# Computational Physics Lab HW1

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October 2022

## 1 $\pi$ Calculation

In this paragraph, we'll calculate $\pi$ by using the area of the rectangular to approximate the area of the circle.

$$S = \pi R^2 \approx 2 \sum_{n=1}^{N} H_n \frac{2R}{N} \tag{1}$$

where $R$ is the radius of the circle, $N$ is the partition times and $H_n$ is the correspond rectangular of $n$. After reorganizing the eq(1), setting $R = 1$ and plugging the function of the circle into the eq(1), we could get the following eq(2).

$$\pi_{approximation} \approx \frac{4}{N} \sum_{n=1}^{N} H(x_n) \tag{2}$$

$$H(x_n) = \sqrt{1 - x_n^2} \tag{3}$$

We could get the approximate $\pi_{approximation}$ value by plugging the eq(2) into our program.

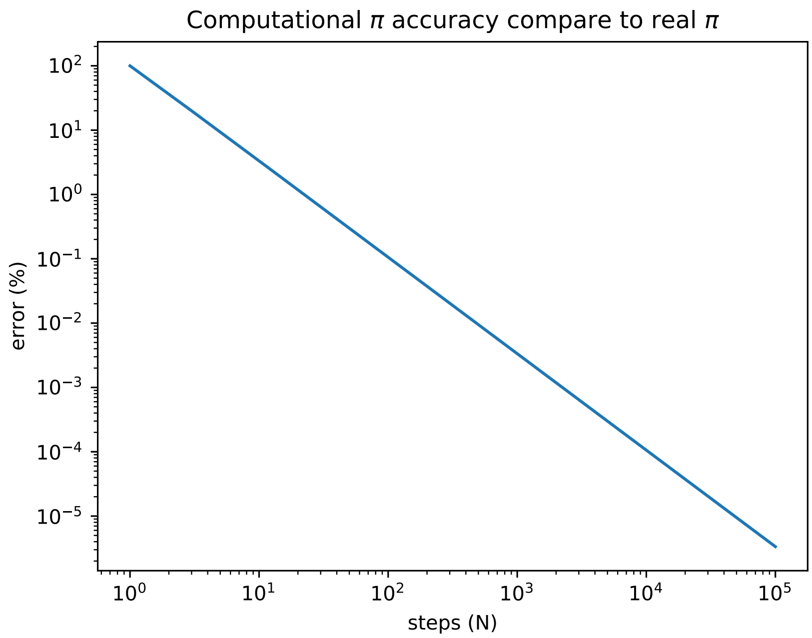


Figure 1: The error between $\pi_{approximation}$ and $\pi_{real}$ versus different partition under log-scale

From Figure 1, we could find the error decay to $10^{-5}$ while the partition times reaches $10^5$. Also, we could observe that the accuracy improves slowly while the partition times becomes large.

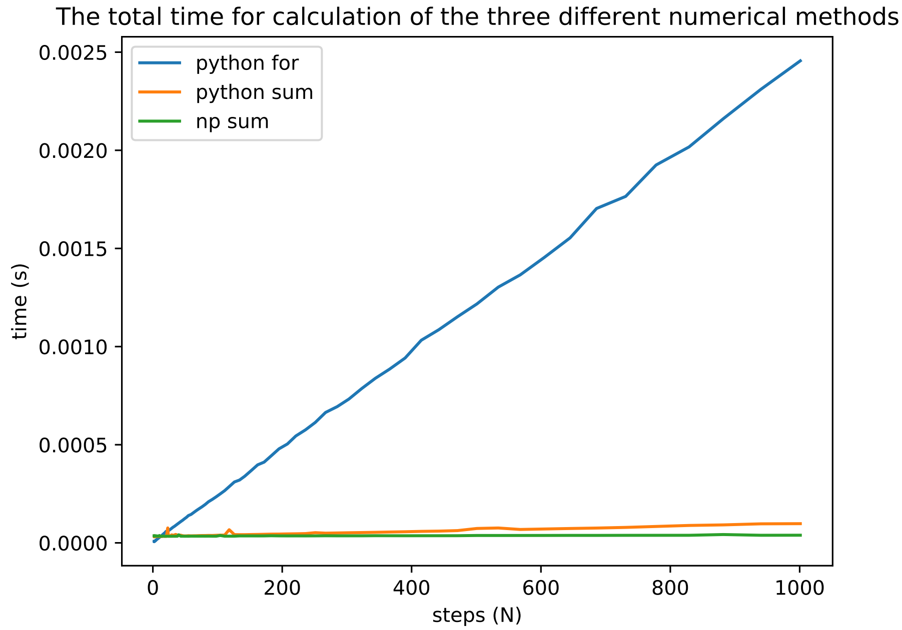


Figure 2: The time consumption of the three numerical ways

From Figure 1, we could obviously find the python built-in for loop method is much slower than the python built-in sum method and the numpy sum method. Also, the numpy sum method is little faster than the python built-in sum method.

## 2 Stefan-Boltzmann Constant

In this paragraph, we'll calculate Stefan-Boltzmann constant $\sigma_B$. We could get $\sigma_B$ from the following formula,

$$\frac{\sigma_B T^4}{\pi} = \int_0^\infty B_v(T) dv \tag{4}$$

where

$$B_v(T) = \frac{2hv^3}{c^2(e^{\frac{hv}{kT}} - 1)} \tag{5}$$

is the Planck function. We choose $T = 6000K$ here to integrate the Planck function. Just like what we did in $\pi$ calculation task, we'll use the rectangular area to approximate the area below the Planck function.

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The Stefan-Boltzmann constant of my program: 5.651587e-08
The experimental Stefan-Boltzmann constant: 5.670367e-08
The difference between them is 0.33%
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Figure 3: The results of Stefan-Boltzmann Constant

From the Figure 3, we could find that the difference of the Stefan-Boltzmann constant $\sigma_B$ between my computation and experiment is only 0.33%. The results reveal that my program is acceptable.

# 3 Projection with Drag Force

In this paragraph, we'll calculate the landing spot of the bird with the drag force. First, we have the following formula of the drag force,

$$F = -K\eta v \tag{6}$$

where $K$ is the drag coefficient, $\eta$ is the coefficient of viscosity and $v$ is the current velocity of the bird. Assuming the bird is a sphere so the drag coefficient could express as the following eq(7),

$$K = 6\pi R \tag{7}$$

where $R = 0.3m$ is the radius of the sphere. Also, we have the gravity,

$$F = mg \tag{8}$$

where $m = 5\ kg$ is the mass of the bird and $g = 9.8\ m/s^2$ is the gravitational acceleration. Therefore, we could get the net force acting on the bird through combining the drag force with the gravity. Also we could calculate the acceleration of the bird by the Newton's Second Law.

$$F = ma \tag{9}$$

In my program, I'll use the Euler method as the algorithm to simulate the projection. Furthermore, I'll find the proper initial velocity and inclination angle which could hit the pig at $20m$ away from the origin under the two different values of viscosity which are $\eta = 0.0002$ and $\eta = 0.01$.

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Initial velocity = 14.003 m/s, inclination angle = 45 deg, the distance of the projection is 20.00 m, hit the pig!
Initial velocity = 15.048 m/s, inclination angle = 60 deg, the distance of the projection is 20.00 m, hit the pig!
Initial velocity = 14.108 m/s, inclination angle = 45 deg, the distance of the projection is 20.00 m, hit the pig!
Initial velocity = 15.197 m/s, inclination angle = 60 deg, the distance of the projection is 20.00 m, hit the pig!
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Figure 4: The results of Stefan-Boltzmann Constant