HW2

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FNCE 7170: Financial Derivatives

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1 Part 1

Question A

The plot of WorldCom shares to be exchanged for each MCI share under the terms of the offer is going to be dependent on how many shares of WCOM shares we get for each MCI share.

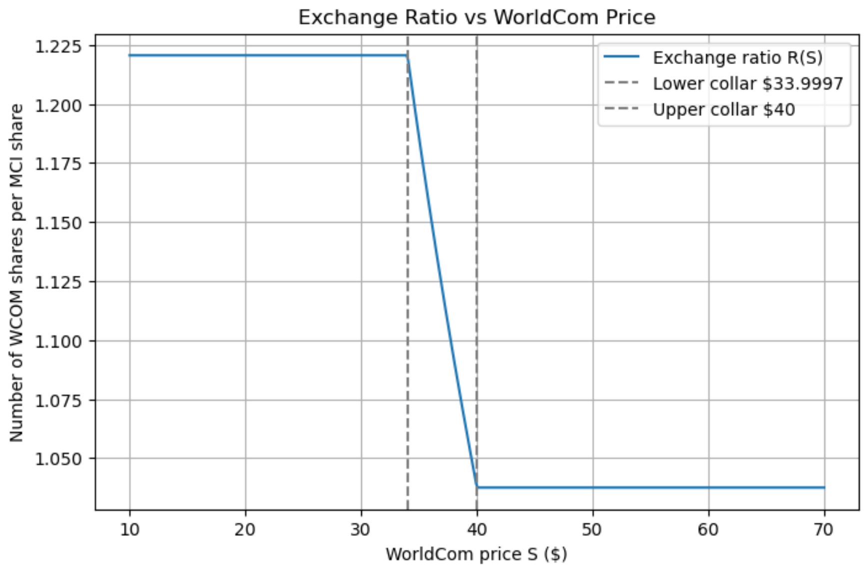
Let us first define a function R(S), that represents the number of shares of WCOM an MCI shareholder get on a given date, when the price of WCOM is closing at S.

$$R(S) = \begin{cases} 1.2206, & \text{if } S < 33.9997, \\ \dfrac{41.50}{S} & \text{if } 33.9997 \leq S \leq 40, \\ 1.0375 & \text{if } S > 40 \end{cases}$$

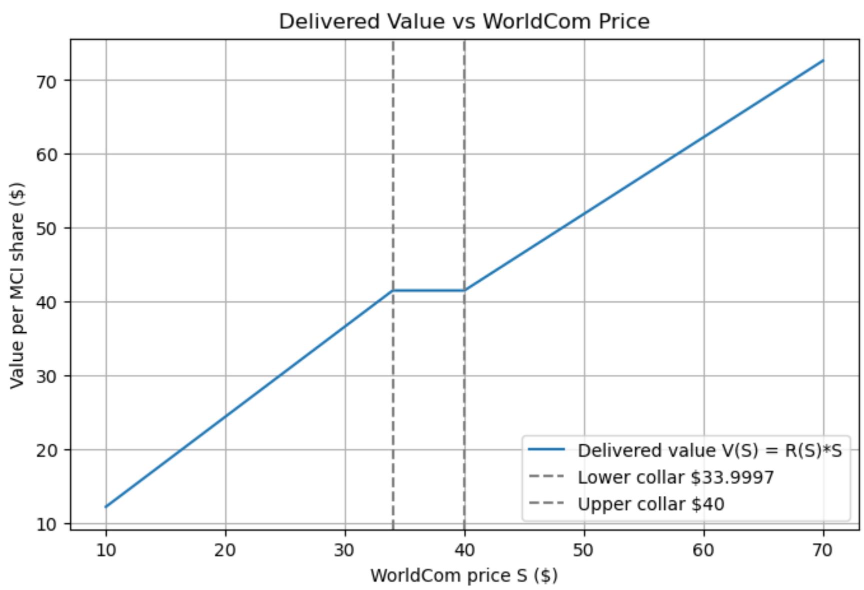
Let us also define the value of the WCOM shares that a MCI shareholder would receive, by V(S). V(S) = R(S) * S, and thus:

$$V(S) = \begin{cases} 1.2206 \cdot S, & \text{if } S < 33.9997, \\ 41.50 & \text{if } 33.9997 \leq S \leq 40, \\ 1.0375 \cdot S & \text{if } S > 40 \end{cases}$$

