0.1 Question 1: Using Linear Algebra for Optimization

In recommender system module, low-rank matrix factorization was used to execute latent factor modeling of movie ratings data.

Specifically, we calculated matrices U and V to solve the following optimization problem (if all ratings were given):

$$\min_{U,V} f(U,V) = \min_{U,V} \|R - VU^T\|_F^2 = \min_{U,V} \left\{ \sum_{m=1}^{M} \sum_{i=1}^{I} I_{mi}(r_{mi} - v_m u_i^T)^2 \right\},$$

where

$$I_{mi} = \begin{cases} 1, \text{ if } r_{mi} \text{ is observed} \\ 0, \text{ if } r_{mi} \text{ is missing.} \end{cases}$$

The best U and V were calculated iteratively by improving on current estimates:

$$u_i^{\text{new}} = u_i + 2\alpha(r_{mi} - v_m u_i^T) \cdot v_m$$
$$v_m^{\text{new}} = v_m + 2\alpha(r_{mi} - v_m u_i^T) \cdot u_i,$$

where α is the step-size that is to be chosen by the user. (We won't discuss the role in this class, but treat it as an arbitrary, but given, parameter)

We can make calculating the updates more efficient by calculating them with matrix operations. For example, instead of calculating each deviation $\gamma_{mi} = r_{mi} - v_m u_i^T$ separately for all $m = 1, 2, \ldots, M$ and $i = 1, 2, \ldots, I$, matrix Γ of all deviations can be computed together using matrix operation *(verify for yourself)*:

$$\Gamma = R - VU^T$$

Similarly, updating U and V can be combined into matrix calculations which makes the optimization procedure more efficient.

First, note that updates for u_i, $i = 1, 2, \ldots, I$ can be rewritten as

$$\begin{aligned} u_1^{\text{new}} &= u_1 + 2\alpha\gamma_{m1} \cdot v_m \\ u_2^{\text{new}} &= u_2 + 2\alpha\gamma_{m2} \cdot v_m \\ \vdots \qquad & \qquad \vdots \\ u_I^{\text{new}} &= u_I + 2\alpha\gamma_{mI} \cdot v_m. \end{aligned}$$

Stacking all I equations into a matrix form,

$$U^{\text{new}} = U + 2\alpha\Gamma_{m-}^T v_m,$$

where Γ_{m-} is the m-th row of Γ (use the notation Γ_{-i} for the i-th column). When evaluating U^{new}, the latest updated values of U, V, and Γ are used.

Note that there are M such update equations (one for each $m = 1, 2, \ldots, M$) that can also be combined into one matrix update equation involving matrices U, V, Γ and scalars. As stated earlier, since α is assumed to be an arbitrary step-size parameter, we can replace α/M with α.

0.1.1 Question 1a: Using Linear Algebra for Optimization

Complete the following update equations:

$$U^{\text{new}} = U + 2\alpha[\text{some function of } \Gamma][\text{some function of } V]$$
$$V^{\text{new}} = V + 2\alpha[\text{some function of } \Gamma][\text{some function of } U]$$

SOLUTION

To complete the update equations, let express the terms involving Γ and V in terms of matrix operations. Recall that $\Gamma = R - VU^T$, and we'll use this information to find the expressions:

$$U^{\text{new}} = U + 2\alpha[\text{some function of } \Gamma][\text{some function of } V]$$
$$V^{\text{new}} = V + 2\alpha[\text{some function of } \Gamma][\text{some function of } U]$$

For U^{new}:

$$\begin{aligned} U^{\text{new}} &= U + 2\alpha[\text{some function of } \Gamma][\text{some function of } V] \\ &= U + 2\alpha(\Gamma V) \end{aligned}$$

Explanation: We use ΓV to get a matrix whose i-th column is $\Gamma_{m-}^T v_m$. The multiplication by 2α is then applied element-wise.

For V^{new}:

$$\begin{aligned} V^{\text{new}} &= V + 2\alpha[\text{some function of } \Gamma][\text{some function of } U] \\ &= V + 2\alpha(\Gamma^T U) \end{aligned}$$

Explanation: We use $\Gamma^T U$ to get a matrix whose m-th column is $\Gamma_{m-}^T u_m$. The multiplication by 2α is then applied element-wise.

So, the complete update equations are:

$$U^{\text{new}} = U + 2\alpha(\Gamma V)$$
$$V^{\text{new}} = V + 2\alpha(\Gamma^T U)$$

0.1.2 Question 1d: Interpret Diagnostic Plots

Following figures tell us if the optimization algorithm is working properly.

