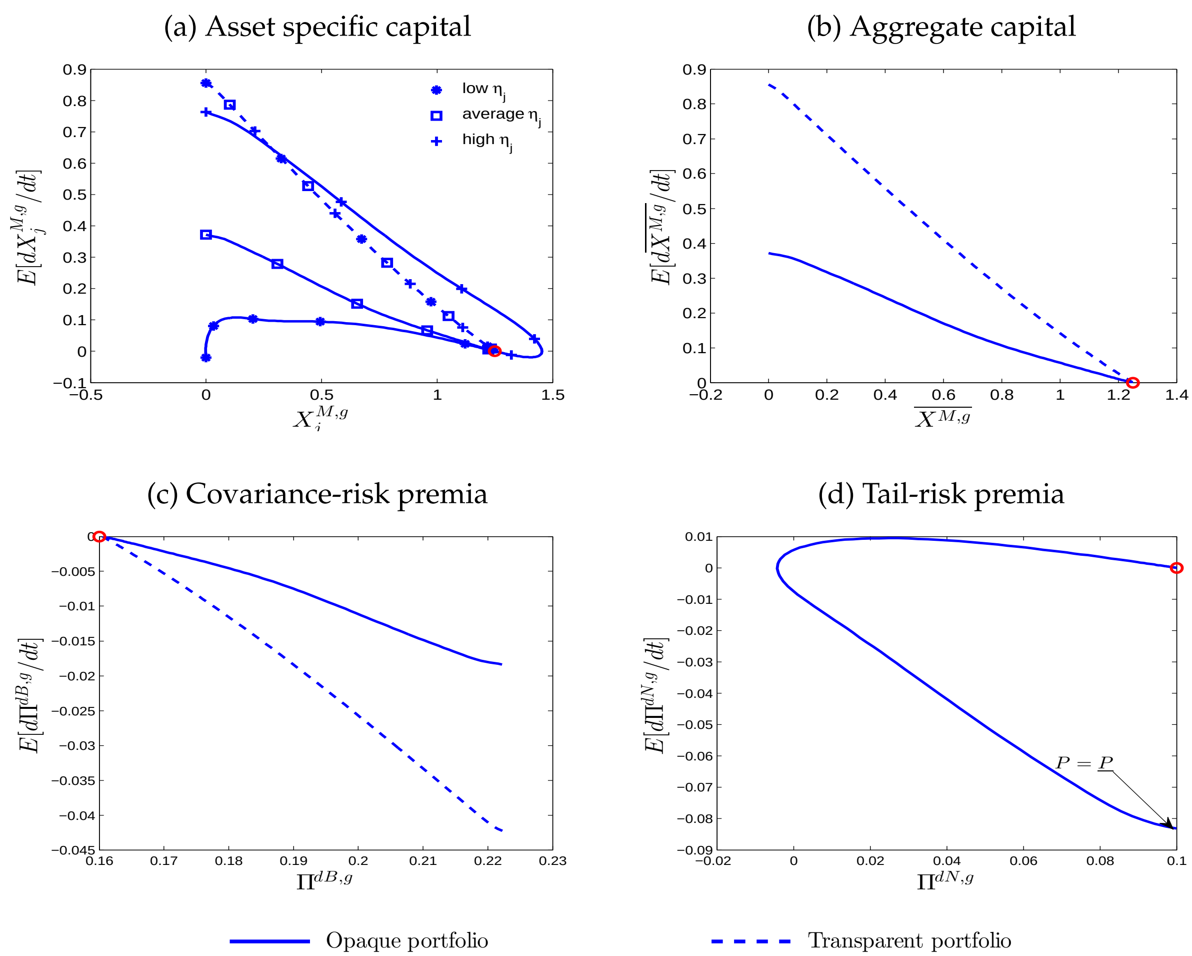
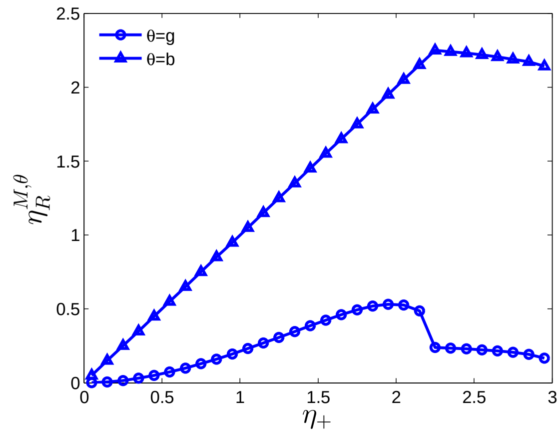


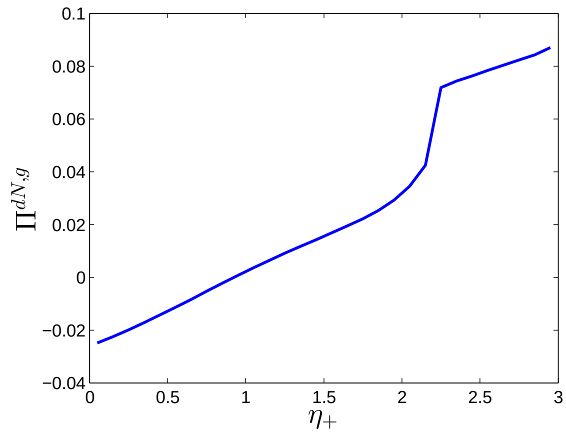
Figure 5: Financial innovation

Tail-risk taking, tail-risk premia, covariance-risk premia, and covariance-risk premia convergence rate as a function of the the degree of financial innovation η_+ in the local market. Values reported for reputation P=0.7.

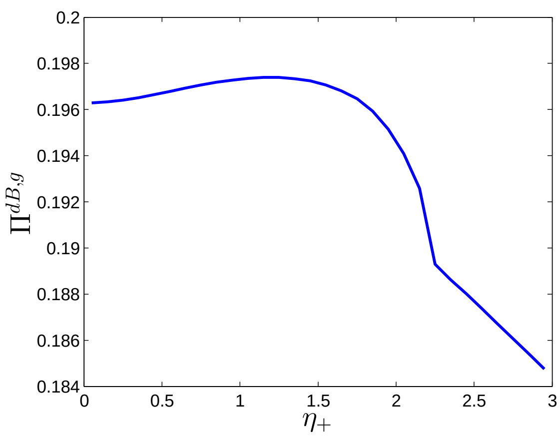
(a) Tail-risk exposure



(b) Tail-risk premia



(c) Covariance-risk premia



(d) Covergence rate

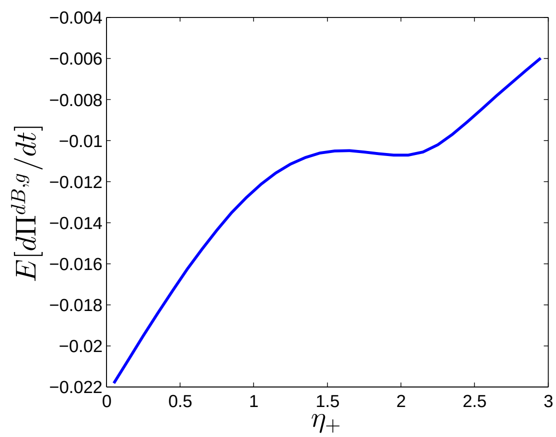


Figure 6: Investor sophistication and amplification

Incentives and tail-risk premia as function of the sophsitication of investor′s beliefs.

