POLITECNICO DI MILANO

    

HELID
Rotorcraft system identification: an integrated time-frequency domain approach

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Helicopter model identification

- The dynamics of a rotorcraft during a stationary maneuver (e.g., hover, forward flight)

can be well described using a MIMO LTI continuous-time system

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
$$y(t) = C(\theta)x(t) + D(\theta)u(t)$$

where the system matrices depend on unknown parameters

- The objective is to estimate the unknown parameters θ

State of the art

- Frequency-domain approaches (e.g., CIFER)
  - Advantage: computationally **fast** (few data-points)
  - Advantage: deal with unstable system in a very natural way (phase signs)
  - Drawback: long and **expensive** experiments (frequency sweeps)

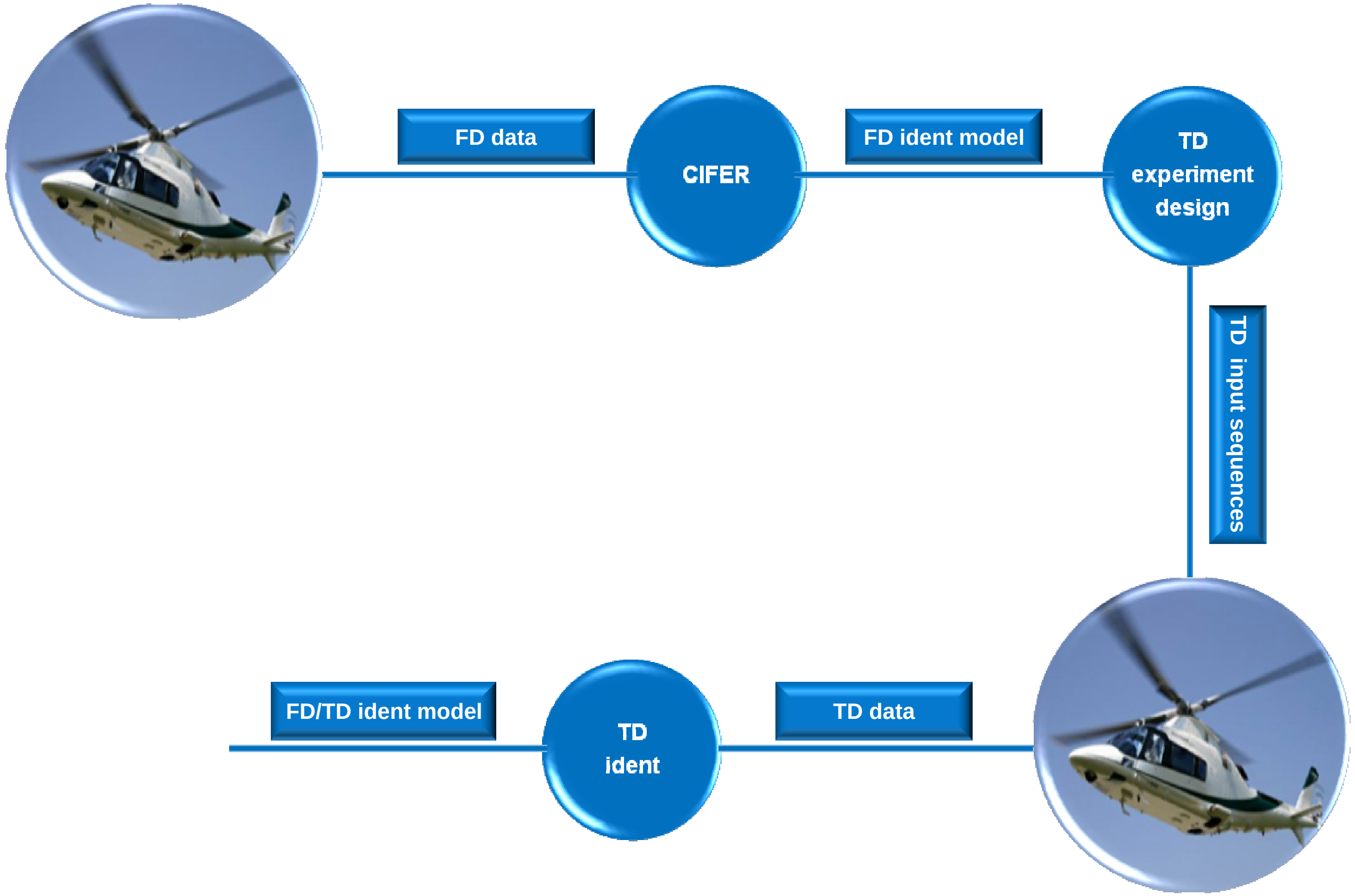
- Iterative time-domain approaches (e.g., OE, EE, etc.)
  - Advantage: shorter, **cheaper**, and **safer** experiments (3211 sequences)
  - Drawback: computationally **slow** (a lot of data-points)
  - Drawback: some tricks are needed in order to deal with unstable system

- NON-iterative time-domain approaches (e.g., subspace methods)
  - Advantage: computationally **efficient and robust**
  - Advantage: shorter, **cheaper**, and **safer** experiments (3211 sequences)
  - Drawback: no control on state space basis of identified models.

