Forall-x: Calgary, Model-Generated Solutions

David Llewellyn-Jones

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This document presents proofs for all of the exercises and examples in the Forall-x: Calgary book, covering Truth Functional Logic and First Order Logic. The Modal Logic proofs are not covered.

All of these proofs were generated by the Qwen3.5-35B-A3B-UD model quantised to Q8_K_XL format with inference running on an Nvidia A100 or H100 GPU.

The proofs were generated and checked to be correct using a proof checking tool and have been transcribed directly into this document. Consequently although all of the proofs are correct and give the expected result, they may not be the most efficient or attractive solutions to the questions.

In some cases – for example where a question asks for an equivalence to be proven – the solutions may be split into multiple parts.

Each proof is presented using Fitch notation and is followed by the specific commands needed to enter the proof into Daeducer, the proof tool they were created with.

Proofs

Chapter 17, Example p.134

Proof presented using Fitch notation:

1	$((P \wedge Q) \vee (P \wedge R))$	PR
2	$\quad (P \wedge Q)$	AS
3	$\quad P$	$\wedge$E, 2
4	$\quad (P \wedge R)$	AS
5	$\quad P$	$\wedge$E, 4
6	P	$\vee$E, 1, 2–3, 4–5

Daeducer commands needed to generate the proof:

```
premise ((P ^ Q) v (P ^ R))
assumption (P ^ Q)
and_elim_left 2
discharge
assumption (P ^ R)
and_elim_left 5
discharge
or_elim 1, 2, 3, 5, 6
qed
```

Chapter 17, Example p.134

Proof presented using Fitch notation:

1	$(A \wedge (B \vee C))$	PR
2	A	$\wedge$E, 1
3	$(B \vee C)$	$\wedge$E, 1
4	$\quad B$	AS
5	$\quad (A \wedge B)$	$\wedge$I, 2, 4
6	$\quad ((A \wedge B) \vee (A \wedge C))$	$\vee$E, 5
7	$\quad C$	AS
8	$\quad (A \wedge C)$	$\wedge$I, 2, 7
9	$\quad ((A \wedge B) \vee (A \wedge C))$	$\vee$E, 8
10	$((A \wedge B) \vee (A \wedge C))$	$\vee$E, 3, 4–6, 7–9

Daeducer commands needed to generate the proof:

```
premise (A ^ (B v C))
and_elim_left 1
and_elim_right 1
assumption B
and_intro 2, 4
or_intro_left 5, (A ^ C)
discharge
assumption C
and_intro 2, 8
or_intro_right 9, (A ^ B)
discharge
or_elim 3, 4, 6, 8, 10
qed
```

Chapter 17, Example p.137

Proof presented using Fitch notation:

1	D	PR
2	$\quad \neg D$	AS
3	$\quad \bot$	$\neg$E, 2, 1
4	$\neg\neg D$	$\neg$I, 2–3

Daeducer commands needed to generate the proof:

```
premise D
assumption !D
not_elim 2, 1
discharge
not_intro 2, 3
qed
```

Chapter 17, Example p.138

Proof presented using Fitch notation:

1	$\neg\neg D$	PR
2	$\quad \neg D$	AS
3	$\quad \bot$	$\neg$E, 1, 2
4	D	IP, 2–3

Daeducer commands needed to generate the proof:

```
premise !!D
assumption !D
not_elim 1, 2
discharge
indirect 2, 3
qed
```

Chapter 17, Exercise A.1

Proof presented using Fitch notation:

1	$(A \wedge (B \wedge C))$	PR
2	$((B \vee C) \rightarrow D)$	PR
3	$(B \wedge C)$	$\wedge$E, 1
4	B	$\wedge$E, 3
5	$(B \vee C)$	$\vee$E, 4
6	D	$\rightarrow$E, 2, 5

Daeducer commands needed to generate the proof:

```
premise (A ^ (B ^ C))
premise ((B v C) -> D)
and_elim_right 1
and_elim_left 3
or_intro_left 4, C
imp_elim 2, 5
qed
```

Chapter 17, Exercise B.1

Proof presented using Fitch notation:

1	$(P \wedge S)$	PR
2	$(S \rightarrow R)$	PR
3	S	$\wedge$E, 1
4	R	$\rightarrow$E, 2, 3
5	$(R \vee E)$	$\vee$E, 4

Daeducer commands needed to generate the proof:

```
premise (P ^ S)
premise (S -> R)
and_elim_right 1
imp_elim 2, 3
or_intro_left 4, E
qed
```

Chapter 17, Exercise B.2

Proof presented using Fitch notation:

1	$(A \rightarrow D)$	PR
2	$\quad (A \wedge B)$	AS
3	$\quad A$	$\wedge$E, 2
4	$\quad D$	$\rightarrow$E, 1, 3
5	$\quad (D \vee E)$	$\vee$E, 4
6	$((A \wedge B) \rightarrow (D \vee E))$	$\rightarrow$I, 2, 5

Daeducer commands needed to generate the proof:

```
premise (A -> D)
assumption (A ^ B)
and_elim_left 2
imp_elim 1, 3
or_intro_left 4, E
discharge
imp_intro 2, 5
qed
```

Chapter 17, Exercise B.3

Proof presented using Fitch notation:

1	$(\neg L \to (J \vee L))$	PR
2	$\neg L$	PR
3	$(J \vee L)$	$\to$E, 1, 2
4	$\quad J$	AS
5	$\quad J$	R, 4
6	$\quad L$	AS
7	$\quad \bot$	$\neg$E, 2, 6
8	$\quad J$	X, 7
9	J	$\vee$E, 3, 4–5, 6–8

Daeducer commands needed to generate the proof:

```
premise (!L -> (J v L))
premise !L
imp_elim 1, 2
assumption J
reiteration 4
discharge
assumption L
not_elim 2, 7
explosion 8, J
discharge
or_elim 3, 4, 5, 7, 9
qed
```

Chapter 17, Exercise C.1

Proof presented using Fitch notation:

1	$(J \to \neg J)$	PR
2	$\quad J$	AS
3	$\quad \neg J$	$\to$E, 1, 2
4	$\quad \bot$	$\neg$E, 3, 2
5	$\neg J$	$\neg$I, 2–4

Daeducer commands needed to generate the proof:

```
premise (J -> !J)
assumption J
imp_elim 1, 2
not_elim 3, 2
discharge
not_intro 2, 4
qed
```

Chapter 17, Exercise C.2

Proof presented using Fitch notation:

1	$(Q \to (Q \land \neg Q))$	PR
2	$\quad Q$	AS
3	$\quad (Q \land \neg Q)$	$\to$E, 1, 2
4	$\quad \neg Q$	$\land$E, 3
5	$\quad \bot$	$\neg$E, 4, 2
6	$\neg Q$	$\neg$I, 2–5

Daeducer commands needed to generate the proof:

```
premise (Q -> (Q ^ !Q))
assumption Q
imp_elim 1, 2
and_elim_right 3
not_elim 4, 2
discharge
not_intro 2, 5
qed
```

Chapter 17, Exercise C.3

Proof presented using Fitch notation:

1	$(A \to (B \to C))$	PR
2	$\quad (A \land B)$	AS
3	$\quad A$	$\land$E, 2
4	$\quad B$	$\land$E, 2
5	$\quad (B \to C)$	$\to$E, 1, 3
6	$\quad C$	$\to$E, 5, 4
7	$((A \land B) \to C)$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise (A -> (B -> C))
assumption (A ^ B)
and_elim_left 2
and_elim_right 2
imp_elim 1, 3
imp_elim 5, 4
discharge
imp_intro 2, 6
qed
```

Chapter 17, Exercise C.4

Proof presented using Fitch notation:

1	$(K \wedge L)$	PR
2	K	$\wedge$E, 1
3	L	$\wedge$E, 1
4	$\quad K$	AS
5	$\quad L$	R, 3
6	$(K \rightarrow L)$	$\rightarrow$I, 4, 5
7	$\quad L$	AS
8	$\quad K$	R, 2
9	$(L \rightarrow K)$	$\rightarrow$I, 7, 8
10	$((K \rightarrow L) \wedge (L \rightarrow K))$	$\wedge$I, 6, 9

Daeducer commands needed to generate the proof:

```
premise (K ^ L)
and_elim_left 1
and_elim_right 1
assumption K
reiteration 3
discharge
imp_intro 4, 5
assumption L
reiteration 2
discharge
imp_intro 8, 9
and_intro 7, 11
qed
```

Chapter 17, Exercise C.5

Proof presented using Fitch notation:

1	$((C \wedge D) \vee E)$	PR
2	$\quad (C \wedge D)$	AS
3	$\quad D$	$\wedge$E, 2
4	$\quad (E \vee D)$	$\vee$E, 3
5	$\quad E$	AS
6	$\quad (E \vee D)$	$\vee$E, 5
7	$(E \vee D)$	$\vee$E, 1, 2–4, 5–6

Daeducer commands needed to generate the proof:

```
premise ((C ^ D) v E)
assumption (C ^ D)
and_elim_right 2
or_intro_right 3, E
discharge
assumption E
```

```
or_intro_left 6, D
discharge
or_elim 1, 2, 4, 6, 7
qed
```

Chapter 17, Exercise C.6

Proof presented using Fitch notation:

1	$((A \to B) \land (B \to A))$	PR
2	$((B \to C) \land (C \to B))$	PR
3	$(A \to B)$	$\land$E, 1
4	$(B \to C)$	$\land$E, 2
5	$\quad A$	AS
6	$\quad B$	$\to$E, 3, 5
7	$\quad C$	$\to$E, 4, 6
8	$(A \to C)$	$\to$I, 5, 7
9	$(B \to A)$	$\land$E, 1
10	$(C \to B)$	$\land$E, 2
11	$\quad C$	AS
12	$\quad B$	$\to$E, 10, 11
13	$\quad A$	$\to$E, 9, 12
14	$(C \to A)$	$\to$I, 11, 13
15	$((A \to C) \land (C \to A))$	$\land$I, 8, 14

Daeducer commands needed to generate the proof:

```
premise ((A -> B) ^ (B -> A))
premise ((B -> C) ^ (C -> B))
and_elim_left 1
and_elim_left 2
assumption A
imp_elim 3, 5
imp_elim 4, 6
discharge
imp_intro 5, 7
and_elim_right 1
and_elim_right 2
assumption C
imp_elim 11, 12
imp_elim 10, 13
discharge
imp_intro 12, 14
and_intro 9, 16
qed
```

Chapter 17, Exercise C.7

Proof presented using Fitch notation:

1	$(\neg F \to G)$	PR
2	$(F \to H)$	PR
3	$\quad \neg(G \vee H)$	AS
4	$\quad\quad F$	AS
5	$\quad\quad H$	$\to$E, 2, 4
6	$\quad\quad (G \vee H)$	$\vee$E, 5
7	$\quad\quad \bot$	$\neg$E, 3, 6
8	$\quad \neg F$	$\neg$I, 4–7
9	$\quad G$	$\to$E, 1, 8
10	$\quad (G \vee H)$	$\vee$E, 9
11	$\quad \bot$	$\neg$E, 3, 10
12	$(G \vee H)$	IP, 3–11

Daeducer commands needed to generate the proof:

```
premise (!F -> G)
premise (F -> H)
assumption !(G v H)
assumption F
imp_elim 2, 4
or_intro_right 5, G
not_elim 3, 6
discharge
not_intro 4, 7
imp_elim 1, 9
or_intro_left 10, H
not_elim 3, 11
discharge
indirect 3, 12
qed
```

Chapter 17, Exercise C.8

Proof presented using Fitch notation:

1	$((Z \wedge K) \vee (K \wedge M))$	PR
2	$(K \rightarrow D)$	PR
3	$\quad (Z \wedge K)$	AS
4	$\quad K$	$\wedge$E, 3
5	$\quad D$	$\rightarrow$E, 2, 4
6	$\quad (K \wedge M)$	AS
7	$\quad K$	$\wedge$E, 6
8	$\quad D$	$\rightarrow$E, 2, 7
9	D	$\vee$E, 1, 3–5, 6–8

Daeducer commands needed to generate the proof:

```
premise ((Z ^ K) v (K ^ M))
premise (K -> D)
assumption (Z ^ K)
and_elim_right 3
imp_elim 2, 4
discharge
assumption (K ^ M)
and_elim_left 7
imp_elim 2, 8
discharge
or_elim 1, 3, 5, 7, 9
qed
```

Chapter 17, Exercise C.9

Proof presented using Fitch notation:

1	$(P \wedge (Q \vee R))$	PR
2	$(P \rightarrow \neg R)$	PR
3	P	$\wedge$E, 1
4	$(Q \vee R)$	$\wedge$E, 1
5	$\neg R$	$\rightarrow$E, 2, 3
6	$\quad Q$	AS
7	$\quad (Q \vee E)$	$\vee$E, 6
8	$\quad R$	AS
9	$\quad \perp$	$\neg$E, 5, 8
10	$\quad (Q \vee E)$	X, 9
11	$(Q \vee E)$	$\vee$E, 4, 6–7, 8–10

Daeducer commands needed to generate the proof:

```
premise (P ^ (Q v R))
```

```
premise (P -> !R)
and_elim_left 1
and_elim_right 1
imp_elim 2, 3
assumption Q
or_intro_left 6, E
discharge
assumption R
not_elim 5, 9
explosion 10, (Q v E)
discharge
or_elim 4, 6, 7, 9, 11
qed
```

Chapter 17, Exercise C.10

Proof presented using Fitch notation:

1	$((S \to T) \land (T \to S))$	PR
2	$(S \to T)$	$\land$E, 1
3	$(T \to S)$	$\land$E, 1
4	$\quad S$	AS
5	$\quad (T \lor S)$	$\lor$E, 4
6	$(S \to (T \lor S))$	$\to$I, 4, 5
7	$\quad (T \lor S)$	AS
8	$\quad\quad T$	AS
9	$\quad\quad S$	$\to$E, 3, 8
10	$\quad\quad S$	AS
11	$\quad\quad S$	R, 10
12	$\quad S$	$\lor$E, 7, 8–9, 10–11
13	$((T \lor S) \to S)$	$\to$I, 7, 12
14	$((S \to (T \lor S)) \land ((T \lor S) \to S))$	$\land$I, 6, 13

Daeducer commands needed to generate the proof:

```
premise ((S -> T) ^ (T -> S))
and_elim_left 1
and_elim_right 1
assumption S
or_intro_right 4, T
discharge
imp_intro 4, 5
assumption (T v S)
assumption T
imp_elim 3, 9
discharge
assumption S
reiteration 12
discharge
or_elim 8, 9, 10, 12, 13
discharge
imp_intro 8, 15
and_intro 7, 17
qed
```

Chapter 17, Exercise C.11

Proof presented using Fitch notation:

1	$\neg(P \to Q)$	PR
2	$\quad Q$	AS
3	$\quad\quad P$	AS
4	$\quad\quad Q$	R, 2
5	$\quad (P \to Q)$	$\to$I, 3, 4
6	$\quad \bot$	$\neg$E, 1, 5
7	$\neg Q$	$\neg$I, 2–6

Daeducer commands needed to generate the proof:

```
premise !(P -> Q)
assumption Q
assumption P
reiteration 2
discharge
imp_intro 3, 4
not_elim 1, 6
discharge
not_intro 2, 7
qed
```

Chapter 17, Exercise C.12

Proof presented using Fitch notation:

1	$\neg(P \to Q)$	PR
2	$\quad \neg P$	AS
3	$\quad\quad P$	AS
4	$\quad\quad \bot$	$\neg$E, 2, 3
5	$\quad\quad Q$	X, 4
6	$\quad (P \to Q)$	$\to$I, 3, 5
7	$\quad \bot$	$\neg$E, 1, 6
8	P	IP, 2–7

Daeducer commands needed to generate the proof:

```
premise !(P -> Q)
assumption !P
assumption P
not_elim 2, 3
explosion 4, Q
discharge
imp_intro 3, 5
not_elim 1, 7
discharge
indirect 2, 8
qed
```

Chapter 18, Example p.149

Proof presented using Fitch notation:

1	$((A \wedge B) \vee (A \wedge C))$	PR
2	$\quad (A \wedge B)$	AS
3	$\quad A$	$\wedge$E, 2
4	$\quad B$	$\wedge$E, 2
5	$\quad (B \vee C)$	$\vee$E, 4
6	$\quad (A \wedge (B \vee C))$	$\wedge$I, 3, 5
7	$\quad (A \wedge C)$	AS
8	$\quad A$	$\wedge$E, 7
9	$\quad C$	$\wedge$E, 7
10	$\quad (B \vee C)$	$\vee$E, 9
11	$\quad (A \wedge (B \vee C))$	$\wedge$I, 8, 10
12	$(A \wedge (B \vee C))$	$\vee$E, 1, 2–6, 7–11

Daeducer commands needed to generate the proof:

```
premise ((A ^ B) v (A ^ C))
assumption (A ^ B)
and_elim_left 2
and_elim_right 2
or_intro_left 4, C
and_intro 3, 5
discharge
assumption (A ^ C)
and_elim_left 8
and_elim_right 8
or_intro_right 10, B
and_intro 9, 11
discharge
or_elim 1, 2, 6, 8, 12
qed
```

Chapter 18, Example p.153

Proof presented using Fitch notation:

1	$(A \to B)$	AS
2	$\quad \neg B$	AS
3	$\quad\quad A$	AS
4	$\quad\quad B$	$\to$E, 1, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad \neg A$	$\neg$I, 3–5
7	$(\neg B \to \neg A)$	$\to$I, 2, 6
8	$((A \to B) \to (\neg B \to \neg A))$	$\to$I, 1, 7

Daeducer commands needed to generate the proof:

```
assumption (A -> B)
assumption !B
assumption A
imp_elim 1, 3
not_elim 2, 4
discharge
not_intro 3, 5
discharge
imp_intro 2, 7
discharge
imp_intro 1, 9
qed
```

Chapter 18, Example p.156

Proof presented using Fitch notation:

1	$(A \vee B)$	PR
2	$\neg A$	PR
3	$\quad A$	AS
4	$\quad \bot$	$\neg$E, 2, 3
5	$\quad B$	X, 4
6	$\quad B$	AS
7	$\quad B$	R, 6
8	B	$\vee$E, 1, 3–5, 6–7

Daeducer commands needed to generate the proof:

```
premise (A v B)
premise !A
assumption A
not_elim 2, 3
explosion 4, B
discharge
assumption B
reiteration 7
discharge
```

```
or_elim 1, 3, 5, 7, 8
qed
```

Chapter 18, Example p.158

Proof presented using Fitch notation:

1	$\neg(A \vee \neg A)$	AS
2	$\quad A$	AS
3	$\quad (A \vee \neg A)$	$\vee$E, 2
4	$\quad \bot$	$\neg$E, 1, 3
5	$\neg A$	$\neg$I, 2–4
6	$\quad \neg A$	AS
7	$\quad (A \vee \neg A)$	$\vee$E, 6
8	$\quad \bot$	$\neg$E, 1, 7
9	$\neg\neg A$	$\neg$I, 6–8
10	$\bot$	$\neg$E, 9, 5
11	$(A \vee \neg A)$	IP, 1–10

Daeducer commands needed to generate the proof:

```
assumption !(A v !A)
assumption A
or_intro_left 2, !A
not_elim 1, 3
discharge
not_intro 2, 4
assumption !A
or_intro_right 7, A
not_elim 1, 8
discharge
not_intro 7, 9
not_elim 11, 6
discharge
indirect 1, 12
qed
```

Chapter 18, Exercise A.1

Proof presented using Fitch notation:

1	$(A \to B)$	PR
2	$(A \to C)$	PR
3	$\quad A$	AS
4	$\quad B$	$\to$E, 1, 3
5	$\quad C$	$\to$E, 2, 3
6	$\quad (B \wedge C)$	$\wedge$I, 4, 5
7	$(A \to (B \wedge C))$	$\to$I, 3, 6

Daeducer commands needed to generate the proof:

```
premise (A -> B)
premise (A -> C)
assumption A
imp_elim 1, 3
imp_elim 2, 3
and_intro 4, 5
discharge
imp_intro 3, 6
qed
```

Chapter 18, Exercise A.2

Proof presented using Fitch notation:

1	$((A \wedge B) \to C)$	PR
2	$\quad A$	AS
3	$\quad\quad B$	AS
4	$\quad\quad (A \wedge B)$	$\wedge$I, 2, 3
5	$\quad\quad C$	$\to$E, 1, 4
6	$\quad (B \to C)$	$\to$I, 3, 5
7	$(A \to (B \to C))$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise ((A ^ B) -> C)
assumption A
assumption B
and_intro 2, 3
imp_elim 1, 4
discharge
imp_intro 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 18, Exercise A.3

Proof presented using Fitch notation:

1	$(A \to (B \to C))$	PR
2	$\quad (A \to B)$	AS
3	$\quad\quad A$	AS
4	$\quad\quad (B \to C)$	$\to$E, 1, 3
5	$\quad\quad B$	$\to$E, 2, 3
6	$\quad\quad C$	$\to$E, 4, 5
7	$\quad (A \to C)$	$\to$I, 3, 6
8	$((A \to B) \to (A \to C))$	$\to$I, 2, 7

Daeducer commands needed to generate the proof:

```
premise (A -> (B -> C))
assumption (A -> B)
assumption A
imp_elim 1, 3
imp_elim 2, 3
imp_elim 4, 5
discharge
imp_intro 3, 6
discharge
imp_intro 2, 8
qed
```

Chapter 18, Exercise A.4

Proof presented using Fitch notation:

1	$(A \vee (B \wedge C))$	PR
2	$\quad A$	AS
3	$\quad (A \vee B)$	$\vee$E, 2
4	$\quad (A \vee C)$	$\vee$E, 2
5	$\quad ((A \vee B) \wedge (A \vee C))$	$\wedge$I, 3, 4
6	$\quad (B \wedge C)$	AS
7	$\quad B$	$\wedge$E, 6
8	$\quad (A \vee B)$	$\vee$E, 7
9	$\quad C$	$\wedge$E, 6
10	$\quad (A \vee C)$	$\vee$E, 9
11	$\quad ((A \vee B) \wedge (A \vee C))$	$\wedge$I, 8, 10
12	$((A \vee B) \wedge (A \vee C))$	$\vee$E, 1, 2–5, 6–11

Daeducer commands needed to generate the proof:

```
premise (A v (B ^ C))
```

```
assumption A
or_intro_left 2, B
or_intro_left 2, C
and_intro 3, 4
discharge
assumption (B ^ C)
and_elim_left 7
or_intro_right 8, A
and_elim_right 7
or_intro_right 10, A
and_intro 9, 11
discharge
or_elim 1, 2, 5, 7, 12
qed
```

Chapter 18, Exercise A.5

Proof presented using Fitch notation:

1	$((A \wedge B) \vee (A \wedge C))$	PR
2	$\quad (A \wedge B)$	AS
3	$\quad A$	$\wedge$E, 2
4	$\quad B$	$\wedge$E, 2
5	$\quad (B \vee C)$	$\vee$E, 4
6	$\quad (A \wedge (B \vee C))$	$\wedge$I, 3, 5
7	$\quad (A \wedge C)$	AS
8	$\quad A$	$\wedge$E, 7
9	$\quad C$	$\wedge$E, 7
10	$\quad (B \vee C)$	$\vee$E, 9
11	$\quad (A \wedge (B \vee C))$	$\wedge$I, 8, 10
12	$(A \wedge (B \vee C))$	$\vee$E, 1, 2–6, 7–11

Daeducer commands needed to generate the proof:

```
premise ((A ^ B) v (A ^ C))
assumption (A ^ B)
and_elim_left 2
and_elim_right 2
or_intro_left 4, C
and_intro 3, 5
discharge
assumption (A ^ C)
and_elim_left 8
and_elim_right 8
or_intro_right 10, B
and_intro 9, 11
discharge
or_elim 1, 2, 6, 8, 12
qed
```

Chapter 18, Exercise A.6

Proof presented using Fitch notation:

1	$(A \vee B)$	PR
2	$(A \to C)$	PR
3	$(B \to D)$	PR
4	$\quad A$	AS
5	$\quad C$	$\to$E, 2, 4
6	$\quad (C \vee D)$	$\vee$E, 5
7	$\quad B$	AS
8	$\quad D$	$\to$E, 3, 7
9	$\quad (C \vee D)$	$\vee$E, 8
10	$(C \vee D)$	$\vee$E, 1, 4–6, 7–9

Daeducer commands needed to generate the proof:

```
premise (A v B)
premise (A -> C)
premise (B -> D)
assumption A
imp_elim 2, 4
or_intro_left 5, D
discharge
assumption B
imp_elim 3, 8
or_intro_right 9, C
discharge
or_elim 1, 4, 6, 8, 10
qed
```

Chapter 18, Exercise A.7

Proof presented using Fitch notation:

1	$(\neg A \vee \neg B)$	PR
2	$\quad (A \wedge B)$	AS
3	$\quad\quad \neg A$	AS
4	$\quad\quad A$	$\wedge$E, 2
5	$\quad\quad \bot$	$\neg$E, 3, 4
6	$\quad\quad \neg B$	AS
7	$\quad\quad B$	$\wedge$E, 2
8	$\quad\quad \bot$	$\neg$E, 6, 7
9	$\quad \bot$	$\vee$E, 1, 3–5, 6–8
10	$\neg(A \wedge B)$	$\neg$I, 2–9

Daeducer commands needed to generate the proof:

```
premise (!A v !B)
assumption (A ^ B)
assumption !A
and_elim_left 2
not_elim 3, 4
discharge
assumption !B
and_elim_right 2
not_elim 7, 8
discharge
or_elim 1, 3, 5, 7, 9
discharge
not_intro 2, 11
qed
```

Chapter 18, Exercise A.8

Proof presented using Fitch notation:

1	$(A \wedge \neg B)$	PR
2	$\quad (A \rightarrow B)$	AS
3	$\quad A$	$\wedge$E, 1
4	$\quad \neg B$	$\wedge$E, 1
5	$\quad B$	$\rightarrow$E, 2, 3
6	$\quad \bot$	$\neg$E, 4, 5
7	$\neg(A \rightarrow B)$	$\neg$I, 2–6

Daeducer commands needed to generate the proof:

```
premise (A ^ !B)
assumption (A -> B)
and_elim_left 1
and_elim_right 1
imp_elim 2, 3
not_elim 4, 5
discharge
not_intro 2, 6
qed
```

Chapter 18, Exercise C.1

Proof presented using Fitch notation:

1	$\neg A$	AS
2	$\quad A$	AS
3	$\quad \bot$	$\neg$E, 1, 2
4	$(A \to \bot)$	$\to$I, 2, 3
5	$(\neg A \to (A \to \bot))$	$\to$I, 1, 4

Daeducer commands needed to generate the proof:

```
assumption !A
assumption A
not_elim 1, 2
discharge
imp_intro 2, 3
discharge
imp_intro 1, 5
qed
```

Chapter 18, Exercise C.2

Proof presented using Fitch notation:

1	$(A \wedge \neg A)$	AS
2	A	$\wedge$E, 1
3	$\neg A$	$\wedge$E, 1
4	$\bot$	$\neg$E, 3, 2
5	$\neg(A \wedge \neg A)$	$\neg$I, 1–4

Daeducer commands needed to generate the proof:

```
assumption (A ^ !A)
and_elim_left 1
and_elim_right 1
not_elim 3, 2
discharge
not_intro 1, 4
qed
```

Chapter 18, Exercise C.3

Proof presented using Fitch notation:

1	$((A \to C) \land (B \to C))$	AS
2	$(A \lor B)$	AS
3	$(A \to C)$	$\land$E, 1
4	$(B \to C)$	$\land$E, 1
5	A	AS
6	C	$\to$E, 3, 5
7	B	AS
8	C	$\to$E, 4, 7
9	C	$\lor$E, 2, 5–6, 7–8
10	$((A \lor B) \to C)$	$\to$I, 2, 9
11	$(((A \to C) \land (B \to C)) \to ((A \lor B) \to C))$	$\to$I, 1, 10

Daeducer commands needed to generate the proof:

```
assumption ((A -> C) ^ (B -> C))
assumption (A v B)
and_elim_left 1
and_elim_right 1
assumption A
imp_elim 3, 5
discharge
assumption B
imp_elim 4, 8
discharge
or_elim 2, 5, 6, 8, 9
discharge
imp_intro 2, 11
discharge
imp_intro 1, 13
qed
```

Chapter 18, Exercise C.4

Proof presented using Fitch notation:

1	$\neg(A \to B)$	AS
2	$\quad \neg A$	AS
3	$\quad\quad A$	AS
4	$\quad\quad \bot$	$\neg$E, 2, 3
5	$\quad\quad B$	X, 4
6	$\quad (A \to B)$	$\to$I, 3, 5
7	$\quad \bot$	$\neg$E, 1, 6
8	A	IP, 2–7
9	$\quad B$	AS
10	$\quad\quad A$	AS
11	$\quad\quad B$	R, 9
12	$\quad (A \to B)$	$\to$I, 10, 11
13	$\quad \bot$	$\neg$E, 1, 12
14	$\neg B$	$\neg$I, 9–13
15	$(A \wedge \neg B)$	$\wedge$I, 8, 14
16	$(\neg(A \to B) \to (A \wedge \neg B))$	$\to$I, 1, 15

Daeducer commands needed to generate the proof:

