




NEURAL SCALING LAWS

Are we fundamentally limited and how can we learn
from this?



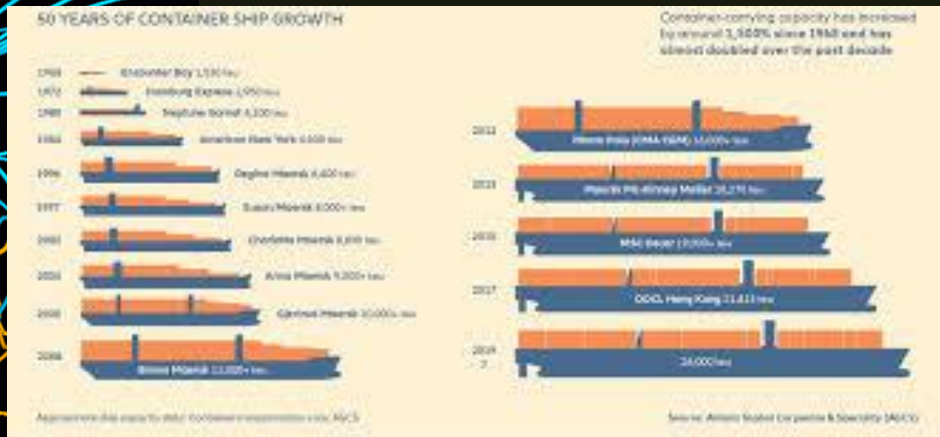
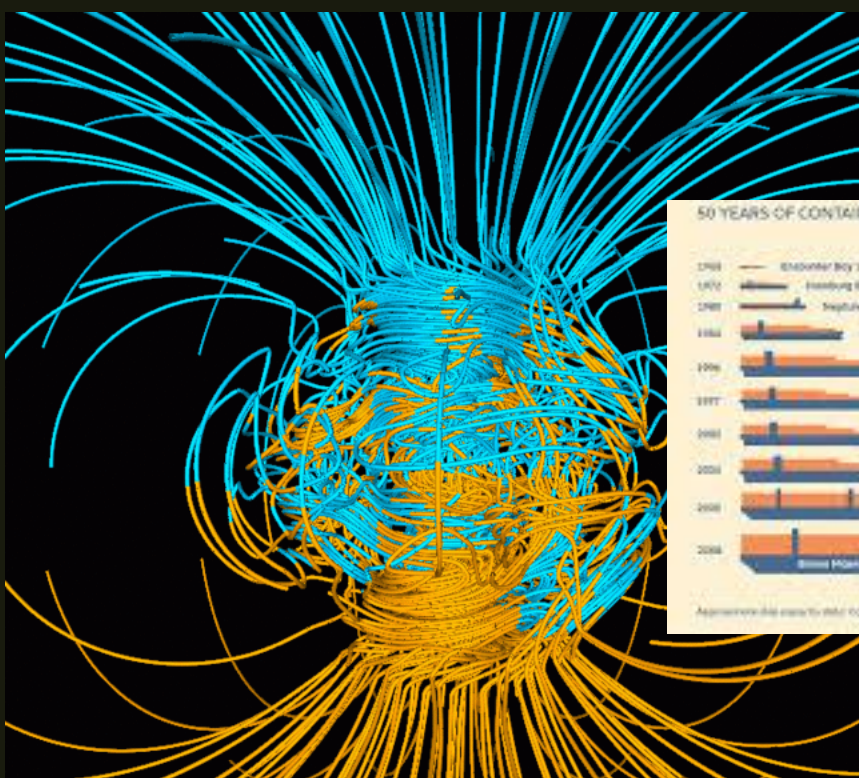
THIS TALK

- What do we mean by (neural) scaling laws?
- Compute, parameters and data: why these three?
- Parameter law
- Data law
- Joint scaling laws
- Compute law
- Can we learn to be compute optimal from this?
- Are we fundamentally limited? (Posturing on why these scaling laws exist)

WHAT EVEN ARE SCALING LAWS?

