System-Time entanglement in a discrete time model

A. Boette, R. Rossignoli, N. Gigena, M. Cerezo

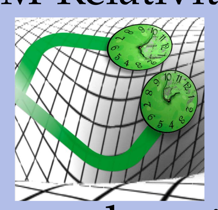
Dto. Física IFLP, FCE, Universidad Nacional de La Plata

Motivations:

Time

external continuos classical parameter

Conection QM-Relativity



Quantum description of time $\Longrightarrow$ Quantum Information description of evolution

Formalism:

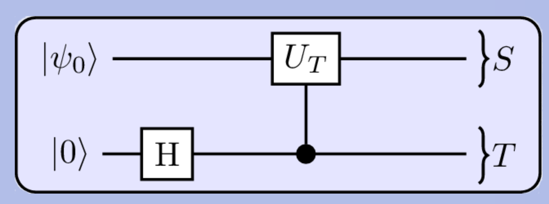
History State: $|\Psi\rangle = \frac{1}{\sqrt{N}}\sum_{t=0}^{N-1}|\psi_t\rangle|t\rangle$

Orthonormal basis of T $\{|t\rangle,\ t=0,\ldots,N-1\}$

Arbitrary pure states of S $\{|\psi_t\rangle,\ t=0,\ldots,N-1\}$

$|\psi_t\rangle = U_t|\psi_0\rangle,\ \ t=0,\ldots,N-1$

Can be generated with the circuit:



recover state at time t: $|\psi_t\rangle \propto \langle t|\Psi\rangle$

"Static" eigenstate of the super-operator:

$\mathcal{U} = \sum_{t=1}^{N} U_{t,t-1}\otimes|t\rangle\langle t-1|$

cyclic conditions $\mathcal{U}|\Psi\rangle = |\Psi\rangle$ (for any $|\Psi_0\rangle$)

invariant under global translations $S+T$

Remaining eigenstates of $\mathcal{U}$

$|\Psi_k\rangle = \frac{1}{\sqrt{N}}\sum_{t=0}^{N-1}e^{i2\pi kt/N}|\psi_t\rangle|t\rangle$ $\quad k$ integer

Represent the evolution associated with operators $U_t^k = e^{i2\pi kt/N}U_t$

$\mathcal{U}|\Psi_k\rangle = e^{-i2\pi k/N}|\Psi_k\rangle,\ \ k=0,\ldots,N-1$

for general U_t $\quad \mathcal{U} = \exp[-i\mathcal{J}]$

Discrete Wheeler-DeWitt equation: $\mathcal{J}|\Psi\rangle = 0$

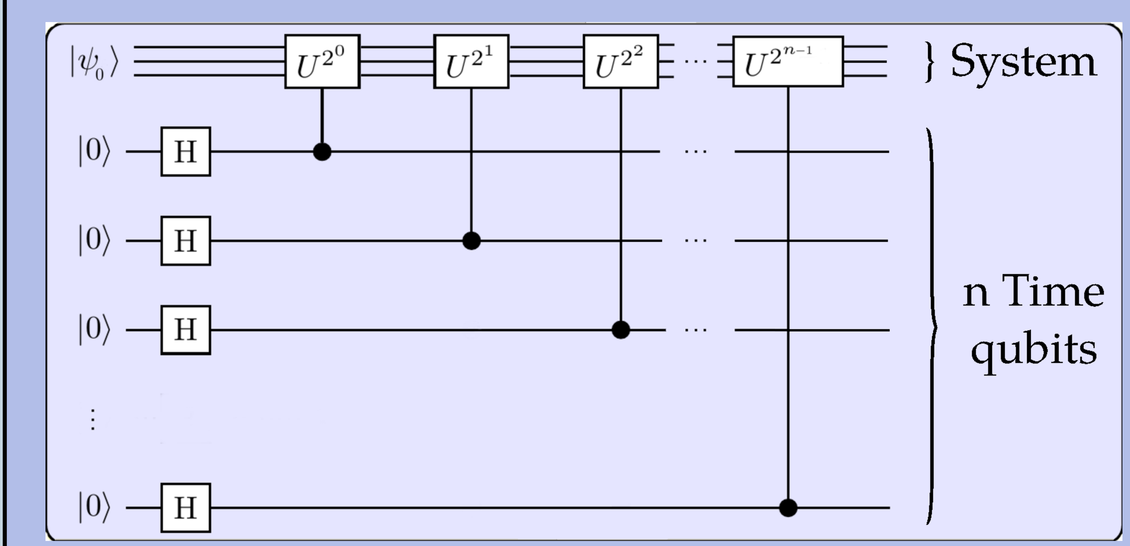
$\mathcal{J}$ hermitian and satisfying $\mathcal{J}|\Psi_k\rangle = 2\pi\frac{k}{N}|\Psi_k\rangle$, $k=0,\ldots,N-1$.

