Computational Physics Lab HW4

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1 Written Assignments

$$u_{xx} + u_{yy} = -\rho(x, y) \tag{1}$$

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h^2} = -\rho_{i,j} \tag{2}$$

$$\begin{aligned}
4u_{11} - u_{01} - u_{21} - u_{10} - u_{12} = \rho_{11}h^2\\
4u_{21} - u_{11} - u_{31} - u_{20} - u_{22} = \rho_{21}h^2\\
4u_{31} - u_{21} - u_{41} - u_{30} - u_{32} = \rho_{31}h^2\\
4u_{12} - u_{02} - u_{22} - u_{11} - u_{13} = \rho_{12}h^2\\
4u_{22} - u_{12} - u_{32} - u_{21} - u_{23} = \rho_{22}h^2\\
4u_{32} - u_{22} - u_{42} - u_{31} - u_{33} = \rho_{32}h^2\\
4u_{13} - u_{03} - u_{23} - u_{12} - u_{14} = \rho_{13}h^2\\
4u_{23} - u_{13} - u_{33} - u_{22} - u_{24} = \rho_{23}h^2\\
4u_{33} - u_{23} - u_{43} - u_{32} - u_{34} = \rho_{33}h^2
\end{aligned} \tag{3}$$

$$A = \begin{bmatrix}
4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
-1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0\\
-1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0\\
0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0\\
0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1\\
0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0\\
0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1\\
0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4
\end{bmatrix} \tag{4}$$

$$b = \begin{bmatrix}
\rho_{11}h^2\\
\rho_{21}h^2\\
\rho_{31}h^2\\
\rho_{12}h^2\\
\rho_{22}h^2\\
\rho_{32}h^2\\
\rho_{13}h^2\\
\rho_{23}h^2\\
\rho_{33}h^2
\end{bmatrix} \tag{5}$$

2 Programming Assignments

2.1 Solve the System's Potential with $\rho_{22} = 1$

Given that $\rho_{22} = 1$ and $h = 1$, the solution potential u for Written Assignments will be:

$$u = \begin{bmatrix} -0.0625 \\ -0.125 \\ -0.0625 \\ -0.125 \\ -0.375 \\ -0.125 \\ -0.0625 \\ -0.125 \\ -0.0625 \end{bmatrix} \tag{6}$$

Plot the solution with plt.imshow():

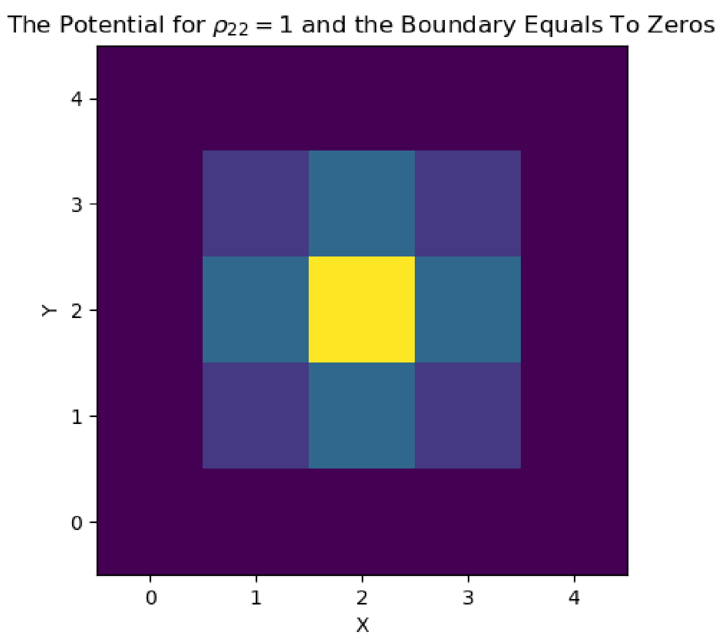

Figure 1: The Solution Potential u for Written Assignments

2.2 Draw Color and Contour plots of the Source Function and the Solution Potential.

In the task, we'll plot the following source function:

$$\rho(x, y) = e^{-1.25r_1^2} + 1.5e^{-r_2^2} \tag{7}$$

where $r_1 = (x+1.5)^2+y^2$ and $r_2 = (x-1.5)^2+y^2$. We'll consider the region $-5 < X < 5$ and $-5 < Y < 5$.

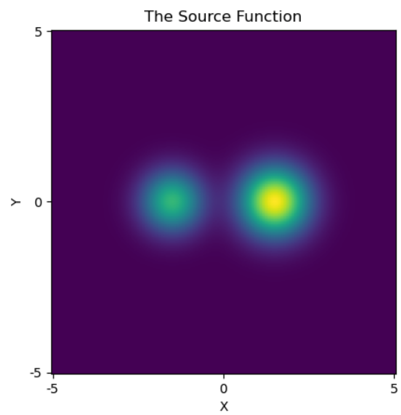

Figure 2: The Source Function

Also we'll plot the solution potential for this source function:

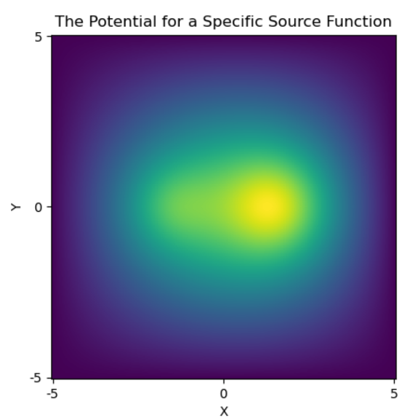

Figure 3: The Solution Potential for This Source Function

2.3 Plots Errors versus Iterations for Jacobi, GS and SOR method

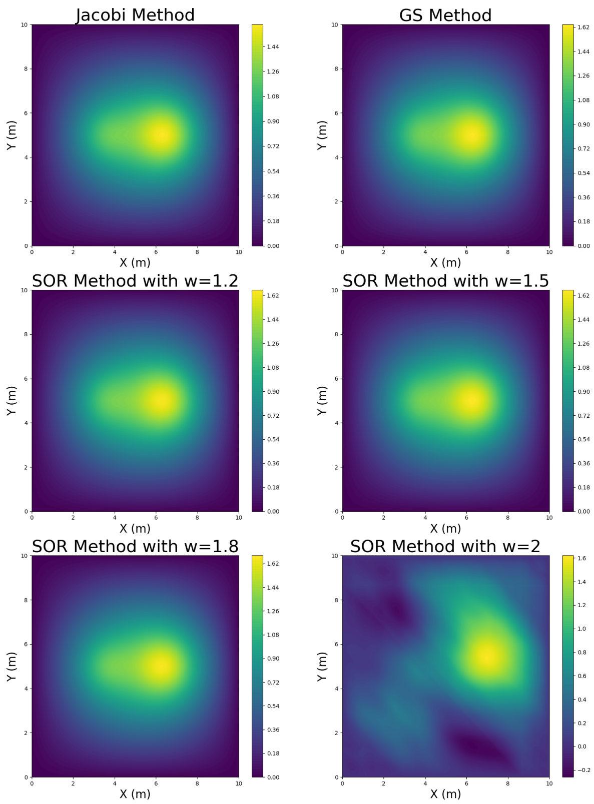

Figure 4: The Solution Potential for six methods

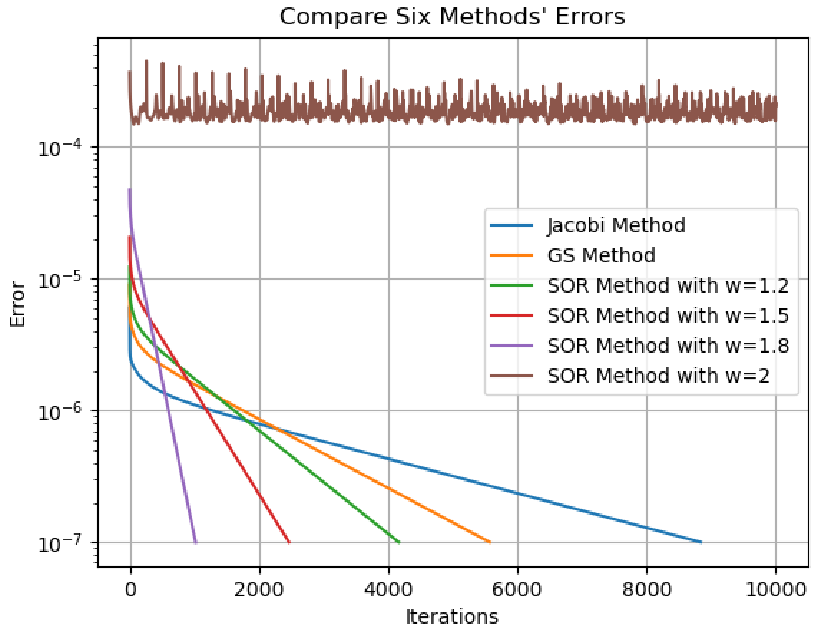

Figure 5: The errors as functions of iterations for six methods

From the Figure 5, we can find the SOR method is faster than the GS method and the Jacobi method. However, the SOR method doesn't always converge as our test in the SOR method with $w = 2$.

2.4 Plot the Computing Times versus Grid Resolutions for Sparse Matrix Solver, Jacobi, GS and SOR method

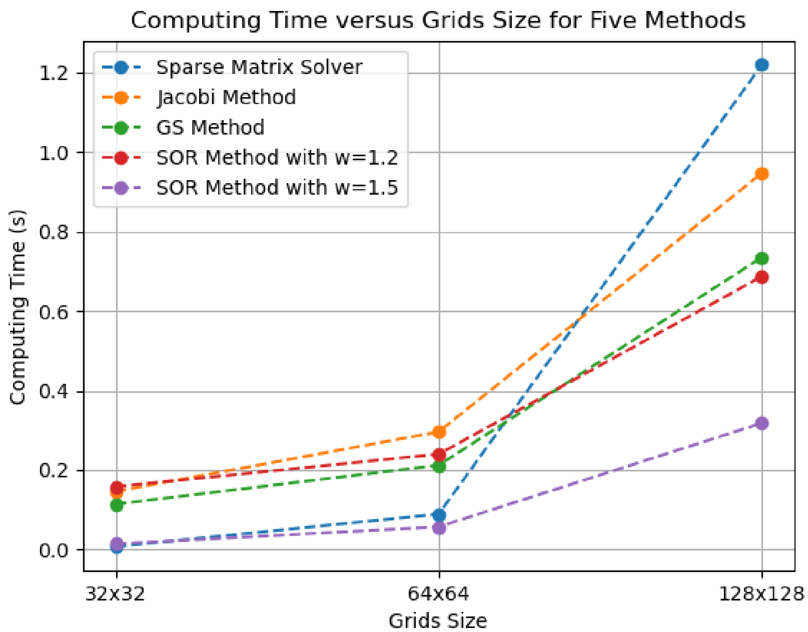

Figure 6: Computing Time versus Grids Size for Five Methods

From the Figure 6, we can find the Sparse Matrix Solver and the SOR method with an optimized $w = 1.5$ are good choice for us when we consider smaller grids size. However, the computing time of the Sparse Matrix Solver blows up as we increase the grids size. Overall, the SOR method with an optimized w performs well even we increase the grids size.