From physics to neural architecture





So what are scaling laws?

- Relationships between two (or more) quantities that describe how one grows in terms of the others.
- In physics (and other natural sciences) these are typically observed, derived, then explained.

E.g.:

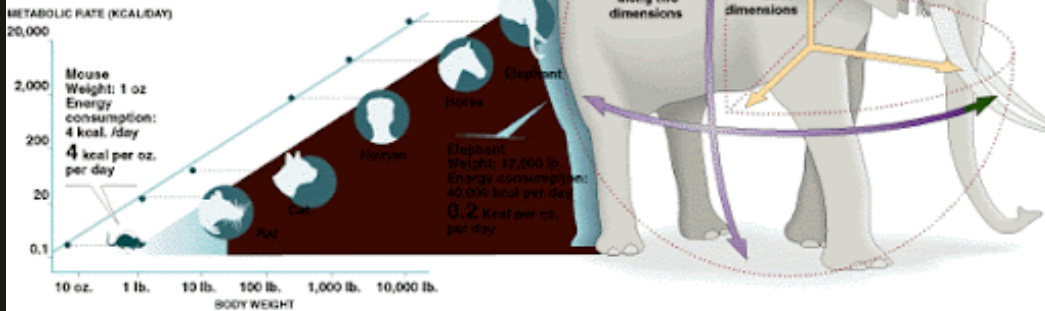
- Strength of an electromagnetic field.
- Size vs. surface area of boats.
- Size of mammals.

$$F(x) = Ax^c$$

From the Small to the Huge

Three scientists have proposed a novel theory to explain how characteristics like body size and energy consumption differ from species to species along fixed scales. Their theory derives from analysis of the circulatory system.

An Example of Scaling: Metabolic Rate



Size and Efficiency

The average elephant weighs 220,000 times as much as the average mouse, but requires only about 10,000 times as much energy in the form of food calories to sustain itself. The

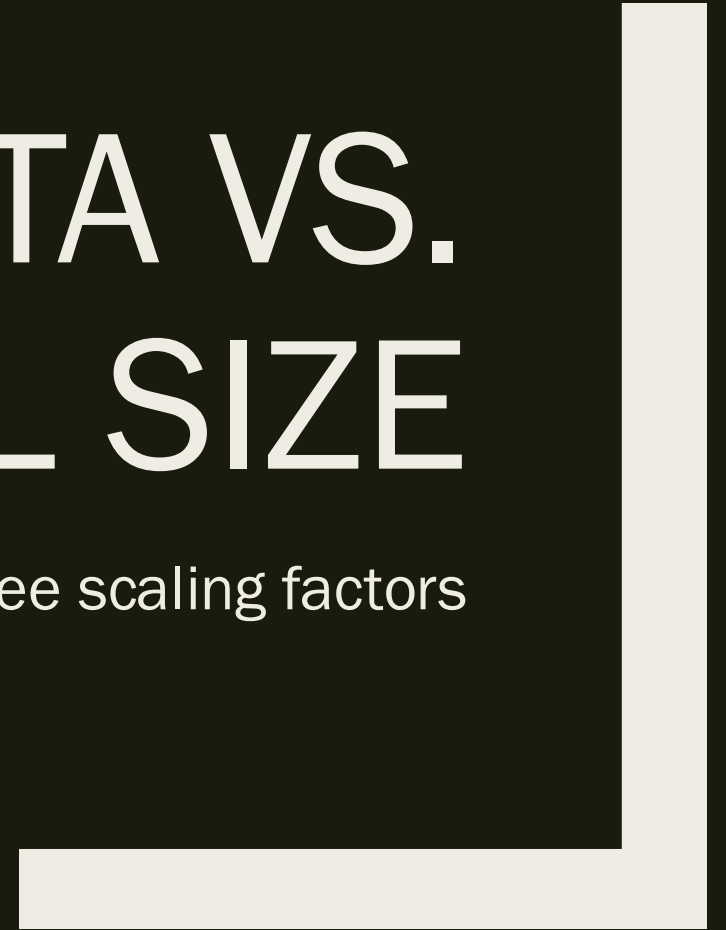
reason lies in the mathematical and geometric nature of networks that distribute nutrients and carry away wastes and heat. The bigger the animal, the more efficiently it uses energy.

