Derivatives HW 1

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Questions

(a) What was the value on March 1, 2024 of the Vanilla Notes in the term sheet in Figure 1?

(b) What was the value on March 1, 2024 of a claim to the dividend payments on one share of XYZ's stock over the next 3 years?

(c) What was the fair 3-year forward price of XYZ's stock on March 1, 2024?

(d) What was the value on March 1, 2024 of a 3-year forward contract on XYZ's stock with delivery price equal to \$250 and unit size?

(e) What was the fair quarterly strike on March 1, 2024 of a 3-year dividend swap on XYZ's stock?

(f) What was the value on March 1, 2024 of a 3-year dividend swap on XYZ's stock with a quarterly strike of \$1.00 and unit size?

(g) What position in the Vanilla Notes in the term sheet in Figure 1, together with XYZ derivatives and borrowing or lending at the risk-free rate, could have been used on March 1, 2024 to replicate the cash flows of the Mandatory Convertible Notes in the term sheet in Figure 2?

(h) What was the value on March 1, 2024 of the Mandatory Convertible Notes?

(i) What position in the Vanilla Notes in the term sheet in Figure 1, together with XYZ equity derivatives and borrowing or lending at the risk-free rate, could have been used on March 1, 2024 to replicate the cash flows of the Contingent Coupon Notes in the term sheet in Figure 3?

(j) What was the value on March 1, 2024 of the Contingent Coupon Notes?

Background: Structured notes

Bonds with coupons that depend on price of an underlying asset.

Combine a vanilla note + derivatives.

- Credit risk is negligible
- risk free rates are in fig 4.
- XYZ stock on 03/01/2024 is \$250 / share
- As of 03/01/2024, XYZ pay dividend on 03/01, 06/01, 09/01, 12/01, equal to 0.4% of cum-dividend share price per share on each date.

Dividend payments coincide with ex-dividend dates.

The vanilla note is structured as the following:

1. General Terms

Description	3 year Vanilla Notes
Issuer	XYZ Corporation (XYZ)
Issue Date	March 1, 2024
Maturity Date	March 1, 2027
Nominal Amount	\$10,000 per Note

2. Principal Payment

Payment Date	March 1, 2027
Principal Amount	100% of Nominal Amount

3. Coupon Payments

Payment Dates	The 1st day in March, June, September and December, from and including June 1, 2024 to and including the Maturity Date
Interest Amount	Interest shall accrue on the Nominal Amount at the applicable Coupon Rate on the basis of a 360-day year of twelve 30-day months
Coupon Rate	4% per annum

Rates:

Tenor	Rate
3 month	5.36056%
6 month	5.17513%
9 month	5.01501%
12 month	4.87682%
15 month	4.75763%
18 month	4.65493%
21 month	4.56655%
24 month	4.49061%
27 month	4.42551%
30 month	4.36983%
33 month	4.32237%
36 month	4.28209%

a. Let us denote the value of the note on 03/01/2024 as F_0.

Since we want to find present value of notes, let us discount the payments of the bonds and also the notional amount.

Define: S_0: nominal amount of the note

r_t: continuously compounded risk free interest rate at time t.

T: time of maturity

PV: present value.

C: coupon

$$F_0 = PV(Div_{0,T}) + S_0 e^{-r(T-t)}$$

$$F_0 = \sum_{i=1}^{12} C e^{-r_t(T-t)} + [C+S_0] e^{-r_t(T-t)}$$

Each coupon: $C = 10000 \cdot 0.04 \cdot 0.25 = \$100$

Now

$$F_0 = \sum_{i=1}^{11} 100 e^{-r_t(t-0)} + 10100 e^{-0.0428209 \cdot (3)}$$

$$= 98.668795 + 97.4456 + \cdots + 88.79275 + 8882.408288$$

$$= \$9910.249459$$

$$= \boxed{\$9910.25}$$

b. To compute the claim of the dividend payments, we use present stock price and we know it pays at a constant rate of 0.4% of cum-dividend price.