Constant Evolution Operator

$$U_{t,t-1} = U\ \forall\, t \Longrightarrow U_t = (U)^t = \exp[-iHt] \quad N=2^n$$

$$t=0,\ldots,N-1$$

The history state can be generated with the circuit:



n Control gates $U^t|\psi_0\rangle = \prod_{j=1}^{n}U^{t_j 2^{j-1}}|\psi_0\rangle$

n Hadamard gates

$$|0\rangle_T \equiv \otimes_{j=1}^{n}|0_j\rangle \to \otimes_{j=1}^{n}\frac{|0_j\rangle+|1_j\rangle}{\sqrt{2}} = \frac{1}{\sqrt{N}}\sum_{t=0}^{N-1}|t\rangle$$

A measurment of the time qubits with result t makes S collapse to the state: $|\psi_t\rangle = e^{-iHt}|\psi_0\rangle$

System-Time entanglement

- Quantify distinguishable evolution of a pure state

minimum time $\tau_m = 2^{E(S,T)}-1$

Schmidt decomposition

$$|\Psi\rangle = \sum_k\sqrt{p_k}|k\rangle_S|k\rangle_T \quad \rho_{S(T)} = \mathrm{Tr}_{T(S)}|\Psi\rangle\langle\Psi| = \sum_k p_k|k\rangle_{S(T)}$$

$$E(S,T) = S(\rho_S) = S(\rho_T) = -\sum_k p_k\log_2 p_k$$

- Stationary history state $|\Psi\rangle = |\psi_0\rangle(\frac{1}{\sqrt{N}}\sum_t e^{i\gamma_t}|t\rangle)$ $\Longrightarrow E(S,T)=0 \Longrightarrow \tau_m = 0$
- N orthogonal states $|\psi_t\rangle$, $|\Psi\rangle$ is *maximally entangled* $\Longrightarrow E(S,T) = E_{\max}(S,T) = \log_2 N$, $\tau_m = N-1$

Relation with energy spread

In the constant case evolution: $|\psi_0\rangle = \sum_k c_k|k\rangle$ con $H|k\rangle = E_k|k\rangle$ (eigenstates)

$$|\psi_t\rangle = \sum_k c_k e^{-iE_kt}|k\rangle \Longrightarrow |\Psi\rangle = \frac{1}{\sqrt{N}}\sum_{k,t}c_ke^{-iE_kt}|k\rangle|t\rangle$$

$$= \sum_k c_k|k\rangle|\tilde{k}\rangle_T$$

In the cyclic case $U^N = I$ $\quad E_k = 2\pi k/N,\ k=0,\ldots,N-1$ — is the *Schmidt decomposition* $p_k = |c_k|^2$

In general $E(S,T) \leq -\sum_k|c_k|^2\log_2|c_k|^2$ $\quad \{|c_k|^2\}\prec\{p_k\}$

For any entropic form $E_f(S,T) = \sum_k f(p_k) \leq \sum_k f(|c_k|^2)$

system-time generated by *any* Hamiltonianian diagonal in $|k\rangle$ is bounded by the entropy of the spread over eigenstates of H

The highest for a given distribution $\{|c_k|^2\}$ $\Longrightarrow$ cyclic evolution, equally spaced spectrum $E_k = 2\pi k/N \in [0,2\pi]$