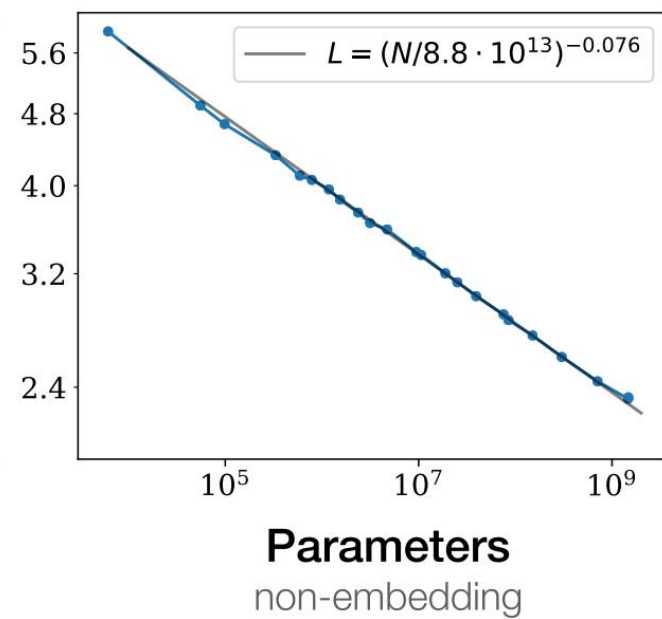
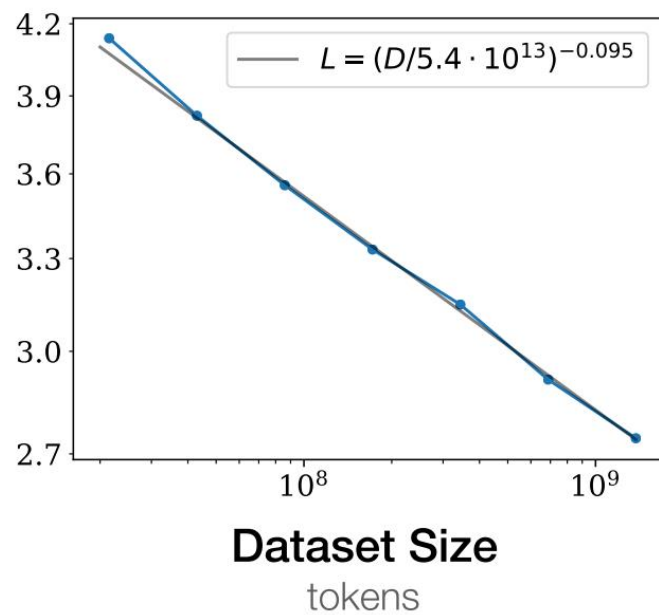
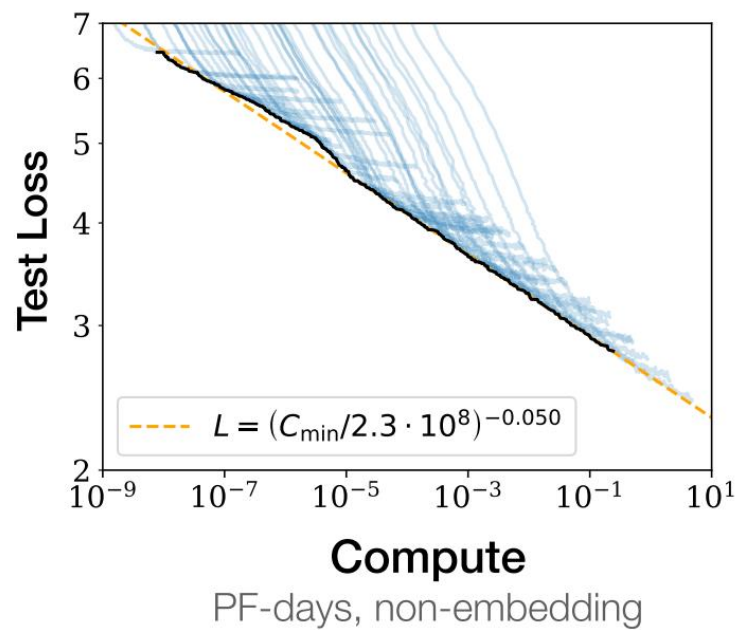
So why should we care here?

