Model description for the implementation of System Identification methodologies on the rotorcraft SH09

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List of symbols

α	Angle of attack	[deg]

1 Introduction

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2 Model description

2.1 F-16 model

This section aims to present the reduced equations for the dynamics of the F-16 fixed-wing aircraft which simulated in a non-linear manner using SIDPAC software.

2.1.1 Mathematical models of aircrafts

The notation for the aircraft is the one shown in Figure 1.

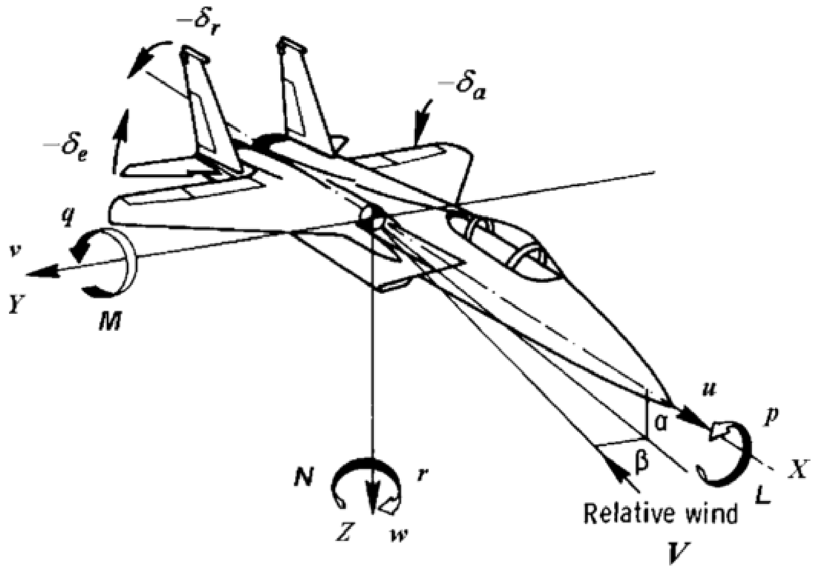

Figure 1: Airplane notation and sign conventions: u, v, w 5 body-axis components of aircraft velocity relative to Earth axes; p, q, r 5 body-axis components of aircraft angular velocity; X, Y, Z 5 body-axis components of aerodynamic force acting on the aircraft; and L, M, N 5 body-axis components of aerodynamic moment acting on the aircraft.

The components of the aerodynamic forces and moments, are the following:
Forces:

$$\text{Body axes} \qquad \text{Stability axes} \tag{1}$$

$$X = \bar{q}SC_X \qquad D = \bar{q}SC_D \tag{2}$$

$$Z = \bar{q}SC_Z \qquad L = \bar{q}SC_L \tag{3}$$

$$Y = \bar{q}SC_Y \qquad Y = \bar{q}SC_Y \tag{4}$$

Moments:

$$L = \bar{q}bSC_l \tag{5}$$

$$M = \bar{q}\bar{c}SC_m \tag{6}$$

$$N = \bar{q}bSC_n \tag{7}$$

where $\bar{q} = 1/2\rho V^2$ is the dynamic pressure, ρ is the air density, V is the airspeed, S is the wing reference area, b is the wing span and $\bar{c}$ is the mean aerodynamic chord (MAC).

The forces expressed in the wind axis systems as shown in set Equations 8.

$$
\begin{aligned}
C_{\mathrm{L}} &= -C_{\mathrm{Z}} \cos\alpha + C_{\mathrm{X}} \sin\alpha \\
C_{\mathrm{D}} &= -C_{\mathrm{X}} \cos\alpha - C_{\mathrm{Z}} \sin\alpha
\end{aligned} \tag{8}
$$

The Taylor expansion for the longitudinal motion of the aircraft are expressed in set of Equations 9.