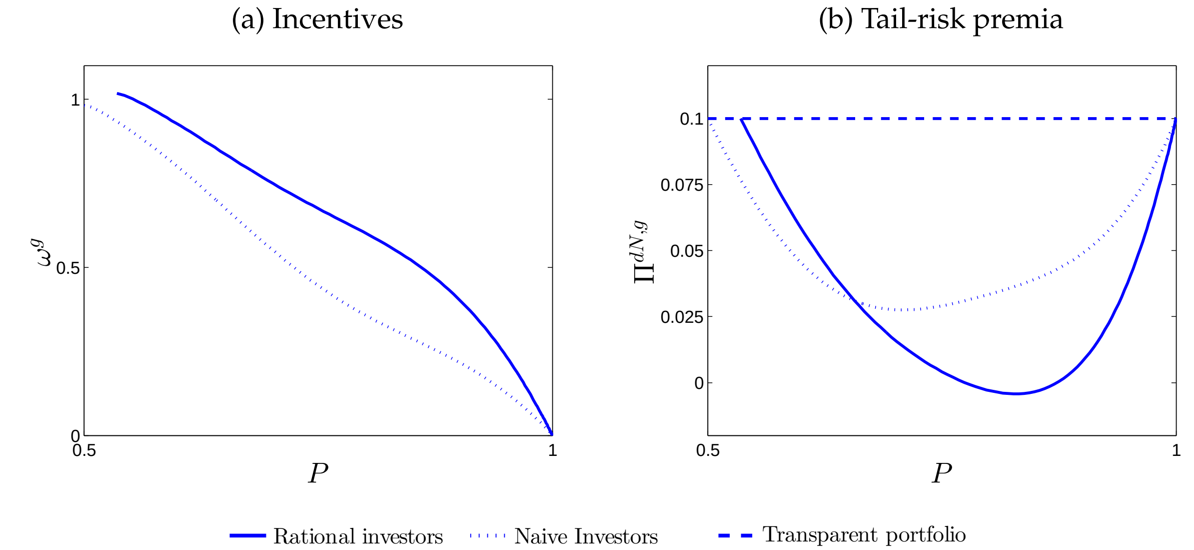


Figure 7: Intermediation booms

Tail-risk premia and covariance-risk premia as a function of the size of investor base relative to the size of local market (m). Values reported for reputation P=0.7.

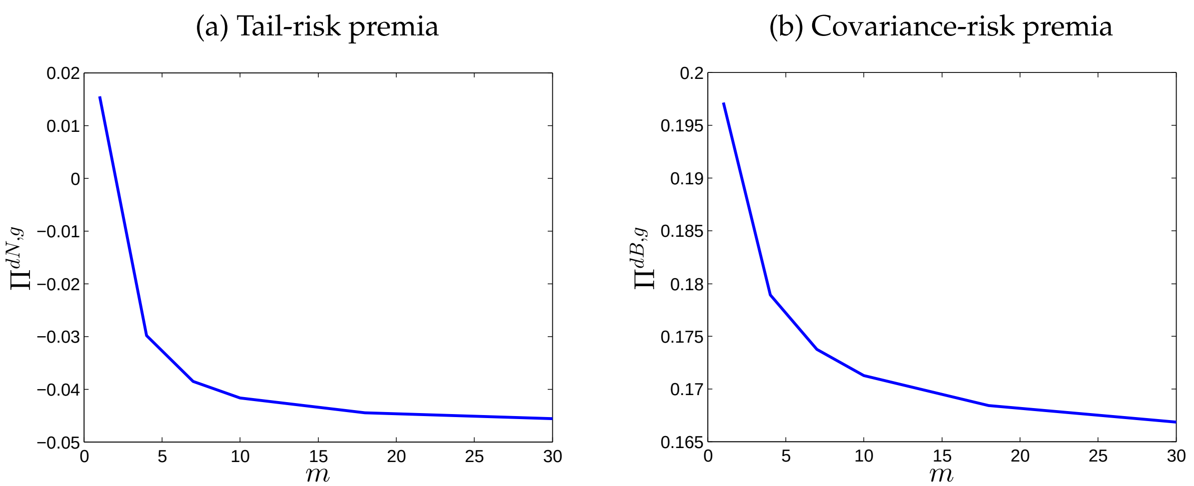
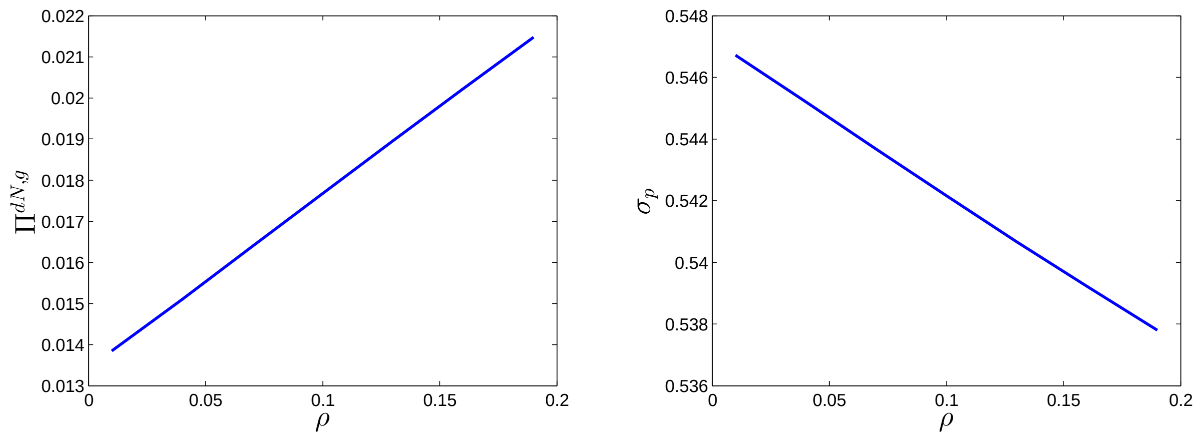


Figure 8: Monetary policy

Tail-risk premia and reputational-performance sensitivity as a function of the risk-free interest rate ρ. Values reported for reputation P=0.7.

