TP1

1)

a_ $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2} \rightarrow \langle\psi|\psi\rangle = \frac{(\langle 0| + \langle 1|)}{\sqrt{2}} \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} = \frac{1}{2}(\langle 0|0\rangle + \langle 0|1\rangle + \langle 1|0\rangle + \langle 1|1\rangle)$

$= \frac{1}{2} 2 = \boxed{1}$ ✓

Posible

b) $(\sqrt{3}|0\rangle - |1\rangle)/2$

$\langle\psi|\psi\rangle = \frac{(\sqrt{3}\langle 0| - \langle 1|)}{2} \cdot \frac{(\sqrt{3}|0\rangle - |1\rangle)}{2} = \frac{\sqrt{3}.\sqrt{3} + 1}{2} = 2$ X No esta normalizado

c) $0.7|0\rangle + 0.3|1\rangle$

$\langle\psi|\psi\rangle = (0.7\langle 0| + 0.3\langle 1|)(0,7|0\rangle + 0.3|1\rangle) = 0,7^2 + 0,3^2 \neq 1$ X

d) $\langle\psi|\psi\rangle = (0,8\langle 0| + 0,6\langle 1|)(0,8|0\rangle + 0,6|1\rangle) = 0,8^2 + 0,6^2 = 0,64 + 0,36 = \boxed{1}$ ✓

e) $\langle\psi|\psi\rangle = (\cos\theta\langle 0| - i\,\mathrm{sen}\,\theta\langle 1|)(\cos\theta|0\rangle + i\,\mathrm{sen}\,\theta|1\rangle) =$

$= \cos^2\theta + \mathrm{sen}^2\theta = \boxed{1}$ ✓

f) $(\cos^2\theta\langle 0| - \mathrm{sen}^2\theta\langle 1|)(\cos^2\theta|0\rangle - \mathrm{sen}^2\theta|1\rangle) =$

$= \cos^4\theta + \mathrm{sen}^4\theta \rightarrow$ Solo vale uno cuando $\theta = \frac{k\pi}{2}$

Probabilidades

$P(n) = \langle\psi|M_n^\dagger M_n|\psi\rangle$; $M_0 = |0\rangle\langle 0| = \begin{pmatrix}1&0\\0&0\end{pmatrix} = M_0^\dagger$ $\quad M_1 = |1\rangle\langle 1| = \begin{pmatrix}0&0\\0&1\end{pmatrix} = M_1^\dagger$

$M_+ = |+\rangle\langle +| = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\left(\frac{\langle 0| + \langle 1|}{\sqrt{2}}\right) = \frac{\begin{pmatrix}1\\1\end{pmatrix}}{\sqrt{2}} \frac{(1\ 1)}{\sqrt{2}} = \begin{pmatrix}1&1\\1&1\end{pmatrix}/2 = M_+^\dagger$

$M_- = |-\rangle\langle -| = \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\sqrt{2}} \frac{(1\ -1)}{\sqrt{2}} = \begin{pmatrix}1&-1\\-1&1\end{pmatrix}/2 = M_-^\dagger$

$\overbrace{\langle\psi|M_0^\dagger M_0|\psi\rangle}^{P(0)}$

a) $\langle\psi|M_0^\dagger M_0|\psi\rangle = \langle\psi|0\rangle\langle 0|0\rangle\langle 0|\psi\rangle = \langle\psi|0\rangle\langle 0|\psi\rangle = \left(\frac{\langle 0|0\rangle + \langle 1|0\rangle}{\sqrt{2}}\right)\cdot\left(\frac{\langle 0|0\rangle + \langle 0|1\rangle}{\sqrt{2}}\right) =$

$= \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \boxed{\frac{1}{2}}$

$P(1) = \langle\psi|1\rangle\langle 1|\psi\rangle =$

$= \left(\frac{\langle 0|1\rangle + \langle 1|1\rangle}{\sqrt{2}}\right)\left(\frac{\langle 1|0\rangle + \langle 1|1\rangle}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \boxed{\frac{1}{2}}$ $\qquad P(0) + P(1) = 1$

$P(+) = \langle\psi|+\rangle\langle +|\psi\rangle = \langle +|+\rangle\langle +|+\rangle = 1$ $\qquad \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle$

$P(-) = \langle +|-\rangle\langle -|+\rangle = 0$

d) $\langle\psi|0\rangle\langle 0|\psi\rangle = 0{,}8 \cdot 0{,}8 = 0{,}64 = P(0)$