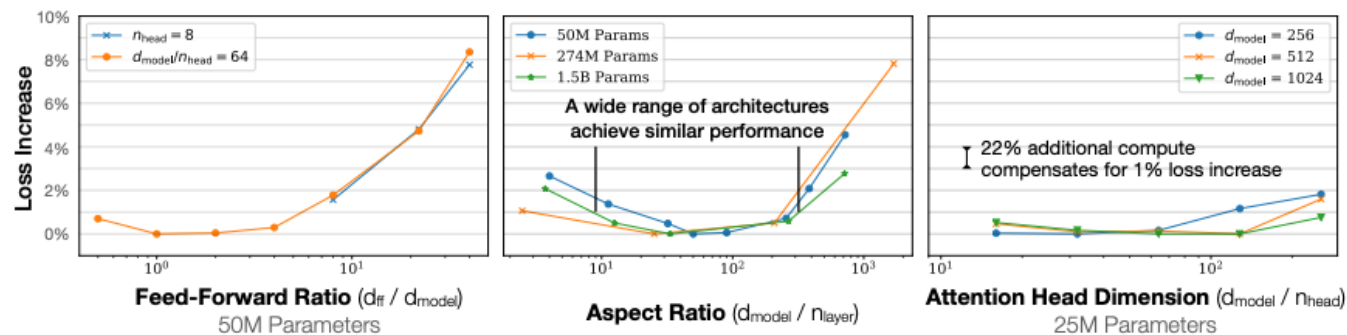
- Training and inference through these models is expensive, especially for larger models.
- If we can define how to be as efficient as possible by scaling our models, compute and dataset size appropriately we reduce inefficiency.
- Further, these scaling laws have been observed, but not fully explained (more on that later).

COMPUTE VS. DATA VS. MODEL SIZE

The big three scaling factors







Why these three?

- Empirically shown that performance depends strongly on scale and weakly on model shape.
- By scale we refer to:
 - Model parameters (N), excluding embeddings.
 - Dataset size (D), measured in tokens.
 - Total amount of compute (C) measured in peta-flop days.
- Depth vs. width and other shape factors have some dependence.