$$
\begin{aligned}
C_D &= C_{D_0} + C_{D_V}\frac{\Delta V}{V_0} + C_{D_\alpha}\Delta\alpha + C_{D_q}\frac{q\bar{c}}{2V_0} + C_{D_{\delta_{\mathrm{e}}}}\Delta\delta_{\mathrm{e}} \\
C_L &= C_{L_0} + C_{L_V}\frac{\Delta V}{V_0} + C_{L_\alpha}\Delta\alpha + C_{L_{\dot\alpha}}\frac{\dot\alpha\bar{c}}{2V_0} + C_{L_q}\frac{q\bar{c}}{2V_0} + C_{L_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}} \\
C_m &= C_{m_0} + C_{m_V}\frac{\Delta V}{V_0} + C_{m_\alpha}\Delta\alpha + C_{m_{\dot\alpha}}\frac{\dot\alpha\bar{c}}{2V_0} + C_{m_q}\frac{q\bar{c}}{2V_0} + C_{m_{\delta_{\mathrm{e}}}}\delta_{\mathrm{e}}
\end{aligned} \tag{9}
$$

The set of equations that describe the motion of the aircraft, obtained from the Taylor series expansion is the one represented in Equation 10.

$$
\begin{aligned}
C_Y &= C_{Y_0} + C_{Y_\beta}\Delta\beta + C_{Y_p}\frac{pb}{2V_0} + C_{Y_r}\frac{rb}{2V_0} + C_{Y_{\delta_{\mathrm{a}}}}\Delta\delta_{\mathrm{a}} + C_{Y_{\delta_{\mathrm{r}}}}\Delta\delta_{\mathrm{r}} \\
C_l &= C_{l_0} + C_{l_\beta}\Delta\beta + C_{l_p}\frac{pb}{2V_0} + C_{l_r}\frac{rb}{2V_0} + C_{l_{\delta_{\mathrm{a}}}}\Delta\delta_{\mathrm{a}} \qquad (C_{l_{\delta_{\mathrm{r}}}} \ll 1) \\
C_n &= C_{n_0} + C_{n_\beta}\Delta\beta + C_{n_p}\frac{pb}{2V_0} + C_{n_r}\frac{rb}{2V_0} + C_{n_{\delta_{\mathrm{r}}}}\Delta\delta_{\mathrm{r}} \qquad (C_{l_{\delta_{\mathrm{a}}}} \ll 1)
\end{aligned} \tag{10}
$$

2.2 Helicopter model

2.2.1 Model forces and moments coefficients

Forces:

$$X_u, X_v, X_w, X_p, X_q, X_r Y_u, Y_v, Y_w, Y_p, Y_q, Y_r Z_u, Z_v, Z_w, Z_p, Z_q, Z_r$$

Moments:

$$L_u, L_v, L_w, L_p, L_q, L_r M_u, M_v, M_w, M_p, M_q, M_r N_u, N_v, N_w, N_p, N_q, N_r$$

Controllability, **G** matrix: Forces

$$X_{\theta_{\mathrm{lon}}}, X_{\theta_{\mathrm{lat}}}, X_{\theta_{\mathrm{ped}}}, X_{\theta_{\mathrm{col}}} Y_{\theta_{\mathrm{lon}}}, Y_{\theta_{\mathrm{lat}}}, Y_{\theta_{\mathrm{ped}}}, Y_{\theta_{\mathrm{col}}} Z_{\theta_{\mathrm{lon}}}, Z_{\theta_{\mathrm{lat}}}, Z_{\theta_{\mathrm{ped}}}, Z_{\theta_{\mathrm{col}}}$$

Moments

$$M_{\theta_{\mathrm{lon}}}, M_{\theta_{\mathrm{lat}}}, M_{\theta_{\mathrm{ped}}}, M_{\theta_{\mathrm{col}}} N_{\theta_{\mathrm{lon}}}, N_{\theta_{\mathrm{lat}}}, N_{\theta_{\mathrm{ped}}}, N_{\theta_{\mathrm{col}}} N_{\theta_{\mathrm{lon}}}, N_{\theta_{\mathrm{lat}}}, N_{\theta_{\mathrm{ped}}}, N_{\theta_{\mathrm{col}}}$$

Time delays

$$\tau_{\mathrm{lon}}, \tau_{\mathrm{lat}}, \tau_{\mathrm{ped}}, \tau_{\mathrm{col}}$$