(a) Tail-risk premia (b) Reputation-performance sensitivity



Appendix

A Proofs and derivations

A.1 Investor learning and capital allocation policy

Proposition 1. *(Reputation evolution)*

Proof. Consider the following realized return history R_t in an interval t, and let P the perceived probability the intermediary is of type $\theta = g$ in the beginning of the interval. Bayes law implies,

$$P_t = \frac{f(R_t|\theta = g, P_0)P}{f(R_t|\theta = g, P)P + f(R_t|\theta = b, P)(1-P)}, \tag{27}$$

where $f(R|\theta, P_0)$ is the probability distribution of a return history R if the intermediary is of type θ and initial reputation P. In our setting this density is a very complex object since the distribution of realized returns is time-varying due to the time-variation in the intermediary portfolio. However, in the limit dt the problem simplifies as portfolios are approximately constant as the interval becomes arbitrary short. The problem is further simplified because in any interval dt, dN_t equals to zero or one. The learning problem boils down to distinguish between two statistical models is two different observable states. Let me start with normal times, $dN = 0$. In this case, from the investor vantage point returns are distributed as,

$$dR^{\theta} \sim N(\sigma_R \left(\mu_R^{I,\theta} + \lambda_Q \eta_R^{I,\theta}\right) dt, \sigma_R^2 dt). \tag{28}$$

Let $N(dR|\mu, \Sigma^2)$ be the normal pdf and apply equation (14) to get,

$$\begin{aligned} P_{dt} &= \frac{N(dR|\sigma_R \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) dt, \sigma_R^2 dt).P}{N(dR|\sigma_R \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) dt, \sigma_R^2 dt).P + N(dR|\sigma_R \left(\mu_R^{I,b} + \lambda_Q \eta_R^{I,b}\right) dt, \sigma_R^2 dt).(1-P)} \\ &= \frac{N(\frac{dR}{\sigma_R}| \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) dt, dt).P}{N(\frac{dR}{\sigma_R}| \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) dt, dt).P + N(\frac{dR}{\sigma_R}| \left(\mu_R^{I,b} + \lambda_Q \eta_R^{I,b}\right) dt, dt).(1-P)} \end{aligned} \tag{29}$$

Now lets go to log-likelihood space, define

$$p_{dt} = ln\left(\frac{P_{dt}}{1-P_{dt}}\right) \tag{30}$$

and substitute equation (16) to get,

$$\begin{aligned} p_{dt} &= ln\left(N(\frac{dR}{\sigma_R}| \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) dt, dt).P\right) - ln\left(N(\frac{dR}{\sigma_R}| \left(\mu_R^{I,b} + \lambda_Q \eta_R^{I,b}\right) dt, dt).(1-P)\right) \\ &= p + ln\left(\exp\left(-(\frac{dR}{\sigma_R} - \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) dt)^2 (2dt)^{-1}\right)\right) - \\ &\quad ln\left(\exp\left(-(\frac{dR}{\sigma_R} - \left(\mu_R^{I,b} + \lambda_Q \eta_R^{I,b}\right) dt)^2 (2dt)^{-1}\right)\right) \\ &= p - \frac{(\frac{dR}{\sigma_R} - \left(\mu_R^{I,b} + \lambda_Q \eta_R^{I,b}\right) dt)^2}{2dt} + \frac{(\frac{dR}{\sigma_R} - \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) dt)^2}{2dt^2} \\ &= p + \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g} - \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right)\right) \left(\frac{dR}{\sigma_R} - \frac{\left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) + \left(\mu_R^{I,b} + \lambda_Q \eta_R^{I,b}\right)}{2} dt\right) \end{aligned}$$

Now realize that $dR = \sigma_R \mu_R^{M,\theta} dt + \sigma_R dZ_t$ in any interval without a tail event. Plugging back in the above expression we have,

$$p_{dt} = p_0 + \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g} - \left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right)\right) \times \tag{31}$$

$$\left(\left(\mu_R^{M,\theta} + \lambda_Q \eta_R^{M,g}\right) dt - \frac{\left(\mu_R^{I,g} + \lambda_Q \eta_R^{I,g}\right) + \left(\mu_R^{I,b} + \lambda_Q \eta_R^{I,b}\right)}{2} dt + dZ_t\right). \tag{32}$$

Now lets focus on instants with a tail event realization, $dN = 1$, and $dR^{\theta} \sim N(-\sigma_R\sqrt{\nu}\eta_R^{I,\theta}, \nu\sigma_R^2)$.Bayes law implies,

$$P_{dt} = \frac{N(dR| - \sigma_R\sqrt{\nu}\eta_R^{I,g}, \nu\sigma_R^2).P}{N(dR| - \sigma_R\sqrt{\nu}\eta_R^{I,g}, \nu\sigma_R^2).P + N(dR| - \sigma_R\sqrt{\nu}\eta_R^{I,b}, \nu\sigma_R^2).(1-P)}.$$

Repeating exactly the same algebra as in the $dN = 0$ case we obtain,

$$p_{dt} = p - \frac{\left(\eta_R^{I,g} - \eta_R^{I,b}\right)}{\sqrt{\nu}}\left(\frac{dR}{\sqrt{\nu}\sigma_R} + \frac{\eta_R^{I,g} + \eta_R^{I,b}}{2\sqrt{\nu}}\right).$$

In instants with tail events realized returns are given by, $dR_t = -\eta^{I,\theta}\sigma_R\sqrt{\nu} - \sigma_R\sqrt{\nu}y_t$, where y_t is a standard normal.

$$\begin{aligned} p_{dt} &= p - \frac{\left(\eta_R^{I,g} - \eta_R^{I,b}\right)}{\sqrt{\nu}}\left(-y_t - \eta^{M,\theta} + \frac{\eta_R^{I,g} + \eta_R^{I,b}}{2\sqrt{\nu}}\right) \\ &= p + \frac{\left(\eta_R^{I,g} - \eta_R^{I,b}\right)}{\sqrt{\nu}}\left(\eta^{M,\theta} - \frac{\eta_R^{I,g} + \eta_R^{I,b}}{2\sqrt{\nu}} + y_t\right). \end{aligned} \tag{33}$$

Equation (18) and (19) together imply

$$p_{dt} = \left(\mu_R^{I,g} + \lambda_Q\eta_R^{I,g} - \left(\mu_R^{I,g} + \lambda_Q\eta_R^{I,g}\right)\right) \times \tag{34}$$

$$\left(\left(\mu_R^{M,\theta} + \lambda_Q\eta_R^{M,g}\right)dt - \frac{\left(\mu_R^{I,g} + \lambda_Q\eta_R^{I,g}\right) + \left(\mu_R^{I,b} + \lambda_Q\eta_R^{I,b}\right)}{2}dt + dZ_t\right) \tag{35}$$

$$+\frac{\left(\eta_R^{I,g} - \eta_R^{I,g}\right)}{\sqrt{\nu}}\left(\eta_R^{M,\theta} - \frac{\eta_R^{I,g} + \eta_R^{I,b}}{2\sqrt{\nu}} + y_t\right)dN_t. \tag{36}$$

Since this argument is valid for any interval dt, equation (7) follows. To verify that p_{dt} is a Martingale under the investor's information set it is enough to realize that in lo-likelihood space we have,

$$E^I[\mu_R^{M,\theta}] = E^I[\mu_R^{I,\theta}] = \frac{\left(\mu_R^{I,g} + \lambda_Q\eta_R^{I,g}\right) + \left(\mu_R^{I,b} + \lambda_Q\eta_R^{I,b}\right)}{2},$$

and identical argument follows for $E^I[\eta_R^{M,\theta}]$.

