TP2

1) a)

$$|\phi_{AB}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$$

$$\rho_{AB} = |\phi_{AB}\rangle\langle\phi_{AB}| = \frac{1}{2}\left(|00\rangle\langle 00| \pm |00\rangle\langle 11| \pm |11\rangle\langle 00| + |11\rangle\langle 11|\right) = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & \pm 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 0 & 1 \end{bmatrix}$$

b) $|\psi_{AB}\rangle = \frac{|00\rangle + |10\rangle - |01\rangle - |11\rangle}{2}$ $\rho_{AB} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ +1 \\ -1 \end{pmatrix}\frac{(1 + + 1)}{2} = \begin{pmatrix} 1 & -1 & +1 & -1 \\ -1 & 1 & -1 & +1 \\ +1 & -1 & 1 & -1 \\ -1 & +1 & -1 & 1 \end{pmatrix}$

I.2

a) $\rho_A = Tr_B(\rho_{AB}) = Tr_B\left(\frac{1}{2}\overset{=}{\overbrace{|0\overset{AB}{0}\rangle\langle\overset{AB}{0}0|}} \pm |00\rangle\langle 11| \pm |11\rangle\langle 00| + \overbrace{|11\rangle\langle 11|}\right) = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$

$$S = -\frac{1}{2}\log\frac{1}{2} + \left(-\frac{1}{2}\right)\log\frac{1}{2} = \frac{1}{2} + \frac{1}{2} = \boxed{1}$$ ← Maximamente entrelazado

b) $\rho_A = Tr_B(\rho_{AB}) = \begin{bmatrix} \frac{1}{2} & +\frac{1}{2} \\ +\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\right)$

(haciendo la traza de los submatrices)

$$= \frac{1}{2}\left[(|0\rangle + |1\rangle)\langle 0| + (|0\rangle + |1\rangle)\langle 1|\right]$$

$$= \frac{1}{2}(|0\rangle + |1\rangle)(\langle 0| + \langle 1|) = 1|+\rangle\langle +|$$

$$S = -1\log 1 = 0$$

I.3

a) Ya está en su descomposición de Schmidt

b) $\frac{1}{2}\left(|00\rangle + |10\rangle - |01\rangle - |11\rangle\right) = \frac{1}{2}\left((|0\rangle + |1\rangle)|0\rangle - (|0\rangle + |1\rangle)|1\rangle\right) =$

$\frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle - |1\rangle) = |+-\rangle$ ← Desc de Schmidt

I4 $|\alpha|^2+|\beta|^2=1$ $|\psi_{AB}\rangle=\alpha\frac{|00\rangle+|11\rangle}{\sqrt{2}}+\beta\frac{|01\rangle+|10\rangle}{\sqrt{2}}$

$$=\frac{\alpha}{\sqrt{2}}|00\rangle+\frac{\beta}{\sqrt{2}}|01\rangle+\frac{\beta}{\sqrt{2}}|10\rangle+\frac{\alpha}{\sqrt{2}}|11\rangle$$

$$C=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}\frac{1}{\sqrt{2}}$$

Si $\alpha=0$ y $\beta=1$ y viceversa
El estado será separable

$$C^{\dagger}C=\frac{\begin{bmatrix}\alpha^* & \beta^*\\ \beta^* & \alpha^*\end{bmatrix}}{\sqrt{2}}\frac{\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}}{\sqrt{2}}=\begin{bmatrix}|\alpha|^2+|\beta|^2 & \alpha^*\beta+\beta^*\alpha\\ \beta^*\alpha+\alpha^*\beta & |\alpha|^2+|\beta|^2\end{bmatrix}$$

$$\mathrm{Det}(C^{\dagger}C-I\lambda)=0$$

$$(|\alpha|^2+|\beta|^2-\lambda)(|\alpha|^2+|\beta|^2-\lambda)-(\alpha^*\beta+\beta^*\alpha)^2=0$$

$$(|\alpha|^2+|\beta|^2-\lambda)^2-(\alpha^*\beta+\beta^*\alpha)^2=0$$

$$(1-\lambda)^2-(\alpha^*\beta+\beta^*\alpha)^2=0$$

$$|1-\lambda|=|\alpha^*\beta+\beta^*\alpha|$$

$$1-\lambda= \begin{cases}-(\alpha^*\beta+\beta^*\alpha)\\ \alpha^*\beta+\beta^*\alpha\end{cases}$$

$$\lambda_1=1+\alpha^*\beta+\beta^*\alpha$$
$$\lambda_2=1-\alpha^*\beta+\beta^*\alpha$$

$$\sigma=\sqrt{\lambda(C^{\dagger}C)}$$

$$\sigma^2=\lambda(C^{\dagger}C)$$

Desc. Schmidt

$$|\psi_{AB}\rangle=\sqrt{(1+\alpha^*\beta+\alpha\beta^*)}\,|01\rangle+\sqrt{1-\alpha^*\beta+\alpha\beta^*}\,|10\rangle$$

I 5

$$\rho_\psi = |\psi_{AB}\rangle\langle\psi_{AB}| = \frac{1}{2}\Big(|01\rangle\langle 01| + \overbrace{|01\rangle\langle 10| + |10\rangle\langle 01|}^{\text{La traza se diferencia en esta parte.}} + |10\rangle\langle 10|\Big) \rightarrow \rho_{\psi_A} = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}/2$$

b) $O_A = X$

$$\langle O\rangle = Tr(\rho O) = Tr(\rho_A(O_A \otimes I_B)) = Tr(\overset{X}{O_A}\rho_A)$$

$$= Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}/2\right) = Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}/2\right) = 0 = Tr(O_A\rho_{\psi_A})$$