$$PV(Div_{0:T}) = S_t \left(1 - \left(\prod_{i=1}^{12}(1-\delta_i)\right)\right) \text{ from notes.}$$

And so, $PV(Div_0) = 250 \cdot \left[1 - \left(\prod_{i=1}^{12}(1-0.004)\right)\right]$

$$= 250\left[1 - (1-0.004)^{12}\right]$$

$$= \$11.7394885$$

$$= \boxed{\$11.74}$$

c. Fair price of forward is the future value of dividend payment and the future value of the stock price.

$$F_t = \left[S_t \prod_{i=1}^{12}(1-\delta_i)\right] e^{r_t(T-t)}$$

$$F_0 = \left[250 \prod_{i=1}^{12}(1-0.004)\right] e^{0.0428209(3)}$$

$$= 270.92102559$$

$$= \boxed{\$270.92} \text{ per contract.}$$

d. We already computed the fair forward price in the last part to be \$270.92

Therefore, the value of forward contract

$$FWD_t = (F_t - K)e^{-r(T-t)}$$

$$FWD_0 = (270.92 - 250)e^{-0.0428209 \cdot 3}$$

$$FWD_0 = 18.39892021$$

$$= \boxed{\$18.40 \text{ per share}}$$

e. The fair quarterly strike is based on the Dividend leg and the number of payments. Since XYZ pays out every quarter, we only have 1 payment on each leg.

We already computed the fair forward price $F_0 = \$270.92$

And K, strike, $K = \dfrac{DivLeg_0}{N_s \times \sum_{i=1}^{n} e^{-r_{0,t_i} \cdot t_i}}$

$$DivLeg_0 = \left[S_0 + FV(Div_{t_{i-1},t_i}) - F_0 e^{-r_{0,T}(T-t)}\right]$$

$$= \left[S_0 - F_0 e^{-r_{0,T}(T-0)}\right]$$

$$= \left[250 - 270.92 e^{-0.0428209 \times 3}\right] = 11.7394885$$

$$K = \frac{DivLeg_0}{N_s \cdot \sum_{i=1}^{n} e^{-r_{0,t_i} \cdot t_i}} = \frac{11.7394885}{\left[e^{-0.0536 \cdot 0.25} + e^{-0.0517 \cdot 0.5} + \cdots e^{-0.0428 \times 3}\right]} = \frac{11.7394884}{11.15785848} = 1.05212738$$

$$K = \boxed{\$1.05}$$

F.

$$Swap_0 = DivLeg_0 - FixedLeg_0$$

$DivLeg_0 = 1 \times 11.739488$ from last part

$$FixedLeg_0 = 1 \times 1 \times \sum_{i=1}^{12} e^{-r_{0,t_i}(t_i)} = 1 \times \left[e^{-0.0536 \times 0.25} + e^{-0.0517 \cdot 0.5} + \cdots + e^{-0.0428 \cdot 3}\right] = 11.1578$$

$$Swap_0 = 11.739488 - 11.1578 = 0.58163002$$

$$= \boxed{\$0.58 \text{ per contract.}}$$

G. Terms of the MCN:

1. General Terms

Description	3 year Mandatory Convertible Notes
Issuer	XYZ Corporation (XYZ)
Issue Date	March 1, 2024
Maturity Date	March 1, 2027
Nominal Amount	$10,000 per Note

2. Mandatory Conversion

Conversion Date	March 1, 2027
Conversion	On the Conversion Date, the Notes will automatically convert into shares of XYZ's common stock at a conversion price of $250 per share (equivalent to 40 shares per Note). This share settle~~ment will~~ constitute full satisfaction of the Issuer's obligation to repay the principal amount of the Notes

3. Coupon Payments

Payment Dates	The 1st day in March, June, September and December, from and including June 1, 2024 to and including the Maturity Date
Interest Amount	Interest shall accrue on the Nominal Amount at the applicable Coupon Rate on the basis of a 360-day year of twelve 30-day months
Coupon Rate	4% per annum

1. on the day of conversion, delivery is 40 shares of XYZ at $250 / share.