□

Proposition 2. *(Portfolio size and liquidation threshold)*

Proof. Taking as given how investor's beliefs evolve as a function of reputation, $x^{I,\theta}(p)$, aggregate capital allocation K to each asset, and how much volatility σ_R the manager chooses, the first order condition of the household problem implies

$$w = \frac{(1-\kappa)P\alpha_{H,g}\sigma_R x^{I,g}(p)\Sigma\left(e-K\right)^{\top} - f\sigma_R}{\alpha_I(1-\kappa)^2\sigma_R^2}$$

In equilibrium, the aggregate capital allocation must be $K = wx^{I,g}\sigma_R$. Exploiting the fact that the asset supply elasticity is proportional to Σ we have that

$$A = w\sigma_R = \frac{P\alpha_{H,g}x^{I,g}(p)\Sigma e^{\top} - f(1-\kappa)^{-1}}{\left(P\alpha_{H,g} + \alpha_I(1-\kappa)\right)},$$

where $A = w\sigma_R$ is the dollar instantaneous volatility of the portfolio, what I refer as the portfolio size as it is a combination of the dollar invested by investors (w), and the amount of leverage taken by the managers.

Manager liquidation is pinned-down by $A = 0$,

$$(1-\kappa)\underline{P}\alpha_{H,g}x^{I,g}(0)\Sigma e^{\top} \quad = \quad f(1-\kappa)^{-1}$$

where $\underline{P}$ is the liquidation threshold. Solving for $\underline{P}$ we get,

$$\underline{P} = \frac{f}{(1-\kappa)^2\alpha_{H,g}x^{I,g}(0)\Sigma e^{\top}}.$$

□

A.2 Fund manager portfolio policy

Proposition 3. *(Hamilton-Jacobi-Bellman equation)*

Proof. Proposition 2 is a application from Ito's lemma to the intermediary value-function. Let $x^{M,\theta}, \sigma_R$ be control process. Conjecture that $A_t = A(p)$ is only a function of reputation. Furthermore, conjecture that the remaining equilibrium objects can be written as a function of the state variable p_t, and the manager is liquidated at threshold $p = 0$. Given this conjecture the manager problem can be written as

$$
\begin{aligned}
V(p) &= \max_{\sigma_R, x} \quad E[\int_0^\tau e^{-\rho t} u(\Gamma^M(dR))|\theta, p] \\
s.t. & \\
\tau &= \quad \min_{t \geq 0}\{t|p_t \leq 0\} \\
dR_t &= \quad (r + \sigma_R \mu_R(x,p))\, dt + \sigma_R dZ_{t-} - \sigma_R \left(\eta_R(x) + \sqrt{\nu} y_t\right) dN_t \\
dp_t &= \quad \sigma_p(p) \left(\mu_p(x,p)dt + dZ_{t-}\right) + \sigma_p^N(p) \left(\mu_p^N(x,p) + y_t\right) dN_t
\end{aligned}
$$

where the evolution of dR and dp makes it explicit that reputation-performance sensitivities depend exclusively of equilibrium beliefs, and hence only the state variable p since our equilibria is Markovian, while reputation and return drifts depend directly on the choice x. Note that for any t, $E_t[u(\Gamma^M(dR_t))] = A(p) \left(\kappa x^\top \mu_E^{M,\theta}(p) + f\right)$. We are now ready to apply Theorem 3.1 of Øksendal and Sulem [2005] to obtain equation (6), where our notation maps to theirs as follows: $\{x, \sigma_R\}$ maps to u, p maps to Y, V maps to J, $E_t[u(\Gamma^M(dR_t))]$ maps to $f(\cdot)$, 0 maps to $g(0)$, $\sigma_p(p)\mu_p(x,p)$ maps to $b(\cdot)$,$\sigma_p(p)$ maps to $\sigma(\cdot)$, and $\sigma_p^N(p) \left(\mu_p^N(x,p) + y_t\right)$ maps to $\gamma(\cdot)$. □

Result 1. *(Derivation of Equations* (12) *and* (13)*)*

Proof. Taking the derivative of equation (11) with respect to x we obtain the key first order condition for the high-quality intermediary (I will omit the θ superscript in what follows),

$$
\kappa A \mu_E + V_p \sigma_p \left(\mu_E + (1+\gamma)\lambda\eta\right) + \lambda \sigma_p^N E_t^M \left[V_p \left(p + \sigma_p^N \left(y + \mu_p^N\right)\right)\right] \eta - \xi \Sigma x = 0 \tag{37}
$$

where ξ is the Lagrange multiplier associated with the constraint $x'\Sigma x = 1$. Portfolio opaqueness shows up in equation (23) as intermediaries take as given household beliefs when considering deviation. In the portfolio was transparent, intermediaries would internalize the effect of portfolio changes in equilibrium household behavior. Rearranging we obtain

$$
x = \xi^{-1}\Sigma^{-1} \left(\mu_E + \frac{(1+\gamma)V_p\sigma_p + \sigma_p^N E_t^M \left[V_p \left(p + \sigma_p^N \left(y + \mu_p^N\right)\right)\right]}{\kappa A + V_p \sigma_p} \lambda \eta\right)
$$

substituting in the constraint we get

$$
x = \frac{\Sigma^{-1} y}{\sqrt{y' \Sigma^{-1} y}} \tag{38}
$$

where $y = \left(\mu_E + \frac{(1+\gamma)V_p\sigma_p + \sigma_p^N E_t^M\left[V_p\left(p+\sigma_p^N\left(y+\mu_p^N\right)\right)\right]}{\kappa A_\sigma + V_p\sigma_p}\lambda\eta\right)$. Substituting equilibrium conditions for $\sigma_p, \sigma_p^N, \mu_p^N, \mu_E$, and A we obtain equation (13), which in it's full glory is given by

$$\omega^\theta(p, x^M, x^{I,g}, x^{I,b}) = \frac{\left(\mu_{R,t}^{I,g} - \mu_{R,t}^{I,g}\right)V_p^\theta + \frac{\eta_{R,t}^{I,g} - \eta_{R,t}^{I,b}}{\nu^2} E^{I,\theta}\left[V_p^\theta\left(p + \frac{\eta_{R,t}^{I,g} - \eta_{R,t}^{I,b}}{\nu}\left(y + \frac{1}{\nu}\left(\eta_{R,t}^{M,\theta} - \frac{\eta_{R,t}^{I,g} + \eta_{R,t}^{I,b}}{2}\right)\right)\right)\right]}{\kappa A + \left(\mu_{R,t}^{I,g} - \mu_{R,t}^{I,b}\right)V_p^\theta},$$

where $\mu_{R,t}^{i,\theta} = x_t^{i,\theta}\left(\mu_E^{i,\theta} + \lambda(1+\gamma)\eta\right)$, $\eta_{R,t}^{i,\theta} = x_t^{i,\theta}\eta$, and $A = \frac{P\alpha_{H,g}x^{I,g}(p)\Sigma e^\top - f(1-\kappa)^{-1}}{\left(P\alpha_{H,g} + \alpha_I(1-\kappa)\right)}$. □