2.2.2 Equations of motion for a linearised model

The linearisation of forces and moments by means of the small perturbation theory about the body axes centre leads to:

$$
\begin{aligned}
X &= m\{\dot{u} + qW_e - d_x(q^2 + r^2) + d_y(pq - \dot{r}) + d_z(pr + \dot{q})\} && (11)\\
Y &= m\{\dot{v} - pW_e + rU_e + d_x(pq + \dot{r}) - d_y(p^2 + r^2) + d_z(qr - \dot{p})\} && (12)\\
Z &= m\{\dot{w} - qU_e + d_x(pr - \dot{q}) + d_y(qr + \dot{p}) + d_z(p^2 + q^2)\} && (13)\\
L &= I_{xx}\dot{p} - I_{xz}\dot{r} - Yd_z + Zd_y && (14)\\
M &= I_{yy}\dot{q} - Zd_x + Xd_z && (15)\\
N &= I_{zz}\dot{r} - I_{xz}\dot{p} - Xd_y + Yd_x && (16)
\end{aligned}
$$

And, considering the forces and moments arise from aerodynamic, gravitational and control sources, it can be written that:

$$
\begin{aligned}
X &= u\mathring{X}_u + w\mathring{X}_w + q\mathring{X}_q - \theta mg\cos\theta_e + \theta_{\text{lon}}\mathring{X}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{X}_{\theta_{\text{col}}} && (17)\\
Y &= v\mathring{Y}_v + p\mathring{Y}_p + r\mathring{Y}_r + \phi mg\sin\theta_e + \psi mg\cos\theta_e + \theta_{\text{lat}}\mathring{Y}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{Y}_{\theta_{\text{ped}}} && (18)\\
Z &= u\mathring{Z}_u + w\mathring{Z}_w + q\mathring{Z}_q - \theta mg\sin\theta_e + \theta_{\text{lon}}\mathring{Z}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{Z}_{\theta_{\text{col}}} && (19)\\
L &= v\mathring{L}_v + p\mathring{L}_p + r\mathring{L}_r + \theta_{\text{lat}}\mathring{L}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{L}_{\theta_{\text{ped}}} + \theta_{\text{lon}}\mathring{L}_{\theta_{\text{lon}}} && (20)\\
M &= u\mathring{M}_u + w\mathring{M}_w + q\mathring{M}_q + \theta_{\text{lon}}\mathring{M}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{M}_{\theta_{\text{col}}} + \theta_{\text{ped}}\mathring{M}_{\theta_{\text{ped}}} + \theta_{\text{lat}}\mathring{M}_{\theta_{\text{lat}}} && (21)\\
N &= v\mathring{N}_v + p\mathring{N}_p + r\mathring{N}_r + \theta_{\text{lat}}\mathring{N}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{N}_{\theta_{\text{ped}}} && (22)
\end{aligned}
$$

2.2.3 Longitudinal and lateral dynamics from the linearized model

The linearized model for longitudinal dynamics results on:

$$
\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \mathbf{A} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \mathbf{B} \begin{bmatrix} \theta_{\text{col}} \\ \theta_{\text{lon}} \end{bmatrix} \tag{23}
$$

$$
\mathbf{y} = \mathbf{C} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \mathbf{D} \begin{bmatrix} \theta_{\text{col}} \\ \theta_{\text{lon}} \end{bmatrix} \tag{24}
$$

Ultimately, this will result in the following characteristic equation:

$$(T_1 s + 1)(T_2 s + 1)/(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0$$

And for the lateral, results on:

$$
\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \mathbf{A} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \mathbf{B} \begin{bmatrix} \theta_{\text{lat}} \\ \theta_{\text{ped}} \end{bmatrix} \tag{25}
$$

$$
\mathbf{y} = \mathbf{C} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \mathbf{D} \begin{bmatrix} \theta_{\text{lat}} \\ \theta_{\text{ped}} \end{bmatrix} \tag{26}
$$

Ultimately, this will result in the following characteristic equation for the lateral/directional motion:

$$
(T_1 s + 1)(T_2 s + 1)/(s^2 + 2\zeta\omega_n s + \omega_n^2)s = 0
$$

3 Theory of Helicopters

3.1 Equations of Motion for Rigid Airframe

As shown in [?]...

The axes to be used are the helicopter body axes (O, x, y, z) fixed in the helicopter and with its origin at the body axes centre. The components of velocity and force along the Ox, Oy and Oz axes are U, V, W and X, Y, Z, respectively. The components of the rates of rotation about the same axes are p, q and r and the moments L, M and N.