Time- and frequency-domain methods: possible synergies

- Iterative TD refinement of FD models
- Iterative FD refinement of TD models

HELID 2012:
Frequency-domain/Time-domain identification



Objective and a solution: Subspace model identification methods

- A Time-domain/Frequency-domain approach in which **only** the **advantages** of both methods is our **goal**

- **Approach:** gray-box model identification through black-box modelling

- Subspace model identification methods are **computationally efficient** algorithms able to identify **black-box** (i.e., no physical meaning) models using **time-domain data**

- BUT, they have been studied extensively just for **discrete-time** models, i.e.,

$$x(k+1) = \tilde{A}x(k) + \tilde{B}u(k)$$
$$y(k) = \tilde{C}x(k) + \tilde{D}u(k)$$

- Several advantages (e.g., **non-iterative solution**, natural extension to **MIMO system**, unbiased solution with **closed-loop data**, etc.), but **two new issues**:
  - continuous-time to discrete-time model conversion;
  - black-box to grey-box model transformation.

Continuous-time predictor-based subspace identification algorithm (CT-PBSID$_o$)

Data collection

Signals/Laguerre basis correlation

Discrete-time data

$$(\tilde{u}(k), \tilde{y}(k))$$

SMI algorithm

$$\xi(k+1) = A_o\xi(k) + B_o\tilde{u}(k)$$
$$\tilde{y}(k) = C_o\xi(k) + D_o\tilde{u}(k)$$

Discrete-time identified model

-1

Signals/Laguerre basis correlation

$$\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t)$$
$$y(t) = \hat{C}x(t) + \hat{D}u(t)$$

Black-box continuous-time identified model

Black-box to grey-box model transformation in the frequency-domain

- Black-box identified model

$$\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t)$$
$$y(t) = \hat{C}x(t) + \hat{D}u(t)$$

- Black-box identified model

$$\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t)$$
$$y(t) = \hat{C}x(t) + \hat{D}u(t)$$

- Grey-box model structure

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
$$y(t) = C(\theta)x(t) + D(\theta)u(t)$$

Black-box to grey-box model transformation in the frequency-domain

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$$\Rightarrow \quad \hat{G}_{ns}(s)$$

- Grey-box model structure

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
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$$\Rightarrow \quad G_s(s;\theta)$$

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$$\Rightarrow \quad G_s(s;\theta)$$

- H_{inf} approach in frequency-domain

$$\theta^\star = \arg\min_{\theta} \|\hat{G}_{ns}(s) - G_s(s;\theta)\|_\infty$$

CT-PBSID$_o$: comments

- The computation of the signals transformations

$$\tilde{u}(k) = \int_0^\infty \ell_k(t)u(t)dt \quad \tilde{y}(k) = \int_0^\infty \ell_k(t)y(t)dt$$

allows to deal with **non uniform sampling**.

- Data from **different experiments** can be naturally **merged** in the identification procedure.

- The identification algorithm is based on standard **QR** or **SVD** solvers (very efficient implementations are available in Matlab).

H_{inf} approach: comments

- The optimization of the H_{inf} norm can be performed using a grid of frequencies and an optimization genetic algorithm
- The non-smooth non-convex optimization problem can be solved using some recent algorithms available in literature (and in Matlab R2012a)
- Frequency-domain data (if available) can be included in the optimization problem

BO-105 Example Problem
Introduction

- The BO-105 is a light, twin-engine, multi-purpose utility helicopter
- It is considered in forward flight at 80 knots (unstable dynamics)
- Identification of a continuous-time state-space model with test data extracted from a nine-DOF simulator (taken from Tischler and Caufmann 1992)

- 4 inputs, 11 outputs, 12 state variables
  47 unknown parameters

- 4 inputs, 11 outputs, 12 state variables
  47 unknown parameters

	Simulator				Identified Model (CT-PBSID$_o$)			
	Real	Imag	Omega	Zeta	Real	Imag	Omega	Zeta
Pitch phugoid	0.119	0.278	0.302	-0.394	0.119	0.278	0.302	-0.394
Dutch roll	-0.571	2.546	2.609	0.219	-0.571	2.546	2.609	0.219
Roll/flapping	-9.904	7.740	12.569	0.788	-9.901	7.7399	12.568	0.788
Lead-Lag	-0.868	15.567	15.592	0.0557	-0.867	15.566	15.590	0.0556
Spiral	-0.0510				-0.0507			
Pitch$_1$	-0.448				-0.448			
Pitch$_2$	-5.843				-5.844			
Long. flapping	-15.930				-15.901			