A.3 Equilibrium

Proposition 4. *(Incentives and capital allocation)*

Proof. Define $B(\mu_E) = \sqrt{(\mu_E + \omega\lambda\eta)^\top\Sigma^{-1}(\mu_E + \omega\lambda\eta)^\top}$. Plugging Equation (6) into Equation(13), one obtains

$$x = \frac{(\alpha_{H,\theta}\Sigma(e - x^{M,\theta}A)^\top + \omega\lambda\eta)^\top\Sigma^{-1}}{B(\alpha_{H,\theta}\Sigma(e - x^{M,\theta}A)^\top)}.$$

Impose market clearing, $x = x^{M,\theta}$, and rearrange to get,

$$x = \frac{\alpha_{H,\theta}e + \omega\lambda\eta^\top\Sigma^{-1}}{B(\alpha_{H,\theta}\Sigma(e - x^{M,\theta}A)^\top) + \alpha_{H,\theta}A}.$$

I now use the fact that $x\Sigma x = 1$ and solve for $B(\cdot)$. Using that $e\eta = 0$ and $\eta_+ = \sqrt{\eta^\top\Sigma^{-1}\eta}$, I obtain

$$B(\alpha_{H,\theta}\Sigma(e - x^{M,\theta}A)^\top)) = \sqrt{\alpha_{H,\theta}^2\sigma_e^2 + \omega^2\lambda^2\eta_+^2} - \alpha_{H,\theta}A.$$

Define $\psi^\theta(\omega) = \sqrt{\frac{\alpha_{H,\theta}^2\sigma_e^2}{\alpha_{H,\theta}^2\sigma_e^2 + \omega^2\lambda^2\eta_+^2}}$ and plug $B(\cdot)$ back in the expression for x to get ,

$$x^\theta(\omega) = \frac{\psi^\theta(\omega)}{\alpha_{H,\theta}\sigma_e}\left(\alpha_{H,\theta}e + \omega\lambda\eta^\top\Sigma^{-1}\right).$$

This proves equation (14). □

Proposition 5. *(Bad type)*

Proof. I will omit the $\theta = b$ superscript in what follows. Plugging $\mu_E = 0$ in equation (13), I get

$$x(\omega) = \frac{\omega}{|\omega|}\frac{\Sigma^{-1}\eta}{\sqrt{\eta'\Sigma^{-1}\eta}},$$

If $\omega = 0$, the low type is just indifferent across all feasible portfolios. If $\omega > 0$, then the intermediary chooses the maximum tail-risk portfolio $\frac{\Sigma^{-1}\eta}{\sqrt{\eta^\top\Sigma^{-1}\eta}}$, and if $\omega < 0$, he chooses the minimum tail-risk portfolio $-\frac{\Sigma^{-1}\eta}{\sqrt{\eta^\top\Sigma^{-1}\eta}}$. However, the portfolio$-\frac{\Sigma^{-1}\eta}{\sqrt{\eta^\top\Sigma^{-1}\eta}}$ can never be a solution. Let us write $\omega^b(p,x,x^{I,b},x)$ as function of the perceived Good-type alpha, $\alpha \equiv x^{I,g}\mu_E^{I,g}$ and the perceived difference in tail exposures across intermediaries, $\delta = -\left(x^{I,g}-x\right)^{\top}\eta$,

$$\omega = \frac{(\alpha-\lambda(1+\gamma)\delta)V_p(1+\gamma) - \frac{\delta}{\nu^2}E^M\left[V_p\left(p-\left(\frac{\delta}{\nu}\right)^2-\frac{\delta}{\nu}y\right)\right]}{\kappa A+(\alpha-\lambda(1+\gamma)\delta)V_p}.$$

I can now frame the problem as the Bad type choosing δ subject to the maximum feasible tail exposure, that is $\delta \geq \sqrt{\eta'\Sigma^{-1}\eta}-\left(x^{H,1}\right)^{\top}\eta$. For example if the Good type is believed to chooses $\left(x^{I,g}\right)^{\top}\eta = \sqrt{\eta'\Sigma^{-1}\eta}$, the Bad type can only choose a lower tail exposure, so $\delta \leq 0$.

$$\delta^* = \min\left\{\frac{\omega}{|\omega|}\frac{\eta'\Sigma^{-1}\eta}{\sqrt{\eta'\Sigma^{-1}\eta}}, \sqrt{\eta'\Sigma^{-1}\eta}\right\} - \left(x^{I,g}\right)^{\top}\eta$$

Because reputation is a good, $V_p > 0$, for any $\delta < 0$, $\omega < 0$, what would imply $\delta^* = \frac{\eta'\Sigma^{-1}\eta}{\sqrt{\eta'\Sigma^{-1}\eta}} - \left(x^{H,1}\right)^{\top}\eta > 0$, what is inconsistent with the conjecture that $\delta < 0$. if $\delta \in \left(0, \frac{\eta'\Sigma^{-1}\eta}{\sqrt{\eta'\Sigma^{-1}\eta}} - \left(x^{I,g}\right)^{\top}\eta\right)$,then it must be $\omega = 0$. After some manipulation I get

$$\alpha = \frac{\delta}{V_p}\left(\lambda(1+\gamma)V_p + \frac{1}{\nu^2}\frac{E^I\left[V_p\left(p-\left(\frac{\delta}{\nu}\right)^2+\frac{\delta}{\nu}y\right)\right]}{(1+\gamma)}\right) \tag{39}$$