$P(1) = \langle\psi|1\rangle\langle 1|\psi\rangle = 0{,}6 \cdot 0{,}6 = 0{,}36$

$P(+) = (0{,}8 ; 0{,}6)\frac{\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}{2}\begin{pmatrix} 0{,}8 \\ 0{,}6 \end{pmatrix} = \frac{(1{,}4 \quad 1{,}4)}{2}\begin{pmatrix} 0{,}8 \\ 0{,}6 \end{pmatrix} = \frac{1{,}4 \cdot 0{,}8 + 1{,}4 \cdot 0{,}6}{2} = 0{,}98$

$P(-) = (0{,}8 \quad 0{,}6)\frac{\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}}{2}\begin{pmatrix} 0{,}8 \\ 0{,}6 \end{pmatrix} = \frac{(0{,}2 \quad -0{,}2)}{2}\begin{pmatrix} 0{,}8 \\ 0{,}6 \end{pmatrix} = \frac{0{,}2 \cdot 0{,}8 - 0{,}2 \cdot 0{,}6}{2} = 0{,}02$

e) $P(0) = \cos^2\theta \qquad P(1) = \operatorname{sen}^2\theta$

f) $P(+) = (\cos\theta - i\operatorname{sen}\theta)\frac{\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}{2}\begin{pmatrix} \cos\theta \\ +i\operatorname{sen}\theta \end{pmatrix} = \frac{(\cos\theta - i\operatorname{sen}\theta \quad \cos\theta - i\operatorname{sen}\theta)}{2}\begin{pmatrix} \cos\theta \\ +i\operatorname{sen}\theta \end{pmatrix} =$

$= \frac{\cos^2\theta - i\operatorname{sen}\theta\cos\theta + i\operatorname{sen}\theta\cos\theta + \operatorname{sen}^2\theta}{2}$

$= \boxed{\frac{1}{2}}$

2) $|\psi\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$

$$\alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) =$$

$$= \left(\frac{\alpha+\beta}{\sqrt{2}}\right)|0\rangle + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)|1\rangle$$

$$\begin{cases} \frac{\alpha+\beta}{\sqrt{2}} = 3/5 \\ \frac{\alpha-\beta}{\sqrt{2}} = \frac{4}{5} \end{cases} \qquad \frac{2\alpha}{\sqrt{2}} = \frac{3}{5}+\frac{4}{5} \to \alpha = \frac{7}{5}\cdot\frac{\sqrt{2}}{2} = \frac{7\sqrt{2}}{10}$$

$$\beta = \frac{3}{5}\sqrt{2} - \frac{7\sqrt{2}}{10} = -\frac{\sqrt{2}}{10}$$

$$\boxed{|\varphi\rangle = \frac{7\sqrt{2}}{10}|+\rangle - \frac{\sqrt{2}}{10}|-\rangle}$$

II

1) Si B es unitaria $\to B B^{\dagger} = I \to \frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -i\\ -i & 1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} I$

B es unitaria

b)
$HSB|0\rangle = U|0\rangle$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & i\\ 1 & -i\end{pmatrix}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix} = \begin{pmatrix}0 & 2i\\ 2 & 0\end{pmatrix}\frac{1}{2} = \begin{pmatrix}0 & i\\ 1 & 0\end{pmatrix}$$

$$\boxed{U = \begin{pmatrix}0 & i\\ 1 & 0\end{pmatrix}}$$

2) $M|0\rangle = ((\alpha|0\rangle + \beta|1\rangle)\overset{1}{\langle 0|} + (\beta^*|0\rangle - \alpha^*|1\rangle)\overset{0}{\langle 1|})|0\rangle =$

$= (\alpha|0\rangle + \beta|1\rangle)$

$M|1\rangle = (\beta^*|0\rangle - \alpha^*|1\rangle))$

b) $M = \begin{pmatrix}\alpha & \beta^*\\ \beta & -\alpha^*\end{pmatrix}$

c) $\begin{pmatrix}\alpha & \beta^*\\ \beta & -\alpha^*\end{pmatrix}\begin{pmatrix}\alpha^* & \beta^*\\ \beta & -\alpha\end{pmatrix} = \begin{pmatrix}|\alpha|^2+|\beta|^2 & 0\\ 0 & |\alpha|^2+|\beta|^2\end{pmatrix} \underset{|\alpha|^2+|\beta|^2=1}{=} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