In [17]:
```
import altair as alt
logscale = alt.Scale(type='log', base=10)
fig_rmse = \
    alt.Chart(output1['rmse'])\
    .mark_line()\
    .encode(
        x='iteration:Q',
        y=alt.Y('rmse:Q', scale=logscale)
    )
fig_max_residual_change = \
    alt.Chart(output1['rmse'])\
    .mark_line()\
    .encode(
        x='iteration:Q',
        y=alt.Y('max residual change:Q', scale=logscale)
    )
fig_updates = \
    alt.Chart(output1['update'])\
    .mark_line()\
    .encode(
        x='iteration:Q',
        y=alt.Y('max update:Q', scale=logscale)
    )
alt.vconcat(
    fig_rmse | fig_max_residual_change,
    fig_updates
)
```

Out[17]:
```
alt.VConcatChart(…)
```

By referring back to the function used to calculate the quantities in each figure, describe what each figure is showing and interpret the behavior of the optimization algorithm.

SOLUTION

0.1.3 Question 1e: Analyze Large Dataset

Following code will analyze a larger dataset:

In [18]:
```
# run on larger dataset: ratings for 100 movies
Rbig = pd.read_pickle('data/ratings_stacked.pkl').unstack().iloc[:100]

np.random.seed(14) # set seed for tests
output3 = compute_UV(Rbig, K=5, alpha=0.001, max_iteration=500)

Rhatbig = output3['V']@output3['U'].T
```

In [19]:
```
fit_vs_obs = pd.concat([
    Rhatbig.rename(columns={'rating':'fit'}),
    Rbig.rename(columns={'rating':'observed'}),
], axis=1).stack().dropna().reset_index()[['fit','observed']]

fit_vs_obs = fit_vs_obs.iloc[np.random.choice(len(fit_vs_obs), 5000)]

alt.Chart(fit_vs_obs).transform_density(
    density='fit',
    bandwidth=0.01,
    groupby=['observed'],
    extent= [0, 6]
).mark_bar().encode(
    alt.X('value:Q'),
    alt.Y('density:Q'),
    alt.Row('observed:N')
).properties(width=800, height=50)
```

Out[19]:
```
alt.Chart(…)
```

In [20]:
```
fit_vs_obs = pd.concat([
    Rhatbig.rename(columns={'rating':'fit'}),
    Rbig.rename(columns={'rating':'observed'}),
], axis=1).stack().dropna().reset_index()[['fit','observed']]

fit_vs_obs = fit_vs_obs.iloc[np.random.choice(len(fit_vs_obs), 5000)]

alt.Chart(fit_vs_obs).transform_density(
    density='fit',
    bandwidth=0.01,
    groupby=['observed'],
    extent= [0, 6],
    counts=True
).mark_bar().encode(
    alt.X('value:Q'),
```

```
        alt.Y('density:Q'),
        alt.Row('observed:N')
    ).properties(width=800, height=50)
```

Out[20]: alt.Chart(…)

Consider the above plot. By reading the code, comment on what the plot is illustrating. What happens when you add `counts=True` to `transform_density`? What can you conclude?

SOLUTION

With counts=True, the second plot provides additional information on the data distribution by showing the count of data points contributing to each density estimate. This can be useful to assess the reliability and significance of the density estimates. Higher counts indicate more data points in that region, providing a sense of the data's density and distribution.

0.1.4 Question 1f: Make Recommendation

What movies would you recommend to `user id` 601? Do you see any similarities to movies the user rated high?

SOLUTION

```
In [21]: # Extract latent factors for user 601
         #user_601_latent_factors = output3['U'].xs(key=601, level='user id', axis=0, drop_level=False)

         # Get the indices of the top-k latent factors (let's say top 5 for this example)
         #top_k_indices = user_601_latent_factors.iloc[0].argsort()[-5:][::-1]

         # Retrieve movie titles corresponding to the top-k latent factors
         #recommended_movies = output3['V'].iloc[top_k_indices].index.get_level_values('movie title')

         # Display the recommended movies
         #recommended_movies

         where_not_rated = Rbig.droplevel(level=0,axis=1).loc[:,601]#.index.levels[1]
         not_rated = where_not_rated[where_not_rated.isna()].index.get_level_values(1)
         Rbig.iloc[:,601]
         idx = pd.IndexSlice
         Rhatbig.droplevel(level=0, axis=1).loc[idx[:,not_rated],601].nlargest(10)
```

```
Out[21]: movie id  movie title
         13        Mighty Aphrodite (1995)          4.000717
         48        Hoop Dreams (1994)               3.844658
         19        Antonia's Line (1995)            3.792223
         14        Postino, Il (1994)               3.745982
         26        Brothers McMullen, The (1995)    3.698932
         59        Three Colors: Red (1994)         3.690157
         23        Taxi Driver (1976)               3.661859
         45        Eat Drink Man Woman (1994)       3.281692
         61        Three Colors: White (1994)       3.266549
         20        Angels and Insects (1995)        3.225736
         Name: 601, dtype: float64
```

Top 5 Movies:

(1) Mighty Aphrodite
(2) Hoop Dreams
(3) Antonia's Line
(4) Postino, Il
(5) Brothers McMullen

Similarities: movies were made between 1994 and 1995. all explore interpersonal dynamics, according to ChatGPT

0.1.5 Question 2a: Derive New Gradients and Update Rules

Based on the new objective function $g(U, V)$, derive its gradients and update rules for U^{new} and V^{new}.

