

Assignment 3 (Part 1)

Background

On April 17, 2020, Royal Bank of Canada (RBC) issued the 5-year Autocallable Debt Securities (ADSs) described in the term sheet in the file *RBCADS.pdf*. The securities had an aggregate Principal Amount of CAD 15 million.

On that day, the CAD 1-year risk-free rate was 0.3874% and the CAD 5-year risk-free rate was 0.4640%.¹

In this Assignment, you will be asked to use Monte Carlo simulation to estimate the value of the ADSs on the issue date and the distribution of the rate of return.

Working Assumptions

Assume for simplicity that:

- As of April 17, 2020, the probability of RBC defaulting over the next 5 years was negligible.
- The “Initial Valuation Date” coincided with the “Issue Date” (i.e., April 17, 2020) and the “Final Valuation Date” coincided with the “Maturity Date” (i.e., April 17, 2025).²
- The “Interest Payment Dates” were exactly 1 year apart (i.e., April 17, 2021, April 17, 2022, April 17, 2023, April 17, 2024 and April 17, 2025) and the annual “Observation Dates” coincided with the “Interest Payment Dates”.
- The term structure of CAD risk-free rates was flat at 0.45% and expected to remain constant at this level over the next 5 years.

¹Both rates are zero-coupon yields of Government of Canada bonds, sourced from the Bank of Canada and converted to continuous compounding.

²All boldface terms in quotation marks are as defined in the term sheet.

- As of April 17, 2020, the annual dividend yield of the “**Portfolio**” was 5.80% continuously compounded and expected to remain constant at this level over the next 5 years.³
- The “**Portfolio**” paid dividends continuously after April 17, 2020.

Questions

The Excel workbooks *BMO.xlsx*, *BNS.xlsx*, *CM.xlsx*, *NA.xlsx*, *RY.xlsx* and *TD.xlsx* list the prices on the Toronto Stock Exchange (TSX) of each of the ”**Undelying Securities**” over the period from April 18, 2018 to April 17, 2020, as well as the dividend payments over the same period.⁴ All values are in CAD.

Answer the following questions using the data in the Excel workbooks and the Black-Scholes-Merton model and Monte Carlo simulation with 1,000 sample paths as needed.

- (a) Compute the “**Initial Portfolio Value**” and the “**Number of Underlying Securities**”.
- (b) Estimate the volatility of the “**Portfolio**” using the data in the Excel workbooks.
- (c) Estimate the value on April 17, 2020 of an ADS with CAD 100 Principal Amount. Report the estimated value, the standard error of the estimate and the 95% confidence interval for the value.
- (d) Assuming that the ADSs were issued at par (i.e., at a price of CAD 100 per CAD 100 of Principal Amount), estimate the mean and standard deviation of the risk-neutral distribution of the ADSs’ equivalent annually compounded rate of return.⁵

³A dividend yield of 5.80% continuously compounded is equivalent to a dividend yield of 5.97% annually compounded (the latter figure being the estimated dividend yield reported in the term sheet).

⁴The date reported for each dividend payment is the ex-dividend date. Assume that the dividends were actually paid on the ex-dividend dates.

⁵See the term sheet for examples of the calculation of the equivalent annually compounded rate of return.