III

1) a. $X \otimes I = \begin{bmatrix} 0 & \begin{matrix}1&0\\0&1\end{matrix} \\ \begin{matrix}1&0\\0&1\end{matrix} & 0 \end{bmatrix}$ b) $I \otimes X = \begin{bmatrix} \begin{matrix}0&1\\1&0\end{matrix} & 0 \\ 0 & \begin{matrix}0&1\\1&0\end{matrix} \end{bmatrix}$

c) $X \otimes X = \begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix}$ d) $U_X = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$

$= \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{bmatrix} \quad \begin{matrix} I & 0 \\ 0 & X \end{matrix}$

$X . X^{\dagger} = \begin{bmatrix} 0&1 \\ 1&0 \end{bmatrix} \begin{bmatrix} 0&1 \\ 1&0 \end{bmatrix} = \begin{bmatrix} 1&0 \\ 0&1 \end{bmatrix} = I$ $X^{-1} = \dfrac{\begin{bmatrix} 0&-1 \\ -1&0 \end{bmatrix}}{-1} = \begin{bmatrix} 0&1 \\ 1&0 \end{bmatrix}$

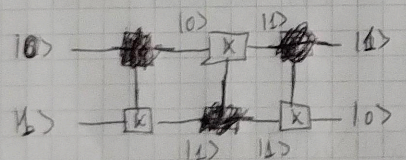
$Y . Y^{\dagger} = \begin{bmatrix} 0&-i \\ i&0 \end{bmatrix} \begin{bmatrix} 0&-i \\ i&0 \end{bmatrix} = \begin{bmatrix} 1&0 \\ 0&1 \end{bmatrix} = I$ $Y^{-1} = \dfrac{\begin{bmatrix} 0&i \\ -i&0 \end{bmatrix}}{-1} = \begin{bmatrix} 0&-i \\ i&0 \end{bmatrix}$

$Z . Z^{\dagger} = \begin{bmatrix} 1&0 \\ 0&-1 \end{bmatrix} \begin{bmatrix} 1&0 \\ 0&-1 \end{bmatrix} = \begin{bmatrix} 1&0 \\ 0&1 \end{bmatrix} = I$ $Z^{-1} = \dfrac{\begin{bmatrix} -1&0 \\ 0&1 \end{bmatrix}}{-1} = \begin{bmatrix} 1&0 \\ 0&-1 \end{bmatrix}$

2)

$U_S = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \\ 0&1&0&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix} = \begin{pmatrix} 1&0&0&0 \\ 0&0&0&1 \\ 0&1&0&0 \\ 0&0&1&0 \end{pmatrix} \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix} = \begin{pmatrix} 1&0&0&0 \\ 0&0&1&0 \\ 0&1&0&0 \\ 0&0&0&1 \end{pmatrix}$

$|00\rangle \to |00\rangle$
$|01\rangle \to |10\rangle$
$|10\rangle \to |01\rangle$
$|11\rangle \to |11\rangle$



II)

3) $H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}}$ $\qquad W = U_x (H \otimes I)$ $\qquad H \otimes I = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \frac{1}{\sqrt{2}}$

$$U_x \cdot (H \otimes I) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \cancel{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = W \cdot |\psi\rangle}$$

$$\cancel{W|\psi\rangle} = \cancel{\alpha|00\rangle} + \cancel{\beta|1\rangle}$$

$$W|00\rangle = \frac{\cancel{\;\;}}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{\cancel{\;\;}}{\sqrt{2}} \left[\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right] = \frac{\cancel{\;\;}(|00\rangle + |11\rangle)}{\sqrt{2}} = |\beta_{00}\rangle$$

$$W|01\rangle = \frac{\cancel{\;\;}}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \frac{\cancel{\;\;}}{\sqrt{2}}(|01\rangle + |10\rangle) = |\beta_{01}\rangle$$

$$W|10\rangle = \frac{\cancel{\;\;}}{\sqrt{2}} \begin{bmatrix} \; \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} = \frac{\cancel{\;\;}}{\sqrt{2}}(|00\rangle - |11\rangle) = |\beta_{10}\rangle$$

$$W|11\rangle = \frac{\cancel{\;\;}}{\sqrt{2}} \begin{bmatrix} \; \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} = \frac{\cancel{\;\;}}{\sqrt{2}}(|01\rangle - |10\rangle) = |\beta_{11}\rangle$$