SOLUTION

In a vector form, gradients are

$$\frac{\partial}{\partial u_i} g(u_i, v_m) = ...$$
$$\frac{\partial}{\partial v_m} g(u_i, v_m) = ...$$

By stacking equations, gradients can be expressed in a matrix form as

$$\frac{\partial}{\partial U} g(U, V) = ...$$
$$\frac{\partial}{\partial V} g(U, V) = ...$$

Finally, the update rules are (in a matrix form)

$$U^{\text{new}} = ...$$
$$V^{\text{new}} = ...$$

SOLUTION

In a vector form, gradients are

$$\frac{\partial}{\partial u_i} g(u_i, v_m) = -2((r_{mi} - v_m u_i^T))v_m - \lambda u_i)$$
$$\frac{\partial}{\partial v_m} g(u_i, v_m) = -2((r_{mi} - v_m u_i^T))u_i - \lambda v_m)$$

By stacking equations, gradients can be expressed in a matrix form as

$$\frac{\partial}{\partial U} g(U, V) = -2 \times (\Gamma^T V - \lambda U)$$
$$\frac{\partial}{\partial V} g(U, V) = -2 \times (\Gamma U - \lambda V)$$

Finally, the update rules are (in a matrix form)

$$U^{\text{new}} = U + 2 \times \alpha(\Gamma^T V - \lambda U)$$
$$V^{\text{new}} = V + 2 \times (\Gamma U - \lambda V)$$

1

1.0.1 Question 2d: Investigating the Effects of Regularization

Adding the regularization terms to the objective function will affect the estimates of U and V. Here, we consider comparing the user matrix U.

Using the dataset *Rsmall*, obtain two estimated user matrices, say $\hat{U}$ for a non-regularized model and $\hat{U}_{\text{reg}}$ for a regularized model. Select $K = 20$ and $\lambda = 5$. Come up with an effective visualization for comparing $\hat{U}$ and $\hat{U}_{\text{reg}}$, and describe any differences you notice. Additionally, analyze whether the observed differences in patterns align with the concept of regularization.

Provide reasoning supported by evidence, such as code implementation and results.

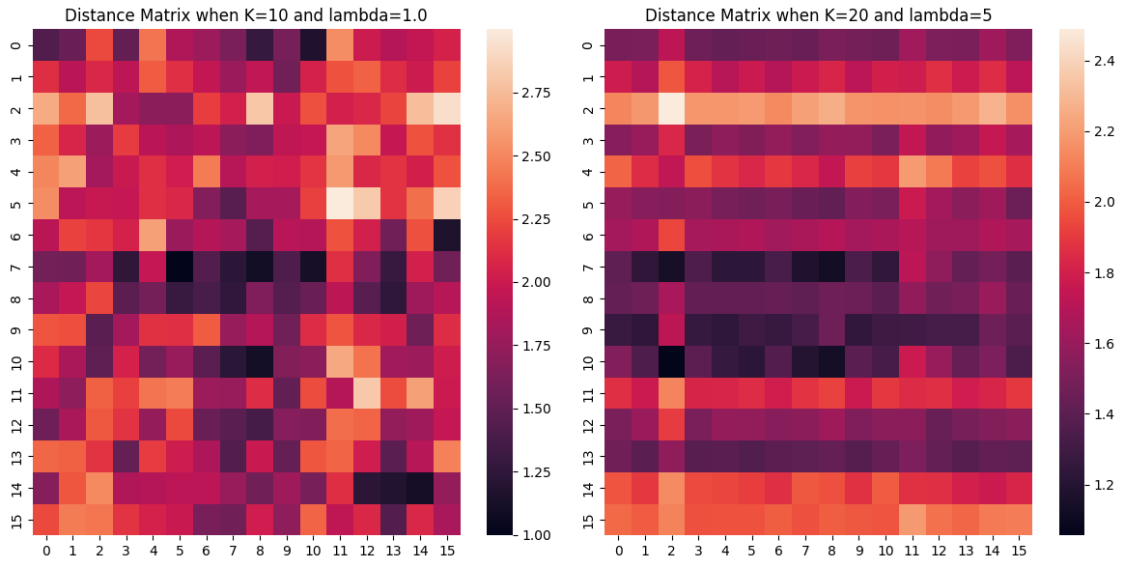
SOLUTION

In [27]:
```
# np.random.seed(134) # set seed for tests
output_noreg = compute_UV(Rsmall, K=10, alpha=0.001)
output_reg = compute_UV_reg(Rsmall, K=10, lam=1.0, alpha=0.001)
```

In [ ]:

In [28]:
```
# SELECT K = 20 AND LAMBDA = 5
output2_noreg = compute_UV(Rsmall, K=20, alpha=0.001)
output2_reg = compute_UV_reg(Rsmall, K=20, lam=5, alpha=0.001)
# OUTPUT [1]
Uhat = output_noreg['U']
Ureg = output_reg['U']
# OUTPUT 2
U2hat = output2_noreg['U']
U2reg = output2_reg['U']

from scipy.spatial import distance_matrix
import seaborn as sns
import matplotlib.pyplot as plt
# Side by side plots
fig, axes = plt.subplots(1, 2, figsize=(12, 6))
# Heatmap
sns.heatmap(distance_matrix(Uhat, Ureg), ax=axes[0])
axes[0].set_title('Distance Matrix when K=10 and lambda=1.0')
sns.heatmap(distance_matrix(U2hat, U2reg), ax=axes[1])
axes[1].set_title('Distance Matrix when K=20 and lambda=5')
plt.tight_layout()
```



The interpretation of the results is as follows:

1. **Heatmap Explanation:**
   - The heatmap displays pairwise distances between two vectors in a dataset, with darker colors indicating smaller distances (closeness) and lighter colors indicating larger distances.
   - The vectors being compared are denoted as $\hat{U}$ and regularized $\hat{U}$.
   - The comparison is made for two different parameter settings: $K = 10$ and $\lambda = 1.0$, and $K = 20$ and $\lambda = 5$.
2. **Comparison of Pairwise Distances:**
   - The distances between $\hat{U}$ and regularized $\hat{U}$ are compared for the two parameter settings.
   - The result indicates that when $K = 20$ and $\lambda = 5$, the pairwise distances are smaller compared to when $K = 10$ and $\lambda = 1.0$.
3. **Interpretation of Heatmaps:**
   - The rightmost heatmap (associated with $K = 20$ and $\lambda = 5$) has a larger concentration of dark colors, particularly grouped by the y-axis.
   - This suggests that the distance between $\hat{U}$ and regularized $\hat{U}$ for $K = 20$ and $\lambda = 5$ is smaller, especially in the y-axis range of 4.5 to 10.5.
4. **Overall Observation:**
   - In general, for both heatmaps, pairwise distances between $\hat{U}$ and regularized $\hat{U}$ are smaller when the y-axis (or $\hat{U}$) value is between 5.5 and 10.5.
5. **Positive Differences and Model Performance:**
   - All the differences between $\hat{U}$ and regularized $\hat{U}$ are positive for both parameter conditions (i.e., $K = 10$ and $\lambda = 1.0$, and $K = 20$ and $\lambda = 5$).
   - The positive differences suggest that the regularized model performs better than the non-regularized model.

- This aligns with the understanding that regularized models are introduced to prevent overfitting, and a higher performance is associated with higher accuracy and lower error.

In summary, the results suggest that the regularized model with $K = 20$ and $\lambda = 5$ performs better than the non-regularized model with $K = 10$ and $\lambda = 1.0$, as evidenced by smaller pairwise distances in the specified y-axis range.

1.0.2 Question 2e: Practical Aspects

In the previous question, a specific values for K and λ were provided. Now, try applying various K's and λ's. Specifically, try the following:

- While keeping K constant, experiment with different values of λ. What do you notice? Why do you think this happens?
- While keeping λ constant, experiment with different values of K. What do you notice? Why do you think this happens?

If your optimization algorithm is correctly implemented, you will notice that the choice of K and λ has a significant impact on the final estimates. Hence, selecting appropriate values for K and λ is crucial when applying the recommendation algorithm in practice. As a practitioner, how would you approach choosing K and λ?

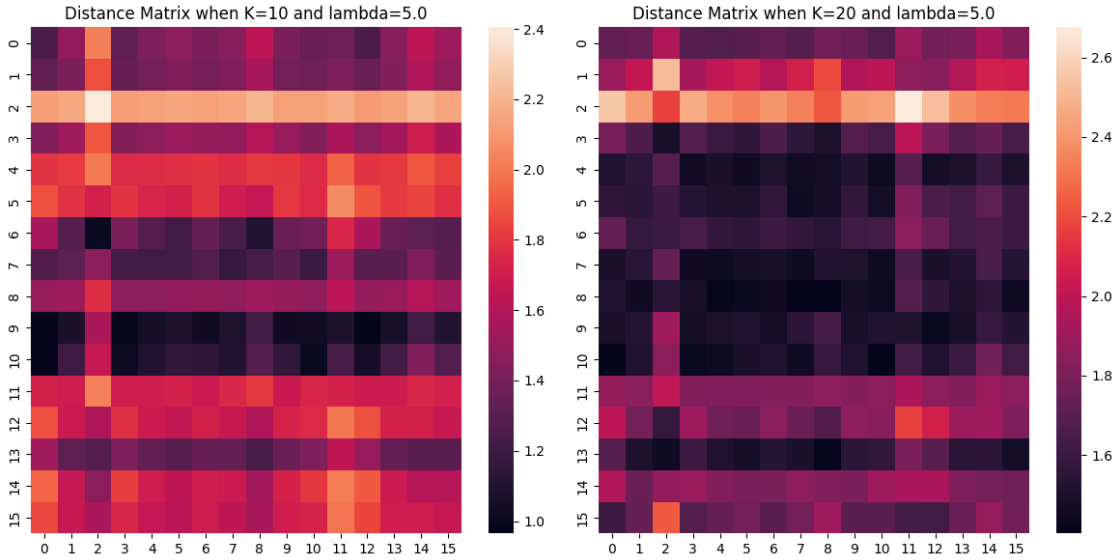
Provide reasoning supported by evidence, such as code implementation and results.