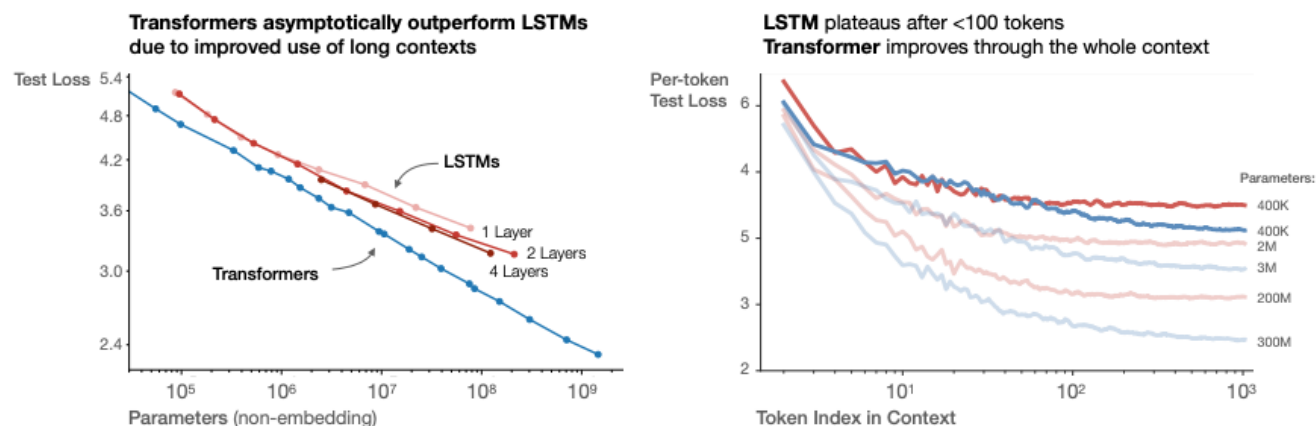
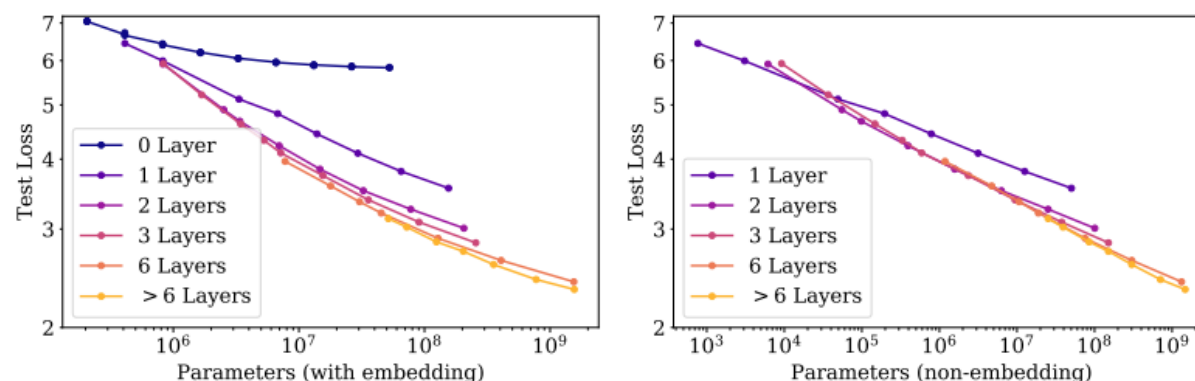


Figure 7

THE MODEL SCALING LAW



The law itself

- Notation:

- $L(\cdot)$: Loss, measured in nats.
- N : Number of parameters in the model (excluding embeddings).
- $N_c = 8.8 \times 10^{13}$: scale.
- $\alpha_N = 0.76$: power law.

$$L(N) = \left(\frac{N_c}{N} \right)^{\alpha_N}$$

- This makes the assumption of ‘infinite’ data and compute.

THE DATA SCALING LAW



The law itself

- Notation:

- $L(\cdot)$: Loss, measured in nats.
- D : Dataset size in tokens.
- $D_c = 5.4 \times 10^{13}$: scale.
- $\alpha_D = 0.095$: power law.

$$L(D) = \left(\frac{D_c}{D} \right)^{\alpha_D}$$

- This makes the assumption of ‘infinite’ model size and early stopping compute.

JOINT LAWS



Parameters and Data

- This gives how we should expect the loss to scale when we jointly scale the number of parameters and the data.
- Capturing a more realistic scenario than holding one constant and scaling the other.

$$L(N, D) = \left[\left(\frac{N_c}{N} \right)^{\frac{\alpha_N}{\alpha_D}} + \frac{D_c}{D} \right]^{\alpha_D}$$

- Assuming the compute stops early here.

