

## Assignment 3 (Part 1)

### Background

On April 17, 2020, Royal Bank of Canada (RBC) issued the 5-year Autocallable Debt Securities (ADSs) described in the term sheet in the file *RBCADS.pdf*. The securities had an aggregate Principal Amount of CAD 15 million.

On that day, the CAD 1-year risk-free rate was 0.3874% and the CAD 5-year risk-free rate was 0.4640%.<sup>1</sup>

In this Assignment, you will be asked to use Monte Carlo simulation to estimate the value of the ADSs on the issue date and the distribution of the rate of return.

### Working Assumptions

Assume for simplicity that:

- As of April 17, 2020, the probability of RBC defaulting over the next 5 years was negligible.
- The “**Initial Valuation Date**” coincided with the “**Issue Date**” (i.e., April 17, 2020) and the “**Final Valuation Date**” coincided with the “**Maturity Date**” (i.e., April 17, 2025).<sup>2</sup>
- The “**Interest Payment Dates**” were exactly 1 year apart (i.e., April 17, 2021, April 17, 2022, April 17, 2023, April 17, 2024 and April 17, 2025) and the annual “**Observation Dates**” coincided with the “**Interest Payment Dates**”.
- The term structure of CAD risk-free rates was flat at 0.45% and expected to remain constant at this level over the next 5 years.

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<sup>1</sup>Both rates are zero-coupon yields of Government of Canada bonds, sourced from the Bank of Canada and converted to continuous compounding.

<sup>2</sup>All boldface terms in quotation marks are as defined in the term sheet.

- As of April 17, 2020, the annual dividend yield of the “**Portfolio**” was 5.80% continuously compounded and expected to remain constant at this level over the next 5 years.<sup>3</sup>
- The “**Portfolio**” paid dividends continuously after April 17, 2020.

## Questions

The Excel workbooks *BMO.xlsx*, *BNS.xlsx*, *CM.xlsx*, *NA.xlsx*, *RY.xlsx* and *TD.xlsx* list the prices on the Toronto Stock Exchange (TSX) of each of the “**Undelying Securities**” over the period from April 18, 2018 to April 17, 2020, as well as the dividend payments over the same period.<sup>4</sup> All values are in CAD.

Answer the following questions using the data in the Excel workbooks and the Black-Scholes-Merton model and Monte Carlo simulation with 1,000 sample paths as needed.

- (a) Compute the “**Initial Portfolio Value**” and the “**Number of Underlying Securities**”.
- (b) Estimate the volatility of the “**Portfolio**” using the data in the Excel workbooks.
- (c) Estimate the value on April 17, 2020 of an ADS with CAD 100 Principal Amount. Report the estimated value, the standard error of the estimate and the 95% confidence interval for the value.
- (d) Assuming that the ADSs were issued at par (i.e., at a price of CAD 100 per CAD 100 of Principal Amount), estimate the mean and standard deviation of the risk-neutral distribution of the ADSs’ equivalent annually compounded rate of return.<sup>5</sup>

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<sup>3</sup>A dividend yield of 5.80% continuously compounded is equivalent to a dividend yield of 5.97% annually compounded (the latter figure being the estimated dividend yield reported in the term sheet).

<sup>4</sup>The date reported for each dividend payment is the ex-dividend date. Assume that the dividends were actually paid on the ex-dividend dates.

<sup>5</sup>See the term sheet for examples of the calculation of the equivalent annually compounded rate of return.