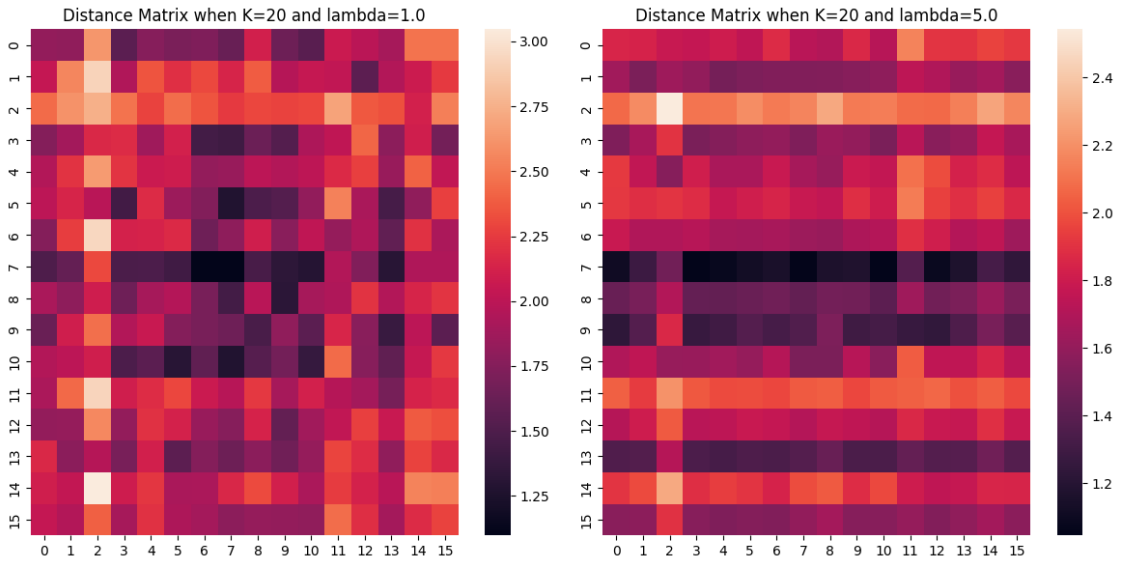
SOLUTION

```
In [29]: np.random.seed(134) # set seed for tests
         # Keeping lambda constant, experiment with different values of K
         # K=10, lambda=1.0
         output5_noreg = compute_UV(Rsmall, K=10, alpha=0.001)
         output5_reg = compute_UV_reg(Rsmall, K=10, lam=5.0, alpha=0.001)
         # K=20, lambda=1.0
         output6_noreg = compute_UV(Rsmall, K=20, alpha=0.001)
         output6_reg = compute_UV_reg(Rsmall, K=20, lam=5.0, alpha=0.001)
         # U5hat and U5reg
         U5hat = output5_noreg['U']
         U5reg = output5_reg['U']
         # U6hat and U6reg
         U6hat = output6_noreg['U']
         U6reg = output6_reg['U']
         # Side by side plots
         fig, axes = plt.subplots(1, 2, figsize=(12, 6))
         # Heatmap
         sns.heatmap(distance_matrix(U5hat, U5reg), ax=axes[0])
         axes[0].set_title('Distance Matrix when K=10 and lambda=5.0')
         sns.heatmap(distance_matrix(U6hat, U6reg), ax=axes[1])
         axes[1].set_title('Distance Matrix when K=20 and lambda=5.0')
         plt.tight_layout()

         np.random.seed(134) # set seed for tests
         # Keeping K constant, experiment with different values of lambda
         # K=20, lambda=1.0
         output3_noreg = compute_UV(Rsmall, K=20, alpha=0.001)
         output3_reg = compute_UV_reg(Rsmall, K=20, lam=1, alpha=0.001)
```

```
# K=20, lambda=5.0
output4_noreg = compute_UV(Rsmall, K=20, alpha=0.001)
output4_reg = compute_UV_reg(Rsmall, K=20, lam=5, alpha=0.001)
# U3hat and U3reg
U3hat = output3_noreg['U']
U3reg = output3_reg['U']
# U4hat and U4reg
U4hat = output4_noreg['U']
U4reg = output4_reg['U']
# Side by side plots
fig, axes = plt.subplots(1, 2, figsize=(12, 6))
# Heatmap
sns.heatmap(distance_matrix(U3hat, U3reg), ax=axes[0])
axes[0].set_title('Distance Matrix when K=20 and lambda=1.0')
sns.heatmap(distance_matrix(U4hat, U4reg), ax=axes[1])
axes[1].set_title('Distance Matrix when K=20 and lambda=5.0')
plt.tight_layout()
```