Indistinguible con observable local (La traza parcial ρ_A es igual)

$$\langle O\rangle = Tr(\rho_{AB}(O_A \otimes O_B)) =$$

$$= Tr((O_A \otimes O_B)\rho_{AB})$$

$$Tr\left(X_A \otimes X_B\left(\frac{1}{2}\left(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 10|\right)\right)\right) =$$

$$= Tr\left(\frac{1}{2}\left(|10\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 01| + |01\rangle\langle 10|\right)\right) = \frac{1}{2} + \frac{1}{2} = \boxed{1}$$

$$\langle O\rangle = Tr\left((X_A \otimes X_B)\rho\right) = \frac{1}{2}\left(|01\rangle\langle 01| + |10\rangle\langle 10|\right)$$

$$= Tr\left(\frac{1}{2}\left(|10\rangle\langle 01| + |01\rangle\langle 10|\right)\right) = \boxed{0} \longrightarrow \text{Distinguibles.}$$

$_A(O_A \rho_A)$

Observable local depende de A

Entonces en 2 ρ_A iguales que provienen de ρ_{AB} distintos, no se pueden distinguir.

Para diferenciar se hace $O = O_A \otimes O_B$

y $Tr((O_A \otimes O_B)\rho_{AB})$ $\quad O_A$ y $O_B \to$ Matrices de Pauli

(II)

1) • $\rho = |\psi\rangle\langle\psi| \to \rho^2 = |\psi\rangle\overbrace{\langle\psi|\psi\rangle}^{1}\langle\psi| = |\psi\rangle\langle\psi| = \rho$

• $\rho = \sum^{rs} \sigma_k^2 |k\rangle\langle k|$

$\rho \cdot \rho = \sum_{k,k'} \sigma_k^2 \sigma_{k'}^2 |k\rangle\overbrace{\langle k|k'\rangle}^{\delta kk'}\langle k'|$

$\rho^2 = \sum_k \sigma_k^2 |k\rangle\langle k| = \rho$

② $Tr(\rho) = 1$, $\rho^+ = \rho$ $\lambda(\rho) \geqslant 0$; $\rho = \sum p_i \rho_i = \sum p_i |\psi\rangle\langle\psi_i|$

$\to Tr(\rho) = Tr(\sum p_i |\psi_i\rangle\langle\psi_i|) = \underbrace{\sum p_i}_{1} \underbrace{Tr(|\psi_i\rangle\langle\psi_i|)}_{1} = 1$

$\to$ Si $\rho_i = \rho_i^+ \to \rho = \rho^+$

$\to \langle\psi|\rho|\psi\rangle = \langle\psi|\sum p_k \rho_k|\psi\rangle$ $\quad |\psi\rangle = \sum \alpha_i |i\rangle$

$= \sum \alpha_j^+ \langle j|\sum p_k |\psi_k\rangle\langle\psi_k|\sum_i \alpha_i |i\rangle$

$= \sum_{i,j,k} \alpha_i \alpha_j^+ p_k \underbrace{\langle j|\psi_k\rangle}_{\delta_{jk}} \underbrace{\langle\psi_k|i\rangle}_{\delta_{ki}}$

$= \sum_k |\alpha_k|^2 p_k \geqslant 0$

③
$\sqrt{q}\,|a_i\rangle = \sum u_{ii} \sqrt{p_i}\,|i\rangle$

$\sqrt{q}\,|\alpha\rangle = u_{11}\sqrt{p}\,|0\rangle + u_{12}\sqrt{(1-p)}\,|1\rangle$

$\sqrt{1-q}\,|\beta\rangle = u_{21}\sqrt{p}\,|0\rangle + u_{22}\sqrt{1-p}\,|1\rangle$

$\sqrt{q}\cdot\sqrt{q}\,\langle k|k\rangle = q$

$\sqrt{1-q}\,\sqrt{1-q}\,\langle\beta|\beta\rangle = 1-q$

$U = \begin{pmatrix} u_{11} , & u_{12} \\ u_{21} & u_{22} \end{pmatrix}$

(5)

- $\rho = x|\phi\rangle\langle\phi| + (1-x)\frac{I_d}{d}$

$Tr\,\rho = x + (1-x)\frac{d}{d} =$

$= x + 1 - x = 1 \rightarrow$ Para cualquier valor de $x \rightarrow Tr\,\rho = 1$

No nos da nada

- $\langle\phi|\rho|\phi\rangle = x\overbrace{\langle\phi|\phi\rangle}^{1}\overbrace{\langle\phi|\phi\rangle}^{1} + (1-x)\overbrace{\langle\phi|\frac{I_d}{d}|\phi\rangle}^{1} \geq 0$

$x + \frac{(1-x)}{d} \geq 0$

$xd + 1 - x \geq 0$

$x(d-1) \geq -1$

$x \geq \frac{1}{1-d}$

Para que sea un op. densidad

Si $x = 1 \rightarrow$ Puro

Si no, tiene ruido $\left((1-x)\frac{I_d}{d}\right)$