```
assumption !(A -> B)
assumption !A
assumption A
not_elim 2, 3
explosion 4, B
discharge
imp_intro 3, 5
not_elim 1, 7
discharge
indirect 2, 8
assumption B
assumption A
reiteration 11
discharge
imp_intro 12, 13
not_elim 1, 15
discharge
not_intro 11, 16
and_intro 10, 18
discharge
imp_intro 1, 19
qed
```

Chapter 18, Exercise C.5

Proof presented using Fitch notation:

1	$(\neg A \vee B)$	AS
2	$\quad A$	AS
3	$\quad\quad \neg A$	AS
4	$\quad\quad \bot$	$\neg$E, 3, 2
5	$\quad\quad B$	X, 4
6	$\quad\quad B$	AS
7	$\quad\quad B$	R, 6
8	$\quad B$	$\vee$E, 1, 3–5, 6–7
9	$(A \to B)$	$\to$I, 2, 8
10	$((\neg A \vee B) \to (A \to B))$	$\to$I, 1, 9

Daeducer commands needed to generate the proof:

```
assumption (!A v B)
assumption A
assumption !A
not_elim 3, 2
explosion 4, B
discharge
assumption B
reiteration 7
discharge
or_elim 1, 3, 5, 7, 8
discharge
imp_intro 2, 10
discharge
imp_intro 1, 12
qed
```

Chapter 18, Exercise D.1

Proof presented using Fitch notation:

1	$\quad \neg\neg A$	AS
2	$\quad\quad \neg A$	AS
3	$\quad\quad \bot$	$\neg$E, 1, 2
4	$\quad A$	IP, 2–3
5	$(\neg\neg A \to A)$	$\to$I, 1, 4

Daeducer commands needed to generate the proof:

```
assumption !!A
assumption !A
not_elim 1, 2
discharge
indirect 2, 3
discharge
imp_intro 1, 5
qed
```

Chapter 18, Exercise D.2

Proof presented using Fitch notation:

1	$(\neg A \to \neg B)$	PR
2	$\quad B$	AS
3	$\quad\quad \neg A$	AS
4	$\quad\quad \neg B$	$\to$E, 1, 3
5	$\quad\quad \bot$	$\neg$E, 4, 2
6	$\quad A$	IP, 3–5
7	$(B \to A)$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise (!A -> !B)
assumption B
assumption !A
imp_elim 1, 3
not_elim 4, 2
discharge
indirect 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 18, Exercise D.3

Proof presented using Fitch notation:

1	$(A \to B)$	PR
2	$\quad \neg(\neg A \vee B)$	AS
3	$\quad\quad A$	AS
4	$\quad\quad B$	$\to$E, 1, 3
5	$\quad\quad (\neg A \vee B)$	$\vee$E, 4
6	$\quad\quad \bot$	$\neg$E, 2, 5
7	$\quad \neg A$	$\neg$I, 3–6
8	$\quad (\neg A \vee B)$	$\vee$E, 7
9	$\quad \bot$	$\neg$E, 2, 8
10	$(\neg A \vee B)$	IP, 2–9

Daeducer commands needed to generate the proof:

```
premise (A -> B)
assumption !(!A v B)
assumption A
imp_elim 1, 3
or_intro_right 4, !A
not_elim 2, 5
discharge
not_intro 3, 6
or_intro_left 8, B
not_elim 2, 9
discharge
indirect 2, 10
qed
```

Chapter 18, Exercise D.4

Proof presented using Fitch notation:

1	$\neg(A \wedge B)$	AS
2	$\neg(\neg A \vee \neg B)$	AS
3	$\neg A$	AS
4	$(\neg A \vee \neg B)$	$\vee$E, 3
5	$\bot$	$\neg$E, 2, 4
6	A	IP, 3–5
7	$\neg B$	AS
8	$(\neg A \vee \neg B)$	$\vee$E, 7
9	$\bot$	$\neg$E, 2, 8
10	B	IP, 7–9
11	$(A \wedge B)$	$\wedge$I, 6, 10
12	$\bot$	$\neg$E, 1, 11
13	$(\neg A \vee \neg B)$	IP, 2–12
14	$(\neg(A \wedge B) \rightarrow (\neg A \vee \neg B))$	$\rightarrow$I, 1, 13

Daeducer commands needed to generate the proof:

```
assumption !(A ^ B)
assumption !(!A v !B)
assumption !A
or_intro_left 3, !B
not_elim 2, 4
discharge
indirect 3, 5
assumption !B
or_intro_right 8, !A
not_elim 2, 9
discharge
indirect 8, 10
and_intro 7, 12
not_elim 1, 13
discharge
indirect 2, 14
discharge
imp_intro 1, 16
qed
```

Chapter 18, Exercise D.5

Proof presented using Fitch notation:

1	$(A \to (B \vee C))$	PR
2	$\quad \neg((A \to B) \vee (A \to C))$	AS
3	$\quad\quad (A \to B)$	AS
4	$\quad\quad ((A \to B) \vee (A \to C))$	$\vee$E, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad \neg(A \to B)$	$\neg$I, 3–5
7	$\quad\quad (A \to C)$	AS
8	$\quad\quad ((A \to B) \vee (A \to C))$	$\vee$E, 7
9	$\quad\quad \bot$	$\neg$E, 2, 8
10	$\quad \neg(A \to C)$	$\neg$I, 7–9
11	$\quad\quad \neg A$	AS
12	$\quad\quad\quad A$	AS
13	$\quad\quad\quad \bot$	$\neg$E, 11, 12
14	$\quad\quad\quad B$	X, 13
15	$\quad\quad (A \to B)$	$\to$I, 12, 14
16	$\quad\quad \bot$	$\neg$E, 6, 15
17	$\quad A$	IP, 11–16
18	$\quad (B \vee C)$	$\to$E, 1, 17
19	$\quad\quad B$	AS
20	$\quad\quad\quad A$	AS
21	$\quad\quad\quad B$	R, 19
22	$\quad\quad (A \to B)$	$\to$I, 20, 21
23	$\quad\quad \bot$	$\neg$E, 6, 22
24	$\quad \neg B$	$\neg$I, 19–23
25	$\quad\quad C$	AS
26	$\quad\quad\quad A$	AS
27	$\quad\quad\quad C$	R, 25
28	$\quad\quad (A \to C)$	$\to$I, 26, 27
29	$\quad\quad \bot$	$\neg$E, 10, 28
30	$\quad \neg C$	$\neg$I, 25–29
31	$\quad \bot$	$\vee$E, 18, 19–23, 25–29
32	$((A \to B) \vee (A \to C))$	IP, 2–31

Daeducer commands needed to generate the proof:

```
premise (A -> (B v C))
assumption !((A -> B) v (A -> C))
assumption (A -> B)
or_intro_left 3, (A -> C)
not_elim 2, 4
discharge
not_intro 3, 5
assumption (A -> C)
or_intro_right 8, (A -> B)
not_elim 2, 9
discharge
not_intro 8, 10
assumption !A
assumption A
not_elim 13, 14
explosion 15, B
discharge
imp_intro 14, 16
not_elim 7, 18
discharge
indirect 13, 19
imp_elim 1, 21
assumption B
assumption A
reiteration 23
discharge
imp_intro 24, 25
not_elim 7, 27
discharge
not_intro 23, 28
assumption C
assumption A
reiteration 31
discharge
imp_intro 32, 33
not_elim 12, 35
discharge
not_intro 31, 36
or_elim 22, 23, 28, 31, 36
discharge
indirect 2, 39
qed
```

Chapter 18, Exercise D.6

Proof presented using Fitch notation:

1	$\neg((A \to B) \lor (B \to A))$	AS
2	$\quad (A \to B)$	AS
3	$\quad ((A \to B) \lor (B \to A))$	$\lor$E, 2
4	$\quad \bot$	$\neg$E, 1, 3
5	$\neg(A \to B)$	$\neg$I, 2–4
6	$\quad (B \to A)$	AS
7	$\quad ((A \to B) \lor (B \to A))$	$\lor$E, 6
8	$\quad \bot$	$\neg$E, 1, 7
9	$\neg(B \to A)$	$\neg$I, 6–8
10	$\quad \neg A$	AS
11	$\quad\quad A$	AS
12	$\quad\quad \neg A$	R, 10
13	$\quad\quad \bot$	$\neg$E, 12, 11
14	$\quad\quad B$	X, 13
15	$\quad (A \to B)$	$\to$I, 11, 14
16	$\quad \bot$	$\neg$E, 5, 15
17	A	IP, 10–16
18	$\quad B$	AS
19	$\quad\quad A$	AS
20	$\quad\quad B$	R, 18
21	$\quad (A \to B)$	$\to$I, 19, 20
22	$\quad \bot$	$\neg$E, 5, 21
23	$\neg B$	$\neg$I, 18–22
24	$\quad \neg B$	AS
25	$\quad\quad B$	AS
26	$\quad\quad \neg B$	R, 24
27	$\quad\quad \bot$	$\neg$E, 26, 25
28	$\quad\quad A$	X, 27
29	$\quad (B \to A)$	$\to$I, 25, 28
30	$\quad \bot$	$\neg$E, 9, 29
31	B	IP, 24–30
32	$\quad A$	AS

33	B	AS
34	A	R, 32
35	$(B \to A)$	$\to$I, 33, 34
36	$\bot$	$\neg$E, 9, 35
37	$\neg A$	$\neg$I, 32–36
38	$\bot$	$\neg$E, 37, 17
39	$((A \to B) \vee (B \to A))$	IP, 1–38

Daeducer commands needed to generate the proof:

```
assumption !((A -> B) v (B -> A))
assumption (A -> B)
or_intro_left 2, (B -> A)
not_elim 1, 3
discharge
not_intro 2, 4
assumption (B -> A)
or_intro_right 7, (A -> B)
not_elim 1, 8
discharge
not_intro 7, 9
assumption !A
assumption A
reiteration 12
not_elim 14, 13
explosion 15, B
discharge
imp_intro 13, 16
not_elim 6, 18
discharge
indirect 12, 19
assumption B
assumption A
reiteration 22
discharge
imp_intro 23, 24
not_elim 6, 26
discharge
not_intro 22, 27
assumption !B
assumption B
reiteration 30
not_elim 32, 31
explosion 33, A
discharge
imp_intro 31, 34
not_elim 11, 36
discharge
indirect 30, 37
assumption A
assumption B
reiteration 40
discharge
imp_intro 41, 42
not_elim 11, 44
discharge
not_intro 40, 45
not_elim 47, 21
discharge
indirect 1, 48
qed
```

Chapter 18, Exercise D.7

Proof presented using Fitch notation:

1	$((A \to B) \to A)$	AS
2	$\quad \neg A$	AS
3	$\quad\quad A$	AS
4	$\quad\quad \bot$	¬E, 2, 3
5	$\quad\quad B$	X, 4
6	$\quad (A \to B)$	→I, 3, 5
7	$\quad A$	→E, 1, 6
8	$\quad \bot$	¬E, 2, 7
9	A	IP, 2–8
10	$(((A \to B) \to A) \to A)$	→I, 1, 9

Daeducer commands needed to generate the proof:

```
assumption ((A -> B) -> A)
assumption !A
assumption A
not_elim 2, 3
explosion 4, B
discharge
imp_intro 3, 5
imp_elim 1, 7
not_elim 2, 8
discharge
indirect 2, 9
discharge
imp_intro 1, 11
qed
```

Chapter 19, Example p.163

Proof presented using Fitch notation:

1	$(A \vee B)$	PR
2	$\neg A$	PR
3	$\quad A$	AS
4	$\quad \bot$	$\neg$E, 2, 3
5	$\quad B$	X, 4
6	$\quad B$	AS
7	$\quad B$	R, 6
8	B	$\vee$E, 1, 3–5, 6–7

Daeducer commands needed to generate the proof:

```
premise (A v B)
premise !A
assumption A
not_elim 2, 3
explosion 4, B
discharge
assumption B
reiteration 7
discharge
or_elim 1, 3, 5, 7, 8
qed
```

Chapter 19, Example p.163

Proof presented using Fitch notation:

1	$(A \vee B)$	PR
2	$\neg B$	PR
3	$\quad A$	AS
4	$\quad A$	R, 3
5	$\quad B$	AS
6	$\quad \bot$	$\neg$E, 2, 5
7	$\quad A$	X, 6
8	A	$\vee$E, 1, 3–4, 5–7

Daeducer commands needed to generate the proof:

```
premise (A v B)
premise !B
assumption A
reiteration 3
discharge
assumption B
not_elim 2, 6
explosion 7, A
discharge
or_elim 1, 3, 4, 6, 8
```

```
qed
```

Chapter 19, Example p.164

Proof presented using Fitch notation:

1	$(A \to B)$	PR
2	$\neg B$	PR
3	$\quad A$	AS
4	$\quad B$	$\to$E, 1, 3
5	$\quad \bot$	$\neg$E, 2, 4
6	$\neg A$	$\neg$I, 3–5

Daeducer commands needed to generate the proof:

```
premise (A -> B)
premise !B
assumption A
imp_elim 1, 3
not_elim 2, 4
discharge
not_intro 3, 5
qed
```

Chapter 19, Example p.164

Proof presented using Fitch notation:

1	$\neg\neg A$	PR
2	$\quad \neg A$	AS
3	$\quad \bot$	$\neg$E, 1, 2
4	A	IP, 2–3

Daeducer commands needed to generate the proof:

```
premise !!A
assumption !A
not_elim 1, 2
discharge
indirect 2, 3
qed
```

Chapter 19, Example p.165

Proof presented using Fitch notation:

1	$(A \to B)$	PR
2	$(\neg A \to B)$	PR
3	$\quad \neg B$	AS
4	$\quad\quad A$	AS
5	$\quad\quad B$	$\to$E, 1, 4
6	$\quad\quad \bot$	$\neg$E, 3, 5
7	$\quad \neg A$	$\neg$I, 4–6
8	$\quad\quad \neg A$	AS
9	$\quad\quad B$	$\to$E, 2, 8
10	$\quad\quad \bot$	$\neg$E, 3, 9
11	$\quad \neg\neg A$	$\neg$I, 8–10
12	$\quad \bot$	$\neg$E, 11, 7
13	B	IP, 3–12

Daeducer commands needed to generate the proof:

```
premise (A -> B)
premise (!A -> B)
assumption !B
assumption A
imp_elim 1, 4
not_elim 3, 5
discharge
not_intro 4, 6
assumption !A
imp_elim 2, 9
not_elim 3, 10
discharge
not_intro 9, 11
not_elim 13, 8
discharge
indirect 3, 14
qed
```

Chapter 19, Example p.166

Proof presented using Fitch notation:

1	P	PR
2	$\quad \neg(D \vee \neg D)$	AS
3	$\quad\quad D$	AS
4	$\quad\quad (D \vee \neg D)$	$\vee$E, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad \neg D$	$\neg$I, 3–5
7	$\quad (D \vee \neg D)$	$\vee$E, 6
8	$\quad \bot$	$\neg$E, 2, 7
9	$(D \vee \neg D)$	IP, 2–8
10	$\quad D$	AS
11	$\quad (P \wedge D)$	$\wedge$I, 1, 10
12	$\quad ((P \wedge D) \vee (P \wedge \neg D))$	$\vee$E, 11
13	$\quad \neg D$	AS
14	$\quad (P \wedge \neg D)$	$\wedge$I, 1, 13
15	$\quad ((P \wedge D) \vee (P \wedge \neg D))$	$\vee$E, 14
16	$((P \wedge D) \vee (P \wedge \neg D))$	$\vee$E, 9, 10–12, 13–15

Daeducer commands needed to generate the proof:

```
premise P
assumption !(D v !D)
assumption D
or_intro_left 3, !D
not_elim 2, 4
discharge
not_intro 3, 5
or_intro_right 7, D
not_elim 2, 8
discharge
indirect 2, 9
assumption D
and_intro 1, 12
or_intro_left 13, (P ^ !D)
discharge
assumption !D
and_intro 1, 16
or_intro_right 17, (P ^ D)
discharge
or_elim 11, 12, 14, 16, 18
qed
```

Chapter 19, Exampe p.166

Proof presented using Fitch notation:

1	$(A \to \neg A)$	PR
2	$\quad A$	AS
3	$\quad \neg A$	$\to$E, 1, 2
4	$\quad \bot$	$\neg$E, 3, 2
5	$\neg A$	$\neg$I, 2–4

Daeducer commands needed to generate the proof:

```
premise (A -> !A)
assumption A
imp_elim 1, 2
not_elim 3, 2
discharge
not_intro 2, 4
qed
```

Chapter 19, Example p.167

Proof presented using Fitch notation:

1	$\neg(A \wedge B)$	PR
2	$\quad \neg(\neg A \vee \neg B)$	AS
3	$\quad\quad \neg A$	AS
4	$\quad\quad (\neg A \vee \neg B)$	$\vee$E, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad \neg\neg A$	$\neg$I, 3–5
7	$\quad\quad \neg B$	AS
8	$\quad\quad (\neg A \vee \neg B)$	$\vee$E, 7
9	$\quad\quad \bot$	$\neg$E, 2, 8
10	$\quad \neg\neg B$	$\neg$I, 7–9
11	$\quad\quad \neg A$	AS
12	$\quad\quad \bot$	$\neg$E, 6, 11
13	$\quad A$	IP, 11–12
14	$\quad\quad \neg B$	AS
15	$\quad\quad \bot$	$\neg$E, 10, 14
16	$\quad B$	IP, 14–15
17	$\quad (A \wedge B)$	$\wedge$I, 13, 16
18	$\quad \bot$	$\neg$E, 1, 17
19	$(\neg A \vee \neg B)$	IP, 2–18

Daeducer commands needed to generate the proof:

```
premise !(A ^ B)
assumption !(!A v !B)
assumption !A
or_intro_left 3, !B
not_elim 2, 4
discharge
not_intro 3, 5
assumption !B
or_intro_right 8, !A
not_elim 2, 9
discharge
not_intro 8, 10
assumption !A
not_elim 7, 13
discharge
indirect 13, 14
assumption !B
not_elim 12, 17
discharge
indirect 17, 18
and_intro 16, 20
not_elim 1, 21
discharge
indirect 2, 22
qed
```

Chapter 19, Example p.167

Proof presented using Fitch notation:

1	$(\neg A \vee \neg B)$	PR
2	$\quad (A \wedge B)$	AS
3	$\quad A$	$\wedge$E, 2
4	$\quad B$	$\wedge$E, 2
5	$\quad\quad \neg A$	AS
6	$\quad\quad \bot$	$\neg$E, 5, 3
7	$\quad\quad \neg B$	AS
8	$\quad\quad \bot$	$\neg$E, 7, 4
9	$\quad \bot$	$\vee$E, 1, 5–6, 7–8
10	$\neg(A \wedge B)$	$\neg$I, 2–9

Daeducer commands needed to generate the proof:

```
premise (!A v !B)
assumption (A ^ B)
and_elim_left 2
and_elim_right 2
assumption !A
not_elim 5, 3
discharge
assumption !B
not_elim 8, 4
discharge
or_elim 1, 5, 6, 8, 9
discharge
not_intro 2, 11
qed
```

Chapter 19, Example p.167

Proof presented using Fitch notation:

1	$\neg(A \vee B)$	PR
2	$\quad A$	AS
3	$\quad (A \vee B)$	$\vee$E, 2
4	$\quad \bot$	$\neg$E, 1, 3
5	$\neg A$	$\neg$I, 2–4
6	$\quad B$	AS
7	$\quad (A \vee B)$	$\vee$E, 6
8	$\quad \bot$	$\neg$E, 1, 7
9	$\neg B$	$\neg$I, 6–8
10	$(\neg A \wedge \neg B)$	$\wedge$I, 5, 9

Daeducer commands needed to generate the proof:

```
premise !(A v B)
assumption A
or_intro_left 2, B
not_elim 1, 3
discharge
not_intro 2, 4
assumption B
or_intro_right 7, A
not_elim 1, 8
discharge
not_intro 7, 9
and_intro 6, 11
qed
```

Chapter 19, Example p.167

Proof presented using Fitch notation:

1	$(\neg A \wedge \neg B)$	PR
2	$\neg A$	$\wedge$E, 1
3	$\neg B$	$\wedge$E, 1
4	$\quad (A \vee B)$	AS
5	$\quad\quad A$	AS
6	$\quad\quad \bot$	$\neg$E, 2, 5
7	$\quad\quad B$	AS
8	$\quad\quad \bot$	$\neg$E, 3, 7
9	$\quad \bot$	$\vee$E, 4, 5–6, 7–8
10	$\neg(A \vee B)$	$\neg$I, 4–9

Daeducer commands needed to generate the proof:

```
premise (!A ^ !B)
and_elim_left 1
and_elim_right 1
assumption (A v B)
assumption A
not_elim 2, 5
discharge
assumption B
not_elim 3, 8
discharge
or_elim 4, 5, 6, 8, 9
discharge
not_intro 4, 11
qed
```

Chapter 19, Exercise A.1

Proof presented using Fitch notation:

1	$(W \to \neg B)$	PR
2	$(A \wedge W)$	PR
3	$(B \vee (J \wedge K))$	PR
4	W	$\wedge$E, 2
5	$\neg B$	$\to$E, 1, 4
6	$\quad B$	AS
7	$\quad \bot$	$\neg$E, 5, 6
8	$\quad K$	X, 7
9	$\quad (J \wedge K)$	AS
10	$\quad K$	$\wedge$E, 9
11	K	$\vee$E, 3, 6–8, 9–10

Daeducer commands needed to generate the proof:

```
premise (W -> !B)
premise (A ^ W)
premise (B v (J ^ K))
and_elim_right 2
imp_elim 1, 4
assumption B
not_elim 5, 6
explosion 7, K
discharge
assumption (J ^ K)
and_elim_right 10
discharge
or_elim 3, 6, 8, 10, 11
qed
```

Chapter 19, Exercise A.2

Proof presented using Fitch notation:

1	$((L \to \neg O) \land (\neg O \to L))$	PR
2	$(L \lor \neg O)$	PR
3	$(L \to \neg O)$	$\land$E, 1
4	$(\neg O \to L)$	$\land$E, 1
5	$\quad L$	AS
6	$\quad L$	R, 5
7	$\quad \neg O$	AS
8	$\quad L$	$\to$E, 4, 7
9	L	$\lor$E, 2, 5–6, 7–8

Daeducer commands needed to generate the proof:

```
premise ((L -> !O) ^ (!O -> L))
premise (L v !O)
and_elim_left 1
and_elim_right 1
assumption L
reiteration 5
discharge
assumption !O
imp_elim 4, 8
discharge
or_elim 2, 5, 6, 8, 9
qed
```

Chapter 19, Exercise A.3

Proof presented using Fitch notation:

1	$(Z \to (C \land \neg N))$	PR
2	$(\neg Z \to (N \land \neg C))$	PR
3	$\quad \neg(N \lor C)$	AS
4	$\quad\quad N$	AS
5	$\quad\quad (N \lor C)$	$\lor$E, 4
6	$\quad\quad \bot$	$\neg$E, 3, 5
7	$\quad \neg N$	$\neg$I, 4–6
8	$\quad\quad C$	AS
9	$\quad\quad (N \lor C)$	$\lor$E, 8
10	$\quad\quad \bot$	$\neg$E, 3, 9
11	$\quad \neg C$	$\neg$I, 8–10
12	$\quad\quad Z$	AS
13	$\quad\quad (C \land \neg N)$	$\to$E, 1, 12
14	$\quad\quad C$	$\land$E, 13
15	$\quad\quad \bot$	$\neg$E, 11, 14
16	$\quad \neg Z$	$\neg$I, 12–15
17	$\quad (N \land \neg C)$	$\to$E, 2, 16
18	$\quad N$	$\land$E, 17
19	$\quad \bot$	$\neg$E, 7, 18
20	$(N \lor C)$	IP, 3–19

Daeducer commands needed to generate the proof:

```
premise (Z -> (C ^ !N))
premise (!Z -> (N ^ !C))
assumption !(N v C)
assumption N
or_intro_left 4, C
not_elim 3, 5
discharge
not_intro 4, 6
assumption C
or_intro_right 9, N
not_elim 3, 10
discharge
not_intro 9, 11
assumption Z
imp_elim 1, 14
and_elim_left 15
not_elim 13, 16
discharge
not_intro 14, 17
imp_elim 2, 19
and_elim_left 20
not_elim 8, 21
discharge
indirect 3, 22
```

```
qed
```

Chapter 19, Exercise B.1

Proof presented using Fitch notation:

1	$(E \vee F)$	PR
2	$(F \vee G)$	PR
3	$\neg F$	PR
4	$\quad E$	AS
5	$\quad E$	R, 4
6	$\quad F$	AS
7	$\quad \bot$	¬E, 3, 6
8	$\quad E$	X, 7
9	E	∨E, 1, 4–5, 6–8
10	$\quad F$	AS
11	$\quad \bot$	¬E, 3, 10
12	$\quad G$	X, 11
13	$\quad G$	AS
14	$\quad G$	R, 13
15	G	∨E, 2, 10–12, 13–14
16	$(E \wedge G)$	∧I, 9, 15

Daeducer commands needed to generate the proof:

```
premise (E v F)
premise (F v G)
premise !F
assumption E
reiteration 4
discharge
assumption F
not_elim 3, 7
explosion 8, E
discharge
or_elim 1, 4, 5, 7, 9
assumption F
not_elim 3, 12
explosion 13, G
discharge
assumption G
reiteration 16
discharge
or_elim 2, 12, 14, 16, 17
and_intro 11, 19
qed
```

Chapter 19, Exercise B.2

Proof presented using Fitch notation:

1	$(M \vee (N \to M))$	PR
2	$\quad \neg M$	AS
3	$\quad\quad M$	AS
4	$\quad\quad \bot$	$\neg$E, 2, 3
5	$\quad\quad \neg N$	X, 4
6	$\quad\quad (N \to M)$	AS
7	$\quad\quad\quad N$	AS
8	$\quad\quad\quad M$	$\to$E, 6, 7
9	$\quad\quad\quad \bot$	$\neg$E, 2, 8
10	$\quad\quad \neg N$	$\neg$I, 7–9
11	$\quad \neg N$	$\vee$E, 1, 3–5, 6–10
12	$(\neg M \to \neg N)$	$\to$I, 2, 11

Daeducer commands needed to generate the proof:

```
premise (M v (N -> M))
assumption !M
assumption M
not_elim 2, 3
explosion 4, !N
discharge
assumption (N -> M)
assumption N
imp_elim 7, 8
not_elim 2, 9
discharge
not_intro 8, 10
discharge
or_elim 1, 3, 5, 7, 12
discharge
imp_intro 2, 14
qed
```

Chapter 19, Exercise B.3

Proof presented using Fitch notation:

1	$((M \vee N) \wedge (O \vee P))$	PR
2	$(N \rightarrow P)$	PR
3	$\neg P$	PR
4	$(M \vee N)$	$\wedge$E, 1
5	$(O \vee P)$	$\wedge$E, 1
6	$\quad N$	AS
7	$\quad P$	$\rightarrow$E, 2, 6
8	$\quad \perp$	$\neg$E, 3, 7
9	$\neg N$	$\neg$I, 6–8
10	$\quad M$	AS
11	$\quad M$	R, 10
12	$\quad N$	AS
13	$\quad \perp$	$\neg$E, 9, 12
14	$\quad M$	X, 13
15	M	$\vee$E, 4, 10–11, 12–14
16	$\quad O$	AS
17	$\quad O$	R, 16
18	$\quad P$	AS
19	$\quad \perp$	$\neg$E, 3, 18
20	$\quad O$	X, 19
21	O	$\vee$E, 5, 16–17, 18–20
22	$(M \wedge O)$	$\wedge$I, 15, 21

Daeducer commands needed to generate the proof:

```
premise ((M v N) ^ (O v P))
premise (N -> P)
premise !P
and_elim_left 1
and_elim_right 1
assumption N
imp_elim 2, 6
not_elim 3, 7
discharge
not_intro 6, 8
assumption M
reiteration 11
discharge
assumption N
not_elim 10, 14
explosion 15, M
discharge
or_elim 4, 11, 12, 14, 16
assumption O
reiteration 19
```

```
discharge
assumption P
not_elim 3, 22
explosion 23, 0
discharge
or_elim 5, 19, 20, 22, 24
and_intro 18, 26
qed
```

Chapter 19, Exercise B.4

Proof presented using Fitch notation:

1	$((X \wedge Y) \vee (X \wedge Z))$	PR
2	$\neg(X \wedge D)$	PR
3	$(D \vee M)$	PR
4	$\quad (X \wedge Y)$	AS
5	$\quad X$	$\wedge$E, 4
6	$\quad (X \wedge Z)$	AS
7	$\quad X$	$\wedge$E, 6
8	X	$\vee$E, 1, 4–5, 6–7
9	$\quad D$	AS
10	$\quad (X \wedge D)$	$\wedge$I, 8, 9
11	$\quad \perp$	$\neg$E, 2, 10
12	$\neg D$	$\neg$I, 9–11
13	$\quad D$	AS
14	$\quad \perp$	$\neg$E, 12, 13
15	$\quad M$	X, 14
16	$\quad M$	AS
17	$\quad M$	R, 16
18	M	$\vee$E, 3, 13–15, 16–17

Daeducer commands needed to generate the proof:

```
premise ((X ^ Y) v (X ^ Z))
premise !(X ^ D)
premise (D v M)
assumption (X ^ Y)
and_elim_left 4
discharge
assumption (X ^ Z)
and_elim_left 7
discharge
or_elim 1, 4, 5, 7, 8
assumption D
and_intro 10, 11
not_elim 2, 12
discharge
not_intro 11, 13
assumption D
not_elim 15, 16
```

```
explosion 17, M
discharge
assumption M
reiteration 20
discharge
or_elim 3, 16, 18, 20, 21
qed
```

Chapter 20, Example p.171

Proof presented using Fitch notation:

1	$\quad\mid\ (A \wedge \neg A)$	AS
2	$\quad\mid\ A$	$\wedge$E, 1
3	$\quad\mid\ \neg A$	$\wedge$E, 1
4	$\quad\mid\ \bot$	$\neg$E, 3, 2
5	$\neg(A \wedge \neg A)$	$\neg$I, 1–4

Daeducer commands needed to generate the proof:

```
assumption (A ^ !A)
and_elim_left 1
and_elim_right 1
not_elim 3, 2
discharge
not_intro 1, 4
qed
```

Chapter 20, Exercise A.1

Proof presented using Fitch notation:

1	$\quad\mid\ O$	AS
2	$\quad\mid\ O$	R, 1
3	$(O \rightarrow O)$	$\rightarrow$I, 1, 2

Daeducer commands needed to generate the proof:

```
assumption O
reiteration 1
discharge
imp_intro 1, 2
qed
```

Chapter 20, Exercise A.2

Proof presented using Fitch notation:

1	$\neg(N \vee \neg N)$	AS
2	N	AS
3	$(N \vee \neg N)$	$\vee$E, 2
4	$\bot$	$\neg$E, 1, 3
5	$\neg N$	$\neg$I, 2–4
6	$(N \vee \neg N)$	$\vee$E, 5
7	$\bot$	$\neg$E, 1, 6
8	$(N \vee \neg N)$	IP, 1–7

Daeducer commands needed to generate the proof:

```
assumption !(N v !N)
assumption N
or_intro_left 2, !N
not_elim 1, 3
discharge
not_intro 2, 4
or_intro_right 6, N
not_elim 1, 7
discharge
indirect 1, 8
qed
```

Chapter 20, Exercise A.3

Proof presented using Fitch notation:

1	J	AS
2	$(J \vee (L \wedge \neg L))$	$\vee$E, 1
3	$(J \to (J \vee (L \wedge \neg L)))$	$\to$I, 1, 2
4	$(J \vee (L \wedge \neg L))$	AS
5	J	AS
6	J	R, 5
7	$(L \wedge \neg L)$	AS
8	L	$\wedge$E, 7
9	$\neg L$	$\wedge$E, 7
10	$\bot$	$\neg$E, 9, 8
11	J	X, 10
12	J	$\vee$E, 4, 5–6, 7–11
13	$((J \vee (L \wedge \neg L)) \to J)$	$\to$I, 4, 12
14	$((J \to (J \vee (L \wedge \neg L))) \wedge ((J \vee (L \wedge \neg L)) \to J))$	$\wedge$I, 3, 13

Daeducer commands needed to generate the proof:

```
assumption J
or_intro_left 1, (L ^ !L)
discharge
imp_intro 1, 2
assumption (J v (L ^ !L))
assumption J
reiteration 6
discharge
assumption (L ^ !L)
and_elim_left 9
and_elim_right 9
not_elim 11, 10
explosion 12, J
discharge
or_elim 5, 6, 7, 9, 13
discharge
imp_intro 5, 15
and_intro 4, 17
qed
```

Chapter 20, Exercise A.4

Proof presented using Fitch notation:

1	$((A \to B) \to A)$	AS
2	$\neg A$	AS
3	A	AS
4	$\bot$	$\neg$E, 2, 3
5	B	X, 4
6	$(A \to B)$	$\to$I, 3, 5
7	A	$\to$E, 1, 6
8	$\bot$	$\neg$E, 2, 7
9	A	IP, 2–8
10	$(((A \to B) \to A) \to A)$	$\to$I, 1, 9

Daeducer commands needed to generate the proof:

```
assumption ((A -> B) -> A)
assumption !A
assumption A
not_elim 2, 3
explosion 4, B
discharge
imp_intro 3, 5
imp_elim 1, 7
not_elim 2, 8
discharge
indirect 2, 9
discharge
imp_intro 1, 11
qed
```

Chapter 20, Exercise B.1

Proof presented using Fitch notation:

1	$(C \to (E \land G))$	PR
2	$(\neg C \to G)$	PR
3	$\quad \neg G$	AS
4	$\quad\quad C$	AS
5	$\quad\quad (E \land G)$	$\to$E, 1, 4
6	$\quad\quad G$	$\land$E, 5
7	$\quad\quad \bot$	$\neg$E, 3, 6
8	$\quad \neg C$	$\neg$I, 4–7
9	$\quad G$	$\to$E, 2, 8
10	$\quad \bot$	$\neg$E, 3, 9
11	G	IP, 3–10

Daeducer commands needed to generate the proof:

```
premise (C -> (E ^ G))
premise (!C -> G)
assumption !G
assumption C
imp_elim 1, 4
and_elim_right 5
not_elim 3, 6
discharge
not_intro 4, 7
imp_elim 2, 9
not_elim 3, 10
discharge
indirect 3, 11
qed
```

Chapter 20, Exercise B.2

Proof presented using Fitch notation:

1	$(M \wedge (\neg N \to \neg M))$	PR
2	M	$\wedge$E, 1
3	$(\neg N \to \neg M)$	$\wedge$E, 1
4	$\quad \neg N$	AS
5	$\quad \neg M$	$\to$E, 3, 4
6	$\quad \bot$	$\neg$E, 5, 2
7	N	IP, 4–6
8	$(N \wedge M)$	$\wedge$I, 7, 2
9	$((N \wedge M) \vee \neg M)$	$\vee$E, 8

Daeducer commands needed to generate the proof:

```
premise (M ^ (!N -> !M))
and_elim_left 1
and_elim_right 1
assumption !N
imp_elim 3, 4
not_elim 5, 2
discharge
indirect 4, 6
and_intro 8, 2
or_intro_left 9, !M
qed
```

Chapter 20, Exercise B.3

Proof presented using Fitch notation:

1	$(((Z \wedge K) \to (Y \wedge M)) \wedge ((Y \wedge M) \to (Z \wedge K)))$	PR
2	$(D \wedge (D \to M))$	PR
3	D	$\wedge$E, 2
4	$(D \to M)$	$\wedge$E, 2
5	M	$\to$E, 4, 3
6	$\quad Y$	AS
7	$\quad ((Z \wedge K) \to (Y \wedge M))$	$\wedge$E, 1
8	$\quad ((Y \wedge M) \to (Z \wedge K))$	$\wedge$E, 1
9	$\quad (Y \wedge M)$	$\wedge$I, 6, 5
10	$\quad (Z \wedge K)$	$\to$E, 8, 9
11	$\quad Z$	$\wedge$E, 10
12	$(Y \to Z)$	$\to$I, 6, 11

Daeducer commands needed to generate the proof:

```
premise (((Z ^ K) -> (Y ^ M)) ^ ((Y ^ M) -> (Z ^ K)))
premise (D ^ (D -> M))
and_elim_left 2
and_elim_right 2
imp_elim 4, 3
assumption Y
and_elim_left 1
and_elim_right 1
and_intro 6, 5
imp_elim 8, 9
and_elim_left 10
discharge
imp_intro 6, 11
qed
```

Chapter 20, Exercise B.4

Proof presented using Fitch notation:

1	$((W \vee X) \vee (Y \vee Z))$	PR
2	$(X \rightarrow Y)$	PR
3	$\neg Z$	PR
4	$\quad (W \vee X)$	AS
5	$\quad\quad W$	AS
6	$\quad\quad (W \vee Y)$	$\vee$E, 5
7	$\quad\quad X$	AS
8	$\quad\quad Y$	$\rightarrow$E, 2, 7
9	$\quad\quad (W \vee Y)$	$\vee$E, 8
10	$\quad (W \vee Y)$	$\vee$E, 4, 5–6, 7–9
11	$\quad (Y \vee Z)$	AS
12	$\quad\quad Y$	AS
13	$\quad\quad (W \vee Y)$	$\vee$E, 12
14	$\quad\quad Z$	AS
15	$\quad\quad \perp$	$\neg$E, 3, 14
16	$\quad\quad (W \vee Y)$	X, 15
17	$\quad (W \vee Y)$	$\vee$E, 11, 12–13, 14–16
18	$(W \vee Y)$	$\vee$E, 1, 4–10, 11–17

Daeducer commands needed to generate the proof:

```
premise ((W v X) v (Y v Z))
premise (X -> Y)
premise !Z
assumption (W v X)
assumption W
or_intro_left 5, Y
discharge
assumption X
imp_elim 2, 8
or_intro_right 9, W
discharge
```

```
or_elim 4, 5, 6, 8, 10
discharge
assumption (Y v Z)
assumption Y
or_intro_right 15, W
discharge
assumption Z
not_elim 3, 18
explosion 19, (W v Y)
discharge
or_elim 14, 15, 16, 18, 20
discharge
or_elim 1, 4, 12, 14, 22
qed
```

Chapter 20, Exercise C.1.a

Proof presented using Fitch notation:

1	$((R \to E) \land (E \to R))$	PR
2	$(R \to E)$	$\land$E, 1
3	$(E \to R)$	$\land$E, 1
4	$((E \to R) \land (R \to E))$	$\land$I, 3, 2

Daeducer commands needed to generate the proof:

```
premise ((R -> E) ^ (E -> R))
and_elim_left 1
and_elim_right 1
and_intro 3, 2
qed
```

Chapter 20, Exercise C.1.b

Proof presented using Fitch notation:

1	$((E \to R) \land (R \to E))$	PR
2	$(E \to R)$	$\land$E, 1
3	$(R \to E)$	$\land$E, 1
4	$((R \to E) \land (E \to R))$	$\land$I, 3, 2

Daeducer commands needed to generate the proof:

```
premise ((E -> R) ^ (R -> E))
and_elim_left 1
and_elim_right 1
and_intro 3, 2
qed
```

Chapter 20, Exercise C.2.a

Proof presented using Fitch notation:

1	G	PR
2	$\quad \neg G$	AS
3	$\quad \bot$	$\neg$E, 2, 1
4	$\neg\neg G$	$\neg$I, 2–3
5	$\quad \neg\neg\neg G$	AS
6	$\quad \bot$	$\neg$E, 5, 4
7	$\neg\neg\neg\neg G$	$\neg$I, 5–6

Daeducer commands needed to generate the proof:

```
premise G
assumption !G
not_elim 2, 1
discharge
not_intro 2, 3
assumption !!!G
not_elim 6, 5
discharge
not_intro 6, 7
qed
```

Chapter 20, Exercise C.2.b

Proof presented using Fitch notation:

1	$\neg\neg\neg\neg G$	PR
2	$\quad \neg\neg\neg G$	AS
3	$\quad \bot$	$\neg$E, 1, 2
4	$\neg\neg G$	IP, 2–3
5	$\quad \neg G$	AS
6	$\quad \bot$	$\neg$E, 4, 5
7	G	IP, 5–6

Daeducer commands needed to generate the proof:

```
premise !!!!G
assumption !!!G
not_elim 1, 2
discharge
indirect 2, 3
assumption !G
not_elim 5, 6
discharge
indirect 6, 7
qed
```

Chapter 20, Exercise C.3.a

Proof presented using Fitch notation:

1	$(T \to S)$	PR
2	$\quad \neg S$	AS
3	$\quad\quad T$	AS
4	$\quad\quad S$	$\to$E, 1, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad \neg T$	$\neg$I, 3–5
7	$(\neg S \to \neg T)$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise (T -> S)
assumption !S
assumption T
imp_elim 1, 3
not_elim 2, 4
discharge
not_intro 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 20, Exercise C.3.b

Proof presented using Fitch notation:

1	$(\neg S \to \neg T)$	PR
2	$\quad T$	AS
3	$\quad\quad \neg S$	AS
4	$\quad\quad \neg T$	$\to$E, 1, 3
5	$\quad\quad \bot$	$\neg$E, 4, 2
6	$\quad S$	IP, 3–5
7	$(T \to S)$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise (!S -> !T)
assumption T
assumption !S
imp_elim 1, 3
not_elim 4, 2
discharge
indirect 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 20, Exercise C.4.a

Proof presented using Fitch notation:

1	$(U \to I)$	PR
2	$\quad (U \land \neg I)$	AS
3	$\quad U$	$\land$E, 2
4	$\quad \neg I$	$\land$E, 2
5	$\quad I$	$\to$E, 1, 3
6	$\quad \bot$	$\neg$E, 4, 5
7	$\neg(U \land \neg I)$	$\neg$I, 2–6

Daeducer commands needed to generate the proof:

```
premise (U -> I)
assumption (U ^ !I)
and_elim_left 2
and_elim_right 2
imp_elim 1, 3
not_elim 4, 5
discharge
not_intro 2, 6
qed
```

Chapter 20, Exercise C.4.b

Proof presented using Fitch notation:

1	$\neg(U \land \neg I)$	PR
2	$\quad U$	AS
3	$\quad\quad \neg I$	AS
4	$\quad\quad (U \land \neg I)$	$\land$I, 2, 3
5	$\quad\quad \bot$	$\neg$E, 1, 4
6	$\quad I$	IP, 3–5
7	$(U \to I)$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise !(U ^ !I)
assumption U
assumption !I
and_intro 2, 3
not_elim 1, 4
discharge
indirect 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 20, Exercise C.5.a

Proof presented using Fitch notation:

1	$\neg(C \to D)$	PR
2	$\quad \neg C$	AS
3	$\quad\quad C$	AS
4	$\quad\quad \bot$	$\neg$E, 2, 3
5	$\quad\quad D$	X, 4
6	$\quad (C \to D)$	$\to$I, 3, 5
7	$\quad \bot$	$\neg$E, 1, 6
8	C	IP, 2–7
9	$\quad D$	AS
10	$\quad\quad C$	AS
11	$\quad\quad D$	R, 9
12	$\quad (C \to D)$	$\to$I, 10, 11
13	$\quad \bot$	$\neg$E, 1, 12
14	$\neg D$	$\neg$I, 9–13
15	$(C \wedge \neg D)$	$\wedge$I, 8, 14

Daeducer commands needed to generate the proof:

```
premise !(C -> D)
assumption !C
assumption C
not_elim 2, 3
explosion 4, D
discharge
imp_intro 3, 5
not_elim 1, 7
discharge
indirect 2, 8
assumption D
assumption C
reiteration 11
discharge
imp_intro 12, 13
not_elim 1, 15
discharge
not_intro 11, 16
and_intro 10, 18
qed
```

Chapter 20, Exercise C.5.b

Proof presented using Fitch notation:

1	$(C \wedge \neg D)$	PR
2	C	$\wedge$E, 1
3	$\neg D$	$\wedge$E, 1
4	$\quad (C \rightarrow D)$	AS
5	$\quad D$	$\rightarrow$E, 4, 2
6	$\quad \bot$	$\neg$E, 3, 5
7	$\neg(C \rightarrow D)$	$\neg$I, 4–6

Daeducer commands needed to generate the proof:

```
premise (C ^ !D)
and_elim_left 1
and_elim_right 1
assumption (C -> D)
imp_elim 4, 2
not_elim 3, 5
discharge
not_intro 4, 6
qed
```

Chapter 20, Exercise C.6.a

Proof presented using Fitch notation:

1	$((\neg G \to H) \land (H \to \neg G))$	PR
2	$\quad ((G \to H) \land (H \to G))$	AS
3	$\quad (G \to H)$	$\land$E, 2
4	$\quad (H \to G)$	$\land$E, 2
5	$\quad (\neg G \to H)$	$\land$E, 1
6	$\quad (H \to \neg G)$	$\land$E, 1
7	$\quad\quad H$	AS
8	$\quad\quad \neg G$	$\to$E, 6, 7
9	$\quad\quad G$	$\to$E, 4, 7
10	$\quad\quad \bot$	$\neg$E, 8, 9
11	$\quad \neg H$	$\neg$I, 7–10
12	$\quad\quad \neg G$	AS
13	$\quad\quad H$	$\to$E, 5, 12
14	$\quad\quad \bot$	$\neg$E, 11, 13
15	$\quad G$	IP, 12–14
16	$\quad H$	$\to$E, 3, 15
17	$\quad \bot$	$\neg$E, 11, 16
18	$\neg((G \to H) \land (H \to G))$	$\neg$I, 2–17

Daeducer commands needed to generate the proof:

```
premise ((!G -> H) ^ (H -> !G))
assumption ((G -> H) ^ (H -> G))
and_elim_left 2
and_elim_right 2
and_elim_left 1
and_elim_right 1
assumption H
imp_elim 6, 7
imp_elim 4, 7
not_elim 8, 9
discharge
not_intro 7, 10
assumption !G
imp_elim 5, 13
not_elim 12, 14
discharge
indirect 13, 15
imp_elim 3, 17
not_elim 12, 18
discharge
not_intro 2, 19
qed
```

Chapter 20, Exercise C.6.b

Proof presented using Fitch notation:

1	$\neg((G \to H) \land (H \to G))$	PR
2	$\quad \neg G$	AS
3	$\quad\quad \neg H$	AS
4	$\quad\quad\quad G$	AS
5	$\quad\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad\quad\quad H$	X, 5
7	$\quad\quad (G \to H)$	$\to$I, 4, 6
8	$\quad\quad\quad H$	AS
9	$\quad\quad\quad \bot$	$\neg$E, 3, 8
10	$\quad\quad\quad G$	X, 9
11	$\quad\quad (H \to G)$	$\to$I, 8, 10
12	$\quad\quad ((G \to H) \land (H \to G))$	$\land$I, 7, 11
13	$\quad\quad \bot$	$\neg$E, 1, 12
14	$\quad H$	IP, 3–13
15	$(\neg G \to H)$	$\to$I, 2, 14
16	$\quad H$	AS
17	$\quad\quad G$	AS
18	$\quad\quad\quad G$	AS
19	$\quad\quad\quad H$	R, 16
20	$\quad\quad (G \to H)$	$\to$I, 18, 19
21	$\quad\quad\quad H$	AS
22	$\quad\quad\quad G$	R, 17
23	$\quad\quad (H \to G)$	$\to$I, 21, 22
24	$\quad\quad ((G \to H) \land (H \to G))$	$\land$I, 20, 23
25	$\quad\quad \bot$	$\neg$E, 1, 24
26	$\quad \neg G$	$\neg$I, 17–25
27	$(H \to \neg G)$	$\to$I, 16, 26
28	$((\neg G \to H) \land (H \to \neg G))$	$\land$I, 15, 27

Daeducer commands needed to generate the proof:

```
premise !((G -> H) ^ (H -> G))
assumption !G
assumption !H
assumption G
not_elim 2, 4
explosion 5, H
discharge
imp_intro 4, 6
```

```
assumption H
not_elim 3, 9
explosion 10, G
discharge
imp_intro 9, 11
and_intro 8, 13
not_elim 1, 14
discharge
indirect 3, 15
discharge
imp_intro 2, 17
assumption H
assumption G
assumption G
reiteration 20
discharge
imp_intro 22, 23
assumption H
reiteration 21
discharge
imp_intro 26, 27
and_intro 25, 29
not_elim 1, 30
discharge
not_intro 21, 31
discharge
imp_intro 20, 33
and_intro 19, 35
qed
```

Chapter 21, Example p.176

Proof presented using Fitch notation:

1	$(A \to B)$	PR
2	$\neg B$	PR
3	$\quad A$	AS
4	$\quad B$	$\to$E, 1, 3
5	$\quad \bot$	$\neg$E, 2, 4
6	$\neg A$	$\neg$I, 3–5

Daeducer commands needed to generate the proof:

```
premise (A -> B)
premise !B
assumption A
imp_elim 1, 3
not_elim 2, 4
discharge
not_intro 3, 5
qed
```

Chapter 21, Example p.177

Proof presented using Fitch notation:

1	$\neg\neg A$	PR
2	$\quad \neg A$	AS
3	$\quad \bot$	¬E, 1, 2
4	A	IP, 2–3

Daeducer commands needed to generate the proof:

```
premise !!A
assumption !A
not_elim 1, 2
discharge
indirect 2, 3
qed
```

Chapter 21, Example p.178

Proof presented using Fitch notation:

1	$(A \to B)$	PR
2	$(\neg A \to B)$	PR
3	$\quad \neg B$	AS
4	$\quad\quad A$	AS
5	$\quad\quad B$	→E, 1, 4
6	$\quad\quad \bot$	¬E, 3, 5
7	$\quad \neg A$	¬I, 4–6
8	$\quad B$	→E, 2, 7
9	$\quad \bot$	¬E, 3, 8
10	B	IP, 3–9

Daeducer commands needed to generate the proof:

```
premise (A -> B)
premise (!A -> B)
assumption !B
assumption A
imp_elim 1, 4
not_elim 3, 5
discharge
not_intro 4, 6
imp_elim 2, 8
not_elim 3, 9
discharge
indirect 3, 10
qed
```

Chapter 21, Example p.180

Proof presented using Fitch notation:

1	$\neg(A \wedge B)$	PR
2	$\quad \neg(\neg A \vee \neg B)$	AS
3	$\quad\quad \neg A$	AS
4	$\quad\quad (\neg A \vee \neg B)$	$\vee$E, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad A$	IP, 3–5
7	$\quad\quad \neg B$	AS
8	$\quad\quad (\neg A \vee \neg B)$	$\vee$E, 7
9	$\quad\quad \bot$	$\neg$E, 2, 8
10	$\quad B$	IP, 7–9
11	$\quad (A \wedge B)$	$\wedge$I, 6, 10
12	$\quad \bot$	$\neg$E, 1, 11
13	$(\neg A \vee \neg B)$	IP, 2–12

Daeducer commands needed to generate the proof:

```
premise !(A ^ B)
assumption !(!A v !B)
assumption !A
or_intro_left 3, !B
not_elim 2, 4
discharge
indirect 3, 5
assumption !B
or_intro_right 8, !A
not_elim 2, 9
discharge
indirect 8, 10
and_intro 7, 12
not_elim 1, 13
discharge
indirect 2, 14
qed
```

Chapter 21, Example p.180

Proof presented using Fitch notation:

1	$(\neg A \vee \neg B)$	PR
2	$\quad (A \wedge B)$	AS
3	$\quad A$	$\wedge$E, 2
4	$\quad B$	$\wedge$E, 2
5	$\quad\quad \neg A$	AS
6	$\quad\quad \bot$	$\neg$E, 5, 3
7	$\quad\quad \neg B$	AS
8	$\quad\quad \bot$	$\neg$E, 7, 4
9	$\quad \bot$	$\vee$E, 1, 5–6, 7–8
10	$\neg(A \wedge B)$	$\neg$I, 2–9

Daeducer commands needed to generate the proof:

```
premise (!A v !B)
assumption (A ^ B)
and_elim_left 2
and_elim_right 2
assumption !A
not_elim 5, 3
discharge
assumption !B
not_elim 8, 4
discharge
or_elim 1, 5, 6, 8, 9
discharge
not_intro 2, 11
qed
```

Chapter 21, Exercise A.1

Proof presented using Fitch notation:

1	$\neg(A \vee B)$	PR
2	$\quad A$	AS
3	$\quad (A \vee B)$	$\vee$E, 2
4	$\quad \bot$	$\neg$E, 1, 3
5	$\neg A$	$\neg$I, 2–4
6	$\quad B$	AS
7	$\quad (A \vee B)$	$\vee$E, 6
8	$\quad \bot$	$\neg$E, 1, 7
9	$\neg B$	$\neg$I, 6–8
10	$(\neg A \wedge \neg B)$	$\wedge$I, 5, 9

Daeducer commands needed to generate the proof:

```
premise !(A v B)
assumption A
or_intro_left 2, B
not_elim 1, 3
discharge
not_intro 2, 4
assumption B
or_intro_right 7, A
not_elim 1, 8
discharge
not_intro 7, 9
and_intro 6, 11
qed
```

Chapter 21, Exercise A.2

Proof presented using Fitch notation:

1	$(\neg A \wedge \neg B)$	PR
2	$\neg A$	$\wedge$E, 1
3	$\neg B$	$\wedge$E, 1
4	$\quad (A \vee B)$	AS
5	$\quad\quad A$	AS
6	$\quad\quad \bot$	$\neg$E, 2, 5
7	$\quad\quad B$	AS
8	$\quad\quad \bot$	$\neg$E, 3, 7
9	$\quad \bot$	$\vee$E, 4, 5–6, 7–8
10	$\neg(A \vee B)$	$\neg$I, 4–9

Daeducer commands needed to generate the proof:

```
premise (!A ^ !B)
and_elim_left 1
and_elim_right 1
assumption (A v B)
assumption A
not_elim 2, 5
discharge
assumption B
not_elim 3, 8
discharge
or_elim 4, 5, 6, 8, 9
discharge
not_intro 4, 11
qed
```

Chapter 22, Exercise A.1

Proof presented using Fitch notation:

1	A	AS
2	$((B \wedge C) \vee D)$	AS
3	A	R, 1
4	$(((B \wedge C) \vee D) \rightarrow A)$	$\rightarrow$I, 2, 3
5	$(A \rightarrow (((B \wedge C) \vee D) \rightarrow A))$	$\rightarrow$I, 1, 4

Daeducer commands needed to generate the proof:

```
assumption A
assumption ((B ^ C) v D)
reiteration 1
discharge
imp_intro 2, 3
discharge
imp_intro 1, 5
qed
```

Chapter 22, Exercise A.2

Proof presented using Fitch notation:

1	A	PR
2	$\neg B$	PR
3	$\quad (A \to (A \to B))$	AS
4	$\quad (A \to B)$	$\to$E, 3, 1
5	$\quad B$	$\to$E, 4, 1
6	$\quad \bot$	$\neg$E, 2, 5
7	$\neg(A \to (A \to B))$	$\neg$I, 3–6

Daeducer commands needed to generate the proof:

```
premise A
premise !B
assumption (A -> (A -> B))
imp_elim 3, 1
imp_elim 4, 1
not_elim 2, 5
discharge
not_intro 3, 6
qed
```

Chapter 22, Exercise A.3

Proof presented using Fitch notation:

1	$\neg A$	PR
2	$\quad A$	AS
3	$\quad \neg A$	R, 1
4	$(A \to \neg A)$	$\to$I, 2, 3

Daeducer commands needed to generate the proof:

```
premise !A
assumption A
reiteration 1
discharge
imp_intro 2, 3
qed
```

Chapter 22, Exercise A.4

Proof presented using Fitch notation:

1	$((A \to \neg A) \land (\neg A \to A))$	PR
2	$(A \to \neg A)$	$\land$E, 1
3	$(\neg A \to A)$	$\land$E, 1
4	$\quad \neg A$	AS
5	$\quad A$	$\to$E, 3, 4
6	$\quad \bot$	$\neg$E, 4, 5
7	A	IP, 4–6
8	$\neg A$	$\to$E, 2, 7
9	$\bot$	$\neg$E, 8, 7

Daeducer commands needed to generate the proof:

```
premise ((A -> !A) ^ (!A -> A))
and_elim_left 1
and_elim_right 1
assumption !A
imp_elim 3, 4
not_elim 4, 5
discharge
indirect 4, 6
imp_elim 2, 8
not_elim 9, 8
qed
```

Chapter 22, Exercise A.5.a

Proof presented using Fitch notation:

1	W	PR
2	$\neg J$	PR
3	$\quad (W \to (J \lor J))$	AS
4	$\quad (J \lor J)$	$\to$E, 3, 1
5	$\quad\quad J$	AS
6	$\quad\quad \bot$	$\neg$E, 2, 5
7	$\quad\quad J$	AS
8	$\quad\quad \bot$	$\neg$E, 2, 7
9	$\quad \bot$	$\lor$E, 4, 5–6, 7–8
10	$\neg(W \to (J \lor J))$	$\neg$I, 3–9

Daeducer commands needed to generate the proof:

```
premise W
premise !J
assumption (W -> (J v J))
```

```
imp_elim 3, 1
assumption J
not_elim 2, 5
discharge
assumption J
not_elim 2, 8
discharge
or_elim 4, 5, 6, 8, 9
discharge
not_intro 3, 11
qed
```

Chapter 22, Exercise A.5.b

Proof presented using Fitch notation:

1	W	PR
2	J	PR
3	$\quad \neg(W \to (J \vee J))$	AS
4	$\quad\quad W$	AS
5	$\quad\quad (J \vee J)$	$\vee$E, 2
6	$\quad (W \to (J \vee J))$	$\to$I, 4, 5
7	$\quad \bot$	$\neg$E, 3, 6
8	$\neg\neg(W \to (J \vee J))$	$\neg$I, 3–7

Daeducer commands needed to generate the proof:

```
premise W
premise J
assumption !(W -> (J v J))
assumption W
or_intro_left 2, J
discharge
imp_intro 4, 5
not_elim 3, 7
discharge
not_intro 3, 8
qed
```

Chapter 22, Exercise A.6

Proof presented using Fitch notation:

1	$\neg(\neg(X \vee (Y \vee Z)) \vee (X \vee (Y \vee Z)))$	AS
2	$(X \vee (Y \vee Z))$	AS
3	$(\neg(X \vee (Y \vee Z)) \vee (X \vee (Y \vee Z)))$	$\vee$E, 2
4	$\bot$	$\neg$E, 1, 3
5	$\neg(X \vee (Y \vee Z))$	$\neg$I, 2–4
6	$(\neg(X \vee (Y \vee Z)) \vee (X \vee (Y \vee Z)))$	$\vee$E, 5
7	$\bot$	$\neg$E, 1, 6
8	$(\neg(X \vee (Y \vee Z)) \vee (X \vee (Y \vee Z)))$	IP, 1–7

Daeducer commands needed to generate the proof:

```
assumption !(!(X v (Y v Z)) v (X v (Y v Z)))
assumption (X v (Y v Z))
or_intro_right 2, !(X v (Y v Z))
not_elim 1, 3
discharge
not_intro 2, 4
or_intro_left 6, (X v (Y v Z))
not_elim 1, 7
discharge
indirect 1, 8
qed
```

Chapter 22, Exercise A.7

Proof presented using Fitch notation:

1	$(B \to \neg S)$	AS
2	$\neg\neg B$	AS
3	$\neg B$	AS
4	$\bot$	$\neg$E, 2, 3
5	B	IP, 3–4
6	$\neg S$	$\to$E, 1, 5
7	$(\neg\neg B \to \neg S)$	$\to$I, 2, 6
8	$((B \to \neg S) \to (\neg\neg B \to \neg S))$	$\to$I, 1, 7
9	$(\neg\neg B \to \neg S)$	AS
10	B	AS
11	$\neg B$	AS
12	$\bot$	$\neg$E, 11, 10
13	$\neg\neg B$	$\neg$I, 11–12
14	$\neg S$	$\to$E, 9, 13
15	$(B \to \neg S)$	$\to$I, 10, 14
16	$((\neg\neg B \to \neg S) \to (B \to \neg S))$	$\to$I, 9, 15
17	$(((B \to \neg S) \to (\neg\neg B \to \neg S)) \wedge ((\neg\neg B \to \neg S) \to (B \to \neg S)))$	$\wedge$I, 8, 16

Daeducer commands needed to generate the proof:

```
assumption (B -> !S)
assumption !!B
assumption !B
not_elim 2, 3
discharge
indirect 3, 4
imp_elim 1, 6
discharge
imp_intro 2, 7
discharge
imp_intro 1, 9
assumption (!!B -> !S)
assumption B
assumption !B
not_elim 14, 13
discharge
not_intro 14, 15
imp_elim 12, 17
discharge
imp_intro 13, 18
discharge
imp_intro 12, 20
and_intro 11, 22
qed
```

Chapter 22, Exercise A.8

Proof presented using Fitch notation:

1	X	PR
2	O	PR
3	$\quad ((X \land O) \land \neg(X \lor O))$	AS
4	$\quad \neg(X \lor O)$	$\land$E, 3
5	$\quad (X \lor O)$	$\lor$E, 1
6	$\quad \bot$	$\neg$E, 4, 5
7	$\neg((X \land O) \land \neg(X \lor O))$	$\neg$I, 3–6

Daeducer commands needed to generate the proof:

```
premise X
premise O
assumption ((X ^ O) ^ !(X v O))
and_elim_right 3
or_intro_left 1, O
not_elim 4, 5
discharge
not_intro 3, 6
qed
```

Chapter 22, Exercise A.9

Proof presented using Fitch notation:

1	$\neg(A \lor B)$	PR
2	C	PR
3	$(C \to A)$	PR
4	A	$\to$E, 3, 2
5	$(A \lor B)$	$\lor$E, 4
6	$\bot$	$\neg$E, 1, 5

Daeducer commands needed to generate the proof:

```
premise !(A v B)
premise C
premise (C -> A)
imp_elim 3, 2
or_intro_left 4, B
not_elim 1, 5
qed
```

Chapter 22, Exercise A.10

Proof presented using Fitch notation:

1	$\neg A$	PR
2	$\neg B$	PR
3	$\quad (A \vee B)$	AS
4	$\quad\quad A$	AS
5	$\quad\quad \bot$	$\neg$E, 1, 4
6	$\quad\quad B$	AS
7	$\quad\quad \bot$	$\neg$E, 2, 6
8	$\quad \bot$	$\vee$E, 3, 4–5, 6–7
9	$\neg(A \vee B)$	$\neg$I, 3–8
10	$\quad B$	AS
11	$\quad \bot$	$\neg$E, 2, 10
12	$\quad A$	X, 11
13	$(B \to A)$	$\to$I, 10, 12
14	$(\neg B \wedge (B \to A))$	$\wedge$I, 2, 13
15	$(\neg(A \vee B) \wedge (\neg B \wedge (B \to A)))$	$\wedge$I, 9, 14

Daeducer commands needed to generate the proof:

```
premise !A
premise !B
assumption (A v B)
assumption A
not_elim 1, 4
discharge
assumption B
not_elim 2, 7
discharge
or_elim 3, 4, 5, 7, 8
discharge
not_intro 3, 10
assumption B
not_elim 2, 13
explosion 14, A
discharge
imp_intro 13, 15
and_intro 2, 17
and_intro 12, 18
qed
```

Chapter 22, Exercise A.11

Proof presented using Fitch notation:

1	$\neg(A \vee (B \vee C))$	PR
2	$\quad C$	AS
3	$\quad (B \vee C)$	$\vee$E, 2
4	$\quad (A \vee (B \vee C))$	$\vee$E, 3
5	$\quad \bot$	$\neg$E, 1, 4
6	$\neg C$	$\neg$I, 2–5

Daeducer commands needed to generate the proof:

```
premise !(A v (B v C))
assumption C
or_intro_right 2, B
or_intro_right 3, A
not_elim 1, 4
discharge
not_intro 2, 5
qed
```

Chapter 22, Exercise A.12

Proof presented using Fitch notation:

1	$\neg A$	PR
2	B	PR
3	C	PR
4	$\quad \neg C$	AS
5	$\quad \bot$	$\neg$E, 4, 3
6	$\neg\neg C$	$\neg$I, 4–5
7	$\quad (A \wedge (B \vee C))$	AS
8	$\quad A$	$\wedge$E, 7
9	$\quad \bot$	$\neg$E, 1, 8
10	$\neg(A \wedge (B \vee C))$	$\neg$I, 7–9
11	$(\neg(A \wedge (B \vee C)) \wedge \neg\neg C)$	$\wedge$I, 10, 6

Daeducer commands needed to generate the proof:

```
premise !A
premise B
premise C
assumption !C
not_elim 4, 3
discharge
not_intro 4, 5
assumption (A ^ (B v C))
and_elim_left 8
not_elim 1, 9
discharge
```

```
not_intro 8, 10
and_intro 12, 7
qed
```

Chapter 22, Exercise B.1

Proof presented using Fitch notation:

1	$\quad A$	AS
2	$\quad\quad B$	AS
3	$\quad\quad A$	R, 1
4	$\quad (B \to A)$	$\to$I, 2, 3
5	$(A \to (B \to A))$	$\to$I, 1, 4

Daeducer commands needed to generate the proof:

```
assumption A
assumption B
reiteration 1
discharge
imp_intro 2, 3
discharge
imp_intro 1, 5
qed
```

Chapter 22, Exercise B.2

Proof presented using Fitch notation:

1	N	PR
2	Q	PR
3	$\quad \neg((((N \to Q) \land (Q \to N)) \lor Q) \lor N)$	AS
4	$\quad ((((N \to Q) \land (Q \to N)) \lor Q) \lor N)$	$\lor$E, 1
5	$\quad \bot$	$\neg$E, 3, 4
6	$\neg\neg((((N \to Q) \land (Q \to N)) \lor Q) \lor N)$	$\neg$I, 3–5

Daeducer commands needed to generate the proof:

```
premise N
premise Q
assumption !((((N -> Q) ^ (Q -> N)) v Q) v N)
or_intro_right 1, (((N -> Q) ^ (Q -> N)) v Q)
not_elim 3, 4
discharge
not_intro 3, 5
qed
```

Chapter 22, Exercise B.3.a

Proof presented using Fitch notation:

1	Z	PR
2	$(Z \vee ((\neg Z \to Z) \wedge (Z \to \neg Z)))$	$\vee$E, 1

Daeducer commands needed to generate the proof:

```
premise Z
or_intro_left 1, ((!Z -> Z) ^ (Z -> !Z))
qed
```

Chapter 22, Exercise B.3b

Proof presented using Fitch notation:

1	$\neg Z$	PR
2	$\quad (Z \vee ((\neg Z \to Z) \wedge (Z \to \neg Z)))$	AS
3	$\quad\quad Z$	AS
4	$\quad\quad \bot$	$\neg$E, 1, 3
5	$\quad\quad ((\neg Z \to Z) \wedge (Z \to \neg Z))$	AS
6	$\quad\quad (\neg Z \to Z)$	$\wedge$E, 5
7	$\quad\quad Z$	$\to$E, 6, 1
8	$\quad\quad \bot$	$\neg$E, 1, 7
9	$\quad \bot$	$\vee$E, 2, 3–4, 5–8
10	$\neg(Z \vee ((\neg Z \to Z) \wedge (Z \to \neg Z)))$	$\neg$I, 2–9

Daeducer commands needed to generate the proof:

```
premise !Z
assumption (Z v ((!Z -> Z) ^ (Z -> !Z)))
assumption Z
not_elim 1, 3
discharge
assumption ((!Z -> Z) ^ (Z -> !Z))
and_elim_left 6
imp_elim 7, 1
not_elim 1, 8
discharge
or_elim 2, 3, 4, 6, 9
discharge
not_intro 2, 11
qed
```

Chapter 22, Exercise B.4

Proof presented using Fitch notation:

1	$\quad N$	AS
2	$\quad N$	R, 1
3	$(N \to N)$	$\to$I, 1, 2
4	$\quad L$	AS
5	$\quad\quad (N \to N)$	AS
6	$\quad\quad L$	R, 4
7	$\quad ((N \to N) \to L)$	$\to$I, 5, 6
8	$(L \to ((N \to N) \to L))$	$\to$I, 4, 7
9	$\quad ((N \to N) \to L)$	AS
10	$\quad (N \to N)$	R, 3
11	$\quad L$	$\to$E, 9, 10
12	$(((N \to N) \to L) \to L)$	$\to$I, 9, 11
13	$((L \to ((N \to N) \to L)) \wedge (((N \to N) \to L) \to L))$	$\wedge$I, 8, 12
14	$(((L \to ((N \to N) \to L)) \wedge (((N \to N) \to L) \to L)) \vee H)$	$\vee$E, 13

Daeducer commands needed to generate the proof:

```
assumption N
reiteration 1
discharge
imp_intro 1, 2
assumption L
assumption (N -> N)
reiteration 5
discharge
imp_intro 6, 7
discharge
imp_intro 5, 9
assumption ((N -> N) -> L)
reiteration 4
imp_elim 12, 13
discharge
imp_intro 12, 14
and_intro 11, 16
or_intro_left 17, H
qed
```

Chapter 22, Exercise B.5

Proof presented using Fitch notation:

1	$(((A \to A) \land (A \to A)) \land (B \land \neg B))$	PR
2	$(B \land \neg B)$	$\land$E, 1
3	B	$\land$E, 2
4	$\neg B$	$\land$E, 2
5	$\bot$	$\neg$E, 4, 3

Daeducer commands needed to generate the proof:

```
premise (((A -> A) ^ (A -> A)) ^ (B ^ !B))
and_elim_right 1
and_elim_left 2
and_elim_right 2
not_elim 4, 3
qed
```

Chapter 22, Exercise B.6

Proof presented using Fitch notation:

1	B	PR
2	C	PR
3	$\quad B$	AS
4	$\quad (C \lor B)$	$\lor$E, 3
5	$(B \to (C \lor B))$	$\to$I, 3, 4
6	$\quad (C \lor B)$	AS
7	$\quad B$	R, 1
8	$((C \lor B) \to B)$	$\to$I, 6, 7
9	$((B \to (C \lor B)) \land ((C \lor B) \to B))$	$\land$I, 5, 8

Daeducer commands needed to generate the proof:

```
premise B
premise C
assumption B
or_intro_right 3, C
discharge
imp_intro 3, 4
assumption (C v B)
reiteration 1
discharge
imp_intro 7, 8
and_intro 6, 10
qed
```

Chapter 22, Exercise B.7

Proof presented using Fitch notation:

1	X	AS
2	$(((\neg X \to X) \land (X \to \neg X)) \lor X)$	$\lor$E, 1
3	$(X \to (((\neg X \to X) \land (X \to \neg X)) \lor X))$	$\to$I, 1, 2
4	$(((\neg X \to X) \land (X \to \neg X)) \lor X)$	AS
5	$((\neg X \to X) \land (X \to \neg X))$	AS
6	$(\neg X \to X)$	$\land$E, 5
7	$\neg X$	AS
8	X	$\to$E, 6, 7
9	$\bot$	$\neg$E, 7, 8
10	X	IP, 7–9
11	X	AS
12	X	R, 11
13	X	$\lor$E, 4, 5–10, 11–12
14	$((((\neg X \to X) \land (X \to \neg X)) \lor X) \to X)$	$\to$I, 4, 13
15	$((X \to (((\neg X \to X) \land (X \to \neg X)) \lor X)) \land ((((\neg X \to X) \land (X \to \neg X)) \lor X) \to X))$	$\land$I, 3, 14

Daeducer commands needed to generate the proof:

```
assumption X
or_intro_right 1, ((!X -> X) ^ (X -> !X))
discharge
imp_intro 1, 2
assumption (((!X -> X) ^ (X -> !X)) v X)
assumption ((!X -> X) ^ (X -> !X))
and_elim_left 6
assumption !X
imp_elim 7, 8
not_elim 8, 9
discharge
indirect 8, 10
discharge
assumption X
reiteration 14
discharge
or_elim 5, 6, 12, 14, 15
discharge
imp_intro 5, 17
and_intro 4, 19
qed
```

Chapter 22, Exercise B.8

Proof presented using Fitch notation:

1	F	PR
2	$\neg K$	PR
3	$\neg R$	PR
4	$\quad (F \wedge (K \wedge R))$	AS
5	$\quad F$	$\wedge$E, 4
6	$\quad (K \wedge R)$	$\wedge$E, 4
7	$\quad K$	$\wedge$E, 6
8	$\quad \bot$	$\neg$E, 2, 7
9	$\neg(F \wedge (K \wedge R))$	$\neg$I, 4–8
10	$\quad F$	AS
11	$\quad\quad K$	AS
12	$\quad\quad \bot$	$\neg$E, 2, 11
13	$\quad\quad R$	X, 12
14	$\quad (K \to R)$	$\to$I, 11, 13
15	$\quad\quad R$	AS
16	$\quad\quad \bot$	$\neg$E, 3, 15
17	$\quad\quad K$	X, 16
18	$\quad (R \to K)$	$\to$I, 15, 17
19	$\quad ((K \to R) \wedge (R \to K))$	$\wedge$I, 14, 18
20	$(F \to ((K \to R) \wedge (R \to K)))$	$\to$I, 10, 19
21	$\quad ((K \to R) \wedge (R \to K))$	AS
22	$\quad F$	R, 1
23	$(((K \to R) \wedge (R \to K)) \to F)$	$\to$I, 21, 22
24	$((F \to ((K \to R) \wedge (R \to K))) \wedge (((K \to R) \wedge (R \to K)) \to F))$	$\wedge$I, 20, 23
25	$(\neg(F \wedge (K \wedge R)) \wedge ((F \to ((K \to R) \wedge (R \to K))) \wedge (((K \to R) \wedge (R \to K)) \to F)))$	$\wedge$I, 9, 24

Daeducer commands needed to generate the proof:

```
premise F
premise !K
premise !R
assumption (F ^ (K ^ R))
and_elim_left 4
and_elim_right 4
and_elim_left 6
not_elim 2, 7
discharge
not_intro 4, 8
assumption F
assumption K
not_elim 2, 12
explosion 13, R
```

```
discharge
imp_intro 12, 14
assumption R
not_elim 3, 17
explosion 18, K
discharge
imp_intro 17, 19
and_intro 16, 21
discharge
imp_intro 11, 22
assumption ((K -> R) ^ (R -> K))
reiteration 1
discharge
imp_intro 25, 26
and_intro 24, 28
and_intro 10, 29
qed
```

Chapter 22, Exercise B.9

Proof presented using Fitch notation:

1	$\neg(W \to W)$	PR
2	$(((W \to W) \wedge (W \to W)) \wedge W)$	PR
3	$(E \vee (W \to \neg(E \wedge W)))$	PR
4	$((W \to W) \wedge (W \to W))$	$\wedge$E, 2
5	$(W \to W)$	$\wedge$E, 4
6	$\perp$	$\neg$E, 1, 5

Daeducer commands needed to generate the proof:

```
premise !(W -> W)
premise (((W -> W) ^ (W -> W)) ^ W)
premise (E v (W -> !(E ^ W)))
and_elim_left 2
and_elim_left 4
not_elim 1, 5
qed
```

Chapter 22, Exercise B.10

Proof presented using Fitch notation:

1	C	PR
2	$\neg R$	PR
3	$(\neg R \vee C)$	$\vee$E, 2
4	$\quad (C \wedge R)$	AS
5	$\quad \neg R$	R, 2
6	$((C \wedge R) \rightarrow \neg R)$	$\rightarrow$I, 4, 5
7	$\quad \neg(R \vee C)$	AS
8	$\quad (R \vee C)$	$\vee$E, 1
9	$\quad \bot$	$\neg$E, 7, 8
10	$\quad R$	X, 9
11	$(\neg(R \vee C) \rightarrow R)$	$\rightarrow$I, 7, 10
12	$(((C \wedge R) \rightarrow \neg R) \wedge (\neg(R \vee C) \rightarrow R))$	$\wedge$I, 6, 11
13	$((\neg R \vee C) \wedge (((C \wedge R) \rightarrow \neg R) \wedge (\neg(R \vee C) \rightarrow R)))$	$\wedge$I, 3, 12

Daeducer commands needed to generate the proof:

```
premise C
premise !R
or_intro_left 2, C
assumption (C ^ R)
reiteration 2
discharge
imp_intro 4, 5
assumption !(R v C)
or_intro_right 1, R
not_elim 8, 9
explosion 10, R
discharge
imp_intro 8, 11
and_intro 7, 13
and_intro 3, 14
qed
```

Chapter 22, Exercise B.11

Proof presented using Fitch notation:

1	$\neg\neg((C \to \neg C) \land (\neg C \to C))$	PR
2	$((G \lor G) \lor G)$	PR
3	$\quad ((C \to \neg C) \land (\neg C \to C))$	AS
4	$\quad (C \to \neg C)$	$\land$E, 3
5	$\quad (\neg C \to C)$	$\land$E, 3
6	$\quad\quad C$	AS
7	$\quad\quad \neg C$	$\to$E, 4, 6
8	$\quad\quad \bot$	$\neg$E, 7, 6
9	$\quad \neg C$	$\neg$I, 6–8
10	$\quad\quad \neg C$	AS
11	$\quad\quad C$	$\to$E, 5, 10
12	$\quad\quad \bot$	$\neg$E, 10, 11
13	$\quad \neg\neg C$	$\neg$I, 10–12
14	$\quad \bot$	$\neg$E, 13, 9
15	$\neg((C \to \neg C) \land (\neg C \to C))$	$\neg$I, 3–14
16	$\bot$	$\neg$E, 1, 15
17	$((G \to C) \land G)$	X, 16

Daeducer commands needed to generate the proof:

```
premise !!((C -> !C) ^ (!C -> C))
premise ((G v G) v G)
assumption ((C -> !C) ^ (!C -> C))
and_elim_left 3
and_elim_right 3
assumption C
imp_elim 4, 6
not_elim 7, 6
discharge
not_intro 6, 8
assumption !C
imp_elim 5, 11
not_elim 11, 12
discharge
not_intro 11, 13
not_elim 15, 10
discharge
not_intro 3, 16
not_elim 1, 18
explosion 19, ((G -> C) ^ G)
qed
```

Chapter 22, Exercise B.12

Proof presented using Fitch notation:

1	C	PR
2	L	PR
3	$\quad \neg L$	AS
4	$\quad \bot$	$\neg$E, 3, 2
5	$\neg\neg L$	$\neg$I, 3–4
6	$\quad \neg C$	AS
7	$\quad \bot$	$\neg$E, 6, 1
8	$\neg\neg C$	$\neg$I, 6–7
9	$\quad (C \to \neg L)$	AS
10	$\quad C$	R, 1
11	$((C \to \neg L) \to C)$	$\to$I, 9, 10
12	$(((C \to \neg L) \to C) \wedge \neg\neg C)$	$\wedge$I, 11, 8
13	$(\neg\neg L \wedge (((C \to \neg L) \to C) \wedge \neg\neg C))$	$\wedge$I, 5, 12

Daeducer commands needed to generate the proof:

```
premise C
premise L
assumption !L
not_elim 3, 2
discharge
not_intro 3, 4
assumption !C
not_elim 7, 1
discharge
not_intro 7, 8
assumption (C -> !L)
reiteration 1
discharge
imp_intro 11, 12
and_intro 14, 10
and_intro 6, 15
qed
```

Chapter 24, Exercise A.1

Proof presented using Fitch notation:

1	$\forall x(G(x) \to F(x))$	PR
2	$\forall x(H(x) \to G(x))$	PR
3	$\quad H(y)$	AS
4	$\quad (H(y) \to G(y))$	$\forall$E, 2
5	$\quad G(y)$	$\to$E, 4, 3
6	$\quad (G(y) \to F(y))$	$\forall$E, 1
7	$\quad F(y)$	$\to$E, 6, 5
8	$(H(y) \to F(y))$	$\to$I, 3, 7
9	$\forall x(H(x) \to F(x))$	$\forall$I, 8

Daeducer commands needed to generate the proof:

```
premise forall x (G(x) -> F(x))
premise forall x (H(x) -> G(x))
assumption H(y)
forall_elim 2, y
imp_elim 4, 3
forall_elim 1, y
imp_elim 6, 5
discharge
imp_intro 3, 7
forall_intro 9, y, x
qed
```

Chapter 24, Exercise A.2

Proof presented using Fitch notation:

1	$\forall x(G(x) \to \neg F(x))$	PR
2	$\forall x(H(x) \to G(x))$	PR
3	$\quad H(a)$	AS
4	$\quad (H(a) \to G(a))$	$\forall$E, 2
5	$\quad G(a)$	$\to$E, 4, 3
6	$\quad (G(a) \to \neg F(a))$	$\forall$E, 1
7	$\quad \neg F(a)$	$\to$E, 6, 5
8	$(H(a) \to \neg F(a))$	$\to$I, 3, 7
9	$\forall x(H(x) \to \neg F(x))$	$\forall$I, 8

Daeducer commands needed to generate the proof:

```
premise forall x (G(x) -> !F(x))
premise forall x (H(x) -> G(x))
assumption H(a)
forall_elim 2, a
imp_elim 4, 3
```

```
forall_elim 1, a
imp_elim 6, 5
discharge
imp_intro 3, 7
forall_intro 9, a, x
qed
```

Chapter 24, Exercise A.3

Proof presented using Fitch notation:

1	$\forall x(G(x) \to \neg F(x))$	PR
2	$\exists x(H(x) \land G(x))$	PR
3	$\quad (H(a) \land G(a))$	AS
4	$\quad H(a)$	$\land$E, 3
5	$\quad G(a)$	$\land$E, 3
6	$\quad (G(a) \to \neg F(a))$	$\forall$E, 1
7	$\quad \neg F(a)$	$\to$E, 6, 5
8	$\quad (H(a) \land \neg F(a))$	$\land$I, 4, 7
9	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, 3–9

Daeducer commands needed to generate the proof:

```
premise forall x (G(x) -> !F(x))
premise exists x (H(x) ^ G(x))
assumption (H(a) ^ G(a))
and_elim_left 3
and_elim_right 3
forall_elim 1, a
imp_elim 6, 5
and_intro 4, 7
exists_intro 8, (H(x) ^ !F(x))
discharge
exists_elim 2, 3, 9
qed
```

Chapter 24, Exercise A.4

Proof presented using Fitch notation:

1	$\forall x(G(x) \to F(x))$	PR
2	$\exists x(H(x) \land G(x))$	PR
3	$\quad (H(a) \land G(a))$	AS
4	$\quad H(a)$	$\land$E, 3
5	$\quad G(a)$	$\land$E, 3
6	$\quad (G(a) \to F(a))$	$\forall$E, 1
7	$\quad F(a)$	$\to$E, 6, 5
8	$\quad (H(a) \land F(a))$	$\land$I, 4, 7
9	$\quad \exists x(H(x) \land F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land F(x))$	$\exists$E, 2, 3–9

Daeducer commands needed to generate the proof:

```
premise forall x (G(x) -> F(x))
premise exists x (H(x) ^ G(x))
assumption (H(a) ^ G(a))
and_elim_left 3
and_elim_right 3
forall_elim 1, a
imp_elim 6, 5
and_intro 4, 7
exists_intro 8, (H(x) ^ F(x))
discharge
exists_elim 2, 3, 9
qed
```

Chapter 24, Exercise A.5

Proof presented using Fitch notation:

1	$\forall x(F(x) \to G(x))$	PR
2	$\forall x(H(x) \to \neg G(x))$	PR
3	$(F(a) \to G(a))$	$\forall$E, 1
4	$(H(a) \to \neg G(a))$	$\forall$E, 2
5	$\quad H(a)$	AS
6	$\quad \neg G(a)$	$\to$E, 4, 5
7	$\quad \neg F(a)$	MT, 3, 6
8	$(H(a) \to \neg F(a))$	$\to$I, 5, 7
9	$\forall x(H(x) \to \neg F(x))$	$\forall$I, 8

Daeducer commands needed to generate the proof:

```
premise forall x (F(x) -> G(x))
premise forall x (H(x) -> !G(x))
```

```
forall_elim 1, a
forall_elim 2, a
assumption H(a)
imp_elim 4, 5
modus_tollens 3, 6
discharge
imp_intro 5, 7
forall_intro 9, a, x
qed
```

Chapter 24, Exercise A.6

Proof presented using Fitch notation:

1	$\forall x(F(x) \to \neg G(x))$	PR
2	$\forall x(H(x) \to G(x))$	PR
3	$\quad H(a)$	AS
4	$\quad (H(a) \to G(a))$	$\forall$E, 2
5	$\quad G(a)$	$\to$E, 4, 3
6	$\quad (F(a) \to \neg G(a))$	$\forall$E, 1
7	$\quad\quad F(a)$	AS
8	$\quad\quad \neg G(a)$	$\to$E, 6, 7
9	$\quad\quad \bot$	$\neg$E, 8, 5
10	$\quad \neg F(a)$	$\neg$I, 7–9
11	$(H(a) \to \neg F(a))$	$\to$I, 3, 10
12	$\forall x(H(x) \to \neg F(x))$	$\forall$I, 11

Daeducer commands needed to generate the proof:

```
premise forall x (F(x) -> !G(x))
premise forall x (H(x) -> G(x))
assumption H(a)
forall_elim 2, a
imp_elim 4, 3
forall_elim 1, a
assumption F(a)
imp_elim 6, 7
not_elim 8, 5
discharge
not_intro 7, 9
discharge
imp_intro 3, 11
forall_intro 13, a, x
qed
```

Chapter 24, Exercise A.7

Proof presented using Fitch notation:

1	$\forall x(F(x) \to G(x))$	PR
2	$\exists x(H(x) \land \neg G(x))$	PR
3	$\quad (H(a) \land \neg G(a))$	AS
4	$\quad H(a)$	$\land$E, 3
5	$\quad \neg G(a)$	$\land$E, 3
6	$\quad (F(a) \to G(a))$	$\forall$E, 1
7	$\quad \neg F(a)$	MT, 6, 5
8	$\quad (H(a) \land \neg F(a))$	$\land$I, 4, 7
9	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, 3–9

Daeducer commands needed to generate the proof:

```
premise forall x (F(x) -> G(x))
premise exists x (H(x) ^ !G(x))
assumption (H(a) ^ !G(a))
and_elim_left 3
and_elim_right 3
forall_elim 1, a
modus_tollens 6, 5
and_intro 4, 7
exists_intro 8, (H(x) ^ !F(x))
discharge
exists_elim 2, 3, 9
qed
```

Chapter 24, Exercise A.8

Proof presented using Fitch notation:

1	$\forall x(F(x) \to \neg G(x))$	PR
2	$\exists x(H(x) \land G(x))$	PR
3	$\quad (H(a) \land G(a))$	AS
4	$\quad H(a)$	$\land$E, 3
5	$\quad G(a)$	$\land$E, 3
6	$\quad (F(a) \to \neg G(a))$	$\forall$E, 1
7	$\quad\quad F(a)$	AS
8	$\quad\quad \neg G(a)$	$\to$E, 6, 7
9	$\quad\quad \bot$	$\neg$E, 8, 5
10	$\quad \neg F(a)$	$\neg$I, 7–9
11	$\quad (H(a) \land \neg F(a))$	$\land$I, 4, 10
12	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, 11
13	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, 3–12

Daeducer commands needed to generate the proof:

```
premise forall x (F(x) -> !G(x))
premise exists x (H(x) ^ G(x))
assumption (H(a) ^ G(a))
and_elim_left 3
and_elim_right 3
forall_elim 1, a
assumption F(a)
imp_elim 6, 7
not_elim 8, 5
discharge
not_intro 7, 9
and_intro 4, 11
exists_intro 12, (H(x) ^ !F(x))
discharge
exists_elim 2, 3, 13
qed
```

Chapter 24, Exercise A.9

Proof presented using Fitch notation:

1	$\forall x(G(x) \to F(x))$	PR
2	$\exists x(G(x) \land H(x))$	PR
3	$\quad (G(a) \land H(a))$	AS
4	$\quad G(a)$	$\land$E, 3
5	$\quad H(a)$	$\land$E, 3
6	$\quad (G(a) \to F(a))$	$\forall$E, 1
7	$\quad F(a)$	$\to$E, 6, 4
8	$\quad (H(a) \land F(a))$	$\land$I, 5, 7
9	$\quad \exists x(H(x) \land F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land F(x))$	$\exists$E, 2, 3–9

Daeducer commands needed to generate the proof:

```
premise forall x (G(x) -> F(x))
premise exists x (G(x) ^ H(x))
assumption (G(a) ^ H(a))
and_elim_left 3
and_elim_right 3
forall_elim 1, a
imp_elim 6, 4
and_intro 5, 7
exists_intro 8, (H(x) ^ F(x))
discharge
exists_elim 2, 3, 9
qed
```

Chapter 24, Exercise A.10

Proof presented using Fitch notation:

1	$\exists x(G(x) \land F(x))$	PR
2	$\forall x(G(x) \to H(x))$	PR
3	$\quad (G(a) \land F(a))$	AS
4	$\quad G(a)$	$\land$E, 3
5	$\quad F(a)$	$\land$E, 3
6	$\quad (G(a) \to H(a))$	$\forall$E, 2
7	$\quad H(a)$	$\to$E, 6, 4
8	$\quad (H(a) \land F(a))$	$\land$I, 7, 5
9	$\quad \exists x(H(x) \land F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land F(x))$	$\exists$E, 1, 3–9

Daeducer commands needed to generate the proof:

```
premise exists x (G(x) ^ F(x))
```

```
premise forall x (G(x) -> H(x))
assumption (G(a) ^ F(a))
and_elim_left 3
and_elim_right 3
forall_elim 2, a
imp_elim 6, 4
and_intro 7, 5
exists_intro 8, (H(x) ^ F(x))
discharge
exists_elim 1, 3, 9
qed
```

Chapter 24, Exercise A.11

Proof presented using Fitch notation:

1	$\forall x(G(x) \to \neg F(x))$	PR
2	$\exists x(G(x) \land H(x))$	PR
3	$\quad (G(a) \land H(a))$	AS
4	$\quad G(a)$	$\land$E, 3
5	$\quad H(a)$	$\land$E, 3
6	$\quad (G(a) \to \neg F(a))$	$\forall$E, 1
7	$\quad \neg F(a)$	$\to$E, 6, 4
8	$\quad (H(a) \land \neg F(a))$	$\land$I, 5, 7
9	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, 3–9

Daeducer commands needed to generate the proof:

```
premise forall x (G(x) -> !F(x))
premise exists x (G(x) ^ H(x))
assumption (G(a) ^ H(a))
and_elim_left 3
and_elim_right 3
forall_elim 1, a
imp_elim 6, 4
and_intro 5, 7
exists_intro 8, (H(x) ^ !F(x))
discharge
exists_elim 2, 3, 9
qed
```

Chapter 24, Exercise A.12

Proof presented using Fitch notation:

1	$\exists x(G(x) \land \neg F(x))$	PR
2	$\forall x(G(x) \to H(x))$	PR
3	$\quad (G(a) \land \neg F(a))$	AS
4	$\quad G(a)$	$\land$E, 3
5	$\quad \neg F(a)$	$\land$E, 3
6	$\quad (G(a) \to H(a))$	$\forall$E, 2
7	$\quad H(a)$	$\to$E, 6, 4
8	$\quad (H(a) \land \neg F(a))$	$\land$I, 7, 5
9	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 1, 3–9

Daeducer commands needed to generate the proof:

```
premise exists x (G(x) ^ !F(x))
premise forall x (G(x) -> H(x))
assumption (G(a) ^ !F(a))
and_elim_left 3
and_elim_right 3
forall_elim 2, a
imp_elim 6, 4
and_intro 7, 5
exists_intro 8, (H(x) ^ !F(x))
discharge
exists_elim 1, 3, 9
qed
```

Chapter 24, Exercise A.13

Proof presented using Fitch notation:

1	$\forall x(F(x) \to G(x))$	PR
2	$\forall x(G(x) \to \neg H(x))$	PR
3	$(F(x) \to G(x))$	$\forall$E, 1
4	$(G(x) \to \neg H(x))$	$\forall$E, 2
5	$\quad H(x)$	AS
6	$\quad\quad F(x)$	AS
7	$\quad\quad G(x)$	$\to$E, 3, 6
8	$\quad\quad \neg H(x)$	$\to$E, 4, 7
9	$\quad\quad \bot$	$\neg$E, 8, 5
10	$\quad \neg F(x)$	$\neg$I, 6–9
11	$(H(x) \to \neg F(x))$	$\to$I, 5, 10
12	$\forall x(H(x) \to \neg F(x))$	$\forall$I, 11

Daeducer commands needed to generate the proof:

```
premise forall x (F(x) -> G(x))
premise forall x (G(x) -> !H(x))
forall_elim 1, x
forall_elim 2, x
assumption H(x)
assumption F(x)
imp_elim 3, 6
imp_elim 4, 7
not_elim 8, 5
discharge
not_intro 6, 9
discharge
imp_intro 5, 11
forall_intro 13, x, x
qed
```

Chapter 24, Exercise A.14

Proof presented using Fitch notation:

1	$\exists x(F(x) \land G(x))$	PR
2	$\forall x(G(x) \to H(x))$	PR
3	$\quad (F(a) \land G(a))$	AS
4	$\quad F(a)$	$\land$E, 3
5	$\quad G(a)$	$\land$E, 3
6	$\quad (G(a) \to H(a))$	$\forall$E, 2
7	$\quad H(a)$	$\to$E, 6, 5
8	$\quad (H(a) \land F(a))$	$\land$I, 7, 4
9	$\quad \exists x(H(x) \land F(x))$	$\exists$I, 8
10	$\exists x(H(x) \land F(x))$	$\exists$E, 1, 3–9

Daeducer commands needed to generate the proof:

```
premise exists x (F(x) ^ G(x))
premise forall x (G(x) -> H(x))
assumption (F(a) ^ G(a))
and_elim_left 3
and_elim_right 3
forall_elim 2, a
imp_elim 6, 5
and_intro 7, 4
exists_intro 8, (H(x) ^ F(x))
discharge
exists_elim 1, 3, 9
qed
```

Chapter 24, Exercise A.15

Proof presented using Fitch notation:

1	$\forall x(F(x) \to \neg G(x))$	PR
2	$\exists x(G(x) \land H(x))$	PR
3	$\quad (G(a) \land H(a))$	AS
4	$\quad G(a)$	$\land$E, 3
5	$\quad H(a)$	$\land$E, 3
6	$\quad (F(a) \to \neg G(a))$	$\forall$E, 1
7	$\quad\quad F(a)$	AS
8	$\quad\quad \neg G(a)$	$\to$E, 6, 7
9	$\quad\quad \bot$	$\neg$E, 8, 4
10	$\quad \neg F(a)$	$\neg$I, 7–9
11	$\quad (H(a) \land \neg F(a))$	$\land$I, 5, 10
12	$\quad \exists x(H(x) \land \neg F(x))$	$\exists$I, 11
13	$\exists x(H(x) \land \neg F(x))$	$\exists$E, 2, 3–12

Daeducer commands needed to generate the proof:

```
premise forall x (F(x) -> !G(x))
premise exists x (G(x) ^ H(x))
assumption (G(a) ^ H(a))
and_elim_left 3
and_elim_right 3
forall_elim 1, a
assumption F(a)
imp_elim 6, 7
not_elim 8, 4
discharge
not_intro 7, 9
and_intro 5, 11
exists_intro 12, (H(x) ^ !F(x))
discharge
exists_elim 2, 3, 13
qed
```

Chapter 36, Exercise B.1

Proof presented using Fitch notation:

1	$\forall x \exists y (R(x,y) \vee R(y,x))$	PR
2	$\forall x \neg R(m,x)$	PR
3	$\exists y (R(m,y) \vee R(y,m))$	$\forall$E, 1
4	$\quad (R(m,a) \vee R(a,m))$	AS
5	$\quad \neg R(m,a)$	$\forall$E, 2
6	$\quad R(a,m)$	DS, 4, 5
7	$\quad \exists x R(x,m)$	$\exists$I, 6
8	$\exists x R(x,m)$	$\exists$E, 3, 4–7

Daeducer commands needed to generate the proof:

```
premise forall x exists y (R(x, y) v R(y, x))
premise forall x !R(m, x)
forall_elim 1, m
assumption (R(m, a) v R(a, m))
forall_elim 2, a
or_syllogism_left 4, 5
exists_intro 6, R(x, m)
discharge
exists_elim 3, 4, 7
qed
```

Chapter 36, Exercise B.2

Proof presented using Fitch notation:

1	$\forall x (\exists y L(x,y) \rightarrow \forall z L(z,x))$	PR
2	$L(a,b)$	PR
3	$(\exists y L(a,y) \rightarrow \forall z L(z,a))$	$\forall$E, 1
4	$\exists y L(a,y)$	$\exists$I, 2
5	$\forall z L(z,a)$	$\rightarrow$E, 3, 4
6	$L(x,a)$	$\forall$E, 5
7	$\exists y L(x,y)$	$\exists$I, 6
8	$(\exists y L(x,y) \rightarrow \forall z L(z,x))$	$\forall$E, 1
9	$\forall z L(z,x)$	$\rightarrow$E, 8, 7
10	$L(x,x)$	$\forall$E, 9
11	$\forall x L(x,x)$	$\forall$I, 10

Daeducer commands needed to generate the proof:

```
premise forall x (exists y L(x, y) -> forall z L(z, x))
premise L(a, b)
forall_elim 1, a
exists_intro 2, L(a, y)
```

```
imp_elim 3, 4
forall_elim 5, x
exists_intro 6, L(x, y)
forall_elim 1, x
imp_elim 8, 7
forall_elim 9, x
forall_intro 10, x, x
qed
```

Chapter 36, Exercise B.3

Proof presented using Fitch notation:

1	$\forall x(J(x) \to K(x))$	PR
2	$\exists x \forall y L(x, y)$	PR
3	$\forall x J(x)$	PR
4	$\quad \forall y L(a, y)$	AS
5	$\quad L(a, a)$	$\forall$E, 4
6	$\quad J(a)$	$\forall$E, 3
7	$\quad (J(a) \to K(a))$	$\forall$E, 1
8	$\quad K(a)$	$\to$E, 7, 6
9	$\quad (K(a) \wedge L(a, a))$	$\wedge$I, 8, 5
10	$\quad \exists x(K(x) \wedge L(x, x))$	$\exists$I, 9
11	$\exists x(K(x) \wedge L(x, x))$	$\exists$E, 2, 4–10

Daeducer commands needed to generate the proof:

```
premise forall x (J(x) -> K(x))
premise exists x forall y L(x, y)
premise forall x J(x)
assumption forall y L(a, y)
forall_elim 4, a
forall_elim 3, a
forall_elim 1, a
imp_elim 7, 6
and_intro 8, 5
exists_intro 9, (K(x) ^ L(x, x))
discharge
exists_elim 2, 4, 10
qed
```

Chapter 36, Exercise D.1

Proof presented using Fitch notation:

1	$\exists x H(x)$	PR
2	$\forall x(G(x) \to F(x))$	PR
3	$\exists x(H(x) \to G(x))$	PR
4	$\quad (H(a) \to G(a))$	AS
5	$\quad\quad H(a)$	AS
6	$\quad\quad G(a)$	$\to$E, 4, 5
7	$\quad\quad (G(a) \to F(a))$	$\forall$E, 2
8	$\quad\quad F(a)$	$\to$E, 7, 6
9	$\quad (H(a) \to F(a))$	$\to$I, 5, 8
10	$\quad \exists x(H(x) \to F(x))$	$\exists$I, 9
11	$\exists x(H(x) \to F(x))$	$\exists$E, 3, 4–10

Daeducer commands needed to generate the proof:

```
premise exists x H(x)
premise forall x (G(x) -> F(x))
premise exists x (H(x) -> G(x))
assumption (H(a) -> G(a))
assumption H(a)
imp_elim 4, 5
forall_elim 2, a
imp_elim 7, 6
discharge
imp_intro 5, 8
exists_intro 10, (H(x) -> F(x))
discharge
exists_elim 3, 4, 11
qed
```

Chapter 36, Exercise D.2

Proof presented using Fitch notation:

1	$\exists x H(x)$	PR
2	$\forall x(G(x) \to \neg F(x))$	PR
3	$\exists x(H(x) \to G(x))$	PR
4	$\quad (H(c) \to G(c))$	AS
5	$\quad (G(c) \to \neg F(c))$	$\forall$E, 2
6	$\quad\quad H(c)$	AS
7	$\quad\quad G(c)$	$\to$E, 4, 6
8	$\quad\quad \neg F(c)$	$\to$E, 5, 7
9	$\quad (H(c) \to \neg F(c))$	$\to$I, 6, 8
10	$\quad \exists x(H(x) \to \neg F(x))$	$\exists$I, 9
11	$\exists x(H(x) \to \neg F(x))$	$\exists$E, 3, 4–10

Daeducer commands needed to generate the proof:

```
premise exists x H(x)
premise forall x (G(x) -> !F(x))
premise exists x (H(x) -> G(x))
assumption (H(c) -> G(c))
forall_elim 2, c
assumption H(c)
imp_elim 4, 6
imp_elim 5, 7
discharge
imp_intro 6, 8
exists_intro 10, (H(x) -> !F(x))
discharge
exists_elim 3, 4, 11
qed
```

Chapter 36, Exercise D.3

Proof presented using Fitch notation:

1	$\exists x H(x)$	PR
2	$\forall x(F(x) \to \neg G(x))$	PR
3	$\forall x(H(x) \to G(x))$	PR
4	$\quad H(c)$	AS
5	$\quad (F(c) \to \neg G(c))$	$\forall$E, 2
6	$\quad (H(c) \to G(c))$	$\forall$E, 3
7	$\quad\quad H(c)$	AS
8	$\quad\quad G(c)$	$\to$E, 6, 7
9	$\quad\quad\quad F(c)$	AS
10	$\quad\quad\quad \neg G(c)$	$\to$E, 5, 9
11	$\quad\quad\quad \bot$	$\neg$E, 10, 8
12	$\quad\quad \neg F(c)$	$\neg$I, 9–11
13	$\quad (H(c) \to \neg F(c))$	$\to$I, 7, 12
14	$\quad \exists x(H(x) \to \neg F(x))$	$\exists$I, 13
15	$\exists x(H(x) \to \neg F(x))$	$\exists$E, 1, 4–14

Daeducer commands needed to generate the proof:

```
premise exists x H(x)
premise forall x (F(x) -> !G(x))
premise forall x (H(x) -> G(x))
assumption H(c)
forall_elim 2, c
forall_elim 3, c
assumption H(c)
imp_elim 6, 7
assumption F(c)
imp_elim 5, 9
not_elim 10, 8
discharge
not_intro 9, 11
discharge
imp_intro 7, 13
exists_intro 15, (H(x) -> !F(x))
discharge
exists_elim 1, 4, 16
qed
```

Chapter 36, Exercise D.4

Proof presented using Fitch notation:

1	$\exists x H(x)$	PR
2	$\forall x(F(x) \to G(x))$	PR
3	$\forall x(H(x) \to \neg G(x))$	PR
4	$\quad H(a)$	AS
5	$\quad (F(a) \to G(a))$	$\forall$E, 2
6	$\quad (H(a) \to \neg G(a))$	$\forall$E, 3
7	$\quad\quad H(a)$	AS
8	$\quad\quad \neg G(a)$	$\to$E, 6, 4
9	$\quad\quad \neg F(a)$	MT, 5, 8
10	$\quad (H(a) \to \neg F(a))$	$\to$I, 7, 9
11	$\quad \exists x(H(x) \to \neg F(x))$	$\exists$I, 10
12	$\exists x(H(x) \to \neg F(x))$	$\exists$E, 1, 4–11

Daeducer commands needed to generate the proof:

```
premise exists x H(x)
premise forall x (F(x) -> G(x))
premise forall x (H(x) -> !G(x))
assumption H(a)
forall_elim 2, a
forall_elim 3, a
assumption H(a)
imp_elim 6, 4
modus_tollens 5, 8
discharge
imp_intro 7, 9
exists_intro 11, (H(x) -> !F(x))
discharge
exists_elim 1, 4, 12
qed
```

Chapter 36, Exercise D.5

Proof presented using Fitch notation:

1	$\exists x G(x)$	PR
2	$\forall x(G(x) \to \neg F(x))$	PR
3	$\forall x(G(x) \to H(x))$	PR
4	$\quad G(a)$	AS
5	$\quad (G(a) \to \neg F(a))$	$\forall$E, 2
6	$\quad \neg F(a)$	$\to$E, 5, 4
7	$\quad\quad H(a)$	AS
8	$\quad\quad \neg F(a)$	R, 6
9	$\quad (H(a) \to \neg F(a))$	$\to$I, 7, 8
10	$\quad \exists x(H(x) \to \neg F(x))$	$\exists$I, 9
11	$\exists x(H(x) \to \neg F(x))$	$\exists$E, 1, 4–10

Daeducer commands needed to generate the proof:

```
premise exists x G(x)
premise forall x (G(x) -> !F(x))
premise forall x (G(x) -> H(x))
assumption G(a)
forall_elim 2, a
imp_elim 5, 4
assumption H(a)
reiteration 6
discharge
imp_intro 7, 8
exists_intro 10, (H(x) -> !F(x))
discharge
exists_elim 1, 4, 11
qed
```

Chapter 36, Exercise D.6

Proof presented using Fitch notation:

1	$\exists x G(x)$	PR
2	$\forall x(G(x) \to F(x))$	PR
3	$\forall x(G(x) \to H(x))$	PR
4	$\quad G(c)$	AS
5	$\quad (G(c) \to F(c))$	$\forall$E, 2
6	$\quad F(c)$	$\to$E, 5, 4
7	$\quad\quad H(c)$	AS
8	$\quad\quad F(c)$	R, 6
9	$\quad (H(c) \to F(c))$	$\to$I, 7, 8
10	$\quad \exists x(H(x) \to F(x))$	$\exists$I, 9
11	$\exists x(H(x) \to F(x))$	$\exists$E, 1, 4–10

Daeducer commands needed to generate the proof:

```
premise exists x G(x)
premise forall x (G(x) -> F(x))
premise forall x (G(x) -> H(x))
assumption G(c)
forall_elim 2, c
imp_elim 5, 4
assumption H(c)
reiteration 6
discharge
imp_intro 7, 8
exists_intro 10, (H(x) -> F(x))
discharge
exists_elim 1, 4, 11
qed
```

Chapter 36, Exercise D.7

Proof presented using Fitch notation:

1	$\exists x H(x)$	PR
2	$\forall x(F(x) \to G(x))$	PR
3	$\forall x(G(x) \to \neg H(x))$	PR
4	$\quad H(c)$	AS
5	$\quad (F(c) \to G(c))$	$\forall$E, 2
6	$\quad (G(c) \to \neg H(c))$	$\forall$E, 3
7	$\quad\quad F(c)$	AS
8	$\quad\quad G(c)$	$\to$E, 5, 7
9	$\quad\quad \neg H(c)$	$\to$E, 6, 8
10	$\quad\quad \bot$	$\neg$E, 9, 4
11	$\quad \neg F(c)$	$\neg$I, 7–10
12	$\quad\quad H(c)$	AS
13	$\quad\quad \neg F(c)$	R, 11
14	$\quad (H(c) \to \neg F(c))$	$\to$I, 12, 13
15	$\quad \exists x(H(x) \to \neg F(x))$	$\exists$I, 14
16	$\exists x(H(x) \to \neg F(x))$	$\exists$E, 1, 4–15

Daeducer commands needed to generate the proof:

```
premise exists x H(x)
premise forall x (F(x) -> G(x))
premise forall x (G(x) -> !H(x))
assumption H(c)
forall_elim 2, c
forall_elim 3, c
assumption F(c)
imp_elim 5, 7
imp_elim 6, 8
not_elim 9, 4
discharge
not_intro 7, 10
assumption H(c)
reiteration 12
discharge
imp_intro 13, 14
exists_intro 16, (H(x) -> !F(x))
discharge
exists_elim 1, 4, 17
qed
```

Chapter 36, Exercise D.8

Proof presented using Fitch notation:

1	$\exists x G(x)$	PR
2	$\forall x(F(x) \to \neg G(x))$	PR
3	$\forall x(G(x) \to H(x))$	PR
4	$\quad G(a)$	AS
5	$\quad (F(a) \to \neg G(a))$	$\forall$E, 2
6	$\quad (G(a) \to H(a))$	$\forall$E, 3
7	$\quad H(a)$	$\to$E, 6, 4
8	$\quad\quad F(a)$	AS
9	$\quad\quad \neg G(a)$	$\to$E, 5, 8
10	$\quad\quad \bot$	$\neg$E, 9, 4
11	$\quad \neg F(a)$	$\neg$I, 8–10
12	$\quad\quad H(a)$	AS
13	$\quad\quad \neg F(a)$	R, 11
14	$\quad (H(a) \to \neg F(a))$	$\to$I, 12, 13
15	$\quad \exists x(H(x) \to \neg F(x))$	$\exists$I, 14
16	$\exists x(H(x) \to \neg F(x))$	$\exists$E, 1, 4–15

Daeducer commands needed to generate the proof:

```
premise exists x G(x)
premise forall x (F(x) -> !G(x))
premise forall x (G(x) -> H(x))
assumption G(a)
forall_elim 2, a
forall_elim 3, a
imp_elim 6, 4
assumption F(a)
imp_elim 5, 8
not_elim 9, 4
discharge
not_intro 8, 10
assumption H(a)
reiteration 12
discharge
imp_intro 13, 14
exists_intro 16, (H(x) -> !F(x))
discharge
exists_elim 1, 4, 17
qed
```

Chapter 36, Exercise D.9

Proof presented using Fitch notation:

1	$\exists x F(x)$	PR
2	$\forall x(F(x) \to G(x))$	PR
3	$\forall x(G(x) \to H(x))$	PR
4	$\quad F(a)$	AS
5	$\quad\quad H(a)$	AS
6	$\quad\quad F(a)$	R, 4
7	$\quad (H(a) \to F(a))$	$\to$I, 5, 6
8	$\quad \exists x(H(x) \to F(x))$	$\exists$I, 7
9	$\exists x(H(x) \to F(x))$	$\exists$E, 1, 4–8

Daeducer commands needed to generate the proof:

```
premise exists x F(x)
premise forall x (F(x) -> G(x))
premise forall x (G(x) -> H(x))
assumption F(a)
assumption H(a)
reiteration 4
discharge
imp_intro 5, 6
exists_intro 8, (H(x) -> F(x))
discharge
exists_elim 1, 4, 9
qed
```

Chapter 36, Exercise E.1

Proof presented using Fitch notation:

1	$\quad \forall x F(x)$	AS
2	$\quad F(y)$	$\forall$E, 1
3	$\quad (F(y) \land F(y))$	$\land$I, 2, 2
4	$\quad \forall y(F(y) \land F(y))$	$\forall$I, 3
5	$(\forall x F(x) \to \forall y(F(y) \land F(y)))$	$\to$I, 1, 4

Daeducer commands needed to generate the proof:

```
assumption forall x F(x)
forall_elim 1, y
and_intro 2, 2
forall_intro 3, y, y
discharge
imp_intro 1, 4
qed
```

Chapter 36, Exercise E.2

Proof presented using Fitch notation:

1	$\forall x(A(x) \to B(x))$	PR
2	$\exists x A(x)$	PR
3	$\quad A(a)$	AS
4	$\quad (A(a) \to B(a))$	$\forall$E, 1
5	$\quad B(a)$	$\to$E, 4, 3
6	$\quad \exists x B(x)$	$\exists$I, 5
7	$\exists x B(x)$	$\exists$E, 2, 3–6

Daeducer commands needed to generate the proof:

```
premise forall x (A(x) -> B(x))
premise exists x A(x)
assumption A(a)
forall_elim 1, a
imp_elim 4, 3
exists_intro 5, B(x)
discharge
exists_elim 2, 3, 6
qed
```

Chapter 36, Exercise E.3

Proof presented using Fitch notation:

1	$\forall x(M(x) \leftrightarrow N(x))$	PR
2	$(M(a) \land \exists x R(x, a))$	PR
3	$M(a)$	$\land$E, 2
4	$(M(a) \leftrightarrow N(a))$	$\forall$E, 1
5	$N(a)$	$\leftrightarrow$E, 4, 3
6	$\exists x N(x)$	$\exists$I, 5

Daeducer commands needed to generate the proof:

```
premise forall x (M(x) <-> N(x))
premise (M(a) ^ exists x R(x, a))
and_elim_left 2
forall_elim 1, a
bi_elim 4, 3
exists_intro 5, N(x)
qed
```

Chapter 36, Exercise E.4

Proof presented using Fitch notation:

1	$\forall x \forall y G(x, y)$	PR
2	$\forall y G(a, y)$	$\forall$E, 1
3	$G(a, a)$	$\forall$E, 2
4	$\exists x G(x, x)$	$\exists$I, 3

Daeducer commands needed to generate the proof:

```
premise forall x forall y G(x, y)
forall_elim 1, a
forall_elim 2, a
exists_intro 3, G(x, x)
qed
```

Chapter 36, Exercise E.5

Proof presented using Fitch notation:

1	$\quad \forall x R(x, x)$	AS
2	$\quad R(a, a)$	$\forall$E, 1
3	$\quad \exists y R(a, y)$	$\exists$I, 2
4	$\quad \exists x \exists y R(x, y)$	$\exists$I, 3
5	$(\forall x R(x, x) \rightarrow \exists x \exists y R(x, y))$	$\rightarrow$I, 1, 4

Daeducer commands needed to generate the proof:

```
assumption forall x R(x, x)
forall_elim 1, a
exists_intro 2, R(a, y)
exists_intro 3, exists y R(x, y)
discharge
imp_intro 1, 4
qed
```

Chapter 36, Exercise E.6

Proof presented using Fitch notation:

1	$\quad Q(a)$	AS
2	$\quad Q(a)$	R, 1
3	$(Q(a) \to Q(a))$	$\to$I, 1, 2
4	$\exists x(Q(a) \to Q(x))$	$\exists$I, 3
5	$\forall y \exists x(Q(y) \to Q(x))$	$\forall$I, 4

Daeducer commands needed to generate the proof:

```
assumption Q(a)
reiteration 1
discharge
imp_intro 1, 2
exists_intro 4, (Q(a) -> Q(x))
forall_intro 5, a, y
qed
```

Chapter 36, Exercise E.7

Proof presented using Fitch notation:

1	$(N(a) \to \forall x(M(x) \leftrightarrow M(a)))$	PR
2	$M(a)$	PR
3	$\neg M(b)$	PR
4	$\quad N(a)$	AS
5	$\quad \forall x(M(x) \leftrightarrow M(a))$	$\to$E, 1, 4
6	$\quad (M(b) \leftrightarrow M(a))$	$\forall$E, 5
7	$\quad M(b)$	$\leftrightarrow$E, 6, 2
8	$\quad \bot$	$\neg$E, 3, 7
9	$\neg N(a)$	$\neg$I, 4–8

Daeducer commands needed to generate the proof:

```
premise (N(a) -> forall x (M(x) <-> M(a)))
premise M(a)
premise !M(b)
assumption N(a)
imp_elim 1, 4
forall_elim 5, b
bi_elim 6, 2
not_elim 3, 7
discharge
not_intro 4, 8
qed
```

Chapter 36, Exercise E.8

Proof presented using Fitch notation:

1	$\forall x \forall y (G(x,y) \to G(y,x))$	PR
2	$\forall y (G(a,y) \to G(y,a))$	$\forall$E, 1
3	$(G(a,b) \to G(b,a))$	$\forall$E, 2
4	$\quad G(a,b)$	AS
5	$\quad G(b,a)$	$\to$E, 3, 4
6	$\forall y (G(b,y) \to G(y,b))$	$\forall$E, 1
7	$(G(b,a) \to G(a,b))$	$\forall$E, 6
8	$\quad G(b,a)$	AS
9	$\quad G(a,b)$	$\to$E, 7, 8
10	$(G(a,b) \leftrightarrow G(b,a))$	$\leftrightarrow$I, 4–5, 8–9
11	$\forall y (G(a,y) \leftrightarrow G(y,a))$	$\forall$I, 10
12	$\forall x \forall y (G(x,y) \leftrightarrow G(y,x))$	$\forall$I, 11

Daeducer commands needed to generate the proof:

```
premise forall x forall y (G(x, y) -> G(y, x))
forall_elim 1, a
forall_elim 2, b
assumption G(a, b)
imp_elim 3, 4
discharge
forall_elim 1, b
forall_elim 7, a
assumption G(b, a)
imp_elim 8, 9
discharge
bi_intro 4, 5, 9, 10
forall_intro 12, b, y
forall_intro 13, a, x
qed
```

Chapter 36, Exercise E.9

Proof presented using Fitch notation:

1	$\forall x(\neg M(x) \vee L(j,x))$	PR
2	$\forall x(B(x) \rightarrow L(j,x))$	PR
3	$\forall x(M(x) \vee B(x))$	PR
4	$(M(a) \vee B(a))$	$\forall$E, 3
5	$(\neg M(a) \vee L(j,a))$	$\forall$E, 1
6	$(B(a) \rightarrow L(j,a))$	$\forall$E, 2
7	$\quad M(a)$	AS
8	$\quad\quad \neg M(a)$	AS
9	$\quad\quad \bot$	$\neg$E, 8, 7
10	$\quad\quad L(j,a)$	X, 9
11	$\quad\quad L(j,a)$	AS
12	$\quad\quad L(j,a)$	R, 11
13	$\quad L(j,a)$	$\vee$E, 5, 8–10, 11–12
14	$\quad B(a)$	AS
15	$\quad L(j,a)$	$\rightarrow$E, 6, 14
16	$L(j,a)$	$\vee$E, 4, 7–13, 14–15
17	$\forall x L(j,x)$	$\forall$I, 16

Daeducer commands needed to generate the proof:

```
premise forall x (!M(x) v L(j, x))
premise forall x (B(x) -> L(j, x))
premise forall x (M(x) v B(x))
forall_elim 3, a
forall_elim 1, a
forall_elim 2, a
assumption M(a)
assumption !M(a)
not_elim 8, 7
explosion 9, L(j, a)
discharge
assumption L(j, a)
reiteration 12
discharge
or_elim 5, 8, 10, 12, 13
discharge
assumption B(a)
imp_elim 6, 17
discharge
or_elim 4, 7, 15, 17, 18
forall_intro 20, a, x
qed
```

ch36-exF

Proof presented using Fitch notation:

1	$\exists x \forall y \forall z((L(x,y) \to L(z,y)) \leftrightarrow L(y,z))$	PR
2	$\quad \forall y \forall z((L(a,y) \to L(z,y)) \leftrightarrow L(y,z))$	AS
3	$\quad \forall z((L(a,a) \to L(z,a)) \leftrightarrow L(a,z))$	$\forall$E, 2
4	$\quad ((L(a,a) \to L(a,a)) \leftrightarrow L(a,a))$	$\forall$E, 3
5	$\quad\quad L(a,a)$	AS
6	$\quad\quad L(a,a)$	R, 5
7	$\quad (L(a,a) \to L(a,a))$	$\to$I, 5, 6
8	$\quad L(a,a)$	$\leftrightarrow$E, 4, 7
9	$\quad \exists x L(x,x)$	$\exists$I, 8
10	$\exists x L(x,x)$	$\exists$E, 1, 2–9

Daeducer commands needed to generate the proof:

```
premise exists x forall y forall z ((L(x, y) -> L(z, y)) <-> L(y, z))
assumption forall y forall z ((L(a, y) -> L(z, y)) <-> L(y, z))
forall_elim 2, a
forall_elim 3, a
assumption L(a, a)
reiteration 5
discharge
imp_intro 5, 6
bi_elim 4, 8
exists_intro 9, L(x, x)
discharge
exists_elim 1, 2, 10
qed
```

Chapter 36, Exercise G.1.a

Proof presented using Fitch notation:

1	$\forall x(A(x) \to \neg B(x))$	PR
2	$(A(x) \to \neg B(x))$	$\forall$E, 1
3	$\quad (A(x) \land B(x))$	AS
4	$\quad A(x)$	$\land$E, 3
5	$\quad B(x)$	$\land$E, 3
6	$\quad \neg B(x)$	$\to$E, 2, 4
7	$\quad \bot$	$\neg$E, 6, 5
8	$\neg(A(x) \land B(x))$	$\neg$I, 3–7
9	$\forall x \neg(A(x) \land B(x))$	$\forall$I, 8
10	$\neg\exists x(A(x) \land B(x))$	CQ, 9

Daeducer commands needed to generate the proof:

```
premise forall x (A(x) -> !B(x))
forall_elim 1, x
assumption (A(x) ^ B(x))
and_elim_left 3
and_elim_right 3
imp_elim 2, 4
not_elim 6, 5
discharge
not_intro 3, 7
forall_intro 9, x, x
conv_forall_not 10
qed
```

Chapter 36, Exercise G.1.b

Proof presented using Fitch notation:

1	$\neg\exists x(A(x) \land B(x))$	PR
2	$\quad A(a)$	AS
3	$\quad\quad B(a)$	AS
4	$\quad\quad (A(a) \land B(a))$	$\land$I, 2, 3
5	$\quad\quad \exists x(A(x) \land B(x))$	$\exists$I, 4
6	$\quad\quad \bot$	$\neg$E, 1, 5
7	$\quad \neg B(a)$	$\neg$I, 3–6
8	$(A(a) \to \neg B(a))$	$\to$I, 2, 7
9	$\forall x(A(x) \to \neg B(x))$	$\forall$I, 8

Daeducer commands needed to generate the proof:

```
premise !exists x (A(x) ^ B(x))
assumption A(a)
assumption B(a)
and_intro 2, 3
exists_intro 4, (A(x) ^ B(x))
not_elim 1, 5
discharge
not_intro 3, 6
discharge
imp_intro 2, 8
forall_intro 10, a, x
qed
```

Chapter 36, Exercise G.2.a

Proof presented using Fitch notation:

1	$\forall x(\neg A(x) \to B(d))$	PR
2	$\quad \neg(\forall x A(x) \vee B(d))$	AS
3	$\quad (\neg\forall x A(x) \wedge \neg B(d))$	DeM, 2
4	$\quad \neg\forall x A(x)$	$\wedge$E, 3
5	$\quad \neg B(d)$	$\wedge$E, 3
6	$\quad \exists x \neg A(x)$	CQ, 4
7	$\quad\quad \neg A(y)$	AS
8	$\quad\quad (\neg A(y) \to B(d))$	$\forall$E, 1
9	$\quad\quad B(d)$	$\to$E, 8, 7
10	$\quad B(d)$	$\exists$E, 6, 7–9
11	$\quad \bot$	$\neg$E, 5, 10
12	$(\forall x A(x) \vee B(d))$	IP, 2–11

Daeducer commands needed to generate the proof:

```
premise forall x (!A(x) -> B(d))
assumption !(forall x A(x) v B(d))
demorgan_not_or 2
and_elim_left 3
and_elim_right 3
conv_not_forall 4
assumption !A(y)
forall_elim 1, y
imp_elim 8, 7
discharge
exists_elim 6, 7, 9
not_elim 5, 11
discharge
indirect 2, 12
qed
```

Chapter 36, Exercise G.2.b

Proof presented using Fitch notation:

1	$(\forall x A(x) \vee B(d))$	PR
2	$\quad \neg A(y)$	AS
3	$\quad\quad \forall x A(x)$	AS
4	$\quad\quad A(y)$	$\forall$E, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad\quad B(d)$	X, 5
7	$\quad\quad B(d)$	AS
8	$\quad\quad B(d)$	R, 7
9	$\quad B(d)$	$\vee$E, 1, 3–6, 7–8
10	$(\neg A(y) \rightarrow B(d))$	$\rightarrow$I, 2, 9
11	$\forall x(\neg A(x) \rightarrow B(d))$	$\forall$I, 10

Daeducer commands needed to generate the proof:

```
premise (forall x A(x) v B(d))
assumption !A(y)
assumption forall x A(x)
forall_elim 3, y
not_elim 2, 4
explosion 5, B(d)
discharge
assumption B(d)
reiteration 8
discharge
or_elim 1, 3, 6, 8, 9
discharge
imp_intro 2, 11
forall_intro 13, y, x
qed
```

Chapter 36, Exercise G.3.a

Proof presented using Fitch notation:

1	$(\exists x P(x) \to Q(c))$	PR
2	$\quad P(a)$	AS
3	$\quad \exists x P(x)$	$\exists$I, 2
4	$\quad Q(c)$	$\to$E, 1, 3
5	$(P(a) \to Q(c))$	$\to$I, 2, 4
6	$\forall x(P(x) \to Q(c))$	$\forall$I, 5

Daeducer commands needed to generate the proof:

```
premise (exists x P(x) -> Q(c))
assumption P(a)
exists_intro 2, P(x)
imp_elim 1, 3
discharge
imp_intro 2, 4
forall_intro 6, a, x
qed
```

Chapter 36, Exercise G.3.b

Proof presented using Fitch notation:

1	$\forall x(P(x) \to Q(c))$	PR
2	$\quad \exists x P(x)$	AS
3	$\quad\quad P(a)$	AS
4	$\quad\quad (P(a) \to Q(c))$	$\forall$E, 1
5	$\quad\quad Q(c)$	$\to$E, 4, 3
6	$\quad Q(c)$	$\exists$E, 2, 3–5
7	$(\exists x P(x) \to Q(c))$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise forall x (P(x) -> Q(c))
assumption exists x P(x)
assumption P(a)
forall_elim 1, a
imp_elim 4, 3
discharge
exists_elim 2, 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 36, Exercise H.2

Proof presented using Fitch notation:

1	$\forall x \forall y \forall z B(x, y, z)$	PR
2	$\forall y \forall z B(x, y, z)$	$\forall$E, 1
3	$\forall z B(x, x, z)$	$\forall$E, 2
4	$B(x, x, x)$	$\forall$E, 3
5	$\forall x B(x, x, x)$	$\forall$I, 4

Daeducer commands needed to generate the proof:

```
premise forall x forall y forall z B(x, y, z)
forall_elim 1, x
forall_elim 2, x
forall_elim 3, x
forall_intro 4, x, x
qed
```

Chapter 36, Exercise H.3.b

Proof presented using Fitch notation:

1	$\forall y \forall x D(x, y)$	PR
2	$\forall x D(x, b)$	$\forall$E, 1
3	$D(a, b)$	$\forall$E, 2
4	$\forall y D(a, y)$	$\forall$I, 3
5	$\forall x \forall y D(x, y)$	$\forall$I, 4

Daeducer commands needed to generate the proof:

```
premise forall y forall x D(x, y)
forall_elim 1, b
forall_elim 2, a
forall_intro 3, b, y
forall_intro 4, a, x
qed
```

Chapter 36, Exercise H.4

Proof presented using Fitch notation:

1	$\exists x \forall y D(x, y)$	PR
2	$\quad \forall y D(a, y)$	AS
3	$\quad D(a, z)$	$\forall$E, 2
4	$\quad \exists x D(x, z)$	$\exists$I, 3
5	$\quad \forall y \exists x D(x, y)$	$\forall$I, 4
6	$\forall y \exists x D(x, y)$	$\exists$E, 1, 2–5

Daeducer commands needed to generate the proof:

```
premise exists x forall y D(x, y)
assumption forall y D(a, y)
forall_elim 2, z
exists_intro 3, D(x, z)
forall_intro 4, z, y
discharge
exists_elim 1, 2, 5
qed
```

Chapter 36, Exercise H.5

Proof presented using Fitch notation:

1	$\forall x (R(c, a) \leftrightarrow R(x, a))$	PR
2	$\quad R(c, a)$	AS
3	$\quad (R(c, a) \leftrightarrow R(y, a))$	$\forall$E, 1
4	$\quad R(y, a)$	$\leftrightarrow$E, 3, 2
5	$\quad \forall x R(x, a)$	$\forall$I, 4
6	$\quad \forall x R(x, a)$	AS
7	$\quad R(c, a)$	$\forall$E, 6
8	$(R(c, a) \leftrightarrow \forall x R(x, a))$	$\leftrightarrow$I, 2–5, 6–7

Daeducer commands needed to generate the proof:

```
premise forall x (R(c, a) <-> R(x, a))
assumption R(c, a)
forall_elim 1, y
bi_elim 3, 2
forall_intro 4, y, x
discharge
assumption forall x R(x, a)
forall_elim 7, c
discharge
bi_intro 2, 5, 7, 8
qed
```

Chapter 36, Exercise I.1

Proof presented using Fitch notation:

1	$\exists y \forall x R(x, y)$	PR
2	$\quad \forall x R(x, c)$	AS
3	$\quad R(z, c)$	$\forall$E, 2
4	$\quad \exists y R(z, y)$	$\exists$I, 3
5	$\quad \forall x \exists y R(x, y)$	$\forall$I, 4
6	$\forall x \exists y R(x, y)$	$\exists$E, 1, 2–5

Daeducer commands needed to generate the proof:

```
premise exists y forall x R(x, y)
assumption forall x R(x, c)
forall_elim 2, z
exists_intro 3, R(z, y)
forall_intro 4, z, x
discharge
exists_elim 1, 2, 5
qed
```

Chapter 36, Exercise I.4

Proof presented using Fitch notation:

1	$\forall x(S(x) \to T(a))$	PR
2	$S(d)$	PR
3	$(S(d) \to T(a))$	$\forall$E, 1
4	$T(a)$	$\to$E, 3, 2

Daeducer commands needed to generate the proof:

```
premise forall x (S(x) -> T(a))
premise S(d)
forall_elim 1, d
imp_elim 3, 2
qed
```

Chapter 36, Exercise I.5

Proof presented using Fitch notation:

1	$\forall x(A(x) \to B(x))$	PR
2	$\forall x(B(x) \to C(x))$	PR
3	$(B(a) \to C(a))$	$\forall$E, 2
4	$\quad A(a)$	AS
5	$\quad (A(a) \to B(a))$	$\forall$E, 1
6	$\quad B(a)$	$\to$E, 5, 4
7	$\quad C(a)$	$\to$E, 3, 6
8	$(A(a) \to C(a))$	$\to$I, 4, 7
9	$\forall x(A(x) \to C(x))$	$\forall$I, 8

Daeducer commands needed to generate the proof:

```
premise forall x (A(x) -> B(x))
premise forall x (B(x) -> C(x))
forall_elim 2, a
assumption A(a)
forall_elim 1, a
imp_elim 5, 4
imp_elim 3, 6
discharge
imp_intro 4, 7
forall_intro 9, a, x
qed
```

Chapter 36, Exercise I.7

Proof presented using Fitch notation:

1	$\forall x \forall y(R(x,y) \vee R(y,x))$	PR
2	$\forall y(R(j,y) \vee R(y,j))$	$\forall$E, 1
3	$(R(j,j) \vee R(j,j))$	$\forall$E, 2
4	$\quad R(j,j)$	AS
5	$\quad R(j,j)$	R, 4
6	$\quad R(j,j)$	AS
7	$\quad R(j,j)$	R, 6
8	$R(j,j)$	$\vee$E, 3, 4–5, 6–7

Daeducer commands needed to generate the proof:

```
premise forall x forall y (R(x, y) v R(y, x))
forall_elim 1, j
forall_elim 2, j
assumption R(j, j)
reiteration 4
discharge
assumption R(j, j)
reiteration 7
```

```
discharge
or_elim 3, 4, 5, 7, 8
qed
```

Chapter 36, Exercise I.10

Proof presented using Fitch notation:

1	$(\exists x M(x) \to \exists x N(x))$	PR
2	$\neg\exists x N(x)$	PR
3	$\neg\exists x M(x)$	MT, 1, 2
4	$\forall x \neg M(x)$	CQ, 3

Daeducer commands needed to generate the proof:

```
premise (exists x M(x) -> exists x N(x))
premise !exists x N(x)
modus_tollens 1, 2
conv_not_exists 3
qed
```

Chapter 37, Example p.318

Proof presented using Fitch notation:

1	$\forall x(A(x) \to B)$	PR
2	$\quad \exists x A(x)$	AS
3	$\quad\quad A(a)$	AS
4	$\quad\quad \forall x(A(x) \to B)$	R, 1
5	$\quad\quad (A(a) \to B)$	$\forall$E, 4
6	$\quad\quad B$	$\to$E, 5, 3
7	$\quad B$	$\exists$E, 2, 3–6
8	$(\exists x A(x) \to B)$	$\to$I, 2, 7

Daeducer commands needed to generate the proof:

```
premise forall x (A(x) -> B)
assumption exists x A(x)
assumption A(a)
reiteration 1
forall_elim 4, a
imp_elim 5, 3
discharge
exists_elim 2, 3, 6
discharge
imp_intro 2, 8
qed
```

Chapter 37, Example p.320

Proof presented using Fitch notation:

1	$(\exists x A(x) \to B)$	PR
2	$\quad A(a)$	AS
3	$\quad \exists x A(x)$	$\exists$I, 2
4	$\quad B$	$\to$E, 1, 3
5	$(A(a) \to B)$	$\to$I, 2, 4
6	$\forall x(A(x) \to B)$	$\forall$I, 5

Daeducer commands needed to generate the proof:

```
premise (exists x A(x) -> B)
assumption A(a)
exists_intro 2, A(x)
imp_elim 1, 3
discharge
imp_intro 2, 4
forall_intro 6, a, x
qed
```

Chapter 37, Exercise A.1

Proof presented using Fitch notation:

1	$(A \to \forall x B(x))$	PR
2	$\quad A$	AS
3	$\quad \forall x B(x)$	$\to$E, 1, 2
4	$\quad B(x)$	$\forall$E, 3
5	$(A \to B(x))$	$\to$I, 2, 4
6	$\forall x(A \to B(x))$	$\forall$I, 5

Daeducer commands needed to generate the proof:

```
premise (A -> forall x B(x))
assumption A
imp_elim 1, 2
forall_elim 3, x
discharge
imp_intro 2, 4
forall_intro 6, x, x
qed
```

Chapter 37, Exercise A.2

Proof presented using Fitch notation:

1	$\exists x(A \to B(x))$	PR
2	$\quad (A \to B(c))$	AS
3	$\quad\quad A$	AS
4	$\quad\quad B(c)$	$\to$E, 2, 3
5	$\quad\quad \exists x B(x)$	$\exists$I, 4
6	$\quad (A \to \exists x B(x))$	$\to$I, 3, 5
7	$(A \to \exists x B(x))$	$\exists$E, 1, 2–6

Daeducer commands needed to generate the proof:

```
premise exists x (A -> B(x))
assumption (A -> B(c))
assumption A
imp_elim 2, 3
exists_intro 4, B(x)
discharge
imp_intro 3, 5
discharge
exists_elim 1, 2, 7
qed
```

Chapter 37, Exercise A.3

Proof presented using Fitch notation:

1	$\forall x(A(x) \land B(x))$	AS
2	$(A(a) \land B(a))$	$\forall$E, 1
3	$A(a)$	$\land$E, 2
4	$\forall x A(x)$	$\forall$I, 3
5	$B(a)$	$\land$E, 2
6	$\forall x B(x)$	$\forall$I, 5
7	$(\forall x A(x) \land \forall x B(x))$	$\land$I, 4, 6
8	$(\forall x(A(x) \land B(x)) \to (\forall x A(x) \land \forall x B(x)))$	$\to$I, 1, 7
9	$(\forall x A(x) \land \forall x B(x))$	AS
10	$\forall x A(x)$	$\land$E, 9
11	$\forall x B(x)$	$\land$E, 9
12	$A(a)$	$\forall$E, 10
13	$B(a)$	$\forall$E, 11
14	$(A(a) \land B(a))$	$\land$I, 12, 13
15	$\forall x(A(x) \land B(x))$	$\forall$I, 14
16	$((\forall x A(x) \land \forall x B(x)) \to \forall x(A(x) \land B(x)))$	$\to$I, 9, 15
17	$(\forall x(A(x) \land B(x)) \leftrightarrow (\forall x A(x) \land \forall x B(x)))$	$\leftrightarrow$I, 1–7, 9–15

Daeducer commands needed to generate the proof:

```
assumption forall x (A(x) ^ B(x))
forall_elim 1, a
and_elim_left 2
forall_intro 3, a, x
and_elim_right 2
forall_intro 5, a, x
and_intro 4, 6
discharge
imp_intro 1, 7
assumption (forall x A(x) ^ forall x B(x))
and_elim_left 10
and_elim_right 10
forall_elim 11, a
forall_elim 12, a
and_intro 13, 14
forall_intro 15, a, x
discharge
imp_intro 10, 16
bi_intro 1, 7, 10, 16
qed
```

Chapter 37, Exercise A.4

Proof presented using Fitch notation:

1	$\exists x(A(x) \vee B(x))$	AS
2	$(A(c) \vee B(c))$	AS
3	$A(c)$	AS
4	$\exists x A(x)$	$\exists$I, 3
5	$(\exists x A(x) \vee \exists x B(x))$	$\vee$E, 4
6	$B(c)$	AS
7	$\exists x B(x)$	$\exists$I, 6
8	$(\exists x A(x) \vee \exists x B(x))$	$\vee$E, 7
9	$(\exists x A(x) \vee \exists x B(x))$	$\vee$E, 2, 3–5, 6–8
10	$(\exists x A(x) \vee \exists x B(x))$	$\exists$E, 1, 2–9
11	$(\exists x(A(x) \vee B(x)) \rightarrow (\exists x A(x) \vee \exists x B(x)))$	$\rightarrow$I, 1, 10
12	$(\exists x A(x) \vee \exists x B(x))$	AS
13	$\exists x A(x)$	AS
14	$A(d)$	AS
15	$(A(d) \vee B(d))$	$\vee$E, 14
16	$\exists x(A(x) \vee B(x))$	$\exists$I, 15
17	$\exists x(A(x) \vee B(x))$	$\exists$E, 13, 14–16
18	$\exists x B(x)$	AS
19	$B(d)$	AS
20	$(A(d) \vee B(d))$	$\vee$E, 19
21	$\exists x(A(x) \vee B(x))$	$\exists$I, 20
22	$\exists x(A(x) \vee B(x))$	$\exists$E, 18, 19–21
23	$\exists x(A(x) \vee B(x))$	$\vee$E, 12, 13–17, 18–22
24	$((\exists x A(x) \vee \exists x B(x)) \rightarrow \exists x(A(x) \vee B(x)))$	$\rightarrow$I, 12, 23
25	$(\exists x(A(x) \vee B(x)) \leftrightarrow (\exists x A(x) \vee \exists x B(x)))$	$\leftrightarrow$I, 1–10, 12–23

Daeducer commands needed to generate the proof:

```
assumption exists x (A(x) v B(x))
assumption (A(c) v B(c))
assumption A(c)
exists_intro 3, A(x)
or_intro_left 4, exists x B(x)
discharge
assumption B(c)
exists_intro 7, B(x)
or_intro_right 8, exists x A(x)
discharge
or_elim 2, 3, 5, 7, 9
discharge
exists_elim 1, 2, 11
discharge
```

```
imp_intro 1, 13
assumption (exists x A(x) v exists x B(x))
assumption exists x A(x)
assumption A(d)
or_intro_left 18, B(d)
exists_intro 19, (A(x) v B(x))
discharge
exists_elim 17, 18, 20
discharge
assumption exists x B(x)
assumption B(d)
or_intro_right 25, A(d)
exists_intro 26, (A(x) v B(x))
discharge
exists_elim 24, 25, 27
discharge
or_elim 16, 17, 22, 24, 29
discharge
imp_intro 16, 31
bi_intro 1, 13, 16, 31
qed
```

Chapter 37, Exercise A.5

Proof presented using Fitch notation:

1	$(A \vee \forall x B(x))$	PR
2	$\quad A$	AS
3	$\quad (A \vee B(y))$	$\vee$E, 2
4	$\quad \forall x(A \vee B(x))$	$\forall$I, 3
5	$\quad \forall x B(x)$	AS
6	$\quad B(y)$	$\forall$E, 5
7	$\quad (A \vee B(y))$	$\vee$E, 6
8	$\quad \forall x(A \vee B(x))$	$\forall$I, 7
9	$\forall x(A \vee B(x))$	$\vee$E, 1, 2–4, 5–8

Daeducer commands needed to generate the proof:

```
premise (A v forall x B(x))
assumption A
or_intro_left 2, B(y)
forall_intro 3, y, x
discharge
assumption forall x B(x)
forall_elim 6, y
or_intro_right 7, A
forall_intro 8, y, x
discharge
or_elim 1, 2, 4, 6, 9
qed
```

Chapter 37, Exercise A.6

Proof presented using Fitch notation:

1	$\forall x(A(x) \to B)$	PR
2	$\quad \exists x A(x)$	AS
3	$\quad\quad A(c)$	AS
4	$\quad\quad (A(c) \to B)$	$\forall$E, 1
5	$\quad\quad B$	$\to$E, 4, 3
6	$\quad B$	$\exists$E, 2, 3–5
7	$(\exists x A(x) \to B)$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise forall x (A(x) -> B)
assumption exists x A(x)
assumption A(c)
forall_elim 1, c
imp_elim 4, 3
discharge
exists_elim 2, 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 37, Exercise A.7

Proof presented using Fitch notation:

1	$\exists x(A(x) \to B)$	PR
2	$\quad \forall x A(x)$	AS
3	$\quad\quad (A(c) \to B)$	AS
4	$\quad\quad A(c)$	$\forall$E, 2
5	$\quad\quad B$	$\to$E, 3, 4
6	$\quad B$	$\exists$E, 1, 3–5
7	$(\forall x A(x) \to B)$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise exists x (A(x) -> B)
assumption forall x A(x)
assumption (A(c) -> B)
forall_elim 2, c
imp_elim 3, 4
discharge
exists_elim 1, 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 37, Exercise A.8

Proof presented using Fitch notation:

1	$\quad A(x)$	AS
2	$\quad \exists y A(y)$	$\exists$I, 1
3	$(A(x) \to \exists y A(y))$	$\to$I, 1, 2
4	$\forall x (A(x) \to \exists y A(y))$	$\forall$I, 3

Daeducer commands needed to generate the proof:

```
assumption A(x)
exists_intro 1, A(y)
discharge
imp_intro 1, 2
forall_intro 4, x, x
qed
```

Chapter 37, Exercise B.1

Proof presented using Fitch notation:

1	$\forall x R(x, x)$	PR
2	$R(a, a)$	$\forall$E, 1
3	$\exists y R(a, y)$	$\exists$I, 2
4	$\forall x \exists y R(x, y)$	$\forall$I, 3

Daeducer commands needed to generate the proof:

```
premise forall x R(x, x)
forall_elim 1, a
exists_intro 2, R(a, y)
forall_intro 3, a, x
qed
```

Chapter 37, Exercise B.2

Proof presented using Fitch notation:

1	$\forall x \forall y \forall z((R(x,y) \land R(y,z)) \to R(x,z))$	PR
2	$\forall y \forall z((R(a,y) \land R(y,z)) \to R(a,z))$	$\forall$E, 1
3	$\forall z((R(a,b) \land R(b,z)) \to R(a,z))$	$\forall$E, 2
4	$((R(a,b) \land R(b,c)) \to R(a,c))$	$\forall$E, 3
5	$\quad R(a,b)$	AS
6	$\quad\quad R(b,c)$	AS
7	$\quad\quad (R(a,b) \land R(b,c))$	$\land$I, 5, 6
8	$\quad\quad R(a,c)$	$\to$E, 4, 7
9	$\quad (R(b,c) \to R(a,c))$	$\to$I, 6, 8
10	$\quad \forall z(R(b,z) \to R(a,z))$	$\forall$I, 9
11	$(R(a,b) \to \forall z(R(b,z) \to R(a,z)))$	$\to$I, 5, 10
12	$\forall y(R(a,y) \to \forall z(R(y,z) \to R(a,z)))$	$\forall$I, 11
13	$\forall x \forall y(R(x,y) \to \forall z(R(y,z) \to R(x,z)))$	$\forall$I, 12

Daeducer commands needed to generate the proof:

```
premise forall x forall y forall z ((R(x, y) ^ R(y, z)) -> R(x, z))
forall_elim 1, a
forall_elim 2, b
forall_elim 3, c
assumption R(a, b)
assumption R(b, c)
and_intro 5, 6
imp_elim 4, 7
discharge
imp_intro 6, 8
forall_intro 10, c, z
discharge
imp_intro 5, 11
forall_intro 13, b, y
forall_intro 14, a, x
qed
```

Chapter 37, Exercise B.3

Proof presented using Fitch notation:

1	$\forall x \forall y \forall z((R(x,y) \land R(y,z)) \to R(x,z))$	PR
2	$\forall x \forall y(R(x,y) \to R(y,x))$	PR
3	$\forall y \forall z((R(b,y) \land R(y,z)) \to R(b,z))$	$\forall$E, 1
4	$\forall z((R(b,a) \land R(a,z)) \to R(b,z))$	$\forall$E, 3
5	$((R(b,a) \land R(a,c)) \to R(b,c))$	$\forall$E, 4
6	$\forall y(R(a,y) \to R(y,a))$	$\forall$E, 2
7	$(R(a,b) \to R(b,a))$	$\forall$E, 6
8	$\quad (R(a,b) \land R(a,c))$	AS
9	$\quad R(a,b)$	$\land$E, 8
10	$\quad R(a,c)$	$\land$E, 8
11	$\quad R(b,a)$	$\to$E, 7, 9
12	$\quad (R(b,a) \land R(a,c))$	$\land$I, 11, 10
13	$\quad R(b,c)$	$\to$E, 5, 12
14	$((R(a,b) \land R(a,c)) \to R(b,c))$	$\to$I, 8, 13
15	$\forall z((R(a,b) \land R(a,z)) \to R(b,z))$	$\forall$I, 14
16	$\forall y \forall z((R(a,y) \land R(a,z)) \to R(y,z))$	$\forall$I, 15
17	$\forall x \forall y \forall z((R(x,y) \land R(x,z)) \to R(y,z))$	$\forall$I, 16

Daeducer commands needed to generate the proof:

```
premise forall x forall y forall z ((R(x, y) ^ R(y, z)) -> R(x, z))
premise forall x forall y (R(x, y) -> R(y, x))
forall_elim 1, b
forall_elim 3, a
forall_elim 4, c
forall_elim 2, a
forall_elim 6, b
assumption (R(a, b) ^ R(a, c))
and_elim_left 8
and_elim_right 8
imp_elim 7, 9
and_intro 11, 10
imp_elim 5, 12
discharge
imp_intro 8, 13
forall_intro 15, c, z
forall_intro 16, b, y
forall_intro 17, a, x
qed
```

Chapter 37, Exercise B.4

Proof presented using Fitch notation:

1	$\forall x \forall y (R(x,y) \to R(y,x))$	PR
2	$\quad (R(a,b) \land R(a,c))$	AS
3	$\quad R(a,b)$	$\land$E, 2
4	$\quad R(a,c)$	$\land$E, 2
5	$\quad \forall y (R(a,y) \to R(y,a))$	$\forall$E, 1
6	$\quad (R(a,b) \to R(b,a))$	$\forall$E, 5
7	$\quad R(b,a)$	$\to$E, 6, 3
8	$\quad \forall y (R(a,y) \to R(y,a))$	$\forall$E, 1
9	$\quad (R(a,c) \to R(c,a))$	$\forall$E, 8
10	$\quad R(c,a)$	$\to$E, 9, 4
11	$\quad (R(b,a) \land R(c,a))$	$\land$I, 7, 10
12	$\quad \exists u (R(b,u) \land R(c,u))$	$\exists$I, 11
13	$((R(a,b) \land R(a,c)) \to \exists u (R(b,u) \land R(c,u)))$	$\to$I, 2, 12
14	$\forall z ((R(a,b) \land R(a,z)) \to \exists u (R(b,u) \land R(z,u)))$	$\forall$I, 13
15	$\forall y \forall z ((R(a,y) \land R(a,z)) \to \exists u (R(y,u) \land R(z,u)))$	$\forall$I, 14
16	$\forall x \forall y \forall z ((R(x,y) \land R(x,z)) \to \exists u (R(y,u) \land R(z,u)))$	$\forall$I, 15

Daeducer commands needed to generate the proof:

```
premise forall x forall y (R(x, y) -> R(y, x))
assumption (R(a, b) ^ R(a, c))
and_elim_left 2
and_elim_right 2
forall_elim 1, a
forall_elim 5, b
imp_elim 6, 3
forall_elim 1, a
forall_elim 8, c
imp_elim 9, 4
and_intro 7, 10
exists_intro 11, (R(b, u) ^ R(c, u))
discharge
imp_intro 2, 12
forall_intro 14, c, z
forall_intro 15, b, y
forall_intro 16, a, x
qed
```

Chapter 37, Exercise B.5

Proof presented using Fitch notation:

1	$\exists x \forall y (A(x,y) \leftrightarrow \neg A(y,y))$	AS
2	$\quad \forall y (A(a,y) \leftrightarrow \neg A(y,y))$	AS
3	$\quad (A(a,a) \leftrightarrow \neg A(a,a))$	$\forall$E, 2
4	$\quad\quad A(a,a)$	AS
5	$\quad\quad \neg A(a,a)$	$\leftrightarrow$E, 3, 4
6	$\quad\quad \bot$	$\neg$E, 5, 4
7	$\quad \neg A(a,a)$	$\neg$I, 4–6
8	$\quad A(a,a)$	$\leftrightarrow$E, 3, 7
9	$\quad \bot$	$\neg$E, 7, 8
10	$\bot$	$\exists$E, 1, 2–9
11	$\neg \exists x \forall y (A(x,y) \leftrightarrow \neg A(y,y))$	$\neg$I, 1–10

Daeducer commands needed to generate the proof:

```
assumption exists x forall y (A(x, y) <-> !A(y, y))
assumption forall y (A(a, y) <-> !A(y, y))
forall_elim 2, a
assumption A(a, a)
bi_elim 3, 4
not_elim 5, 4
discharge
not_intro 4, 6
bi_elim 3, 8
not_elim 8, 9
discharge
exists_elim 1, 2, 10
discharge
not_intro 1, 12
qed
```

Chapter 37, Exercise C.1

Proof presented using Fitch notation:

1	$(\forall x A(x) \to B)$	PR
2	$\quad B$	AS
3	$\quad\quad A(y)$	AS
4	$\quad\quad B$	R, 2
5	$\quad (A(y) \to B)$	$\to$I, 3, 4
6	$\quad \exists x(A(x) \to B)$	$\exists$I, 5
7	$(B \to \exists x(A(x) \to B))$	$\to$I, 2, 6
8	$\quad \neg B$	AS
9	$\quad \neg\forall x A(x)$	MT, 1, 8
10	$\quad \exists x \neg A(x)$	CQ, 9
11	$\quad\quad \neg A(y)$	AS
12	$\quad\quad\quad A(y)$	AS
13	$\quad\quad\quad \bot$	$\neg$E, 11, 12
14	$\quad\quad\quad B$	X, 13
15	$\quad\quad (A(y) \to B)$	$\to$I, 12, 14
16	$\quad\quad \exists x(A(x) \to B)$	$\exists$I, 15
17	$\quad \exists x(A(x) \to B)$	$\exists$E, 10, 11–16
18	$(\neg B \to \exists x(A(x) \to B))$	$\to$I, 8, 17
19	$\exists x(A(x) \to B)$	LEM, 7, 18

Daeducer commands needed to generate the proof:

```
premise (forall x A(x) -> B)
assumption B
assumption A(y)
reiteration 2
discharge
imp_intro 3, 4
exists_intro 6, (A(x) -> B)
discharge
imp_intro 2, 7
assumption !B
modus_tollens 1, 10
conv_not_forall 11
assumption !A(y)
assumption A(y)
not_elim 13, 14
explosion 15, B
discharge
imp_intro 14, 16
exists_intro 18, (A(x) -> B)
discharge
exists_elim 12, 13, 19
discharge
imp_intro 10, 21
excluded_middle 9, 23
qed
```

Chapter 37, Exercise C.2

Proof presented using Fitch notation:

1	$(A \to \exists x B(x))$	PR
2	$\quad A$	AS
3	$\quad \exists x B(x)$	$\to$E, 1, 2
4	$\quad\quad B(c)$	AS
5	$\quad\quad\quad A$	AS
6	$\quad\quad\quad B(c)$	R, 4
7	$\quad\quad (A \to B(c))$	$\to$I, 5, 6
8	$\quad\quad \exists x(A \to B(x))$	$\exists$I, 7
9	$\quad \exists x(A \to B(x))$	$\exists$E, 3, 4–8
10	$(A \to \exists x(A \to B(x)))$	$\to$I, 2, 9
11	$\quad \neg A$	AS
12	$\quad\quad A$	AS
13	$\quad\quad \bot$	$\neg$E, 11, 12
14	$\quad\quad B(c)$	X, 13
15	$\quad (A \to B(c))$	$\to$I, 12, 14
16	$\quad \exists x(A \to B(x))$	$\exists$I, 15
17	$(\neg A \to \exists x(A \to B(x)))$	$\to$I, 11, 16
18	$\exists x(A \to B(x))$	LEM, 10, 17

Daeducer commands needed to generate the proof:

```
premise (A -> exists x B(x))
assumption A
imp_elim 1, 2
assumption B(c)
assumption A
reiteration 4
discharge
imp_intro 5, 6
exists_intro 8, (A -> B(x))
discharge
exists_elim 3, 4, 9
discharge
imp_intro 2, 11
assumption !A
assumption A
not_elim 14, 15
explosion 16, B(c)
discharge
imp_intro 15, 17
exists_intro 19, (A -> B(x))
discharge
imp_intro 14, 20
excluded_middle 13, 22
qed
```

Chapter 37, Exercise C.3

Proof presented using Fitch notation:

1	$\forall x(A \vee B(x))$	PR
2	$\quad A$	AS
3	$\quad (A \vee \forall x B(x))$	$\vee$E, 2
4	$(A \rightarrow (A \vee \forall x B(x)))$	$\rightarrow$I, 2, 3
5	$\quad \neg A$	AS
6	$\quad (A \vee B(c))$	$\forall$E, 1
7	$\quad B(c)$	DS, 6, 5
8	$\quad \forall x B(x)$	$\forall$I, 7
9	$\quad (A \vee \forall x B(x))$	$\vee$E, 8
10	$(\neg A \rightarrow (A \vee \forall x B(x)))$	$\rightarrow$I, 5, 9
11	$(A \vee \forall x B(x))$	LEM, 4, 10

Daeducer commands needed to generate the proof:

```
premise forall x (A v B(x))
assumption A
or_intro_left 2, forall x B(x)
discharge
imp_intro 2, 3
assumption !A
forall_elim 1, c
or_syllogism_left 7, 6
forall_intro 8, c, x
or_intro_right 9, A
discharge
imp_intro 6, 10
excluded_middle 5, 12
qed
```

Chapter 37, Exercise C.4

Proof presented using Fitch notation:

1	$\forall y A(y)$	AS
2	$A(a)$	AS
3	$\forall y A(y)$	R, 1
4	$(A(a) \to \forall y A(y))$	$\to$I, 2, 3
5	$\exists x(A(x) \to \forall y A(y))$	$\exists$I, 4
6	$(\forall y A(y) \to \exists x(A(x) \to \forall y A(y)))$	$\to$I, 1, 5
7	$\neg\forall y A(y)$	AS
8	$\exists y \neg A(y)$	CQ, 7
9	$\neg A(b)$	AS
10	$A(b)$	AS
11	$\bot$	$\neg$E, 9, 10
12	$\forall y A(y)$	X, 11
13	$(A(b) \to \forall y A(y))$	$\to$I, 10, 12
14	$\exists x(A(x) \to \forall y A(y))$	$\exists$I, 13
15	$\exists x(A(x) \to \forall y A(y))$	$\exists$E, 8, 9–14
16	$(\neg\forall y A(y) \to \exists x(A(x) \to \forall y A(y)))$	$\to$I, 7, 15
17	$\exists x(A(x) \to \forall y A(y))$	LEM, 6, 16

Daeducer commands needed to generate the proof:

```
assumption forall y A(y)
assumption A(a)
reiteration 1
discharge
imp_intro 2, 3
exists_intro 5, (A(x) -> forall y A(y))
discharge
imp_intro 1, 6
assumption !forall y A(y)
conv_not_forall 9
assumption !A(b)
assumption A(b)
not_elim 11, 12
explosion 13, forall y A(y)
discharge
imp_intro 12, 14
exists_intro 16, (A(x) -> forall y A(y))
discharge
exists_elim 10, 11, 17
discharge
imp_intro 9, 19
excluded_middle 8, 21
qed
```

Chapter 37, Exercise C.5

Proof presented using Fitch notation:

1	$\exists y A(y)$	AS
2	$A(a)$	AS
3	$\exists y A(y)$	AS
4	$A(a)$	R, 2
5	$(\exists y A(y) \to A(a))$	$\to$I, 3, 4
6	$\exists x(\exists y A(y) \to A(x))$	$\exists$I, 5
7	$\exists x(\exists y A(y) \to A(x))$	$\exists$E, 1, 2–6
8	$(\exists y A(y) \to \exists x(\exists y A(y) \to A(x)))$	$\to$I, 1, 7
9	$\neg\exists y A(y)$	AS
10	$\exists y A(y)$	AS
11	$\bot$	$\neg$E, 9, 10
12	$A(b)$	X, 11
13	$(\exists y A(y) \to A(b))$	$\to$I, 10, 12
14	$\exists x(\exists y A(y) \to A(x))$	$\exists$I, 13
15	$(\neg\exists y A(y) \to \exists x(\exists y A(y) \to A(x)))$	$\to$I, 9, 14
16	$\exists x(\exists y A(y) \to A(x))$	LEM, 8, 15

Daeducer commands needed to generate the proof:

```
assumption exists y A(y)
assumption A(a)
assumption exists y A(y)
reiteration 2
discharge
imp_intro 3, 4
exists_intro 6, (exists y A(y) -> A(x))
discharge
exists_elim 1, 2, 7
discharge
imp_intro 1, 9
assumption !exists y A(y)
assumption exists y A(y)
not_elim 12, 13
explosion 14, A(b)
discharge
imp_intro 13, 15
exists_intro 17, (exists y A(y) -> A(x))
discharge
imp_intro 12, 18
excluded_middle 11, 20
qed
```

Chapter 38, Exercise B.1.a

Proof presented using Fitch notation:

1	$\forall x(A(x) \to \neg B(x))$	PR
2	$\quad \exists x(A(x) \land B(x))$	AS
3	$\quad\quad (A(c) \land B(c))$	AS
4	$\quad\quad A(c)$	$\land$E, 3
5	$\quad\quad B(c)$	$\land$E, 3
6	$\quad\quad (A(c) \to \neg B(c))$	$\forall$E, 1
7	$\quad\quad \neg B(c)$	$\to$E, 6, 4
8	$\quad\quad \bot$	$\neg$E, 7, 5
9	$\quad \bot$	$\exists$E, 2, 3–8
10	$\neg\exists x(A(x) \land B(x))$	$\neg$I, 2–9

Daeducer commands needed to generate the proof:

```
premise forall x (A(x) -> !B(x))
assumption exists x (A(x) ^ B(x))
assumption (A(c) ^ B(c))
and_elim_left 3
and_elim_right 3
forall_elim 1, c
imp_elim 6, 4
not_elim 7, 5
discharge
exists_elim 2, 3, 8
discharge
not_intro 2, 10
qed
```

Chapter 38, Exercise B.1.b

Proof presented using Fitch notation:

1	$\neg\exists x(A(x) \land B(x))$	PR
2	$\forall x\neg(A(x) \land B(x))$	CQ, 1
3	$\quad A(y)$	AS
4	$\quad\quad B(y)$	AS
5	$\quad\quad (A(y) \land B(y))$	$\land$I, 3, 4
6	$\quad\quad \neg(A(y) \land B(y))$	$\forall$E, 2
7	$\quad\quad \bot$	$\neg$E, 6, 5
8	$\quad \neg B(y)$	$\neg$I, 4–7
9	$(A(y) \to \neg B(y))$	$\to$I, 3, 8
10	$\forall x(A(x) \to \neg B(x))$	$\forall$I, 9

Daeducer commands needed to generate the proof:

```
premise !exists x (A(x) ^ B(x))
conv_not_exists 1
assumption A(y)
assumption B(y)
and_intro 3, 4
forall_elim 2, y
not_elim 6, 5
discharge
not_intro 4, 7
discharge
imp_intro 3, 9
forall_intro 11, y, x
qed
```

Chapter 38, Exercise B.2.a

Proof presented using Fitch notation:

1	$\forall x(\neg A(x) \to B(d))$	PR
2	$\quad B(d)$	AS
3	$\quad (\forall x A(x) \vee B(d))$	$\vee$E, 2
4	$(B(d) \to (\forall x A(x) \vee B(d)))$	$\to$I, 2, 3
5	$\quad \neg B(d)$	AS
6	$\quad (\neg A(a) \to B(d))$	$\forall$E, 1
7	$\quad \neg\neg A(a)$	MT, 6, 5
8	$\quad A(a)$	DNE, 7
9	$\quad \forall x A(x)$	$\forall$I, 8
10	$\quad (\forall x A(x) \vee B(d))$	$\vee$E, 9
11	$(\neg B(d) \to (\forall x A(x) \vee B(d)))$	$\to$I, 5, 10
12	$(\forall x A(x) \vee B(d))$	LEM, 4, 11

Daeducer commands needed to generate the proof:

```
premise forall x (!A(x) -> B(d))
assumption B(d)
or_intro_right 2, forall x A(x)
discharge
imp_intro 2, 3
assumption !B(d)
forall_elim 1, a
modus_tollens 7, 6
double_not_elim 8
forall_intro 9, a, x
or_intro_left 10, B(d)
discharge
imp_intro 6, 11
excluded_middle 5, 13
qed
```

Chapter 38, Exercise B.2.b

Proof presented using Fitch notation:

1	$(\forall x A(x) \vee B(d))$	PR
2	$\quad \neg A(a)$	AS
3	$\quad\quad \forall x A(x)$	AS
4	$\quad\quad A(a)$	$\forall$E, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad\quad B(d)$	X, 5
7	$\quad\quad B(d)$	AS
8	$\quad\quad B(d)$	R, 7
9	$\quad B(d)$	$\vee$E, 1, 3–6, 7–8
10	$(\neg A(a) \rightarrow B(d))$	$\rightarrow$I, 2, 9
11	$\forall x(\neg A(x) \rightarrow B(d))$	$\forall$I, 10

Daeducer commands needed to generate the proof:

```
premise (forall x A(x) v B(d))
assumption !A(a)
assumption forall x A(x)
forall_elim 3, a
not_elim 2, 4
explosion 5, B(d)
discharge
assumption B(d)
reiteration 8
discharge
or_elim 1, 3, 6, 8, 9
discharge
imp_intro 2, 11
forall_intro 13, a, x
qed
```

Chapter 38, Exercise C.1.a

Proof presented using Fitch notation:

1	$\forall x(F(x) \land G(a))$	PR
2	$(F(y) \land G(a))$	$\forall$E, 1
3	$F(y)$	$\land$E, 2
4	$\forall x F(x)$	$\forall$I, 3
5	$G(a)$	$\land$E, 2
6	$(\forall x F(x) \land G(a))$	$\land$I, 4, 5

Daeducer commands needed to generate the proof:

```
premise forall x (F(x) ^ G(a))
forall_elim 1, y
and_elim_left 2
forall_intro 3, y, x
and_elim_right 2
and_intro 4, 5
qed
```

Chapter 38, Exercise C.1.b

Proof presented using Fitch notation:

1	$(\forall x F(x) \land G(a))$	PR
2	$\forall x F(x)$	$\land$E, 1
3	$G(a)$	$\land$E, 1
4	$F(x)$	$\forall$E, 2
5	$(F(x) \land G(a))$	$\land$I, 4, 3
6	$\forall x(F(x) \land G(a))$	$\forall$I, 5

Daeducer commands needed to generate the proof:

```
premise (forall x F(x) ^ G(a))
and_elim_left 1
and_elim_right 1
forall_elim 2, x
and_intro 4, 3
forall_intro 5, x, x
qed
```

Chapter 38, Exercise C.2.a

Proof presented using Fitch notation:

1	$\exists x(F(x) \lor G(a))$	PR
2	$\quad (F(c) \lor G(a))$	AS
3	$\quad\quad F(c)$	AS
4	$\quad\quad \exists x F(x)$	$\exists$I, 3
5	$\quad\quad (\exists x F(x) \lor G(a))$	$\lor$E, 4
6	$\quad\quad G(a)$	AS
7	$\quad\quad (\exists x F(x) \lor G(a))$	$\lor$E, 6
8	$\quad (\exists x F(x) \lor G(a))$	$\lor$E, 2, 3–5, 6–7
9	$(\exists x F(x) \lor G(a))$	$\exists$E, 1, 2–8

Daeducer commands needed to generate the proof:

```
premise exists x (F(x) v G(a))
assumption (F(c) v G(a))
assumption F(c)
exists_intro 3, F(x)
or_intro_left 4, G(a)
discharge
assumption G(a)
or_intro_right 7, exists x F(x)
discharge
or_elim 2, 3, 5, 7, 8
discharge
exists_elim 1, 2, 10
qed
```

Chapter 38, Exercise C.2.b

Proof presented using Fitch notation:

1	$(\exists x F(x) \lor G(a))$	PR
2	$\quad \exists x F(x)$	AS
3	$\quad\quad F(y)$	AS
4	$\quad\quad (F(y) \lor G(a))$	$\lor$E, 3
5	$\quad\quad \exists x(F(x) \lor G(a))$	$\exists$I, 4
6	$\quad \exists x(F(x) \lor G(a))$	$\exists$E, 2, 3–5
7	$\quad G(a)$	AS
8	$\quad (F(a) \lor G(a))$	$\lor$E, 7
9	$\quad \exists x(F(x) \lor G(a))$	$\exists$I, 8
10	$\exists x(F(x) \lor G(a))$	$\lor$E, 1, 2–6, 7–9

Daeducer commands needed to generate the proof:

```
premise (exists x F(x) v G(a))
```

```
assumption exists x F(x)
assumption F(y)
or_intro_left 3, G(a)
exists_intro 4, (F(x) v G(a))
discharge
exists_elim 2, 3, 5
discharge
assumption G(a)
or_intro_right 9, F(a)
exists_intro 10, (F(x) v G(a))
discharge
or_elim 1, 2, 7, 9, 11
qed
```

Chapter 38, Exercise C.3.a

Proof presented using Fitch notation:

1	$\forall x(G(a) \to F(x))$	PR
2	$\quad G(a)$	AS
3	$\quad (G(a) \to F(x))$	$\forall$E, 1
4	$\quad F(x)$	$\to$E, 3, 2
5	$\quad \forall x F(x)$	$\forall$I, 4
6	$(G(a) \to \forall x F(x))$	$\to$I, 2, 5

Daeducer commands needed to generate the proof:

```
premise forall x (G(a) -> F(x))
assumption G(a)
forall_elim 1, x
imp_elim 3, 2
forall_intro 4, x, x
discharge
imp_intro 2, 5
qed
```

Chapter 38, Exercise C.3.b

Proof presented using Fitch notation:

1	$(G(a) \to \forall x F(x))$	PR
2	$\quad G(a)$	AS
3	$\quad \forall x F(x)$	$\to$E, 1, 2
4	$\quad F(y)$	$\forall$E, 3
5	$(G(a) \to F(y))$	$\to$I, 2, 4
6	$\forall x(G(a) \to F(x))$	$\forall$I, 5

Daeducer commands needed to generate the proof:

```
premise (G(a) -> forall x F(x))
assumption G(a)
imp_elim 1, 2
forall_elim 3, y
discharge
imp_intro 2, 4
forall_intro 6, y, x
qed
```

Chapter 38, Exercise C.4.a

Proof presented using Fitch notation:

1	$\forall x(F(x) \to G(a))$	PR
2	$\quad \exists x F(x)$	AS
3	$\quad\quad F(c)$	AS
4	$\quad\quad (F(c) \to G(a))$	$\forall$E, 1
5	$\quad\quad G(a)$	$\to$E, 4, 3
6	$\quad G(a)$	$\exists$E, 2, 3–5
7	$(\exists x F(x) \to G(a))$	$\to$I, 2, 6

Daeducer commands needed to generate the proof:

```
premise forall x (F(x) -> G(a))
assumption exists x F(x)
assumption F(c)
forall_elim 1, c
imp_elim 4, 3
discharge
exists_elim 2, 3, 5
discharge
imp_intro 2, 7
qed
```

Chapter 38, Exercise C.4.b

Proof presented using Fitch notation:

1	$(\exists x F(x) \to G(a))$	PR
2	$\quad F(y)$	AS
3	$\quad \exists x F(x)$	$\exists$I, 2
4	$\quad G(a)$	$\to$E, 1, 3
5	$(F(y) \to G(a))$	$\to$I, 2, 4
6	$\forall x(F(x) \to G(a))$	$\forall$I, 5

Daeducer commands needed to generate the proof:

```
premise (exists x F(x) -> G(a))
assumption F(y)
exists_intro 2, F(x)
imp_elim 1, 3
discharge
imp_intro 2, 4
forall_intro 6, y, x
qed
```

Chapter 38, Exercise C.5.a

Proof presented using Fitch notation:

1	$\exists x(G(a) \to F(x))$	PR
2	$\quad G(a)$	AS
3	$\quad\quad (G(a) \to F(y))$	AS
4	$\quad\quad G(a)$	R, 2
5	$\quad\quad F(y)$	$\to$E, 3, 4
6	$\quad\quad \exists x F(x)$	$\exists$I, 5
7	$\quad \exists x F(x)$	$\exists$E, 1, 3–6
8	$(G(a) \to \exists x F(x))$	$\to$I, 2, 7

Daeducer commands needed to generate the proof:

```
premise exists x (G(a) -> F(x))
assumption G(a)
assumption (G(a) -> F(y))
reiteration 2
imp_elim 3, 4
exists_intro 5, F(x)
discharge
exists_elim 1, 3, 6
discharge
imp_intro 2, 8
qed
```

Chapter 38, Exercise C.5.b

Proof presented using Fitch notation:

1	$(G(a) \to \exists x F(x))$	PR
2	$\quad G(a)$	AS
3	$\quad \exists x F(x)$	$\to$E, 1, 2
4	$\quad\quad F(b)$	AS
5	$\quad\quad\quad G(a)$	AS
6	$\quad\quad\quad F(b)$	R, 4
7	$\quad\quad (G(a) \to F(b))$	$\to$I, 5, 6
8	$\quad\quad \exists x(G(a) \to F(x))$	$\exists$I, 7
9	$\quad \exists x(G(a) \to F(x))$	$\exists$E, 3, 4–8
10	$(G(a) \to \exists x(G(a) \to F(x)))$	$\to$I, 2, 9
11	$\quad \neg G(a)$	AS
12	$\quad\quad G(a)$	AS
13	$\quad\quad \bot$	$\neg$E, 11, 12
14	$\quad\quad F(a)$	X, 13
15	$\quad (G(a) \to F(a))$	$\to$I, 12, 14
16	$\quad \exists x(G(a) \to F(x))$	$\exists$I, 15
17	$(\neg G(a) \to \exists x(G(a) \to F(x)))$	$\to$I, 11, 16
18	$\exists x(G(a) \to F(x))$	LEM, 10, 17

Daeducer commands needed to generate the proof:

```
premise (G(a) -> exists x F(x))
assumption G(a)
imp_elim 1, 2
assumption F(b)
assumption G(a)
reiteration 4
discharge
imp_intro 5, 6
exists_intro 8, (G(a) -> F(x))
discharge
exists_elim 3, 4, 9
discharge
imp_intro 2, 11
assumption !G(a)
assumption G(a)
not_elim 14, 15
explosion 16, F(a)
discharge
imp_intro 15, 17
exists_intro 19, (G(a) -> F(x))
discharge
imp_intro 14, 20
excluded_middle 13, 22
qed
```

Chapter 38, Exercise C.6.a

Proof presented using Fitch notation:

1	$\exists x(F(x) \to G(a))$	PR
2	$\quad \forall x F(x)$	AS
3	$\quad\quad (F(c) \to G(a))$	AS
4	$\quad\quad \forall x F(x)$	R, 2
5	$\quad\quad F(c)$	$\forall$E, 4
6	$\quad\quad G(a)$	$\to$E, 3, 5
7	$\quad G(a)$	$\exists$E, 1, 3–6
8	$(\forall x F(x) \to G(a))$	$\to$I, 2, 7

Daeducer commands needed to generate the proof:

```
premise exists x (F(x) -> G(a))
assumption forall x F(x)
assumption (F(c) -> G(a))
reiteration 2
forall_elim 4, c
imp_elim 3, 5
discharge
exists_elim 1, 3, 6
discharge
imp_intro 2, 8
qed
```

Chapter 38, Exercise C.6.b

Proof presented using Fitch notation:

1	$(\forall x F(x) \to G(a))$	PR
2	$\quad G(a)$	AS
3	$\quad\quad F(a)$	AS
4	$\quad\quad G(a)$	R, 2
5	$\quad (F(a) \to G(a))$	$\to$I, 3, 4
6	$\quad \exists x(F(x) \to G(a))$	$\exists$I, 5
7	$(G(a) \to \exists x(F(x) \to G(a)))$	$\to$I, 2, 6
8	$\quad \neg G(a)$	AS
9	$\quad \neg\forall x F(x)$	MT, 1, 8
10	$\quad \exists x \neg F(x)$	CQ, 9
11	$\quad\quad \neg F(c)$	AS
12	$\quad\quad\quad F(c)$	AS
13	$\quad\quad\quad \bot$	$\neg$E, 11, 12
14	$\quad\quad\quad G(a)$	X, 13
15	$\quad\quad (F(c) \to G(a))$	$\to$I, 12, 14
16	$\quad\quad \exists x(F(x) \to G(a))$	$\exists$I, 15
17	$\quad \exists x(F(x) \to G(a))$	$\exists$E, 10, 11–16
18	$(\neg G(a) \to \exists x(F(x) \to G(a)))$	$\to$I, 8, 17
19	$\exists x(F(x) \to G(a))$	LEM, 7, 18

Daeducer commands needed to generate the proof:

```
premise (forall x F(x) -> G(a))
assumption G(a)
assumption F(a)
reiteration 2
discharge
imp_intro 3, 4
exists_intro 6, (F(x) -> G(a))
discharge
imp_intro 2, 7
assumption !G(a)
modus_tollens 1, 10
conv_not_forall 11
assumption !F(c)
assumption F(c)
not_elim 13, 14
explosion 15, G(a)
discharge
imp_intro 14, 16
exists_intro 18, (F(x) -> G(a))
discharge
exists_elim 12, 13, 19
discharge
imp_intro 10, 21
excluded_middle 9, 23
qed
```

Chapter 39, Example p.328

Proof presented using Fitch notation:

1	$\quad a = b$	AS
2	$\quad a = a$	=I
3	$\quad b = a$	=E, 1, 2
4	$(a = b \to b = a)$	→I, 1, 3
5	$\forall y(a = y \to y = a)$	∀I, 4
6	$\forall x \forall y(x = y \to y = x)$	∀I, 5

Daeducer commands needed to generate the proof:

```
assumption a = b
id_intro a
id_elim 1, 2, b = a
discharge
imp_intro 1, 3
forall_intro 5, b, y
forall_intro 6, a, x
qed
```

Chapter 39, Example p.328

Proof presented using Fitch notation:

1	$\quad (x = y \land y = z)$	AS
2	$\quad x = y$	∧E, 1
3	$\quad y = z$	∧E, 1
4	$\quad x = x$	=I
5	$\quad y = x$	=E, 2, 4
6	$\quad x = z$	=E, 5, 3
7	$((x = y \land y = z) \to x = z)$	→I, 1, 6
8	$\forall z((x = y \land y = z) \to x = z)$	∀I, 7
9	$\forall y \forall z((x = y \land y = z) \to x = z)$	∀I, 8
10	$\forall x \forall y \forall z((x = y \land y = z) \to x = z)$	∀I, 9

Daeducer commands needed to generate the proof:

```
assumption (x = y ^ y = z)
and_elim_left 1
and_elim_right 1
id_intro x
id_elim 2, 4, y = x
id_elim 5, 3, x = z
discharge
imp_intro 1, 6
forall_intro 8, z, z
forall_intro 9, y, y
forall_intro 10, x, x
qed
```

Chapter 39, Exercise A.1

Proof presented using Fitch notation:

1	$(P(a) \lor Q(b))$	PR
2	$(Q(b) \to b = c)$	PR
3	$\neg P(a)$	PR
4	$Q(b)$	DS, 1, 3
5	$b = c$	→E, 2, 4
6	$Q(c)$	=E, 5, 4

Daeducer commands needed to generate the proof:

```
premise (P(a) v Q(b))
premise (Q(b) -> b = c)
premise !P(a)
or_syllogism_left 1, 3
imp_elim 2, 4
id_elim 5, 4, Q(c)
qed
```

Chapter 39, Exercise A.2

Proof presented using Fitch notation:

1	$(m = n \lor n = o)$	PR
2	$A(n)$	PR
3	$\quad m = n$	AS
4	$\quad A(n)$	R, 2
5	$\quad (A(n) \lor A(o))$	∨E, 4
6	$\quad n = o$	AS
7	$\quad A(n)$	R, 2
8	$\quad A(o)$	=E, 6, 7
9	$\quad (A(n) \lor A(o))$	∨E, 8
10	$(A(n) \lor A(o))$	∨E, 1, 3–5, 6–9

Daeducer commands needed to generate the proof:

```
premise (m = n v n = o)
premise A(n)
assumption m = n
reiteration 2
or_intro_left 4, A(o)
discharge
assumption n = o
reiteration 2
id_elim 7, 8, A(o)
or_intro_right 9, A(n)
discharge
or_elim 1, 3, 5, 7, 10
qed
```

Chapter 39, Exercise A.3

Proof presented using Fitch notation:

1	$\forall x x = m$	PR
2	$R(m, a)$	PR
3	$a = m$	$\forall$E, 1
4	$R(m, m)$	=E, 3, 2
5	$\exists x R(x, x)$	$\exists$I, 4

Daeducer commands needed to generate the proof:

```
premise forall x x = m
premise R(m, a)
forall_elim 1, a
id_elim 3, 2, R(m, m)
exists_intro 4, R(x, x)
qed
```

Chapter 39, Exercise A.4

Proof presented using Fitch notation:

1	$\forall x \forall y (R(x, y) \to x = y)$	PR
2	$\quad R(a, b)$	AS
3	$\quad \forall y (R(a, y) \to a = y)$	$\forall$E, 1
4	$\quad (R(a, b) \to a = b)$	$\forall$E, 3
5	$\quad a = b$	$\to$E, 4, 2
6	$\quad a = a$	=I
7	$\quad b = a$	=E, 5, 6
8	$\quad R(b, b)$	=E, 5, 2
9	$\quad R(b, a)$	=E, 7, 8
10	$(R(a, b) \to R(b, a))$	$\to$I, 2, 9

Daeducer commands needed to generate the proof:

```
premise forall x forall y (R(x, y) -> x = y)
assumption R(a, b)
forall_elim 1, a
forall_elim 3, b
imp_elim 4, 2
id_intro a
id_elim 5, 6, b = a
id_elim 5, 2, R(b, b)
id_elim 7, 8, R(b, a)
discharge
imp_intro 2, 9
qed
```

Chapter 39, Exercise A.5

Proof presented using Fitch notation:

1	$\neg\exists x \neg x = m$	PR
2	$\forall x \neg\neg x = m$	CQ, 1
3	$\neg\neg a = m$	$\forall$E, 2
4	$a = m$	DNE, 3
5	$\neg\neg b = m$	$\forall$E, 2
6	$b = m$	DNE, 5
7	$\quad P(a)$	AS
8	$\quad P(m)$	=E, 4, 7
9	$\quad P(b)$	=E, 6, 8
10	$(P(a) \to P(b))$	$\to$I, 7, 9
11	$\forall y(P(a) \to P(y))$	$\forall$I, 10
12	$\forall x \forall y(P(x) \to P(y))$	$\forall$I, 11

Daeducer commands needed to generate the proof:

```
premise !exists x !x = m
conv_not_exists 1
forall_elim 2, a
double_not_elim 3
forall_elim 2, b
double_not_elim 5
assumption P(a)
id_elim 4, 7, P(m)
id_elim 6, 8, P(b)
discharge
imp_intro 7, 9
forall_intro 11, b, y
forall_intro 12, a, x
qed
```

Chapter 39, Exercise A.6

Proof presented using Fitch notation:

1	$\exists x J(x)$	PR
2	$\exists x \neg J(x)$	PR
3	$\quad J(a)$	AS
4	$\quad\quad \neg J(b)$	AS
5	$\quad\quad\quad a = b$	AS
6	$\quad\quad\quad J(a)$	R, 3
7	$\quad\quad\quad J(b)$	=E, 5, 6
8	$\quad\quad\quad \neg J(b)$	R, 4
9	$\quad\quad\quad \bot$	¬E, 8, 7
10	$\quad\quad \neg a = b$	¬I, 5–9
11	$\quad\quad \exists y \neg a = y$	∃I, 10
12	$\quad\quad \exists x \exists y \neg x = y$	∃I, 11
13	$\quad \exists x \exists y \neg x = y$	∃E, 2, 4–12
14	$\exists x \exists y \neg x = y$	∃E, 1, 3–13

Daeducer commands needed to generate the proof:

```
premise exists x J(x)
premise exists x !J(x)
assumption J(a)
assumption !J(b)
assumption a = b
reiteration 3
id_elim 5, 6, J(b)
reiteration 4
not_elim 8, 7
discharge
not_intro 5, 9
exists_intro 11, !a = y
exists_intro 12, exists y !x = y
discharge
exists_elim 2, 4, 13
discharge
exists_elim 1, 3, 15
qed
```

Chapter 39, Exercise A.7

Proof presented using Fitch notation:

1	$\forall x(x = n \leftrightarrow M(x))$	PR
2	$\forall x(O(x) \lor \neg M(x))$	PR
3	$(O(n) \lor \neg M(n))$	$\forall$E, 2
4	$(n = n \leftrightarrow M(n))$	$\forall$E, 1
5	$n = n$	=I
6	$M(n)$	$\leftrightarrow$E, 4, 5
7	$\quad O(n)$	AS
8	$\quad O(n)$	R, 7
9	$\quad \neg M(n)$	AS
10	$\quad \bot$	$\neg$E, 9, 6
11	$\quad O(n)$	X, 10
12	$O(n)$	$\lor$E, 3, 7–8, 9–11

Daeducer commands needed to generate the proof:

```
premise forall x (x = n <-> M(x))
premise forall x (O(x) v !M(x))
forall_elim 2, n
forall_elim 1, n
id_intro n
bi_elim 4, 5
assumption O(n)
reiteration 7
discharge
assumption !M(n)
not_elim 10, 6
explosion 11, O(n)
discharge
or_elim 3, 7, 8, 10, 12
qed
```

Chapter 39, Exercise A.8

Proof presented using Fitch notation:

1	$\exists x D(x)$	PR
2	$\forall x(x = p \leftrightarrow D(x))$	PR
3	$\quad D(a)$	AS
4	$\quad (a = p \leftrightarrow D(a))$	$\forall$E, 2
5	$\quad a = p$	$\leftrightarrow$E, 4, 3
6	$\quad D(p)$	=E, 5, 3
7	$D(p)$	$\exists$E, 1, 3–6

Daeducer commands needed to generate the proof:

```
premise exists x D(x)
premise forall x (x = p <-> D(x))
assumption D(a)
forall_elim 2, a
bi_elim 4, 3
id_elim 5, 3, D(p)
discharge
exists_elim 1, 3, 6
qed
```

Chapter 39, Exercise A.9

Proof presented using Fitch notation:

1	$\exists x((K(x) \wedge \forall y(K(y) \rightarrow x = y)) \wedge B(x))$	PR
2	$K(d)$	PR
3	$\quad ((K(c) \wedge \forall y(K(y) \rightarrow c = y)) \wedge B(c))$	AS
4	$\quad (K(c) \wedge \forall y(K(y) \rightarrow c = y))$	$\wedge$E, 3
5	$\quad B(c)$	$\wedge$E, 3
6	$\quad K(c)$	$\wedge$E, 4
7	$\quad \forall y(K(y) \rightarrow c = y)$	$\wedge$E, 4
8	$\quad (K(d) \rightarrow c = d)$	$\forall$E, 7
9	$\quad c = d$	$\rightarrow$E, 8, 2
10	$\quad B(d)$	=E, 9, 5
11	$B(d)$	$\exists$E, 1, 3–10

Daeducer commands needed to generate the proof:

```
premise exists x ((K(x) ^ forall y (K(y) -> x = y)) ^ B(x))
premise K(d)
assumption ((K(c) ^ forall y (K(y) -> c = y)) ^ B(c))
and_elim_left 3
and_elim_right 3
and_elim_left 4
and_elim_right 4
```

```
forall_elim 7, d
imp_elim 8, 2
id_elim 9, 5, B(d)
discharge
exists_elim 1, 3, 10
qed
```

Chapter 39, Exercise A.10

Proof presented using Fitch notation:

1	$P(a)$	AS
2	$y = a$	AS
3	$P(y)$	=E, 2, 1
4	$(P(y) \vee \neg y = a)$	∨E, 3
5	$(y = a \to (P(y) \vee \neg y = a))$	→I, 2, 4
6	$\neg y = a$	AS
7	$(P(y) \vee \neg y = a)$	∨E, 6
8	$(\neg y = a \to (P(y) \vee \neg y = a))$	→I, 6, 7
9	$(P(y) \vee \neg y = a)$	LEM, 5, 8
10	$\forall x(P(x) \vee \neg x = a)$	∀I, 9
11	$(P(a) \to \forall x(P(x) \vee \neg x = a))$	→I, 1, 10

Daeducer commands needed to generate the proof:

```
assumption P(a)
assumption y = a
id_elim 2, 1, P(y)
or_intro_left 3, !y = a
discharge
imp_intro 2, 4
assumption !y = a
or_intro_right 7, P(y)
discharge
imp_intro 7, 8
excluded_middle 6, 10
forall_intro 11, y, x
discharge
imp_intro 1, 12
qed
```

Chapter 39, Exercise B.1.a

Proof presented using Fitch notation:

1	$\exists x((F(x) \land \forall y(F(y) \to x = y)) \land x = n)$	PR
2	$((F(a) \land \forall y(F(y) \to a = y)) \land a = n)$	AS
3	$(F(a) \land \forall y(F(y) \to a = y))$	$\land$E, 2
4	$a = n$	$\land$E, 2
5	$F(a)$	$\land$E, 3
6	$\forall y(F(y) \to a = y)$	$\land$E, 3
7	$F(n)$	=E, 4, 5
8	$(F(w) \to a = w)$	$\forall$E, 6
9	$F(w)$	AS
10	$a = w$	$\to$E, 8, 9
11	$n = w$	=E, 4, 10
12	$(F(w) \to n = w)$	$\to$I, 9, 11
13	$\forall y(F(y) \to n = y)$	$\forall$I, 12
14	$(F(n) \land \forall y(F(y) \to n = y))$	$\land$I, 7, 13
15	$(F(n) \land \forall y(F(y) \to n = y))$	$\exists$E, 1, 2–14

Daeducer commands needed to generate the proof:

```
premise exists x ((F(x) ^ forall y (F(y) -> x = y)) ^ x = n)
assumption ((F(a) ^ forall y (F(y) -> a = y)) ^ a = n)
and_elim_left 2
and_elim_right 2
and_elim_left 3
and_elim_right 3
id_elim 4, 5, F(n)
forall_elim 6, w
assumption F(w)
imp_elim 8, 9
id_elim 4, 10, n = w
discharge
imp_intro 9, 11
forall_intro 13, w, y
and_intro 7, 14
discharge
exists_elim 1, 2, 15
qed
```

Chapter 39, Exercise B.1.b

Proof presented using Fitch notation:

1	$(F(n) \land \forall y(F(y) \to n = y))$	PR
2	$n = n$	=I
3	$((F(n) \land \forall y(F(y) \to n = y)) \land n = n)$	$\land$I, 1, 2
4	$\exists x((F(x) \land \forall y(F(y) \to x = y)) \land x = n)$	$\exists$I, 3

Daeducer commands needed to generate the proof:

```
premise (F(n) ^ forall y (F(y) -> n = y))
id_intro n
and_intro 1, 2
exists_intro 3, ((F(x) ^ forall y (F(y) -> x = y)) ^ x = n)
qed
```

Chapter 39, Exercise C.1.a

Proof presented using Fitch notation:

1	$(\exists x F(x) \land \forall x \forall y((F(x) \land F(y)) \to x = y))$	PR
2	$\exists x F(x)$	$\land$E, 1
3	$\forall x \forall y((F(x) \land F(y)) \to x = y)$	$\land$E, 1
4	$\quad F(a)$	AS
5	$\quad\quad F(z)$	AS
6	$\quad\quad \forall y((F(a) \land F(y)) \to a = y)$	$\forall$E, 3
7	$\quad\quad ((F(a) \land F(z)) \to a = z)$	$\forall$E, 6
8	$\quad\quad (F(a) \land F(z))$	$\land$I, 4, 5
9	$\quad\quad a = z$	$\to$E, 7, 8
10	$\quad (F(z) \to a = z)$	$\to$I, 5, 9
11	$\quad \forall y(F(y) \to a = y)$	$\forall$I, 10
12	$\quad (F(a) \land \forall y(F(y) \to a = y))$	$\land$I, 4, 11
13	$\quad \exists x(F(x) \land \forall y(F(y) \to x = y))$	$\exists$I, 12
14	$\exists x(F(x) \land \forall y(F(y) \to x = y))$	$\exists$E, 2, 4–13

Daeducer commands needed to generate the proof:

```
premise (exists x F(x) ^ forall x forall y ((F(x) ^ F(y)) -> x = y))
and_elim_left 1
and_elim_right 1
assumption F(a)
assumption F(z)
forall_elim 3, a
forall_elim 6, z
and_intro 4, 5
imp_elim 7, 8
discharge
imp_intro 5, 9
```

```
forall_intro 11, z, y
and_intro 4, 12
exists_intro 13, (F(x) ^ forall y (F(y) -> x = y))
discharge
exists_elim 2, 4, 14
qed
```

Chapter 39, Exercise C.1.b

Proof presented using Fitch notation:

1	$\exists x(F(x) \land \forall y(F(y) \to x = y))$	PR
2	$\quad (F(a) \land \forall y(F(y) \to a = y))$	AS
3	$\quad F(a)$	$\land$E, 2
4	$\quad \forall y(F(y) \to a = y)$	$\land$E, 2
5	$\quad\quad F(z)$	AS
6	$\quad\quad (F(z) \to a = z)$	$\forall$E, 4
7	$\quad\quad a = z$	$\to$E, 6, 5
8	$\quad\quad a = z$	AS
9	$\quad\quad F(z)$	$=$E, 8, 3
10	$\quad (F(z) \leftrightarrow a = z)$	$\leftrightarrow$I, 5–7, 8–9
11	$\quad \forall y(F(y) \leftrightarrow a = y)$	$\forall$I, 10
12	$\quad \exists x\forall y(F(y) \leftrightarrow x = y)$	$\exists$I, 11
13	$\exists x\forall y(F(y) \leftrightarrow x = y)$	$\exists$E, 1, 2–12

Daeducer commands needed to generate the proof:

```
premise exists x (F(x) ^ forall y (F(y) -> x = y))
assumption (F(a) ^ forall y (F(y) -> a = y))
and_elim_left 2
and_elim_right 2
assumption F(z)
forall_elim 4, z
imp_elim 6, 5
discharge
assumption a = z
id_elim 9, 3, F(z)
discharge
bi_intro 5, 7, 9, 10
forall_intro 12, z, y
exists_intro 13, forall y (F(y) <-> x = y)
discharge
exists_elim 1, 2, 14
qed
```

Chapter 39, Exercise C.1.c

Proof presented using Fitch notation:

1	$\exists x \forall y (F(y) \leftrightarrow x = y)$	PR
2	$\quad \forall y (F(y) \leftrightarrow c = y)$	AS
3	$\quad (F(c) \leftrightarrow c = c)$	$\forall$E, 2
4	$\quad c = c$	=I
5	$\quad F(c)$	$\leftrightarrow$E, 3, 4
6	$\quad \exists x F(x)$	$\exists$I, 5
7	$\quad\quad (F(a) \wedge F(b))$	AS
8	$\quad\quad F(a)$	$\wedge$E, 7
9	$\quad\quad F(b)$	$\wedge$E, 7
10	$\quad\quad (F(a) \leftrightarrow c = a)$	$\forall$E, 2
11	$\quad\quad c = a$	$\leftrightarrow$E, 10, 8
12	$\quad\quad (F(b) \leftrightarrow c = b)$	$\forall$E, 2
13	$\quad\quad c = b$	$\leftrightarrow$E, 12, 9
14	$\quad\quad a = b$	=E, 11, 13
15	$\quad ((F(a) \wedge F(b)) \rightarrow a = b)$	$\rightarrow$I, 7, 14
16	$\quad \forall y ((F(a) \wedge F(y)) \rightarrow a = y)$	$\forall$I, 15
17	$\quad \forall x \forall y ((F(x) \wedge F(y)) \rightarrow x = y)$	$\forall$I, 16
18	$\quad (\exists x F(x) \wedge \forall x \forall y ((F(x) \wedge F(y)) \rightarrow x = y))$	$\wedge$I, 6, 17
19	$(\exists x F(x) \wedge \forall x \forall y ((F(x) \wedge F(y)) \rightarrow x = y))$	$\exists$E, 1, 2–18

Daeducer commands needed to generate the proof:

```
premise exists x forall y (F(y) <-> x = y)
assumption forall y (F(y) <-> c = y)
forall_elim 2, c
id_intro c
bi_elim 3, 4
exists_intro 5, F(x)
assumption (F(a) ^ F(b))
and_elim_left 7
and_elim_right 7
forall_elim 2, a
bi_elim 10, 8
forall_elim 2, b
bi_elim 12, 9
id_elim 11, 13, a = b
discharge
imp_intro 7, 14
forall_intro 16, b, y
forall_intro 17, a, x
and_intro 6, 18
discharge
exists_elim 1, 2, 19
qed
```

Chapter 39, Exercise D.1

Proof presented using Fitch notation:

1	$\exists x(F(x) \land \forall y(F(y) \to x = y))$	PR
2	$\exists x(G(x) \land \forall y(G(y) \to x = y))$	PR
3	$\forall x \neg(F(x) \land G(x))$	PR
4	$\quad (F(a) \land \forall y(F(y) \to a = y))$	AS
5	$\quad F(a)$	$\land$E, 4
6	$\quad \forall y(F(y) \to a = y)$	$\land$E, 4
7	$\quad\quad (G(b) \land \forall y(G(y) \to b = y))$	AS
8	$\quad\quad G(b)$	$\land$E, 7
9	$\quad\quad \forall y(G(y) \to b = y)$	$\land$E, 7
10	$\quad\quad\quad a = b$	AS
11	$\quad\quad\quad \forall x \neg(F(x) \land G(x))$	R, 3
12	$\quad\quad\quad \neg(F(a) \land G(a))$	$\forall$E, 11
13	$\quad\quad\quad G(a)$	=E, 10, 8
14	$\quad\quad\quad (F(a) \land G(a))$	$\land$I, 5, 13
15	$\quad\quad\quad \bot$	$\neg$E, 12, 14
16	$\quad\quad \neg a = b$	$\neg$I, 10–15
17	$\quad\quad (F(a) \lor G(a))$	$\lor$E, 5
18	$\quad\quad (F(b) \lor G(b))$	$\lor$E, 8
19	$\quad\quad\quad (F(z) \lor G(z))$	AS
20	$\quad\quad\quad\quad F(z)$	AS
21	$\quad\quad\quad\quad (F(z) \to a = z)$	$\forall$E, 6
22	$\quad\quad\quad\quad a = z$	$\to$E, 21, 20
23	$\quad\quad\quad\quad a = a$	=I
24	$\quad\quad\quad\quad z = a$	=E, 22, 23
25	$\quad\quad\quad\quad (z = a \lor z = b)$	$\lor$E, 24
26	$\quad\quad\quad\quad G(z)$	AS
27	$\quad\quad\quad\quad (G(z) \to b = z)$	$\forall$E, 9
28	$\quad\quad\quad\quad b = z$	$\to$E, 27, 26
29	$\quad\quad\quad\quad b = b$	=I
30	$\quad\quad\quad\quad z = b$	=E, 28, 29
31	$\quad\quad\quad\quad (z = a \lor z = b)$	$\lor$E, 30
32	$\quad\quad\quad (z = a \lor z = b)$	$\lor$E, 19, 20–25, 26–31

33	$((F(z) \vee G(z)) \to (z = a \vee z = b))$	$\to$I, 19, 32
34	$\forall z((F(z) \vee G(z)) \to (z = a \vee z = b))$	$\forall$I, 33
35	$((F(b) \vee G(b)) \wedge \forall z((F(z) \vee G(z)) \to (z = a \vee z = b)))$	$\wedge$I, 18, 34
36	$((F(a) \vee G(a)) \wedge ((F(b) \vee G(b)) \wedge \forall z((F(z) \vee G(z)) \to (z = a \vee z = b))))$	$\wedge$I, 17, 35
37	$(\neg a = b \wedge ((F(a) \vee G(a)) \wedge ((F(b) \vee G(b)) \wedge \forall z((F(z) \vee G(z)) \to (z = a \vee z = b)))))$	$\wedge$I, 16, 36
38	$\exists y(\neg a = y \wedge ((F(a) \vee G(a)) \wedge ((F(y) \vee G(y)) \wedge \forall z((F(z) \vee G(z)) \to (z = a \vee z = y)))))$	$\exists$I, 37
39	$\exists x \exists y(\neg x = y \wedge ((F(x) \vee G(x)) \wedge ((F(y) \vee G(y)) \wedge \forall z((F(z) \vee G(z)) \to (z = x \vee z = y)))))$	$\exists$I, 38
40	$\exists x \exists y(\neg x = y \wedge ((F(x) \vee G(x)) \wedge ((F(y) \vee G(y)) \wedge \forall z((F(z) \vee G(z)) \to (z = x \vee z = y)))))$	$\exists$E, 2, 7–39
41	$\exists x \exists y(\neg x = y \wedge ((F(x) \vee G(x)) \wedge ((F(y) \vee G(y)) \wedge \forall z((F(z) \vee G(z)) \to (z = x \vee z = y)))))$	$\exists$E, 1, 4–40
42	$\exists x \exists y(\neg x = y \wedge ((F(x) \vee G(x)) \wedge ((F(y) \vee G(y)) \wedge \forall z((F(z) \vee G(z)) \to (z = x \vee z = y)))))$	$\exists$E, 1, 4–40

Daeducer commands needed to generate the proof:

```
premise exists x (F(x) ^ forall y (F(y) -> x = y))
premise exists x (G(x) ^ forall y (G(y) -> x = y))
premise forall x !(F(x) ^ G(x))
assumption (F(a) ^ forall y (F(y) -> a = y))
and_elim_left 4
and_elim_right 4
assumption (G(b) ^ forall y (G(y) -> b = y))
and_elim_left 7
and_elim_right 7
assumption a = b
reiteration 3
forall_elim 11, a
id_elim 10, 8, G(a)
and_intro 5, 13
not_elim 12, 14
discharge
not_intro 10, 15
or_intro_left 5, G(a)
or_intro_right 8, F(b)
assumption (F(z) v G(z))
assumption F(z)
forall_elim 6, z
imp_elim 22, 21
id_intro a
id_elim 23, 24, z = a
or_intro_left 25, z = b
discharge
assumption G(z)
forall_elim 9, z
imp_elim 29, 28
id_intro b
id_elim 30, 31, z = b
```

```
or_intro_right 32, z = a
discharge
or_elim 20, 21, 26, 28, 33
discharge
imp_intro 20, 35
forall_intro 37, z, z
and_intro 19, 38
and_intro 18, 39
and_intro 17, 40
exists_intro 41, (!a = y ^ ((F(a) v G(a)) ^ ((F(y) v G(y)) ^ forall z ((F(z) v G(z)) -> (z = a
    v z = y)))))
exists_intro 42, exists y (!x = y ^ ((F(x) v G(x)) ^ ((F(y) v G(y)) ^ forall z ((F(z) v G(z))
    -> (z = x v z = y)))))
discharge
exists_elim 2, 7, 43
discharge
exists_elim 1, 4, 45
exists_elim 1, 4, 45
qed
```

Chapter 40, Example p.330

Proof presented using Fitch notation:

1	$\forall x \neg A(x)$	PR
2	$\quad \exists x A(x)$	AS
3	$\quad\quad A(c)$	AS
4	$\quad\quad \neg A(c)$	$\forall$E, 1
5	$\quad\quad \bot$	$\neg$E, 4, 3
6	$\quad \bot$	$\exists$E, 2, 3–5
7	$\neg \exists x A(x)$	$\neg$I, 2–6

Daeducer commands needed to generate the proof:

```
premise forall x !A(x)
assumption exists x A(x)
assumption A(c)
forall_elim 1, c
not_elim 4, 3
discharge
exists_elim 2, 3, 5
discharge
not_intro 2, 7
qed
```

Chapter 40, Example p.331

Proof presented using Fitch notation:

1	$\exists x \neg A(x)$	PR
2	$\quad \neg A(a)$	AS
3	$\quad\quad \forall x A(x)$	AS
4	$\quad\quad A(a)$	$\forall$E, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad \neg\forall x A(x)$	$\neg$I, 3–5
7	$\neg\forall x A(x)$	$\exists$E, 1, 2–6
8	$\neg\forall x A(x)$	$\exists$E, 1, 2–6

Daeducer commands needed to generate the proof:

```
premise exists x !A(x)
assumption !A(a)
assumption forall x A(x)
forall_elim 3, a
not_elim 2, 4
discharge
not_intro 3, 5
discharge
exists_elim 1, 2, 7
exists_elim 1, 2, 7
qed
```

Chapter 40, Exercise A.1

Proof presented using Fitch notation:

1	$\neg\exists x A(x)$	PR
2	$\quad A(y)$	AS
3	$\quad \exists x A(x)$	$\exists$I, 2
4	$\quad \bot$	$\neg$E, 1, 3
5	$\neg A(y)$	$\neg$I, 2–4
6	$\forall x \neg A(x)$	$\forall$I, 5

Daeducer commands needed to generate the proof:

```
premise !exists x A(x)
assumption A(y)
exists_intro 2, A(x)
not_elim 1, 3
discharge
not_intro 2, 4
forall_intro 6, y, x
qed
```

Chapter 40, Exercise A.2

Proof presented using Fitch notation:

1	$\neg\forall x A(x)$	PR
2	$\quad \neg\exists x \neg A(x)$	AS
3	$\quad\quad \neg A(c)$	AS
4	$\quad\quad \exists x \neg A(x)$	$\exists$I, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad A(c)$	IP, 3–5
7	$\quad \forall x A(x)$	$\forall$I, 6
8	$\quad \bot$	$\neg$E, 1, 7
9	$\exists x \neg A(x)$	IP, 2–8
10	$\exists x \neg A(x)$	IP, 2–8

Daeducer commands needed to generate the proof:

```
premise !forall x A(x)
assumption !exists x !A(x)
assumption !A(c)
exists_intro 3, !A(x)
not_elim 2, 4
discharge
indirect 3, 5
forall_intro 7, c, x
not_elim 1, 8
discharge
indirect 2, 9
indirect 2, 9
qed
```