The interpretation of the results is as follows:

1. **Effect of K (Number of Neighbors):**
   - When keeping λ constant at 5.0 and varying the number of neighbors (K), specifically with $K = 10$ and $K = 20$, there is no significant difference between the two conditions.
   - At $K = 10$, there are closer values (smaller distances) between the pairwise vectors $\hat{U}$ and $\hat{U}$ in a narrow space of the heatmap.
   - At $K = 20$, a wider space of close values is observed, though not as significant as the distances marked by the first heatmap.
2. **Effect of λ (Regularization Parameter):**
   - When keeping K constant at 5.0 and experimenting with different values of λ, particularly with $\lambda = 1.0$ and $\lambda = 5.0$, a notable difference is observed.
   - In this condition (constant K), the distances between pairwise vectors are smaller (closer) when $\lambda = 5.0$.
   - At $\lambda = 1.0$, distances are generally more random and scattered, with a fuller and wider space of low matrix difference.
   - At $\lambda = 5.0$, distances are more similarly clustered along the y-axis, with a more concentrated space for low matrix difference.

Key findings: - Varying K (number of neighbors) at a constant λ does not show a significant difference, but there are differences in the distribution of values in the heatmap. - Varying λ (regularization parameter) at a constant K results in noticeable differences, with $\lambda = 5.0$ leading to smaller distances and a more concentrated space of values along the y-axis compared to $\lambda = 1.0$.

1.0.3 Question 3a: Concatenate matrix factors and cluster

Entries in either matrix factors are just points in k-dimensional latent variable space. We will use both U and V for segmentation by combining them into one large clustering problem.

Once clusters are identified, you will qualitatively inspect the users and movies in the cluster and decide on a "representative" movie from each cluster.

Consider concatenating U and V into one large matrix. Since these matrices have arbitrary scaling, it would be a good idea to standardize the columns before concatenating them. Standardize U and V separately, then concatenate with numpy's `concatenate` method. Call this concatenated matrix, `UVstd`.

Apply hierarchical and K-means clustering methods on `UVstd`. For each clustering method, identify 5 clusters. Compare the clustering results by applying three different cluster validation metrics to evaluate the clustering performance.

Which cluster performance metrics can you use? Do we have true labels? Does one performance metric seem to clearly be better than another? Why would you choose one metric over another? What interpretation, if any, does each metric have in the context of our problem? Explain.

SOLUTION

```
In [30]: from sklearn.cluster import AgglomerativeClustering

         from sklearn.cluster import KMeans

         from sklearn.preprocessing import StandardScaler

         output2_reg = compute_UV_reg(Rsmall, K=20, lam=5, alpha=0.001)
         U2reg = output2_reg['U']
         V2reg = output2_reg['V']

         Ustd = StandardScaler().fit_transform(U2reg)
         Vstd = StandardScaler().fit_transform(V2reg)
         # Concatenate the standardized U and V; call it UVstd

         UVstd = np.concatenate((Ustd, Vstd))

         kmeans = KMeans(n_clusters=5, random_state=1).fit(UVstd)
         # K-means clustering label
         kmeans_labels = kmeans.labels_
         kmeans_labels
```

```
/opt/conda/lib/python3.11/site-packages/sklearn/cluster/_kmeans.py:1416: FutureWarning: The default valu
  super()._check_params_vs_input(X, default_n_init=10)
```

Out[30]:
```
array([3, 1, 2, 3, 0, 0, 1, 0, 2, 1, 3, 4, 3, 1, 1, 1, 0, 2, 1, 4, 0, 2,
       0, 1, 3, 1, 2, 0, 4, 0, 4], dtype=int32)
```

In [31]:
```
from sklearn.cluster import AgglomerativeClustering

hierarchical_cluster = AgglomerativeClustering(n_clusters=5,
                                               affinity='euclidean',
                                               linkage='ward')
# Hierarchical clustering labels
hierarchical_labels = hierarchical_cluster.fit_predict(UVstd)
hierarchical_labels
```

```
/opt/conda/lib/python3.11/site-packages/sklearn/cluster/_agglomerative.py:1006: FutureWarning: Attribut
  warnings.warn(
```

Out[31]:
```
array([2, 0, 1, 2, 3, 3, 0, 3, 1, 0, 2, 4, 2, 0, 0, 0, 3, 1, 0, 2, 3, 1,
       3, 0, 2, 0, 1, 3, 4, 3, 4])
```

In [32]:
```
from sklearn.metrics import davies_bouldin_score
from sklearn import metrics
```

