1) A) $(|0\rangle + |1\rangle)/\sqrt{2}$.

$\psi\, \psi^{\dagger} = 1 \qquad \langle \psi | \psi \rangle = 1$

$(\alpha\, |0\rangle + \beta\, |1\rangle) \cdot (\alpha^* \langle 0| + \beta^* \langle 1|) = |\alpha^2| + |\beta^2| = 1$

$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1 \rightarrow$ Es un estado posible

$P(0) = \frac{1}{2} \rightarrow |\alpha|^2 \qquad P(1) = \frac{1}{2} \rightarrow |\beta|^2$

$P(+) = \frac{(\langle 0| + \langle 1|)}{\sqrt{2}} \cdot \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} = \frac{1+1}{2} = 1$

$P(-) = \frac{\langle 0| - \langle 1|}{\sqrt{2}} \cdot \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{1-1}{2} = 0$

b) $\frac{\sqrt{3}}{2} |0\rangle - \frac{1}{2} |1\rangle \qquad \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$ ✓

$P(0) = \frac{3}{4} \qquad P(1) = \frac{1}{4} \qquad P(+) = \frac{\langle 0| + \langle 1|}{\sqrt{2}} \cdot \left(\frac{\sqrt{3}}{2} |0\rangle - \frac{1}{2} |1\rangle\right)$

$\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \left|\frac{\sqrt{3} - 1}{2\sqrt{2}}\right|^2 = 0{,}07$

$P(-) = \left|\frac{\sqrt{3} + 1}{2\sqrt{2}}\right|^2 = 0{,}93$

c) $0{,}7^2 + 0{,}3^2 \neq 1$ X

D) $0{,}8^2 + 0{,}6^2 = 0{,}64 + 0{,}36 = 1$

$P(0) = 0{,}64 \qquad P(1) = 0{,}36 \qquad P(+) = \left| \frac{\langle 0| + \langle 1|}{\sqrt{2}} \right| (0{,}8|0\rangle + 0{,}6\,|1\rangle)$

$\left|\frac{0{,}8 + 0{,}6}{\sqrt{2}}\right|^2 = 0{,}99$

$P(-) = \left|\frac{0{,}8 - 0{,}6}{\sqrt{2}}\right|^2 = 0{,}02$

Escaneado con CamScanner

E) $\cos^2(\theta) + \operatorname{sen}^2(\theta) = 1 \quad \forall\, \theta \quad \therefore$ es un estado posible

$P(0) = \cos^2(\theta) \qquad P(1) = \operatorname{sen}^2(\theta) \qquad P(+) = \frac{\langle 0| + \langle 1|}{\sqrt{2}} \left(\cos(\theta)|0\rangle + i\operatorname{sen}(\theta)|1\rangle\right)$

$$= \left|\frac{\cos(\theta) + i\operatorname{sen}(\theta)}{\sqrt{2}}\right|^2$$

$$= \frac{\cos^2(\theta) + \operatorname{sen}^2(\theta)}{2} = \frac{1}{2}$$

$$P(-) = \frac{|\cos^2(\theta) - i\operatorname{sen}^2(\theta)|}{2} = \frac{1}{2}$$

F) $\cos^4(\theta) + \operatorname{sen}^4(\theta) \to$ No es 1 para todo θ X

2) $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle \qquad |\psi\rangle = \alpha|+\rangle + \beta|-\rangle = \frac{\alpha}{\sqrt{2}}|0\rangle + \frac{\alpha}{\sqrt{2}}|1\rangle + \frac{\beta}{\sqrt{2}}|0\rangle - \frac{\beta}{\sqrt{2}}|1\rangle$

$$\frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$$

$= \frac{\alpha+\beta}{\sqrt{2}} = \frac{3}{5} \qquad \frac{\alpha-\beta}{\sqrt{2}} = \frac{4}{5}$

$= \alpha+\beta = \frac{3\sqrt{2}}{5} \qquad \alpha-\beta = \frac{4}{5}\sqrt{2} \qquad 2\alpha - \beta + \beta = \frac{3}{5}\sqrt{2} + \frac{4}{5}\sqrt{2}$

$$\alpha = \frac{7}{10}\sqrt{2}$$

$$\beta = \frac{3}{5}\sqrt{2} - \frac{7}{10}\sqrt{2} = -\frac{\sqrt{2}}{10}$$

$$|\psi\rangle = \frac{7\sqrt{2}}{10}|+\rangle - \frac{\sqrt{2}}{10}|-\rangle$$

II) 1) A) $\quad B\,B^{\dagger} = I \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \to$ Es unitaria

$U = H \cdot S \cdot B \qquad H.S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\frac{1}{\sqrt{2}}$

$HSB = \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{2}\begin{pmatrix} 0 & 2i \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} = U$

$$\begin{pmatrix}0\\1\end{pmatrix}(1\ 0) = \begin{pmatrix}0&0\\1&0\end{pmatrix}$$

2) $M|0\rangle = (\alpha|0\rangle+\beta|1\rangle)\overset{1}{\langle 0|0\rangle} + (\beta^*|0\rangle - \alpha^*|1\rangle)\overset{0}{\langle 1|0\rangle}$