Therefore, the plot of the number of shares an MCI shareholder would receive would look like the following:



And the plot of the value of the WCOM shares an MCI shareholder would receive would look like the following:



Question B

We use the following formula to compute log returns: $R_{t_{i-1},t_i} = ln(\frac{S_{t_i}}{S_{t_{i-1}}})$ and then python to estimate the variance rate for the period from April 1, 1997 to October 1, 1997. We find first the variance rate $\hat{\sigma_h^2} = 0.0006$. With our assumed $h = 1/252$, then the annualized variance rate $\hat{\sigma^2} = 0.1338$.

Therefore, $\hat{\sigma} = \sqrt{\hat{\sigma^2}} = \sqrt{0.1338} = 0.3725$

Question C

To compute the value of WorldCom's offer per share for MCI we can use the generalized multi-step binomial model for a European call option. We can recall the following:

$c_0 = e^{-r \cdot T} \sum_{j=0}^{N} P^*(S_T = S_{n,j})$ x $max[0, S_{n,j} - K]$

We can use this same formula, with a different payoff function to compute the value of WorldCom's offer per share. Let c_0 be the price per share of WCOM on Octber 1, 1997. We first establish the following parameters for the binomial model:

- T = 0.5 (1/2 year)
- h = 1/252 (given)
- N = T / h = 126
- r = 0.0575 (given)
- $\sigma = 0.3725$ (estimated annualized volatility from previous part)
- $u = e^{\sigma \cdot \sqrt{h}} = 1.0237$
- $d = 1/u = 0.9768$
- $p^* = \frac{e^{-r \cdot h} - d}{u - d} = 0.499$

Instead of the call option payoff function, we will use the function we defined earlier, V(S) as the value of an MCI shareholder's stock, given the current spot price of WCOM.

Thus, the formula now becomes:

$c_0 = e^{-r \cdot T} \sum_{j=0}^{N} P^*(S_T = S_{n,j})$ x $V(S_{n,j})$, where

$$P^*(S_T = S_{n,j}) = \binom{n}{j} \cdot (p^*)^{n-j} \cdot (1 - p^*)^j$$

Therefore, $c_0 = \$38.85$

From this computed price, we notice that Mr. Ebber's claim offering 41.50 of WorldCom common stock per MCI share was not accurate. GTE's offer now stands as an improvement over WorldCom's, as they are offer \$40 per share of MCI, while the true value of the offer per share is **\$38.85** of WorldCom stock.

Question D

In this part, we adjust our values in the previous part, because we now assume that WCOM stock pays quarterly dividends (specifically one on December 1, 1997, and one on

March 1, 1998)

Our scenario models one that has dividends paid in known amounts, which means we have to adjust our model parameters to reflect that. Based on the proof in the notes, we can maintain all of our previous model parameters, but we have to adjust the spot price of the stock on October 1, 1997 for the present value of the dividends. We have:

$$S_0^* = S_0 - PV_{div} = S_0 - (0.50 \cdot e^{-0.0575*2/12} + 0.55 \cdot e^{-0.0575*5/12}) = \$34.343$$

We can still use the formula from the previous part, simply use the adjusted stock price to the prepaid forward price. This yields an offer per share price of **\$37.954**. This is lower than the computed value in the previous part.

2 Part 2

Question A

Let us first define a function R(S), that represents the number of shares of UBSG a MCN shareholder get at maturity, when the price of UBSG is closing at S. For simplicity in the plots, we assume that the face value of the MCN is F = CHF 13000. However, the actual function will be dependent on a variable, defined face value.