Parameters and Training Steps

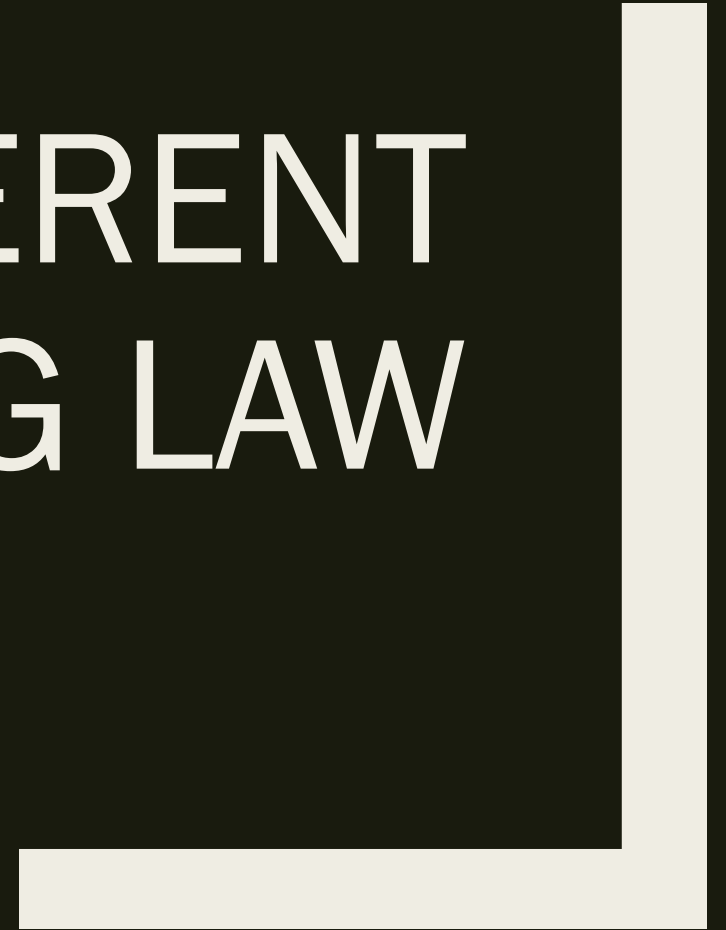
■ Notation:

- $L(\cdot)$: Loss, measured in nats.
- S : Total number of training steps.
- $S_c = 2.1 \times 10^3$: scale.
- B : batch size.
- $S_{min}(S, B)$: estimate of the minimum number of training steps to reach a given value of loss.
- $\alpha_S = 0.76$: power law.

$$L(N, S) = \left(\frac{N_c}{N} \right)^{\alpha_N} + \left(\frac{S_c}{S_{min}(S, B)} \right)^{\alpha_S}$$

- Assuming ‘infinite’ data here, and compute defined by the number of training steps.

A SLIGHTLY DIFFERENT SCALING LAW



An alternative joint scaling law

- By experimenting on Chinchilla a team at google fit this alternative form.

- Notation:

- N : Parameter count.
- D : Dataset size.
- $E = 1.69$
- $A = 406.4$
- $B = 410.7$

$$L(N, D) = E + \frac{A}{N^{0.34}} + \frac{B}{D^{0.28}}$$

- This suggests that both model size and dataset size should be scaled at the same rate for optimality.

THE COMPUTE SCALING LAW



The law itself

■ Notation:

- $L(\cdot)$: Loss, measured in nats.
- C : Amount of compute, in PF-days.
- C_{min} : An estimate of the minimum amount of non-embedding compute to reach a given value of loss.
- $C_c = 1.6 \times 10^7$: scale.
- $C_c^{min} = 3.1 \times 10^8$
- $\alpha_C = 0.057$: power law.
- $\alpha_C^{min} = 0.050$

■ The first law makes assumptions on optimal parameter counts and ‘infinite’ data.

