# Model description for the implementation of System Identification methodologies on the rotorcraft SH09

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## Contents



## List of symbols

| | | |
|---|---|---|
| $\alpha$ | Angle of attack | [deg] |

# 1 Introduction

Bibliography: [1], [2], [3], [4]

# 2 Model description

## 2.1 F-16 model

This section aims to present the reduced equations for the dynamics of the F-16 fixed-wing aircraft which simulated in a non-linear manner using SIDPAC software.

### 2.1.1 Mathematical models of aircrafts

The notation for the aircraft is the one shown in Figure 1.

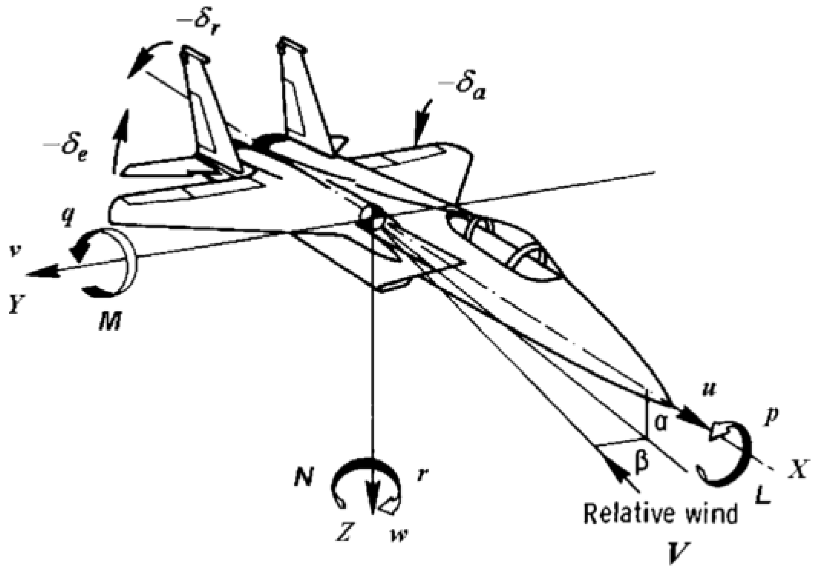


Figure 1: Airplane notation and sign conventions: u, v, w 5 body-axis components of aircraft velocity relative to Earth axes; $p$, $q$, $r$ 5 body-axis components of aircraft angular velocity; $X$, $Y$, $Z$ 5 body-axis components of aerodynamic force acting on the aircraft; and $L$, $M$, $N$ 5 body-axis components of aerodynamic moment acting on the aircraft.

The components of the aerodynamic forces and moments, are the following:
Forces:

$$\text{Body axes} \qquad \text{Stability axes} \tag{1}$$

$$X = \bar{q}SC_X \qquad D = \bar{q}SC_D \tag{2}$$

$$Z = \bar{q}SC_Z \qquad L = \bar{q}SC_L \tag{3}$$

$$Y = \bar{q}SC_Y \qquad Y = \bar{q}SC_Y \tag{4}$$

Moments:

$$L = \bar{q}bSC_l \tag{5}$$

$$M = \bar{q}\bar{c}SC_m \tag{6}$$

$$N = \bar{q}bSC_n \tag{7}$$

where $\bar{q} = 1/2\rho V^2$ is the dynamic pressure, $\rho$ is the air density, $V$ is the airspeed, $S$ is the wing reference area, $b$ is the wing span and $\bar{c}$ is the mean aerodynamic chord (MAC).

The forces expressed in the wind axis systems as shown in set Equations 8.

$$
\begin{aligned}
C_{\mathrm{L}} &= -C_{\mathrm{Z}} \cos\alpha + C_{\mathrm{X}} \sin\alpha \\
C_{\mathrm{D}} &= -C_{\mathrm{X}} \cos\alpha - C_{\mathrm{Z}} \sin\alpha
\end{aligned} \tag{8}
$$

The Taylor expansion for the longitudinal motion of the aircraft are expressed in set of Equations 9.