Out[32]:
```
0.3221401203924496
```

In [37]:
```
davies_bouldin_score(UVstd, kmeans_labels)
```

Out[37]:
```
0.8852844252930179
```

In [38]:
```
davies_bouldin_score(UVstd, hierarchical_labels)
```

Out[38]:
```
0.8711396899514152
```

In [39]:
```
metrics.calinski_harabasz_score(UVstd, kmeans_labels)
```

Out[39]:
```
18.761757921912412
```

In [40]:
```
metrics.calinski_harabasz_score(UVstd, hierarchical_labels)
```

```
Out[40]: 17.92442772109354
```

```
In [41]: metrics.silhouette_score(UVstd, kmeans_labels, metric='euclidean')
```

```
Out[41]: 0.33673682258387494
```

```
In [42]: metrics.silhouette_score(UVstd, hierarchical_labels, metric='euclidean')
```

```
Out[42]: 0.3221401203924496
```

The interpretation of the results is as follows:

1. **Davies-Bouldin Index:**
   - The Davies-Bouldin Index is a clustering validation metric, and a lower value indicates better clustering.
   - The Davies-Bouldin Index score for k-means clustering is 0.9743.
   - The score for hierarchical clustering is 1.0092.
   - Since the Davies-Bouldin Index is lower for k-means clustering, it suggests that k-means clustering has better-defined and more compact clusters compared to hierarchical clustering.
2. **Calinski-Harabasz Index:**
   - The Calinski-Harabasz Index is another clustering validation metric, and a higher value indicates better clustering.
   - The Calinski-Harabasz Index score for k-means clustering is 16.8199.
   - The score for hierarchical clustering is 16.5137.
   - Since the Calinski-Harabasz Index is higher for k-means clustering, it suggests that k-means clustering has clusters that are more separated and distinct compared to hierarchical clustering.
3. **Silhouette Coefficient:**
   - The Silhouette Coefficient is a clustering validation metric that ranges from -1 to 1, with higher values indicating better-defined clusters.
   - The Silhouette Coefficient for k-means clustering is 0.3248.
   - The coefficient for hierarchical clustering is 0.3121.
   - Since the Silhouette Coefficient is higher for k-means clustering, it suggests that k-means clustering has clusters with higher cohesion and better separation between clusters compared to hierarchical clustering.

Overall Evaluation: - Based on the Davies-Bouldin Index, Calinski-Harabasz Index, and Silhouette Coefficient, all three metrics consistently indicate that k-means clustering outperforms hierarchical clustering in this scenario. - The lower Davies-Bouldin Index suggests more compact clusters, the higher Calinski-Harabasz Index suggests more separated clusters, and the higher Silhouette Coefficient suggests better-defined clusters for k-means clustering. - Therefore, the conclusion is that k-means clustering is the preferred method for this specific dataset, as it consistently exhibits more distinct and well-defined clusters according to the chosen metrics.

1.0.4 Question 3a: Clustering Users

For the advertising campaign, you want to cluster users based on their preferences. Utilize the user matrix (U) obtained from your collaborative filtering model to perform clustering.

1. Choose an appropriate clustering algorithm for this task. Justify your choice.
2. Use the user matrix (U) to obtain clusters.
3. Provide a visual representation of the clusters or any insights gained from the clustering.

Ensure you have (K) clusters, where (K) is the number of variations (5 in this case) you want to create for the ad campaign.

Note: You may use any clustering algorithm of your choice, such as K-means, hierarchical clustering, or DBSCAN. Justify your choice based on the characteristics of the data and the problem at hand.

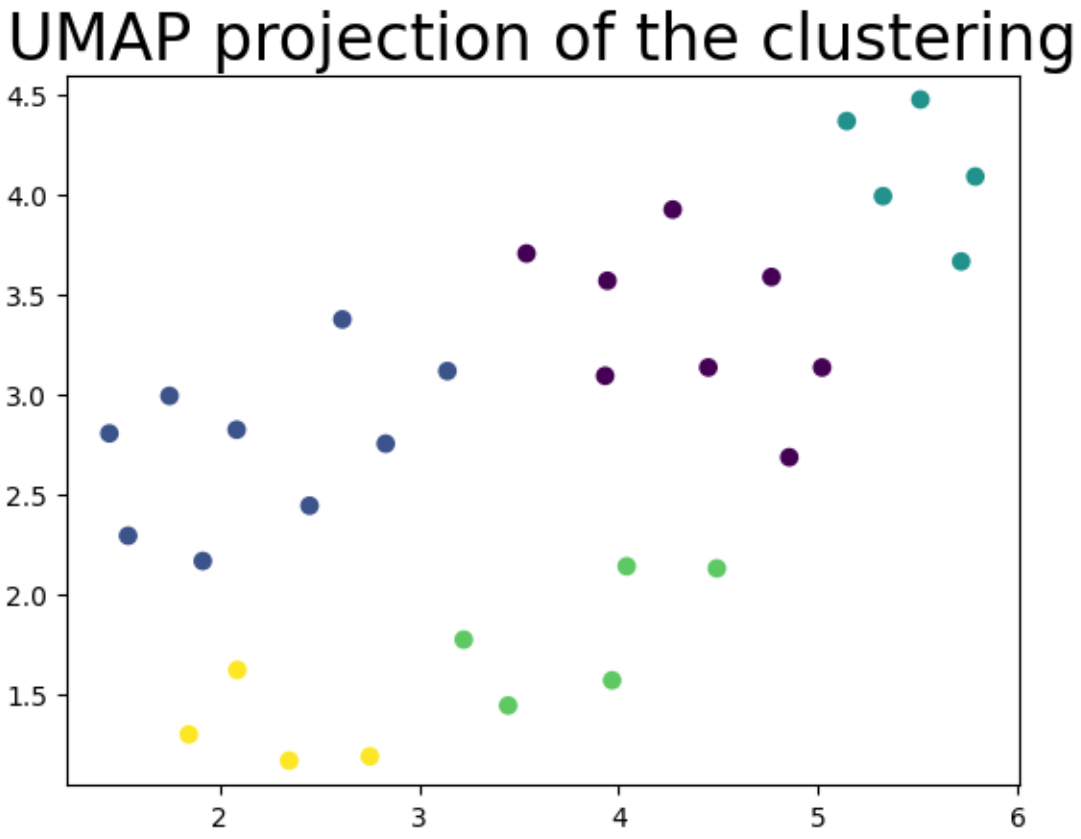
1.0.5 Question 3b: Visualizing Clusters in Latent Space