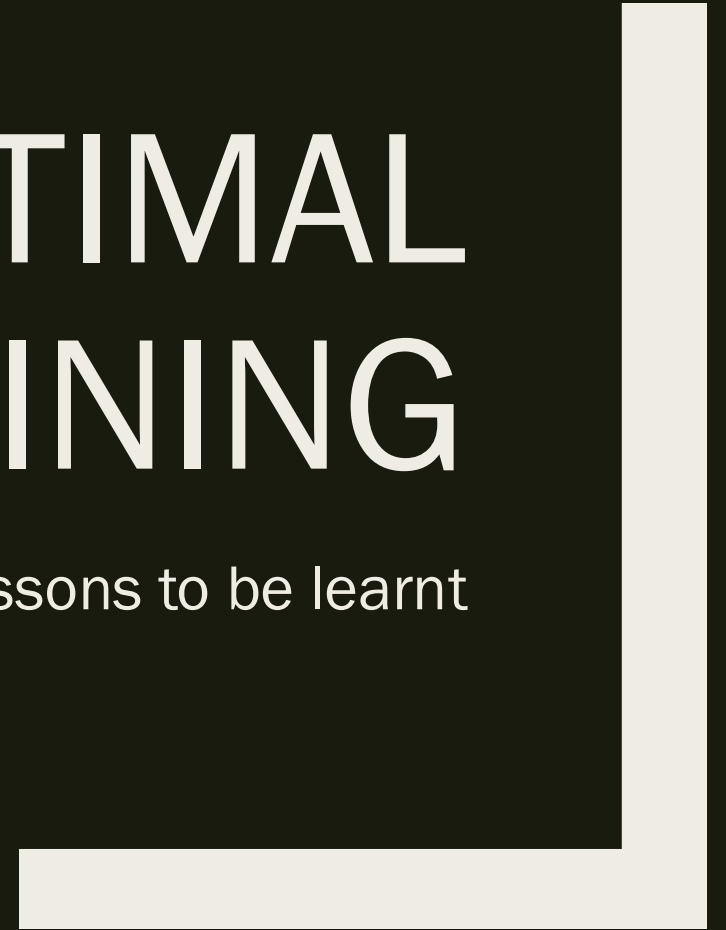
■ The second version assumes optimal parameter and data.

$$L(C) = \left(\frac{C_c}{C} \right)^{\alpha_C}$$

$$L(C_{min}) = \left(\frac{C_c^{min}}{C_{min}} \right)^{\alpha_C^{min}}$$

COMPUTE OPTIMAL SCALING AND TRAINING

Some practical lessons to be learnt



Lessons to be learnt

- Overfitting is easy.
 - Holding either the dataset or the model size constant and scaling the other quickly leads to overfitting and inefficiencies.
- Training curves are dependent on the scaling.
 - This also allows us to predict performance if training was continued.
- Reaching convergence is inefficient, and unnecessary.
 - Assuming no restriction on model or dataset size, using large models and stopping well before convergence is optimal.
- We can forecast compute requirements given an understanding of problem requirements.
- Bigger is better.
 - Somewhat unsurprisingly, bigger models with more data and larger compute do better...

Compute-Efficient Value	Power Law	Scale
$N_{\text{opt}} = N_e \cdot C_{\text{min}}^{p_N}$	$p_N = 0.73$	$N_e = 1.3 \cdot 10^9$ params
$B \ll B_{\text{crit}} = \frac{B_*}{L^{1/\alpha_B}} = B_e C_{\text{min}}^{p_B}$	$p_B = 0.24$	$B_e = 2.0 \cdot 10^6$ tokens
$S_{\text{min}} = S_e \cdot C_{\text{min}}^{p_S}$ (lower bound)	$p_S = 0.03$	$S_e = 5.4 \cdot 10^3$ steps
$D_{\text{opt}} = D_e \cdot C_{\text{min}}^{p_D}$ (1 epoch)	$p_D = 0.27$	$D_e = 2 \cdot 10^{10}$ tokens

Will this help with training?

- Yes and no.
 - We can answer some key questions what we need from compute, model size and dataset size for training here.
 - However there are many more factors one can consider to make a range of improvements. (e.g. see next week's talk).
- We can define optimal parameters for efficient training via these though.

ARE WE FUNDAMENTALLY LIMITED?

Posturing on why we observe these laws

So what's really going on here?

- Empirical observations and fitting of these laws in a natural language domain so far.
- Naturally leads to several follow up questions:
 - *Do these laws apply to other domains?*
 - If not what are the relevant laws in those settings?
 - *We observe these laws but what dictates them?*
- Is there a theoretical framework that can be devised that allows us to give a broader and more consistent picture of the underlying principles governing this behaviour?
 - *Where to start?*
 - *Do we turn to statistical mechanics?*



THANKS FOR
LISTENING!