Some notation:
N: notional of vanilla note.
C: coupon of vanilla note.
T: maturity
S_0: spot price at time 0 (March 1, 2024)

δ_i: dividend rate.
F: forwards price.

The MCN can be constructed in the following way: being long on the

Position	CF on March 1st, 2024	CF at Maturity.
Buy 1 vanilla notes	-9910.25	$+10,000$
Buy 40 XYZ Forwards with $K = 250$	$-40 \cdot 270.92$	$40 \cdot [S_T - 250]$

$= 40 \times S_T$ ✓

To summarize, we can construct the Mandatory convertible Note by:

1. Buy 40 forward contracts
2. Buy 1 vanilla notes.

Value of forwards contract: 18.40 per contract.

And $PV(\text{Vanilla Note}) = -9910.25$

H. The value on March 1, 2024 is simply

$$MCN_0 = \sum_{i=1}^{11} 100\, e^{-r_t(t-0)} + 10100\, e^{-0.0428209 \cdot (3)} - 40 \cdot 18.39892$$

$$= -9910.249499 - 735.9568 = -10646.20631$$

$$\boxed{MCN_0 = 10646.21}$$

J. The value of the contingent coupon note on March 1st, 2024 is simply the discounted notional of the note, 10,000

Value of dividend swap w/ strike $1: 0.58 per contract

$$CCN_0 = -9910.249499 - 0.581629 \times 100$$

$$= -9968.412399$$

$$\boxed{CCN_0 = \$9968.41}$$

I. The format of the Contingent Coupon Notes.

1. General Terms

Description	3 year Contingent Coupon Notes
Issuer	XYZ Corporation
Issue Date	March 1, 2024
Maturity Date	March 1, 2027
Nominal Amount	$10,000 per Note

2. Principal Payment

Payment Date	March 1, 2027
Principal Amount	100% of Nominal Amount

3. Coupon Payments

Payment Dates	The 1st day in March, June, September and December, from and including June 1, 2024 to and including the Maturity Date
Coupon Amount	Each coupon payment shall equal 100 times the relevant Dividend Amount
Dividend Amount	The dividend payment per share of XYZ's common stock in the quarter ending on each coupon Payment Date

So the Contingent Coupon note

- pays out dividends per share of XYZ
- 100x the dividend rate per annum on coupon payment days.

The CCN can be constructed in the following way: Defined terms above.

Position	CF on March 1st, 2024	CF on March 1st, 2027.
Buy 1 vanilla Note	-9910.25	10,000
Buy dividend swap on 100 shares of XYZ. with strike $1	-100×0.58	$+100(1-1)$

$= 10,000$ ✓

To replicate, we should

1. Buy 1 vanilla swap
2. Buy dividend swap on 100 shares of XYZ @ strike $1

Part 2:

Questions

(a) How does the formula in equation (3) in Lecture Note 5 for the net cash flow on the generic payment date t_i to the equity receiver in an equity swap change if the swap is a price return (PR) swap instead of a total return (TR) swap?

(b) What trading strategy could be implemented at a time t with $t_{i-1} \le t < t_i$ in order to replicate the receipt at time t_i of the cash flow in part (a)?

(c) What is the value of a $(t_{i-1} \times t_i)$ PR swaplet at a time t with $t_{i-1} \le t \le t_i$?

(d) What is the value of the PR swaplet at a time t with $0 \le t \le t_i$?

(e) How should the spread over the floating rate be set in order for the value of a PR swap on the trade date to be zero?

(f) On February 20, 2025, the settlement price of the March 2026 E-mini S&P 500 futures contract (ESH6) was 6,366.25. What was the implied dividend yield of the S&P 500 index?