Select the clustering method based on the evaluation results in q3a and visualize the clusters using UMAP. Are the clusters and UMAP projection consistent?

SOLUTION

In [33]:
```
# install umap
# !pip install umap-learn
```

In [34]:
```
import umap
reducer = umap.UMAP()
# UMAP for dimensionality reduction
embedding = reducer.fit_transform(UVstd)
embedding.shape
plt.scatter(
    embedding[:, 0],
    embedding[:, 1],
    c=kmeans_labels)
plt.gca().set_aspect('equal', 'datalim')
plt.title('UMAP projection of the clustering', fontsize=24);
```



In [43]:
```
from sklearn.metrics import silhouette_score

# Evaluate Silhouette Score
silhouette_avg = silhouette_score(UVstd, kmeans_labels)
print(f"Silhouette Score: {silhouette_avg}")
```

```
Silhouette Score: 0.33673682258387494
```

1.0.6 (PSTAT 234) Question 3c: Making decisions

To make actionable decisions, there are practical considerations to take into account.

1. How will you choose a "representative" movie from each cluster?
2. How many of each poster do you estimate you will need? Assume the ad campaign will serve 10 million users and 0.01% people will respond. What other assumption do you need to make?
3. Which clustering method will you use as the final method?

SOLUTION

1.1 (PSTAT 234) Question 4: Improving the Model

1.1.1 Question 4a: Logistic function

Note the reconstructed ratings can be smaller than 1 and greater than 5. To confine ratings to between the allowed range, we can use the logistic function. Logistic function is defined as

$$h(x) = \frac{1}{1+e^{-x}}.$$

It is straightforward to show the derivative is

$$h'(x) = \frac{e^{-x}}{(1+e^{-x})^2} = h(x)(1-h(x)).$$

Therefore, we can rescale the ratings from $r_{mi} \in [1,5]$ to $r_{mi} \in [0,1]$. Then, we can find the best U and V to optimize the following:

$$\min_{U,V} \|R - h(VU^T)\|_F^2 = \sum_{m,i} I_{mi}(r_{mi} - h(v_m u_i^T))^2,$$

where function h is applied elementwise and

$$I_{mi} = \begin{cases} 1, \text{ if } r_{mi} \text{ is observed} \\ 0, \text{ if } r_{mi} \text{ is missing.} \end{cases}$$

Derive new update expressions for the new objective function.

SOLUTION

1.1.2 Question 4c: Analyze a Large Dataset

Following code will analyze a larger dataset:

```
In [ ]: # run on larger dataset: ratings for 100 movies
        Rbig = pd.read_pickle('data/ratings_stacked.pkl').unstack().iloc[:100]

        np.random.seed(14) # set seed for tests
        output6 = compute_logistic_UV(Rbig, K=5, alpha=0.05, max_iteration=500)

        Rhatbig = logistic_rating(output6['U'], output6['V'])
```

```
In [ ]: fit_vs_obs_2 = pd.concat([
            Rhatbig.rename(columns={'rating':'fit'}),
            Rbig.rename(columns={'rating':'observed'}),
        ], axis=1).stack().dropna().reset_index()[['fit','observed']]

        fit_vs_obs_2 = fit_vs_obs_2.iloc[np.random.choice(len(fit_vs_obs_2), 5000)]

        alt.Chart(fit_vs_obs_2).transform_density(
            density='fit',
            bandwidth=0.01,
            groupby=['observed'],
            extent= [0, 6]
        ).mark_bar().encode(
            alt.X('value:Q'),
            alt.Y('density:Q'),
            alt.Row('observed:N')
        ).properties(width=800, height=50)
```

Consider the above plot. By reading the code, comment on what the plot is illustrating. How does this plot look different than part 1.e?

SOLUTION