$$
\begin{aligned}
C_D &= C_{D_0} + C_{D_V}\frac{\Delta V}{V_0} + C_{D_\alpha}\Delta\alpha + C_{D_q}\frac{q\bar{c}}{2V_0} + C_{D_{\delta_\mathrm{e}}}\Delta\delta_\mathrm{e} \\
C_L &= C_{L_0} + C_{L_V}\frac{\Delta V}{V_0} + C_{L_\alpha}\Delta\alpha + C_{L_{\dot\alpha}}\frac{\dot\alpha\bar{c}}{2V_0} + C_{L_q}\frac{q\bar{c}}{2V_0} + C_{L_{\delta_\mathrm{e}}}\delta_\mathrm{e} \\
C_m &= C_{m_0} + C_{m_V}\frac{\Delta V}{V_0} + C_{m_\alpha}\Delta\alpha + C_{m_{\dot\alpha}}\frac{\dot\alpha\bar{c}}{2V_0} + C_{m_q}\frac{q\bar{c}}{2V_0} + C_{m_{\delta_\mathrm{e}}}\delta_\mathrm{e}
\end{aligned} \tag{9}
$$

The set of equations that describe the motion of the aircraft, obtained from the Taylor series expansion is the one represented in Equation 10.

$$
\begin{aligned}
C_Y &= C_{Y_0} + C_{Y_\beta}\Delta\beta + C_{Y_p}\frac{pb}{2V_0} + C_{Y_r}\frac{rb}{2V_0} + C_{Y_{\delta_\mathrm{a}}}\Delta\delta_\mathrm{a} + C_{Y_{\delta_\mathrm{r}}}\Delta\delta_\mathrm{r} \\
C_l &= C_{l_0} + C_{l_\beta}\Delta\beta + C_{l_p}\frac{pb}{2V_0} + C_{l_r}\frac{rb}{2V_0} + C_{l_{\delta_\mathrm{a}}}\Delta\delta_\mathrm{a} \qquad (C_{l_{\delta_\mathrm{r}}} \ll 1) \\
C_n &= C_{n_0} + C_{n_\beta}\Delta\beta + C_{n_p}\frac{pb}{2V_0} + C_{n_r}\frac{rb}{2V_0} + C_{n_{\delta_\mathrm{r}}}\Delta\delta_\mathrm{r} \qquad (C_{l_{\delta_\mathrm{a}}} \ll 1)
\end{aligned} \tag{10}
$$

## 2.2 Helicopter model

### 2.2.1 Model forces and moments coefficients

Forces:

$$X_u, X_v, X_w, X_p, X_q, X_r Y_u, Y_v, Y_w, Y_p, Y_q, Y_r Z_u, Z_v, Z_w, Z_p, Z_q, Z_r$$

Moments:

$$L_u, L_v, L_w, L_p, L_q, L_r M_u, M_v, M_w, M_p, M_q, M_r N_u, N_v, N_w, N_p, N_q, N_r$$

Controllability, **G** matrix: Forces

$$X_{\theta_\mathrm{lon}}, X_{\theta_\mathrm{lat}}, X_{\theta_\mathrm{ped}}, X_{\theta_\mathrm{col}} Y_{\theta_\mathrm{lon}}, Y_{\theta_\mathrm{lat}}, Y_{\theta_\mathrm{ped}}, Y_{\theta_\mathrm{col}} Z_{\theta_\mathrm{lon}}, Z_{\theta_\mathrm{lat}}, Z_{\theta_\mathrm{ped}}, Z_{\theta_\mathrm{col}}$$

Moments

$$M_{\theta_\mathrm{lon}}, M_{\theta_\mathrm{lat}}, M_{\theta_\mathrm{ped}}, M_{\theta_\mathrm{col}} N_{\theta_\mathrm{lon}}, N_{\theta_\mathrm{lat}}, N_{\theta_\mathrm{ped}}, N_{\theta_\mathrm{col}} N_{\theta_\mathrm{lon}}, N_{\theta_\mathrm{lat}}, N_{\theta_\mathrm{ped}}, N_{\theta_\mathrm{col}}$$