(g) ABC Corp. (ABC) is the equity payer in a PR swap on the S&P 500 index with \$10,000,000 notional and quarterly payments on both the equity leg and the floating leg. The swap was traded on February 20, 2025, will terminate on February 20, 2026 and required no upfront payment. What was the swap's fair spread over the floating rate?

(h) Assuming that the spread of ABC's swap was set at the level you determined in part (g) and that as of June 20, 2025 the implied dividend yield of the S&P 500 index was still at the level you determined in part (f), what was the value of ABC's swap position on June 20, 2025?

(i) How does your answer to part (h) change if as of June 20, 2025 the implied dividend yield of the S&P 500 index was 1.4%?

A. TR swap:

Eq 3 in Notes 5 lists out the following:

$$\frac{N}{S_{t_{i-1}}}\left[S_{t_i} + FV_{t_i}\left(Div_{t_{i-1},t_i}\right) - K_i\right]$$

if switched to PR, then only price changes, not dividend changes are going to be counted

The equity leg

$$\text{Equity Leg} = N \times \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}}$$

$$\text{Finance leg} = N \times \left[\left(e^{(r_{t_{i-1}}+s)\times(t_i - t_{i-1})} - 1\right)\right]$$

$$CF_{t_i}^{PR} = N \times \left(\frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}}\right) - N\left[e^{(r_{t_{i-1}}+s)\times(t_i - t_{i-1})} - 1\right]$$

$$CF_{t_i}^{PR} = \boxed{\frac{N}{S_{t_{i-1}}}\left[S_{t_i} - K_i\right], \text{ where } K_i = S_{t_i} e^{(r+s)(t_i - t_{i-1})}}$$

And: N: notional, $S_{t_{i-1}}$: spot price at reset period.

K_i: the strike of the swap at price return.

B. $t_{i-1} \le t \le t_i$ means we have received the dividends

For us to have a no arbitrage scenario, we can replicate cashflows by:

We computed $\rightarrow$ $CF_{t_i}^{PR} = \frac{N}{S_{t_{i-1}}}\left[S_{t_i} - K_i^{PR}\right]$, $K_i^{PR} = S_{t_i} e^{(r_{t_{i-1},t_i} - \delta + s)(t_i - t_{i-1})}$

To replicate, we can buy a $\frac{N}{S_{t_{i-1}}}$ forwards contracts or:

Position	CF at t	CF at t_i
Buy $\frac{N}{S_{t_{i-1}}}$ of index	$\frac{-N}{S_{t_{i-1}}} \cdot S_t e^{-\delta(t_i - t)}$	$+\frac{N}{S_{t_{i-1}}} S_{t_i}$
Lend $PV\left(\frac{N}{S_{t_{i-1}}} \cdot K_i^{PR}\right)$	$+\frac{N}{S_{t_{i-1}}} \cdot K_i \cdot e^{-r_{t,t_i}(t_i - t)}$	$-\left[\frac{N}{S_{t_{i-1}}} \cdot K_i\right]$
Total	$\frac{N}{S_{t_{i-1}}}\left[K_i e^{-r_{t,t_i}(t_i - t)} - S_t e^{-\delta(t_i - t)}\right]$	$\frac{N}{S_{t_{i-1}}}\left[S_{t_i} - K_i\right]$ ✓

1. Buy $\frac{N}{S_{t_{i-1}}}$ of index, reinvesting dividends.
2. Lend $PV\left(\frac{N}{S_{t_{i-1}}} \times K_i\right)$ at risk free

or buy $\frac{N}{S_{t_{i-1}}}$ of forwards contracts at strike K_i^{PR}

C. For $t_{i-1} \le t \le t_i$, we established that buying $\frac{N}{S_{t_{i-1}}}$ forwards contract satisfies the trade. Now, we show cash flows for that trade in previous question.