The optimal δ either solves equation (39) or is given by the maximum feasible difference, $\delta_+ = \frac{\eta'\Sigma^{-1}\eta}{\sqrt{\eta'\Sigma^{-1}\eta}} - \left(x^{I,g}\right)^{\top}\eta$, if $\alpha > \frac{\delta_+}{V_p}\left(\lambda(1+\gamma)V_p + \frac{1}{\nu^2}\frac{E^I\left[V_p\left(p-\left(\frac{\delta_+}{\nu}\right)^2+\frac{\delta_+}{\nu}y\right)\right]}{(1+\gamma)}\right)$. If $V_{pp} < 0$, then the RHS of equation (39) is increasing in δ. This implies that if exists a $\delta \in [0,\delta_+]$ that satisfies equation (39), it must be that $\alpha < \frac{\delta_+}{V_p}\left(\lambda(1+\gamma)V_p + \frac{1}{\nu^2}\frac{E^I\left[V_p\left(p-\left(\frac{\delta_+}{\nu}\right)^2+\frac{\delta_+}{\nu}y\right)\right]}{(1+\gamma)}\right)$, and the solution is unique. ☐

Proposition 6. *(Good type)*

Proof. I have imposed the following equilibrium condition so far: (E1) Investors' optimality in Proposition 2 ; (E4) market clearing in Proposition 4; (E2) manager optimality in Equations (12) and (13); (E3) consistency with Bayes law in the evolution of beliefs in Proposition 1 ;(E3) consistency of investors beliefs with Bad-type behavior in Proposition 5 . Equation (21) imposes consistency of investors' beliefs with Good-type behavior. Equation (21) construct equilibrium incentives reputation by reputation. As such, any function $F(p) : \mathbb{R}_+ \to \mathbb{R}$ that satisfies Equation (21) point-by-point.

Whenever Equation (21) has multiple solutions I focus on incentives that are consistent with equilibrium and maximize the total utility flow. Note that the Bad-type allocation does not impact directly the utility flow of the agents (it only impacts the dynamics), so it is enough to focus in the Good-type case. The total utility flow can be written as

$$\begin{aligned} u_I + u_H + u_{M,g} &= A\left((1-\kappa)x^{I,g}\mu_E^{I,g} - f\right) - \frac{\alpha_I}{2}(1-\kappa)^2A^2 + A\left(e - x^{I,g}\right)\mu_E^{I,g} \\ &\quad -\frac{\alpha_{H,g}}{2}(e - Ax^{I,g})\Sigma\left(e - Ax^{I,g}\right)^\top + A\left(\kappa x^{I,g}\mu_E^{I,g} + f\right) \\ &= -\left(A^2\frac{\alpha_I}{2}(1-\kappa)^2 + \frac{\alpha_{H,g}}{2}(e - Ax^{I,g})\Sigma\left(e - Ax^{I,g}\right)^\top\right) \\ &= -\left(A^2\frac{\alpha_I}{2}(1-\kappa)^2 + \frac{\alpha_{H,g}}{2}\left(\sigma_e^2 + A^2 - 2Ax^{I,g}\Sigma e\right)\right) \\ &= -\left(A^2\frac{\alpha_I}{2}(1-\kappa)^2 + \frac{\alpha_{H,g}}{2}\left(\sigma_e^2 + A^2 - 2A\psi^g(\omega)\sigma_e\right)\right) \\ &= -A^2\left(\frac{\alpha_I(1-\kappa)^2 + \alpha_{H,g}}{2}\right) - \frac{\alpha_{H,g}}{2}\sigma_e^2 + \alpha_{H,g}A\psi^g(\omega)\sigma_e. \end{aligned}$$

Where I substituted out $x^{I,g}$ using Equation (14) in the fifth line. First, note that the above expression is increasing in $\sqrt{\psi^\theta(\omega)}$ and A as long $A < A_{max} = \frac{\alpha_{H,g}\sqrt{\psi^\theta(\omega)}\sigma_e}{\alpha_I(1-\kappa)^2+\alpha_{H,g}}$ (This is the asset value that maximizes the expression). Second, note that the equilibrium assets supplied by investors is $\frac{\Psi(p,\underline{P})\alpha_{H,g}\sqrt{\psi^g(\omega^g)}\sigma_e - f(1-\kappa)^{-1}}{\alpha_I(1-\kappa)+\alpha_{H,g}\Psi(p,\underline{P})}$, which is always strictly lower than A_{max}. Plug the equilibrium expression for A to get

$$\underbrace{\frac{\partial}{\partial\omega^2}\psi^g(\omega)}_{-}\alpha_{H,g}\sigma_e\left(\frac{\Psi(p,\underline{P})}{\alpha_I(1-\kappa)+\alpha_{H,g}\Psi(p,\underline{P})}\underbrace{\left(\alpha_{H,g}\psi^g(\omega)\sigma_e - A\left(\alpha_I(1-\kappa)^2 + \alpha_{H,g}\right)\right)}_{+} + A\right),$$

where the positive term follows that in equilibrium $A < A_{max}$, and the negative term follows from the definition of $\psi^g(\omega)$. This proves the welfare is strictly decreasing in ω^2 . It follows that minimization of ω^2 maximizes total utility flow. □

A.4 Qualitative results

Proposition 7. *(Returns surprises are always good news)*

Proof. This result follows directly from Bad-type optimality. Suppose it does not hold, and $\sigma_p = x^{I,g}\mu_E^{I,g} + (1+\gamma)\lambda(x^{I,g} - x^{I,b})\eta < 0$. In this case, it must be that $\sigma_p^N < 0$ since $x^{I,g}\mu_E^{I,g} > 0$ (otherwise the fund would have been liquidated).Consider the following deviation by the Bad type,

$$x_d^{M,b} = -\epsilon \frac{\eta^\top \Sigma^{-1}}{\eta_+} + x^{I,b}$$

The change in reputational growth is positive state by state,since $\Delta\mu_p = -(1+\gamma)\lambda\epsilon$,$\Delta\mu_p^N = -\frac{\epsilon}{\nu}$, and $\sigma_p, \sigma_p^N < 0$. It follows that $\sigma_p < 0$ cannot be an equilibrium. The argument for $\sigma_p^N > 0$ is analogous. □

Proposition 8. *(No learning during tail events)*

Proof. Starting from case 2. Conjecture that the efficient portfolio $x^{M,g} = \frac{e}{\sigma_e}$ that maximizes risk-adjusted expected returns is an equilibrium. Now lets check the behavior of the Bad type. First check for an interior solution

$$\omega^b = \frac{(\alpha_{H,g}(\sigma_e - A) - \lambda(1+\gamma)\delta)V_p^b(1+\gamma)}{\kappa A + (\alpha_{H,g}(\sigma_e - A) - \lambda(1+\gamma)\delta)V_p^b} = 0,$$

which implies, $\alpha_{H,g}(\sigma_e - A) = \lambda(1+\gamma)\delta$. At portfolio $x^{M,g} = \frac{e}{\sigma_e}$ we have $\eta_R^{M,g} = 0$, so this implies $\eta_R^{M,b} = \frac{\alpha_{H,g}(\sigma_e - A)}{\lambda(1+\gamma)}$. Condition $\sigma_e - \frac{\lambda(1+\gamma)\eta_+}{\alpha_{H,g}} \leq A$ implies that $\frac{\alpha_{H,g}(\sigma_e - A)}{\lambda(1+\gamma)} \leq \eta_+$, so $\eta_R^{M,b} = \frac{\alpha_{H,g}(\sigma_e - A)}{\lambda(1+\gamma)}$ solves the Bad-type problem. It follows that $\sigma_p = 0$ if A is large enough, hence $\omega^g = 0$ and Proposition 4 imply

$$X^{M,g}(0) = \frac{e}{\sigma_e}.$$

It follows that $\frac{e}{\sigma_e}$ is the equilibrium. So once assets are high enough and profitability low enough, learning stops, and the reputation stays constant at this level forever.