BO-105 Example Problem
Grey-box results

- 4 inputs, 11 outputs, 12 state variables
  47 unknown parameters

	Simulator				Identified Model (CT-PBSID$_o$)			
	Real	Imag	Omega	Zeta	Real	Imag	Omega	Zeta
Pitch phugoid	0.119	0.278	0.302	-0.394	0.119	0.278	0.302	-0.394
Dutch roll	-0.571	2.546	2.609	0.219	-0.571	2.546	2.609	0.219
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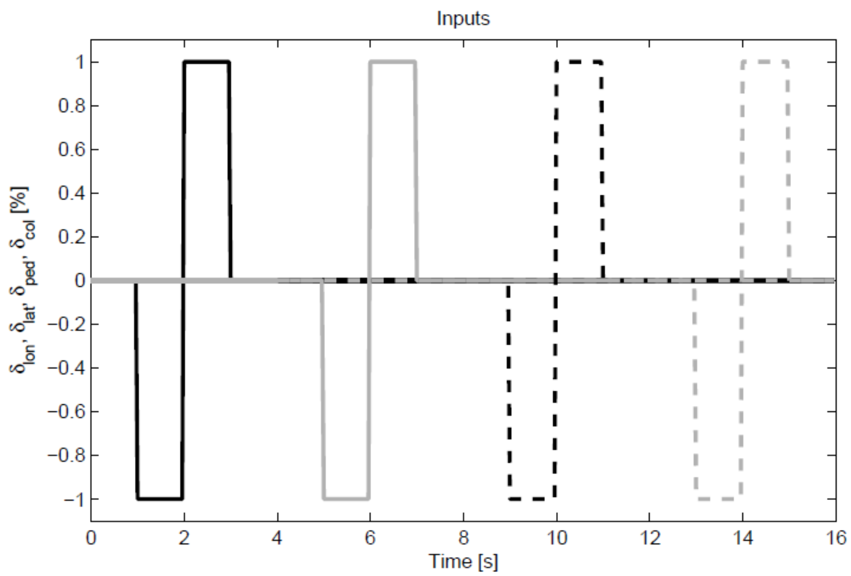
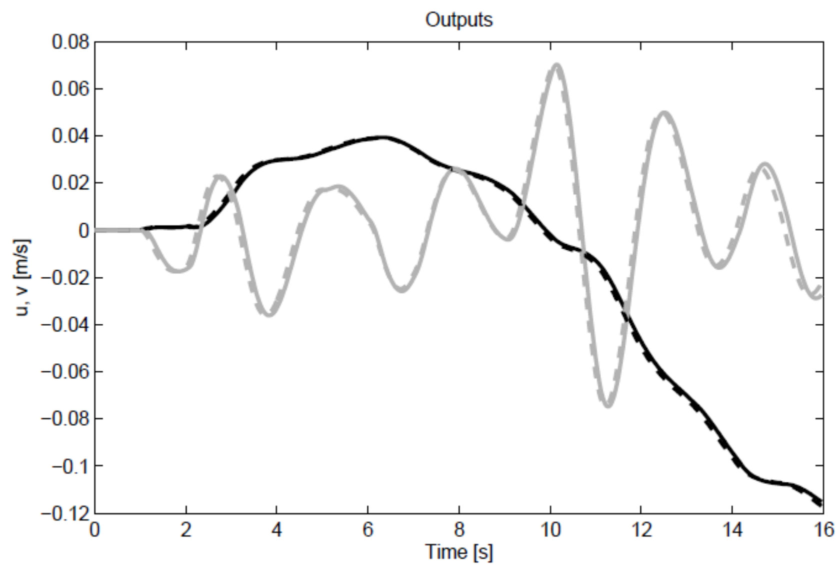
	Simulator				Identified Model (Gray-box)			
	Real	Imag	Omega	Zeta	Real	Imag	Omega	Zeta
Pitch phugoid	0.119	0.278	0.302	-0.394	0.119	0.278	0.302	-0.394
Dutch roll	-0.571	2.546	2.609	0.219	-0.571	2.546	2.609	0.219
Roll/flapping	-9.904	7.740	12.569	0.788	-9.903	7.740	12.568	0.788
Lead-Lag	-0.868	15.567	15.592	0.0557	-0.868	15.566	15.590	0.557
Spiral	-0.0510				-0.0507			
Pitch$_1$	-0.448				-0.448			
Pitch$_2$	-5.843				-5.843			
Long. flapping	-15.930				-15.929			