We also define a few variables as inputs to the function:

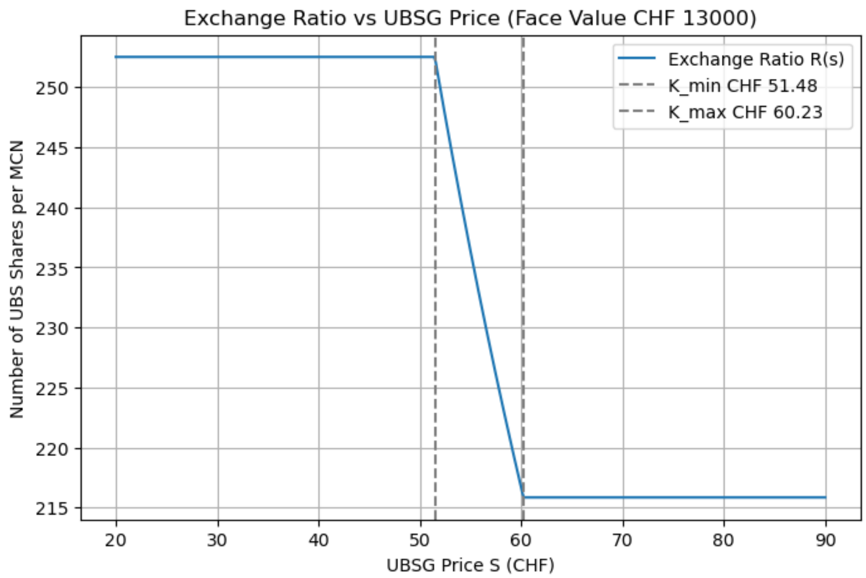
- K_{min} = CHF 51.60 or the minimum conversion price
- K_{max} = CHF 60.23 or the maximum conversion price
- F is the face value of the MCN

$$R_{UBSG}(S) = \begin{cases} \frac{F}{K_{min}}, & \text{if } S < K_{min}, \\ \frac{F}{S} & \text{if } K_{min} \leq S \leq K_{max}, \\ \frac{F}{K_{max}} & \text{if } S > K_{max} \end{cases}$$

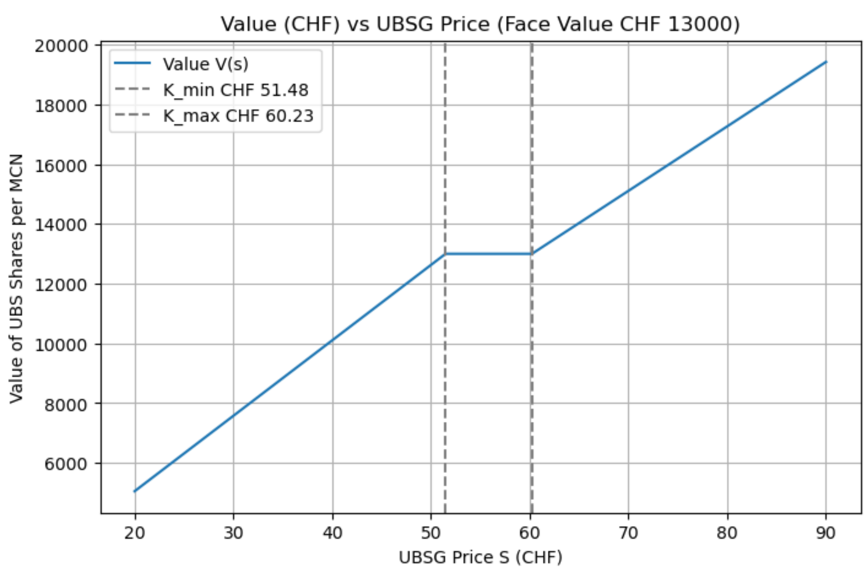
Let us also define the value of the UBSG shares that a MCN holder would receive, by $V_{UBSG}(S)$. $V_{UBSG}(S) = R_{UBSG(S)*S}$, and thus:

$$V_{UBSG}(S) = \begin{cases} \frac{F}{K_{min}} \cdot S, & \text{if } S < K_{min}, \\ F & \text{if } K_{min} \leq S \leq K_{max}, \\ \frac{F}{K_{max}} \cdot S & \text{if } S > K_{max} \end{cases}$$

