# Lecture Notes to a Course on Non-Commutative Algebra

**Taught by Prof. Eli Aljadeff at Technion IIT during Spring 2022**

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March 21, 2022

# Introduction

## Course Information

## Prerequisites

The course will assume undergraduate knowledge in group theory and ring theory.

## Notations & Conventions

- All $k$-algebras are assumed to be associative and unital, but not necessarily commutative.

## Course Goals & Motivation

### Course Goals

During the course we will go over the following topics.

- Category Theory & other algebraic tools.
- Rings & the Jacobson radical.
- The Wedderburn-Artin theorem.
- Central simple algebras.
- Brauer group theory.
- Group cohomology & Galois cohomology.

### Motivation

Let $k$ be a field and consider finite-dimensional $k$-algebras $A$ that are *simple* in the sense that there are no nontrivial two-sided ideals. Assume furthermore that $A$ is *central* in the sense that its centre, $Z(A)$, is $k$. Wedderburn-Artin theorem, any such algebra is a finite product of matrix algebras over division algebras.

**Example.** The following are central simple algebras over a field $K$:

1. $K$.

2. $M_n(K)$ for any $n \in \mathbb{N}$.

3. $$\mathbb{H}_K \coloneqq K\left\langle i, j, k \;\middle|\; \substack{i^2=j^2=k^2 \\ ij=ji=k \\ ik=-ki=-j \\ jk=-kj=i} \right\rangle.$$

   Over $\mathbb{C}$ we have $\mathbb{H}_{\mathbb{C}} \cong M_2(\mathbb{C})$, which is a division algebra contained in $\mathbb{H}_{\mathbb{R}}$, which isn't by itself a division algebra.