Lets turn to case 1. Equation (13) and $\nu^{-1} = 0$ imply

$$\omega^\theta = \frac{V_p^\theta \sigma_p(1+\gamma)}{A\kappa + V_p^\theta \sigma_p}$$

Recognizing that

$$\sigma_p = x'\mu_E^{I,g} + (1+\gamma)\lambda\left(x'\eta - \eta^{I,b}\right)$$

Suppose by contradiction $\sigma_p < 0$,

$$x'\mu_E^{I,g} \leq (1+\gamma)\lambda\left(\eta^{I,b} - x'\eta\right)$$

But it must be that $\eta^{I,b} = -\eta_+$, otherwise the Bad type could increase his short-term reputation drift by choosing a lower tail exposure. This implies $\eta^{I,b} - x'\eta \leq 0$, what implies $x'\mu_E^{I,g} \leq 0$, what implies the fund assets are zero. So there is no equilibria with $\sigma_p < 0$ where the intermediary is alive. This proves reputation is (weakly) increasing in short-term performance.

Since $\sigma_p \geq 0$ and $\frac{\alpha_{H,g}(\sigma_e - A)}{\lambda(1+\gamma)} \geq \eta_+$, it follows that $\sigma_p > 0$. Incentives are $\omega^\theta = \frac{V_p^\theta \sigma_p (1+\gamma)}{\kappa A + V_p^\theta \sigma_p} \in (0, 1+\gamma)$. Proposition 5 implies $\eta^{M,b} = \eta_+$. It remains to prove $\omega^g(P)$ is decreasing, and $\lim_{P\to 1} \omega^g(P) = 0$. Plugging Equations (18),(17), and $\eta^{M,b} = \eta_+$ in σ_p I obtain

$$\sigma_p(p, \omega^g) = \alpha_{H,g} \left(\frac{(1-\kappa)\psi^g(\omega^g)\sigma_e \alpha_I + f}{\alpha_I(1-\kappa) + \alpha_{H,g}\Psi(p, \underline{P})} \right) + (1+\gamma)\lambda\eta_+ \left(\sqrt{1 - (\psi^g(\omega^g))^2} - 1 \right),$$

which is decreasing in p. Equation (16) implies assets are increasing in p and decreasing in ω^g. Plugging all in the fixed point equation I get,

$$\omega^g(p, \omega) = \frac{V_p^g(p)\sigma_p(p, \omega)(1+\gamma)}{\kappa A(p, \omega) + V_p^g(p)\sigma_p(p, \omega)},$$

where the RHS is decreasing in p if $V_{pp} < 0$ (since assets are increasing and $V_p \sigma_p$ decreasing). Note that $\omega^g(p, 0) > 0$, and $\omega^g(p, 1+\gamma) < 1$, so the LHS crosses the RHS from below at least once (both are continuous functions). Furthermore, the first time it crosses pins down the incentive schedule consistent with equilibrium, since is the fixed point consistent with minimum ω^2 (See Proposition 6 for the proof that incentive minimization maximizes welfare). It follows that the equilibrium exists and the RHS crosses the LHS from below. For now assume $V_{pp} < 0$, and let $\hat{\omega}$ be the equilibrium solution of

$$\omega^g(p, \hat{\omega}(p)) = \hat{\omega}(p)$$

Differentiating with respect to p and rearranging I obtain,

$$\frac{\partial \hat{\omega}(p)}{\partial p} = \frac{\overbrace{\frac{\partial \omega^g(p, \hat{\omega}(p))}{\partial p}}^{-}}{\underbrace{1 - \frac{\partial \omega^g(p, \hat{\omega}(p))}{\partial w}}_{+}} < 0.$$

From the previous discussion we have that the numerator is negative. The denominator is positive from the fact that the LHS crosses the RHS from below and the LHS has slope equal to one. So the RHS must have slope lower than 1 at the equilibrium. This proves that Good-type equilibrium incentives are decreasing in reputation. Note that if $\lim_{P\to 1} V_P(P) = 0$ follows that $\lim_{P\to 1} \omega^g(P) = 0$. This concludes the proof. $\square$

Proposition 9. *(Transparent portfolio)*

Proof. When the portfolio is fully transparent, the Bad type chooses the same portfolio as the Good type, as any deviation would reveal his type and imply immediate liquidation. This implies $x^{M,g}\eta = x^{M,b}\eta$. Given this property of the equilibrium, $\sigma_p^N = 0$, and $\sigma_p = x^{M,g}\mu_E^{M,g}$. Substituting and isolating the relevant terms in equation (11) I obtain,

$$\max_x A\left(\kappa\alpha_{H,g}\left(x\Sigma e^\top - A\right) + f\right) + V_p\sigma_p\alpha_{H,g}\left(x\Sigma e^\top - A\right) \tag{40}$$
$$s.t.$$
$$x^\top\Sigma x = 1,$$

Which solution is $x = \frac{e}{\sigma_e}$, since $V_p\sigma_p \geq 0$. $\square$

A.5 Asset pricing

Proposition 10. *(Incentives and expected returns)*

Proof. From equation (10) we have that the cross-sectional of expected returns can be written as,

$$\mathbb{E}_{M,t}\left[dS_t - \rho|\theta,\omega\right] = \alpha_{H,\theta}\Sigma(e - x^{M,\theta}A)^\top + \gamma\lambda\eta.$$

The first term is the risk-adjusted excess return and the second term is the risk premium for the tail-risk exposure. Plugging the expression for the equilibrium portfolio derived above I obtain

$$\begin{aligned}\mathbb{E}_{M,t}\left[dS_t - \rho|\theta,\omega\right] &= \alpha_{H,\theta}\Sigma\left(e - \frac{\psi^\theta(\omega)}{\alpha_{H,\theta}\sigma_e}\left(\alpha_{H,\theta}e + \omega\lambda\eta^\top\Sigma^{-1}\right)A\right)^\top + \gamma\lambda\eta \\ &= \alpha_{H,\theta}\left(1 - \frac{\psi^\theta(\omega)}{\sigma_e}A\right)\Sigma e^\top + \left(\gamma - \frac{\psi^\theta(\omega)}{\sigma_e}\omega A\right)\lambda\eta\end{aligned}$$

This proves the result in equation (23).