Considering the position that the position of the centre of gravity CG is given by the co-ordinates d_x, d_y and d_z relative to the body axes centre, the absolute velocity of CG is given by u', v' and w':

$$u' = U - rd_y + qd_z \qquad v' = V - pd_z + rd_x \qquad w' = W - qd_x + pd_y \tag{27}$$

and similarly, for the accelerations of the CG:

$$a'_x = \dot{U}' - rv' + qw' \qquad a'_y = \dot{v}' - pw' + ru' \qquad a'_z = \dot{w}' - qu' + pv' \tag{28}$$

3.2 Relationship between feathering law and flapping law for hovering

The flapping equation for hover is given by:

$$M^{a,E}_{b,y_{A1}} + k_\beta \beta + I_\beta \left(\frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} + \Omega^2 \beta \right) + x_{\mathrm{GB}} M_P \Omega^2 e \beta = 0,$$

where the aerodynamic moment is given by:

$$M^{a,E}_{b,y_{A1}} = \rho a c \Omega^2 R^4 \left\{ \left[-\frac{1}{6} + \frac{1}{4}\frac{e}{R} - \frac{1}{12}\left(\frac{e}{R}\right)^3 \right] \lambda_i + \left[\frac{1}{8} - \frac{1}{3}\frac{e}{R} ... \right] \right\}, \tag{29}$$

then, substituting $\psi = \Omega t$:

$$\frac{\mathrm{d}^2\beta}{\mathrm{d}\psi^2} + \eta_\beta \frac{\mathrm{d}\beta}{\mathrm{d}\psi} + \lambda_\beta^2 \beta + \delta_\beta \lambda_i - \alpha_\beta \theta = 0, \tag{30}$$

where η_β, δ_β and α_β are function of the Lock number $\gamma = \rho a c R^4 / I_\beta$, the blade eccentricity e and the rotor radius R:

$$\eta_\beta = \frac{\gamma}{8}\left[1 - \frac{8}{3}\frac{e}{R} + 2\left(\frac{e}{R}\right)^2 - \frac{1}{3}\left(\frac{e}{R}\right)^4 \right], \tag{31}$$

$$\delta_\beta = \frac{\gamma}{8}\left[-\frac{4}{3} + 2\frac{e}{R} - \frac{2}{3}\left(\frac{e}{R}\right)^3 \right], \tag{32}$$

$$\alpha_\beta = \frac{\gamma}{8}\left[1 - \frac{4}{3}\frac{e}{R} + \frac{1}{3}\left(\frac{e}{R}\right)^4 \right]. \tag{33}$$

The blade natural damped natural frequency λ_β is given by:

$$\lambda_\beta = \sqrt{1 + \frac{3}{2(1 - e/R)}\frac{e}{R} + 3\frac{k_\beta}{R^3 m_P \Omega^2}\frac{1}{(1 - e/R)^3}}.$$

The feathering control law is given by:

$$\theta(\phi) = \theta_0 + \theta_{1C} \cos\phi + \theta_{1S} \sin\phi$$

Then, it can be assumed that the flapping angle can be reduced to its first harmonic:

$$\beta(\phi) = \beta_0 + \beta_{1C} \cos\phi + \beta_{1S} \sin\phi.$$

Then the constant part β_0 is given by:

$$\beta_0 = \frac{\alpha_\beta \theta_0 - \delta_\beta \lambda_{i0}}{\lambda_\beta^2},$$

and β_{1C} and β_{1C} are given by:

$$\beta_{1C} = \frac{\alpha_\beta}{(\lambda_\beta^2 - 1)^2 + \eta_\beta^2} [(\lambda_\beta^2 - 1)\theta_{1C} - \eta_\beta \theta_{1S}], \tag{34}$$

$$\beta_{1S} = \frac{\alpha_\beta}{(\lambda_\beta^2 - 1)^2 + \eta_\beta^2} [\eta_\beta \theta_{1C} + (\lambda_\beta^2 - 1)\theta_{1S}]. \tag{35}$$

References

[1] V. Klein and E. A. Morelli, "Aircraft System Identification Theory and Practice," 2006.

[2] R. K. Remple and M. B. Tischler, *Aircraft and Rotorcraft System Identification.* 2006.

[3] G. Morelli and S. Derry, "System Identification Methods for Aerodynamic Modeling and Validation using Flight Data," 2011.

[4] G. Morelli, "Aircraft System Identification," 2011.