Time delays

$$\tau_\mathrm{lon}, \tau_\mathrm{lat}, \tau_\mathrm{ped}, \tau_\mathrm{col}$$

### 2.2.2 Equations of motion for a linearised model

The linearisation of forces and moments by means of the small perturbation theory about the body axes centre leads to:

$$
\begin{aligned}
X &= m\{\dot{u} + qW_e - d_x(q^2 + r^2) + d_y(pq - \dot{r}) + d_z(pr + \dot{q})\} && (11)\\
Y &= m\{\dot{v} - pW_e + rU_e + d_x(pq + \dot{r}) - d_y(p^2 + r^2) + d_z(qr - \dot{p})\} && (12)\\
Z &= m\{\dot{w} - qU_e + d_x(pr - \dot{q}) + d_y(qr + \dot{p}) + d_z(p^2 + q^2)\} && (13)\\
L &= I_{xx}\dot{p} - I_{xz}\dot{r} - Yd_z + Zd_y && (14)\\
M &= I_{yy}\dot{q} - Zd_x + Xd_z && (15)\\
N &= I_{zz}\dot{r} - I_{xz}\dot{p} - Xd_y + Yd_x && (16)
\end{aligned}
$$

And, considering the forces and moments arise from aerodynamic, gravitational and control sources, it can be written that:

$$
\begin{aligned}
X &= u\mathring{X}_u + w\mathring{X}_w + q\mathring{X}_q - \theta mg\cos\theta_e + \theta_{\text{lon}}\mathring{X}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{X}_{\theta_{\text{col}}} && (17)\\
Y &= v\mathring{Y}_v + p\mathring{Y}_p + r\mathring{Y}_r + \phi mg\sin\theta_e + \psi mg\cos\theta_e + \theta_{\text{lat}}\mathring{Y}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{Y}_{\theta_{\text{ped}}} && (18)\\
Z &= u\mathring{Z}_u + w\mathring{Z}_w + q\mathring{Z}_q - \theta mg\sin\theta_e + \theta_{\text{lon}}\mathring{Z}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{Z}_{\theta_{\text{col}}} && (19)\\
L &= v\mathring{L}_v + p\mathring{L}_p + r\mathring{L}_r + \theta_{\text{lat}}\mathring{L}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{L}_{\theta_{\text{ped}}} + \theta_{\text{lon}}\mathring{L}_{\theta_{\text{lon}}} && (20)\\
M &= u\mathring{M}_u + w\mathring{M}_w + q\mathring{M}_q + \theta_{\text{lon}}\mathring{M}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{M}_{\theta_{\text{col}}} + \theta_{\text{ped}}\mathring{M}_{\theta_{\text{ped}}} + \theta_{\text{lat}}\mathring{M}_{\theta_{\text{lat}}} && (21)\\
N &= v\mathring{N}_v + p\mathring{N}_p + r\mathring{N}_r + \theta_{\text{lat}}\mathring{N}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{N}_{\theta_{\text{ped}}} && (22)
\end{aligned}
$$

### 2.2.3 Longitudinal and lateral dynamics from the linearized model

The linearized model for longitudinal dynamics results on:

$$
\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \mathbf{A}\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \mathbf{B}\begin{bmatrix} \theta_{\text{col}} \\ \theta_{\text{lon}} \end{bmatrix} \tag{23}
$$

$$
\mathbf{y} = \mathbf{C}\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \mathbf{D}\begin{bmatrix} \theta_{\text{col}} \\ \theta_{\text{lon}} \end{bmatrix} \tag{24}
$$

And for the lateral, results on:

$$
\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \mathbf{A} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \mathbf{B} \begin{bmatrix} \theta_{\text{lat}} \\ \theta_{\text{ped}} \end{bmatrix} \tag{25}
$$

$$
\mathbf{y} = \mathbf{C} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \mathbf{D} \begin{bmatrix} \theta_{\text{lat}} \\ \theta_{\text{ped}} \end{bmatrix} \tag{26}
$$

# 3 Theory of Helicopters

## 3.1 Equations of Motion for Rigid Airframe

As shown in [?]...

The axes to be used are the helicopter body axes $(O, x, y, z)$ fixed in the helicopter and with its origin at the body axes centre. The components of velocity and force along the $Ox$, $Oy$ and $Oz$ axes are $U$, $V$, $W$ and $X$, $Y$, $Z$, respectively. The components of the rates of rotation about the same axes are $p$, $q$ and $r$ and the moments $L$, $M$ and $N$.

