# Proof of Correctedness for the Sum Square Difference

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**Proposition.** *Let $N \in \mathbb{N}_+$ be a positive integer. Then*

$$\left( \sum_{i \in [N]} i \right)^2 - \sum_{i \in [N]} i^2 = 2 \sum_{i,j \in \binom{[N]}{2}} ij.$$

*Proof.* By the multinomial theorem,

$$\left( \sum_{i \in [N]} i \right)^2 = \sum_{\substack{\sum_{i \in [N]} k_i = 2 \\ k_i \geq 0}} \binom{2}{k_1, k_2, \ldots, k_N} \prod_{i \in [N]} i^{k_i}$$

$$= \sum_{i \in [N]} \binom{2}{2} i^2 + \sum_{i,j \in \binom{[N]}{2}} \binom{2}{1} ij$$

$$= \sum_{i \in [N]} i^2 + 2 \sum_{i,j \in \binom{[N]}{2}} ij.$$

$\square$