Proof for Counting Lexicographic Permutations

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We start by stating a well-known theorem.

Theorem 0.1. *The number of permutations of a set of* n *elements are* $n!$*.*

Corollary 0.2. *The number of permutations of a set of* n *elements such that the first element is fixed is* $(n-1)!$*.*

Proof. Permutations of n elements such that the first element is fixed, correspond directly to permutations of the other elements. □

Corollary 0.3. *In order to find the* k^{th} *permutation of the set* $\{0, 1, \ldots, 9\}$*, in lexicographic order, we may follow the following algorithm.*

1. *Let* m *be the number of elements in the set* $\{0, \ldots, 9\}$ *minus one.*
2. *Set the left-most remaining digit to* $\left\lceil \frac{k}{m!} \right\rceil^{th}$ *smallest digit.*
3. *Change the value of* k *to* $k - \left(\left\lceil \frac{k}{m!} \right\rceil - 1\right) \cdot m!$*, and decrease the value of* m *by* 1*.*
4. *If there are more digits to determine, go back to step* 2 *with the rest of the digits available.*

Proof. Each digit appears as the left-most digit in the first $m!$ permutations, where m is the number of digits left to look at. The left-most digit is therefore determined by the number of times $9!$ goes into k.

The next digit is determined similarly by the number of times $8!$ goes into how many permutations are left to count from the first one with the first digit being fixed, etc. □