$\square$

A.6 Naive beliefs

Proposition 11. *(Equilibrium with naive investors) For a given $\tilde{p}$, equilibrium incentives $\tilde{\omega}$ satisfies*

$$\begin{aligned} \tilde{\omega} = \quad & \arg\min_{\omega \in \mathbb{R}} \quad \omega^2 \\ s.t. & \\ \omega \quad & = \quad \frac{(1+\gamma)\sigma_p(p,\omega_\phi(\omega))V_p^g + \frac{\sigma_p^N(p,\omega_\phi(\omega))}{\nu} Z(\omega,p)}{\kappa A(p,\omega_\phi(\omega)) + \sigma_p(p,\omega_\phi(\omega))V_p^g}, \end{aligned} \tag{41}$$

where

$$Z(\omega,p) = \int V_p^g \left(p + \sigma_p^N(p,\omega_\phi(\omega)) \left(\frac{1}{2}\sigma_p^N(p,\omega_\phi(\omega)) + \frac{1}{\nu} \left(\eta_R^{M,g}(\omega) - \eta_R^{M,g}(\omega_\phi(\omega)) \right) - y \right) \right) \mathcal{N}(dy).$$

B Numerical Solution

I apply the finite-difference method to solve the integro partial differential equation (11). To solve for optimal policies I sequentially iterate until the value functions converges. The pair of value functions $\{V^g(p), V^b(p)\}$, and incentive functions $\{\omega^g(p)\ ,\ \omega^b(p)\}$ are determined jointly. The state space consists of manager reputation in log-likelihood space ($p \in \mathbb{R}_+$).

I first discretize the state space as follows. I construct a grid with limits $\{0, \overline{p}\}$ and N grid points. Let $z = (\overline{p}+1)^{1/N}$, I populate the grid by setting $p(j) = z^j - 1$ .Using this grid I discretize Equation (11) using central differences as described in Candler [1998].

First I hold constant the incentive distortion at zero, $\omega^g = \omega^b = 0$, and iterate to find what I denote the efficient solution $\{V^g, V^b\}^{ef}$, which is the solution when the portfolio is transparent. The efficient liquidation threshold is a lower bound to the equilibrium liquidation threshold. Starting from the efficient liquidation policy and value function, I iterate on the HJB, and each time I solve for the par of incentive functions at each step.

More specifically, the iteration procedure can be divided in steps.

1. Given incentives $\left[\omega^g(p), \omega^b(p)\right]^{i-1}$ and solve for $[A(p)]^i$, $[\underline{P}]^i$, $[x^{M,\theta}(p)]^i$, $[\mu_R^{M,\theta}(p)]^i$,$[\eta_R^{M,\theta}(p)]^i$, and $[\mu_E^{M,\theta}(p)]^i$ using Equations (16),(15),(14),(17), (18), (20), and (10) .

2. Solve for $[V^g(p), V^b(p)]^i$ using the discretization of Equation (11) and the boundary condition $V^\theta(0) = 0$.

3. Given $[V^g(p), V^b(p)]^i$ solve for $\left[\omega^g(p), \omega^b(p)\right]^i$ using propositions (5) and (6). (see discussion below)

4. If $|\left(V^{g}\right)^{i}-\left(V^{g}\right)^{i-1}|<\epsilon$, $|\left(V^{b}\right)^{i}-\left(V^{b}\right)^{i-1}|<\epsilon$ and $|\underline{P}^{i}-\underline{P}^{i-1}|<\epsilon$ stop. If not satisfied, repeat .

The procedure converges extremely fast and takes no more than 5 iterations for a typical solution. The typical solution time is 30 seconds. Code is available upon request.

B.1 Fixed point

Step 3 consists of finding the fixed point of equation (21) that is consistent with the minimum incentive distortion, $|\omega^{g}|$.For each reputation p_j grid value I construct a grid with limits $\{\underline{\omega}, 1+\gamma\}$ and N_ω grid points for ω_l. I populate the grid using equal spacing across grid points. This produces $N \times N_\omega$ $\omega_{j,l}$ values. I look for all values of $\omega_{j,l}$, the expression

$$\omega_{j,l}-\omega^{g}\left(p, x^{M,g}(\omega_{j,l}), x^{M,g}(\omega_{j,l}), x^{M,b}\left(p, x^{M,g}(\omega_{j,l})\right)\right)$$

switch sign. This identifies an incentive schedule consistent with Definition 1. If for a given reputation p_j there is only one incentive value $\omega_{j,l}$, this is the equilibrium for this reputation. If there is more than one $\omega_{j,l}$, than the smallest absolute value $|\omega_{j,l}|$ is the equilibrium.

C Parameter choice

The model has five key parameters - $\sigma_e\alpha_{H,g}$, η_+, ν, λ and γ. In what follows I discuss the empirical plausibility of my parameter choice and how the results change with different parameter choices. The scarcity of capital is measured by the Sharpe ratio of the first unit of intermediary capital, $\sigma_e\alpha_{H,g}$. I calibrate $\sigma_e\alpha_{H,g}=1$, which is quite parsimonious given the amount of excess performance claimed by intermediaries such as private equity funds and hedge funds. A higher Sharpe ratio makes the distortions more severe and more costly, as there is more to be lost of capital misallocation. The second key parameter is the degree of financial innovation in the local market η_+, which is the maximum tail-exposure a portfolio of unit-variance can achieve. I set η_+ to 1.5, what implies that in a portfolio with 10% standard deviation, the intermediary can find (hidden) opportunities to have an tail loss of 15%. Given the recent losses experienced in some fixed income markets this number seems plausible.

Tail risk volatility determines how informative tail-event performance is. My baseline calibration uses $\nu=2.5$, which is in line with recent experience. For example, some measures of realized stock market volatility in the Fall of 2008 were 80% (annualized). This compares to an average market volatility of 15% . In fixed income markets, this difference is likely to be larger. As ν grows, the economy converges to the case of Proposition 8 . As ν shrinks to zero, tail event performance becomes completely revealing about the manager portfolio.

Tail-event frequency λ determines by how much tail-risk taking can boost performance during normal-times, but also determine how quickly the long-term arrives. The first effect increases the temptation of the intermediary, the second effect acts in the opposite direction as a disciplining force. Aversion to tail-event risk γ has only the first effect. Increases in γ improve the normal times attractiveness of high-tail-risk assets, without the second attenuating effect. I calibrate λ to 0.25, consistent with a tail event every four years. This number is in line with recent experience. For example, in the last 20 years the world economy experienced the "Tequilla crisis"(1994), the "Asian crisis" (1997-8), the "Tech bubble" bust (2000) , the more recent financial crisis (2007-2009), and the European sovereign debt crisis (2011-2012). I choose $\gamma = 0.1$, what implies agents value tail event consumption 10% more than normal-times consumption.