BO-105 Example Problem
Validation

Inputs



Outputs



Some details

Definitions

- Consider the first order all-pass transfer function

$$w(s) = \frac{s-a}{s+a}, \qquad a > 0$$

- w(s) generates the family of Laguerre filters, defined as

$$\mathcal{L}_i(s) = w^i(s)\mathcal{L}_0(s), \qquad \mathcal{L}_0(s) = \sqrt{2a}\frac{1}{(s+a)}$$

- Denote with $\ell_i(t)$ the impulse response of the *i*-th Laguerre filter. The set

$$\{\ell_0, \ell_1, \ldots, \ell_i, \ldots\}$$

is an orthonormal basis of $L_2(0,\infty)$.

From continuous-time to discrete-time

$$\dot{x}(t) = Ax(t) + Bu(t) + Ke(t)$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

State space matrices transformation

$$A_o = (A - aI)^{-1}(A + aI)$$
$$B_o = \sqrt{2a}(A - aI)^{-1}B$$
$$K_o = \sqrt{2a}(A - aI)^{-1}K$$
$$C_o = -\sqrt{2a}C(A - aI)^{-1}$$
$$D_o = D - C(A - aI)^{-1}B.$$

Signals transformation

$$\tilde{u}(k) = \int_0^{\infty} \ell_k(t)u(t)dt$$
$$\tilde{y}(k) = \int_0^{\infty} \ell_k(t)y(t)dt$$
$$\tilde{e}(k) = \int_0^{\infty} \ell_k(t)e(t)dt,$$

$$\xi(k+1) = A_o\xi(k) + B_o\tilde{u}(k) + K_o\tilde{e}(k),\ \xi(0) = 0$$
$$\tilde{y}(k) = C_o\xi(k) + D_o\tilde{u}(k) + \tilde{e}(k)$$

Discrete index *k*: basis order

Consider the discrete-time system

$$\xi(k+1) = A_o\xi(k) + B_o\tilde{u}(k) + K_o\tilde{e}(k)$$
$$\tilde{y}(k) = C_o\xi(k) + D_o\tilde{u}(k) + \tilde{e}(k)$$

Innovation Form

Closed-loop predictor matrices

$$\bar{A}_o = A_o - K_oC_o$$
$$\bar{B}_o = B_o - K_oD_o$$

$$\tilde{z}(k) = \begin{bmatrix} \tilde{u}(k) \\ \tilde{y}(k) \end{bmatrix}, \quad \tilde{B}_o = \begin{bmatrix} \bar{B}_o & K_o \end{bmatrix}$$

$$\xi(k+1) = \bar{A}_o\xi(k) + \tilde{B}_o\tilde{z}(k)$$
$$\tilde{y}(k) = C_o\xi(k) + D_o\tilde{u}(k) + \tilde{e}(k)$$

Prediction Form

The discrete-time PBSID algorithm: the data equation

Iterating *p-1* times the state equation one gets

$$\xi(k+2) = \bar{A}_o^2 \xi(k) + \begin{bmatrix} \bar{A}_o \tilde{B}_o & \tilde{B}_o \end{bmatrix} \begin{bmatrix} \tilde{z}(k) \\ \tilde{z}(k+1) \end{bmatrix}$$

$$\vdots$$

$$\xi(k+p) = \bar{A}_o^p \xi(k) + \mathcal{K}^p Z^{0,p-1}$$

where

$$\mathcal{K}^p = \begin{bmatrix} \bar{A}_o^{p-1} \tilde{B}_0 & \dots & \tilde{B}_o \end{bmatrix}$$

Extended controllability matrix

and

$$Z^{0,p-1} = \begin{bmatrix} \tilde{z}(k) \\ \vdots \\ \tilde{z}(k+p-1) \end{bmatrix}$$

Input-output "past" data

The discrete-time PBSID algorithm: the data equation

- The predictor is AS by assumption, so

$$\bar{A}_o^p \xi(k) \simeq 0$$

for sufficiently large values of *p* and

$$\xi(k+p) \simeq \mathcal{K}^p Z^{0,p-1}$$

p: past window length
f: future window length

- Then, the input-output behaviour of the system is given by the data equation:

$$\tilde{y}(k+p) \simeq C_o \mathcal{K}^p Z^{0,p-1} + D_o \tilde{u}(k+p) + \tilde{e}(k+p)$$

$$\vdots$$

$$\tilde{y}(k+p+f) \simeq C_o \mathcal{K}^p Z^{f,p+f-1} + D_o \tilde{u}(k+p+f) + \tilde{e}(k+p+f)$$

- Finally, the state space matrices can be recovered from the data equation using Least Squares techniques.