Therefore, the plot of the number of shares a MCN holder would receive would look like the following:



And the plot of the value of the UBSG shares a MCN holder would receive would look like the following:



Question B

To estimate the volatility of the returns of UBSG, we must first account for the dividend payment on April 19, 2007. We use the following formula for log returns: $R_{t_{i-1},t_i} = ln(\frac{S_{t_i}+D_{t_i}}{S_{t_{i-1}}})$

We adjust the close price of UBSG on April 19, 2007 by adding back the dividend to the close price of the stock for that date, then compute the volatility using python. The annualized variance rate is computed to be $\hat{\sigma}^2 = 0.1087$

The historical estimated volatility based on the excel is $\sigma = \sqrt{\hat{\sigma}^2} = \sqrt{0.1087} = \mathbf{0.3297}$
Question C

For this part and the following parts, we use the following input variables to the binomial model:

- T = 2.0 (2 year)
- h = 1/252 (given)
- N = T / h = 504
- r = 0.025 (given)
- $\sigma = 0.3297$ (estimated annualized volatility from previous part)
- $u = e^{\sigma \cdot \sqrt{h}} = 1.021$
- $d = 1/u = 0.9794$
- $p^* = \frac{e^{-r \cdot h} - d}{u - d} = 0.4972$

In part 1, to compute the value of the exchange of WCOM shares, we used the multi-step binomial model for european call options. Since early exercise of the MCN is not assumed in this part, we can apply the same formula. We use our previously defined $V_{UBSG}(S)$ as the payoff function, and simply add the present value of the coupon payments back to the value of the simulated option payoff.

Recall:

$c_0 = e^{-r \cdot T} \sum_{j=0}^{N} P^*(S_T = S_{n,j})$ x $V_{UBSG}(S_{n,j})$, where

$$P^*(S_T = S_{n,j}) = \binom{n}{j} \cdot (p^*)^{n-j} \cdot (1 - p^*)^j$$

To include the present value of the coupon payments we adjust for it to become the following:

$c_0 = e^{-r \cdot T} [\sum_{j=0}^{N} P^*(S_T = S_{n,j})$ x $V_{UBSG}(S_{n,j})] + PV_{\text{coupon}}$, where

$$PV_{\text{coupon}} = 0.09 * Fe^{-r \cdot 1} + 0.09 * Fe^{-r \cdot 2}$$

Therefore, if we assume that the face value of each MCN is CHF 13000, then the value of MCN on March 5, 2008 is **CHF 10152.219**.

Question D

Under the given assumptions, with no dividends paid out to holders of MCN, the value of the MCN on March 5, 2008 with taking into account the early conversion factor is still **CHF 10152.219**.

Question E

Under the given assumptions of no dividends being paid out we computed that the value of the MCN with early conversion taken into account is the same as without taking it into account. Therefore, the optimal policy on early conversion, under these assumptions, is to never convert early. If one converts early, then they will need to forfeit all remaining coupon payments. Furthermore, we can treat the MCN as an option on a non-dividend paying stock conceptually and thus in this situation, the early exercise in american options is never optimal in this set of assumptions.

If we were to retain coupons and paid a dividend during the early conversion period, then there would indeed be a different case for converting during the early conversion period. By retaining the coupons, we recover our potential losses from the present value of the coupons paid. Furthermore, by adding a dividend paid during the early conversion period could make early conversion optimal, as the dividend value could outweigh the remaining value of the option till maturity.