$M|0\rangle = \alpha|0\rangle + \beta|1\rangle$ ✓

$M|1\rangle = (\alpha|0\rangle + \beta|1\rangle)\overset{0}{\langle 0|1\rangle} + (\beta^*|0\rangle - \alpha^*|1\rangle)\langle 1|1\rangle$

$M|1\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$ ✓

$\alpha|0\rangle\langle 0| + \beta|1\rangle\langle 0| + \beta^*|0\rangle\langle 1| - \alpha^*|1\rangle\langle 1|$

$$\begin{pmatrix}\alpha&0\\0&0\end{pmatrix} + \begin{pmatrix}0&0\\\beta&0\end{pmatrix} + \begin{pmatrix}0&\beta^*\\0&0\end{pmatrix} - \begin{pmatrix}0&0\\0&\alpha^*\end{pmatrix} = \begin{pmatrix}\alpha&\beta^*\\\beta&-\alpha^*\end{pmatrix}$$

$M \cdot M^\dagger = I$

$$\begin{pmatrix}\alpha&\beta^*\\\beta&-\alpha^*\end{pmatrix}\cdot\begin{pmatrix}\alpha^*&\beta^*\\\beta&-\alpha\end{pmatrix} = \begin{pmatrix}\alpha\alpha^*+\beta^*\beta & \alpha\beta^*-\beta^*\alpha\\ \beta\alpha^*-\beta\alpha^* & \beta\beta^*+\alpha\alpha^*\end{pmatrix} = \begin{pmatrix}\boxed{|\alpha|^2+|\beta|^2}&0\\0&|\alpha|^2+|\beta|^2\end{pmatrix}$$

$=1$ Porque es un estado

$\begin{pmatrix}1&0\\0&1\end{pmatrix} = I \therefore M$ es unitaria

III) $X\otimes I = \begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} = \begin{pmatrix}0&I\\I&0\end{pmatrix}$

B) $I\otimes X = \begin{pmatrix}X&0\\0&X\end{pmatrix}$

C) $X\otimes X = \begin{pmatrix}0&X\\X&0\end{pmatrix}$

D) $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$

Verificación unitaria $U\cdot U^\dagger = I$

$$\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}\cdot\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = I \text{ Es unitaria}$$

$$\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = I \text{ Es unitaria}$$

$$\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = I \text{ Es unitaria}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$ Es unitaria

Como siempre $U \cdot U^\dagger = I$ y para encontrar la inversa hay que hacer $U \cdot U^{-1} = I$

Entonces $U^\dagger = U^{-1}$ y en estos casos $U^\dagger = U$ ∴ Todas sus inversas son ellas mismas

Entrada $|00\rangle$

$$\left(|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X\right) \cdot |00\rangle = \left(|0\rangle\langle 0| \otimes I\right)\left(|0\rangle \otimes |0\rangle\right) + |1\rangle\langle 1| \otimes X \left(|0\rangle \otimes |0\rangle\right)$$

$$= |0\rangle\langle 0|0\rangle \otimes I|0\rangle + \cancel{|1\rangle\langle 1|0\rangle}^{\,0} \otimes X|0\rangle = |0\rangle \otimes |0\rangle = |00\rangle$$

Entrada $|01\rangle$

$$|0\rangle\langle 0|0\rangle \otimes I|1\rangle + \cancel{|1\rangle\langle 1|0\rangle}^{\,0} \otimes X|1\rangle = |01\rangle$$

Entrada $|10\rangle$

$$|0\rangle\langle 0|1\rangle \otimes I|0\rangle + |1\rangle\langle 1|1\rangle \otimes \overbrace{X|0\rangle}^{|1\rangle} = |11\rangle$$

Entrada $|11\rangle$

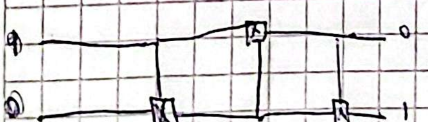
$$|0\rangle\langle 0|1\rangle \otimes I|1\rangle + |1\rangle\langle 1|1\rangle \otimes X|1\rangle = |10\rangle$$

$U_x|00\rangle = |00\rangle$ $\quad U_x|01\rangle = |01\rangle$ $\quad U_x|10\rangle = |11\rangle$ $\quad U_x|11\rangle = |10\rangle$ → Control Not controlado

2)

$$\left.\begin{array}{l} |00\rangle — U_x \xrightarrow{|00\rangle} \bar{U}_x \xrightarrow{|00\rangle} U_x — |00\rangle \\ |01\rangle — U_x \xrightarrow{|01\rangle} \bar{U}_x \xrightarrow{|11\rangle} U_x — |10\rangle \\ |10\rangle — U_x \xrightarrow[\ ]{|11\rangle} \bar{U}_x \xrightarrow[|10\rangle]{|01\rangle} U_x — |01\rangle \\ |11\rangle — U_x \xrightarrow{|10\rangle} \bar{U}_x \xrightarrow{} U_x — |11\rangle \end{array}\right\}$$ Lo que le hace a la base

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ Matriz que representa



✓ circuito

$$U_x \cdot (H \otimes I) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}$$

$$W|00\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \qquad W|01\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \qquad W|10\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$W|11\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$