$$V_{\text{swaplet}} = V_{\text{Equity leg}} - V_{\text{Finance Leg}}$$

$$V_{\text{Equity Leg}} = PV\left(\frac{N}{S_{t_{i-1}}} \cdot S_{t_i}\right) = \frac{N}{S_{t_{i-1}}} \cdot \left[S_t e^{(r_{t,t_i} - \delta)(t_i - t)}\right] e^{-r_{t,t_i}(t_i - t)} = \frac{N}{S_{t_{i-1}}} \cdot \left[S_t e^{-\delta(t_i - t)}\right]$$

$$V_{\text{Finance leg}} = PV\left(\frac{N}{S_{t_{i-1}}} K_i^{PR}\right) = \frac{N}{S_{t_{i-1}}} \cdot K_i^{PR} \cdot e^{-r_{t,t_i}(t_i - t)}$$

$$\text{Swaplet}_{i,t}^{PR} = \frac{N}{S_{t_{i-1}}} \cdot \left[S_t e^{-\delta(t_i - t)}\right] - \frac{N}{S_{t_{i-1}}} \cdot K_i^{PR} \cdot e^{-r_{t,t_i}(t_i - t)}$$

$$= \boxed{\frac{N}{S_{t_{i-1}}}\left[S_t e^{-\delta(t_i - t)} - K_i e^{-r_{t,t_i}(t_i - t)}\right]}$$

D.

To find for $t \le t_{i-1}$, lets compute $t = t_{i-1}$

$$\text{Swaplet}_{i,t}^{PR} = \frac{N}{S_{t_{i-1}}}\left[S_t e^{-\delta(t_i - t)} - K_i e^{-r_{t,t_i}(t_i - t)}\right]$$

$$\text{Swaplet}_{i,t_{i-1}}^{PR} = \frac{N}{S_{t_{i-1}}}\left[S_{t_{i-1}} e^{-\delta(t_i - t)} - K_i e^{-r_{t_{i-1},t_i}(t_i - t_{i-1})}\right]$$

$$= N\left[e^{-\delta(t_i - t_{i-1})} - e^{(r_{t_{i-1},t_i}+s)\times(t_i - t_{i-1})} e^{-r_{t_{i-1},t_i}(t_i - t_{i-1})}\right]$$

$$= N\left[e^{-\delta(t_i - t_{i-1})} - e^{(r_{t_{i-1},t_i})(t_i - t_{i-1}) + s(t_i - t_{i-1}) - r_{t_{i-1},t_i}(t_i - t_{i-1})}\right]$$

$$= N \cdot \left[e^{-\delta(t_i - t_{i-1})} - e^{s(t_i - t_{i-1})}\right]$$

For $t \le t_{i-1}$, we do not know the price of index at t_{i-1}, so we discount until t_{i-1}

$$\therefore \text{Swaplet}_{t_i}^{PR} = \begin{cases} N \cdot \left[e^{-\delta(t_i - t_{i-1})} - e^{s(t_i - t_{i-1})}\right] \cdot e^{-r_{t,t_{i-1}}(t_{i-1} - t)}, & \text{for } 0 \le t \le t_{i-1} \\ \frac{N}{S_{t_{i-1}}}\left[S_t e^{-\delta(t_i - t)} - K_i e^{-r_{t,t_i} \cdot (t_i - t)}\right], & \text{for } t_{i-1} \le t \le t_i \end{cases}$$

