PRACTICA 2

HOJA N

FECHA

1) .1 a) $|\phi_{AB}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

$\rho_{AB} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \cdot \frac{1}{\sqrt{2}}(\langle 00| \pm \langle 11|)$

$= \frac{1}{2}(|00\rangle\langle 00| \pm |11\rangle\langle 00| \pm |00\rangle\langle 11| + |11\rangle\langle 11|)$

$= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & \pm 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 0 & 1 \end{pmatrix}$

b) $|\psi_{AB}\rangle = \frac{|00\rangle + |10\rangle - |01\rangle - |11\rangle}{2}$

$\rho_{AB} = \frac{1}{4}(|00\rangle\langle 00| + |10\rangle\langle 00| - |01\rangle\langle 00| - |11\rangle\langle 00| +$
$|00\rangle\langle 10| + |10\rangle\langle 10| - |01\rangle\langle 10| - |11\rangle\langle 10|$
$- |00\rangle\langle 01| - |10\rangle\langle 01| + |01\rangle\langle 01| + |11\rangle\langle 01|$
$- |00\rangle\langle 11| - |10\rangle\langle 11| + |01\rangle\langle 11| + |11\rangle\langle 11|)$

$= \frac{1}{4}\begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$

COMO UN KET ES UN ESTADO PURO $|\psi_{AB}\rangle$ y $|\phi_{AB}\rangle$ SON PUROS

I.2

a) $\rho_A = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$

$E(A,B) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2 1/2 = 1$

Es maximamente entrelazado

b) $\rho_B = \frac{1}{2}(|0\rangle\langle 0| - |1\rangle\langle 0| - |0\rangle\langle 1| + |1\rangle\langle 1|)$

$E(A,B) = S(\rho_B) = -\frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\log 1/2 + 1/2 \log 1/2 - \frac{1}{2}\log\frac{1}{2}$

$= 0$ No esta entrelazado

I.3

a) Ya es su descomposicion de Smith

b) $\frac{1}{2}(|00\rangle + |10\rangle - |01\rangle - |11\rangle) =$

$\frac{1}{2}((|0\rangle + |1\rangle)|0\rangle - (|0\rangle + |1\rangle)|1\rangle)$

$\frac{1}{2}((|0\rangle + |1\rangle)(|0\rangle - |1\rangle)) = |+-\rangle$

$= |+-\rangle$

FECHA

I.4 $|\psi_{AB}\rangle = \alpha \frac{|00\rangle + |11\rangle}{\sqrt{2}} + \beta \frac{|01\rangle + |10\rangle}{\sqrt{2}}$

$= \frac{\alpha}{\sqrt{2}}|00\rangle + \frac{\alpha}{\sqrt{2}}|11\rangle + \frac{\beta}{\sqrt{2}}|01\rangle + \frac{\beta}{\sqrt{2}}|10\rangle$

$C = \frac{1}{\sqrt{2}}\begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$

$CC^{\dagger} = \frac{1}{2}\begin{pmatrix} \alpha^2 + \beta^2 & \alpha\beta^* + \alpha^*\beta \\ \alpha\beta^* + \alpha^*\beta & \alpha^2 + \beta^2 \end{pmatrix}$

$\mathrm{Det}(CC^{\dagger} - \lambda I) = 0$

$(\underbrace{\alpha^2 + \beta^2}_{1} - \lambda)^2 - (\alpha\beta^* + \alpha^*\beta)^2 = 0$

$(1 - \lambda)^2 = (\alpha\beta^* + \alpha^*\beta)^2$

$|1 - \lambda| = |\alpha\beta^* + \alpha^*\beta|$

$1 - \lambda = \pm(\alpha^*\beta + \alpha\beta^*)$

$\lambda_1 = 1 + \alpha^*\beta + \beta^*\alpha$

$\lambda_2 = 1 - (\alpha^*\beta + \beta^*\alpha)$

$\sigma = \sqrt{\lambda}$

$|\psi_{AB}\rangle = \sqrt{1 + \alpha^*\beta + \beta^*\alpha}\,\underbrace{|00\rangle}_{u_1 \otimes v_1} + \sqrt{1 - (\alpha^*\beta + \alpha\beta^*)}\,\underbrace{|11\rangle}_{u_2 \otimes v_2}$

I.5 $|\Psi_{AB}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$ $\tilde{\rho}_{AB} = \frac{|01\rangle\langle 01| + |10\rangle\langle 10|}{2}$