Considering the position that the position of the centre of gravity $CG$ is given by the co-ordinates $d_x$, $d_y$ and $d_z$ relative to the body axes centre, the absolute velocity of $CG$ is given by $u'$, $v'$ and $w'$:

$$u' = U - rd_y + qd_z \qquad v' = V - pd_z + rd_x \qquad w' = W - qd_x + pd_y \tag{27}$$

and similarly, for the accelerations of the $CG$:

$$a'_x = \dot{U}' - rv' + qw' \qquad a'_y = \dot{v}' - pw' + ru' \qquad a'_z = \dot{w}' - qu' + pv' \tag{28}$$

## 3.2 Relationship between feathering law and flapping law for hovering

The flapping equation for hover is given by:

$$M^{a,E}_{b,y_{A1}} + k_\beta \beta + I_\beta \left( \frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} + \Omega^2 \beta \right) + x_{\mathrm{GB}} M_P \Omega^2 e \beta = 0,$$

where the aerodynamic moment is given by:

$$M^{a,E}_{b,y_{A1}} = \rho a c \Omega^2 R^4 \left\{ \left[ -\frac{1}{6} + \frac{1}{4}\frac{e}{R} - \frac{1}{12}\left(\frac{e}{R}\right)^3 \right] \lambda_i + \left[ \frac{1}{8} - \frac{1}{3}\frac{e}{R} ... \right] \right\}, \tag{29}$$

then, substituting $\psi = \Omega t$:

$$\frac{\mathrm{d}^2\beta}{\mathrm{d}\psi^2} + \eta_\beta \frac{\mathrm{d}\beta}{\mathrm{d}\psi} + \lambda_\beta^2 \beta + \delta_\beta \lambda_i - \alpha_\beta \theta = 0, \tag{30}$$

where $\eta_\beta$, $\delta_\beta$ and $\alpha_\beta$ are function of the Lock number $\gamma = \rho a c R^4 / I_\beta$, the blade eccentricity $e$ and the rotor radius $R$:

$$\eta_\beta = \frac{\gamma}{8} \left[ 1 - \frac{8}{3}\frac{e}{R} + 2\left(\frac{e}{R}\right)^2 - \frac{1}{3}\left(\frac{e}{R}\right)^4 \right], \tag{31}$$

$$\delta_\beta = \frac{\gamma}{8} \left[ -\frac{4}{3} + 2\frac{e}{R} - \frac{2}{3}\left(\frac{e}{R}\right)^3 \right], \tag{32}$$

$$\alpha_\beta = \frac{\gamma}{8} \left[ 1 - \frac{4}{3}\frac{e}{R} + \frac{1}{3}\left(\frac{e}{R}\right)^4 \right]. \tag{33}$$

The blade natural damped natural frequency $\lambda_\beta$ is given by

$$\lambda_\beta = \sqrt{1 + \frac{3}{2(1-e/R)}\frac{e}{R} + 3\frac{k_\beta}{R^3 m_P \Omega^2}\frac{1}{(1-e/R)^3}}$$

The feathering control law is given by:

$$\theta(\phi) = \theta_0 + \theta_{1C}\cos\phi + \theta_{1S}\sin\phi$$

Then, it can be assumed that the flapping angle can be reduced to its first harmonic:

$$\beta(\phi) = \beta_0 + \beta_{1C}\cos\phi + \beta_{1S}\sin\phi.$$

Then the constant part $\beta_0$ is given by:

$$\beta_0 = \frac{\alpha_\beta\theta_0 - \delta_\beta\lambda_{i0}}{\lambda_\beta^2},$$

and $\beta_{1C}$ and $\beta_{1C}$ are given by:

$$\beta_{1C} = \frac{\alpha_\beta}{(\lambda_\beta^2 - 1)^2 + \eta_\beta^2}[(\lambda_\beta^2 - 1)\theta_{1C} - \eta_\beta\theta_{1S}], \tag{34}$$

$$\beta_{1S} = \frac{\alpha_\beta}{(\lambda_\beta^2 - 1)^2 + \eta_\beta^2}[\eta_\beta\theta_{1C} + (\lambda_\beta^2 - 1)\theta_{1S}]. \tag{35}$$

## References

[1] V. Klein and E. A. Morelli, "Aircraft System Identification Theory and Practice," 2006.

[2] R. K. Remple and M. B. Tischler, *Aircraft and Rotorcraft System Identification.* 2006.

[3] G. Morelli and S. Derry, "System Identification Methods for Aerodynamic Modeling and Validation using Flight Data," 2011.

[4] G. Morelli, "Aircraft System Identification," 2011.