e. At $t = t_{i-1}$ we need $V_{\text{swaplet}} = 0$

$$0 = N \cdot \left[e^{-\delta(t_i - t_{i-1})} - e^{s(t_i - t_{i-1})}\right]$$

$$\therefore e^{-\delta(t_i - t_{i-1})} = e^{s(t_i - t_{i-1})}$$

$$\boxed{s = -\delta}$$

F. Based on Yahoo finance, the closing price of S&P Index on February 20th, 2025 was $6117.52

we are assuming that the stock pays dividend yield δ

We are given: $r = 5\%$

and from the notes, we know $F_t = S_t e^{(r-\delta)(T-t)}$

Lets treat 02/20/2025 as time 0,

$F_0 = 6366.25$, $S_0 = 6117.52$, $T = \frac{13}{12}$

$F_0 = S_0 \, e^{(r-\delta)\cdot T}$

$e^{r-\delta} = \frac{F_0}{S_0}$

$\delta = r - \ln\left(\frac{F_0}{S_0}\right)/T$

$\delta = 0.05 - \ln\left(\frac{6366.25}{6117.52}\right)/1.083 = 0.013211$

The implied dividend yield is $\boxed{1.32\%}$

H.

The value of the swap on June 20, 2025 is simply the value of the first swaplet. $\delta = 0.0132$

$t_0 = $ Feb 20, 2025 $S_0 = \$6117.52$ $r = 0.05$

$t_1 = $ Mar 20, 2025 $S_1 = \$5940.46$ $t_i - t = \frac{2}{12} = 0.167$

$t_2 = $ Aug 20, 2025 $S_2 = \$6395.78$ $t_i - t_{i-1} = \frac{3}{12} = 0.25$

$t = $ Jun 20, 2025 $S_t = \$5967.84$

$K_i = S_{t_{i-1}} \cdot e^{(r_{t_{i-1}, t_i})(t_i - t_{i-1})} = 5940.46 \times e^{(0.05-0.132)(0.25)} = 5995.3466$

we computed:

$\text{Swaplet}^{PR}_{t,i} = \frac{N}{S_{t_{i-1}}}\left[S_t\, e^{-\delta(t_i - t)} - K_i\, e^{-r_{t,t_i}(t_i - t)}\right]$

$= \frac{10{,}000{,}000}{5940.46}\left[5967.84 \cdot e^{-0.0132\cdot(0.167)} - 5995.35\, e^{-0.05(0.167)}\right]$

$= 15353.04337$

Since ABC is the equity payer is $\boxed{\$-15353.04}$ value.

G. The maturity of the swap is $T = 1$ year.

$N = 10{,}000{,}000$

$r = 0.05$, $\delta = 0.0132$

we already computed that the PR leg payoff reflects the forward payoff in the equity

and $s = -\delta$ at inception for the swap to have 0 value initially

$\boxed{s = -0.0132}$

I.

Intuitively, the other swaplets would have value, since the swap was traded at a $s = -0.013$

Now, if $\delta = 0.014$, $\text{Swap}_{ABC} = \sum \text{Swaplet}_{ABC} = \text{Swaplet}_{1,t} + \text{Swaplet}_{(t_2 \times t_3)} + \text{Swaplet}_{(t_3 \times t_4)}$

$\text{Swaplet}_{t,i} = \frac{10000000}{5940.46}\left[5967.84\, e^{-0.014(0.167)} - 5940.46 \times e^{(0.05-0.013)(0.25)} \times e^{-0.05\times(0.167)}\right] = 14036.41806$

Since we set spread to be $s = -0.0132$, the future swaplets will not have zero value

For the other swaplets in the future, we use $\text{Swaplet}_{(t_{i-1}, t_i)} = N \times \left[e^{-\delta(t_i - t_{i-1})} - e^{s(t_i - t_{i-1})}\right] e^{-r_{t, t_{i-1}}(t_{i-1} - t)}$

So $\text{Swaplet}_{t_2, t_3} = 10000000\left[e^{-0.014\cdot 0.25} - e^{-0.0132(0.25)}\right] \times e^{-0.05\times(2/12)} = -1947.352571$

and $\text{Swaplet}_{t_3, t_4} = 10000000\left[e^{-0.014\cdot 0.25} - e^{-0.0132(0.25)}\right] e^{-0.05(5/12)} = -1923.162165$

$V_{\text{Swap}} = 14036.42 - 1947.35 - 1923.16 = 10165.90332$

For ABC, as the equity payer, the value is $\boxed{\$-10165.90}$