$\rho_{AB} = |\psi\rangle\langle\psi| = \frac{1}{2}\left(|01\rangle\langle 01| + |10\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 10|\right)$

$= \tilde{\rho}_{AB} + \frac{1}{2}\left(|10\rangle\langle 01| + |01\rangle\langle 10|\right)$

$\langle O\rangle = Tr(O_A \rho_A) = \langle O_A\rangle$

$\tilde{\rho}_A = \rho_A = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$

$\langle O\rangle = Tr\left(\frac{O_A}{2}\right)$ No se puede diferenciar

$O_A = X = O_B$

$\langle O\rangle = Tr\left(\frac{1}{2}\left(|10\rangle\langle 01| + |01\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 10|\right)\right) = 1$

$\langle \tilde{O}\rangle = Tr\left(\frac{1}{2}\left(|10\rangle\langle 01| + |01\rangle\langle 01|\right)\right) = 0$ Si se puede

II.1 $\rho = |\psi\rangle\langle\psi|$

$\rho^2 = |\psi\rangle\langle\psi||\psi\rangle\langle\psi| = |\psi\rangle\langle\psi|$

$\rho = \sum^{n_s} \sigma_k^2 |k\rangle\langle k|$ $\delta_{kk'} \begin{cases} 1 & k = k' \\ 0 & k \neq k' \end{cases}$

$\rho \cdot \rho = \sum_{k,k'} \sigma_k^2 \sigma_{k'}^2 |k\rangle\langle k|k'\rangle\langle k'|$

$\rho^2 = \sum_k \lambda_k^2 |k\rangle\langle k|$ $\lambda = \sigma$

$\sum \lambda |k\rangle\langle k| = \sum \lambda^2 |k\rangle\langle k|$

$\lambda^2 = \lambda$

$\lambda^2 - \lambda = 0$ $\lambda = 1$ Como $Tr(\rho) = 1$

$\lambda(\lambda - 1) = 0$ ↗

II.2

$$\rho = \sum p_i \rho_i \qquad p_i \geq 0$$

$$= \sum p_i |\psi_i\rangle\langle\psi_i|$$

$$\mathrm{Tr}\,\rho = \sum p_i \mathrm{Tr}(|\psi_i\rangle\langle\psi_i|) = 1$$

$\rho_i = \rho_i^+$

$\therefore \rho = \rho^+$

$$\langle\varphi|\rho|\varphi\rangle = \langle\varphi|\sum p_k \rho_k|\varphi\rangle \qquad |\varphi\rangle = \sum \alpha_i |i\rangle$$

$$= \sum_i \alpha_i \langle j| \sum p_k |\psi_k\rangle\langle\psi_k| \sum_j \alpha_j^{\prime} |i\rangle$$

$$= \sum_{i,j,k} \alpha_i \alpha_j^{\prime} p_k \underbrace{\langle i|\psi_k\rangle}_{\delta_{ik}} \underbrace{\langle\psi_k|j\rangle}_{\delta_{kj}}$$

$$= \sum_k |\alpha_k|^2 p_k \geq 0$$

II.3

$$p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1| \qquad p \in (1/2, 1)$$

$$q|\alpha\rangle\langle\alpha| + (1-q)|\beta\rangle\langle\beta| \qquad q \in [1-p, p]$$

$$\sqrt{q}\,|a_i\rangle = \sum u_{ii} \sqrt{p_i}\,|i\rangle$$

$$\sqrt{q}\,|\alpha\rangle = u_{11}\sqrt{p}\,|0\rangle + u_{12}\sqrt{(1-p)}\,|1\rangle$$

$$\sqrt{1-q}\,|\beta\rangle = u_{21}\sqrt{p}\,|0\rangle + u_{22}\sqrt{(1-p)}\,|1\rangle$$

$$\sqrt{q}\sqrt{q}\,\langle a_i|a_i\rangle = q$$

II.5. $\rho = x|\phi\rangle\langle\phi| + (1-x)\frac{\mathbb{I}_d}{d}$

$\mathrm{Tr}\,\rho = x + (1-x)\frac{d}{d} = 1 \;\forall x$

$\rho^{\dagger} = \rho$

$\langle\phi|\rho|\phi\rangle = x\langle\phi|\phi\rangle + \frac{(1-x)}{d}\langle\phi|\mathbb{I}|\phi\rangle \geq 0$

$x + \frac{(1-x)}{d} \geq 0$

$xd + 1 - x \geq 0$

$x \geq \frac{-1}{(d-1)} = \frac{1}{1-d}$

Si $x=1$ es puro, sino es ruidoso