Question F

For this part, we are looking to find values Δ_0 and β_0, which will equal the value of the MCN at time t = 0, or on March 5, 2008. We will find both using the formulas found in the notes, with the following generalizations:

$$\Delta_{i,j} = \frac{c_{i+1,j} - c_{i+1,j+1}}{S_{i,j}(u-d)}$$

$$\beta_{i,j} = \frac{-(c_{i+1,j}d - c_{i+1,j+1}u)}{u-d} \cdot e^{-r \cdot h}$$

We are looking for values to satisfy Δ_0 and β_0. Below are the computations:

$$\begin{aligned} \Delta_0 &= \frac{c_{1,0} - c_{1,1}}{S_0(u-d)} \\ &= \frac{10303.184 - 10004.942}{32.24(1.021 - 0.980)} \\ &= 222.687 \end{aligned}$$

$$\begin{aligned}\beta_0 &= \frac{-(c_{1,0} \cdot d - c_{1,1} \cdot u)}{u - d} \cdot e^{-0.025 \cdot 1/252} \\ &= \frac{-(10303.184 \cdot 0.980 - 10004.942 \cdot 1.021)}{1.021 - 0.980} \cdot e^{-0.025 \cdot 1/252} \\ &= 2972.778\end{aligned}$$

Therefore, on March 5, 2008, to replicate the value of the MCN, we would need to buy **222.687** shares of UBSG and lend **CHF 2972.778** at the risk-free rate.

Question G

When we take into account the new early conversion convention, there will be a change in the value of the MCN, since it could now become optimal to exercise early. This is because instead of the payoff for exercising the MCN early being $F/60.23 * S_t$, the exercise is now F.

At each node, we use the following computations for the continuing value of the node:

$$\text{continueval}_i = (p^* \cdot C_{i+1,j} + (1 - p^*) \cdot C_{i+1,j+1}) * e^{-r*h} + \text{Coupon}_i$$

Where we add on the coupon payments to each node when i = 252 and i = 504 and at all other nodes it is 0.

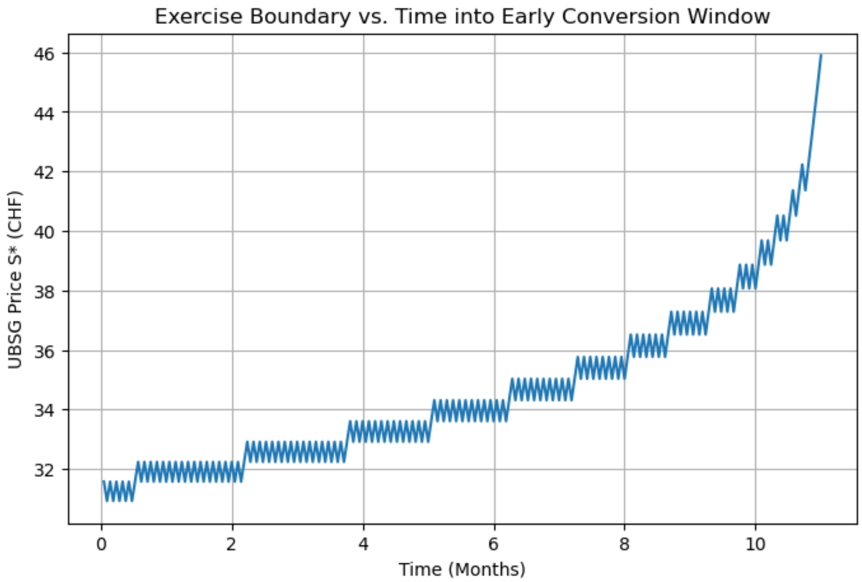
Thus, the value of each node $C_{i,j}$ becomes

$$C_{i,j} = \begin{cases} V(S_{i,j}) & \text{if } i = N, \\ max[\text{continueval}_i, F] & \text{if } i < N \end{cases}$$

The value of the MCN at each node in the binomial model becomes the maximum of the exercise value, and the continuation value. If the early exercise value exceeds the continuation value, then it is optimal to early exercise. By incorporating these values into our binomial model, we get that the fair value of the MCN of face value CHF 13000 is **CHF 14068.989**.

Question H

From the previous part, we derive the S optimal values, the highest price at which we can exercise given a timestep i. We then plot these values against the number of months into the early exercise period, and contribute the following plot:



Exercise Boundary vs. Time into Early Conversion Window
UBSG Price S* (CHF)
Time (Months)
46
44
42
40
38
36
34
32
0
2
4
6
8
10