Energy-time uncertainty relations

In the equally spaced spectrum, we expand the initial state in an orthogonal set

$$|\psi_0\rangle = \sum_{l=0}^{N}\tilde{c}_l|\tilde{l}\rangle_S,\ \ |\tilde{l}\rangle_S = \frac{1}{\sqrt{N}}\sum_k e^{i2\pi kl/N}|k\rangle$$

Fourier Transform $\tilde{c}_l = \frac{1}{\sqrt{N}}\sum_k e^{-i2\pi k/N}c_k$

$$|\Psi\rangle = \sum_{l,t}\tilde{c}_l|\widetilde{l-t}\rangle_S|t\rangle = \sum_l|\tilde{l}\rangle_S(\sum_t\tilde{c}_t|t-l\rangle)$$

determines the distribution of $|t\rangle$ assigned to each $|\tilde{l}\rangle_S$

$$E(S,T)+\tilde{E}(S,T) \geq \log_2 N$$

Localized in energy $|c_k| = \delta_{kk'}$, $E(S,T)=0$ $\Longleftrightarrow$ Maximum time uncertainty $|\tilde{c}_l| = \frac{1}{\sqrt{N}}$, $\tilde{E}(S,T) = \log_2 N$

$n(\{c_k\})\,n(\{\tilde{c}_l\}) \geq N$

$n(\{\alpha_k\})$ number of non-zero α_k's

Examples:

The qubit clock

$$|\Psi\rangle = (|\psi_0\rangle|0\rangle + |\psi_1\rangle|1\rangle)/\sqrt{2}$$

$$= \sqrt{p_+}|++\rangle + \sqrt{p_-}|--\rangle$$

$$p_\pm = (1\pm|\langle\psi_0|\psi_1\rangle|)/2$$

$$E_2(S,T) = 4p_+p_- = 1-|\langle\psi_0|\psi_1\rangle|^2$$

$$E_2(S,T) = S_2(\rho_T) = S_2(\rho_S) = 2(1-\mathrm{Tr}\,\rho_S^2)$$

S=A+B two qubit-system

$$\rho_{Bt} = p|\psi_t^0\rangle\langle\psi_t^0| + q|\psi_t^1\rangle\langle\psi_t^1|,\ \ t=0,1$$

$$|\Psi_j\rangle = \frac{1}{\sqrt{2}}(|\psi_0^j\rangle|0\rangle + |\psi_1^j\rangle|1\rangle)$$

For a pure state B+T

$$E_2(B,T) = C^2(B,T) = 1-F^2(\rho_{B0},\rho_{B1})$$

Fidelity

The continuos limit

S evolves $|\psi_0\rangle = |0\rangle \to |\psi_f\rangle = \cos\phi|0\rangle + \sin\phi|1\rangle$ in N-1 steps

$$|\psi_t\rangle = \cos(\tfrac{\phi t}{N-1})|0\rangle + \sin(\tfrac{\phi t}{N-1})|1\rangle$$

$$E_2(S,T_N) = 1-\frac{\sin^2(\frac{N\phi}{N-1})}{N^2\sin^2(\frac{\phi}{N-1})}$$

Decreases but remains *finite*

$$E_2(S,T_\infty) = 1-\frac{\sin^2\phi}{\phi^2}$$

Conclusions

Parallel-in-time discrete model quantum evolution

clock entangled with system

qubit clock="building block"

- Satisfy: Discrete Wheeler-DeWitt equation: $\mathcal{J}|\Psi\rangle = 0$
- $U_{t,t-1} = U\ \forall\, t$ — implement 2^n times $O(n)$ qubits and control gates
- Energy spread $|\psi_0\rangle$; Energy-time uncertainty
- Continuos limit $E_2(S,T_\infty)$ *Remains finite*

Perspectives

Schmidt decomposition as proper time definition

Interaction between clocks

Simple model of discrete emergent espacio-tiempo

A. Boette, R. Rossignoli, N. Gigena, and M. Cerezo, Phys. Rev. A 93, 062127 (2016).