Quiz 4 - Computational Physics I

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Date: Monday 2 December 2024 **Duration:** 45 minutes
Credits: 20 points (4 questions) **Type of evaluation:** LAB

Please provide concise answers to the following items:

Excellent!

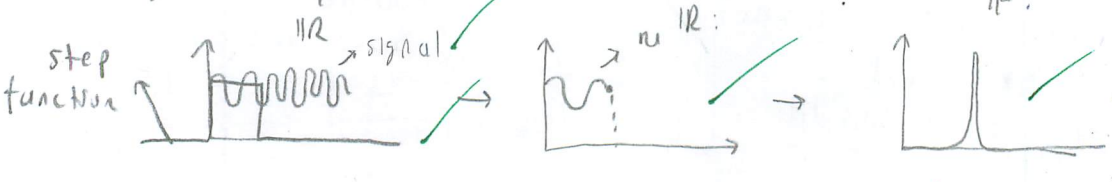
1. **(5 points) Fourier transforms**
   Provide a concise definition of the Fourier Transform and explain its significance for physics.
   Illustrate your answer with the basic formula for the continuous Fourier Transform in 1D.

- Fourier transform is a mathematical tool that allows as to pass from the real space IR to the Fourier space IF, which is actually the reciprocal space of real space or also called k-space. Then, it is also possible return to real space by the inverse F.T., summing all frequencies. ?
- The significance in physics comes from the fact that many measurements (e.g. cosmological) are signals that usually are noised. Therefore, in IF space is easy identify frequencies and amplitud that help us to clean noise and then return to real space a cleaned signal.

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)\, e^{-i2\pi k}\, dx$$

2. **(5 points) Wavelet transforms**
   Explain the process of wavelet decomposition for a 1D signal.
   When do we use Fourier transforms and when wavelet transforms?

- Wavelet decomposition consists ~~the~~ of applying window functions to our original signal that may be non-stationary so that we select a specific range in time or position. Then with F.T we can obtain a frequency or a range of frequencies within this interval.



There is some wavelet function that adapts to our signal, making possible multiresolution analysis.

- We use wavelet transforms when the signal is non-stationary and we want to know the frequency time-evolution since the wavelet transform allow us to pass from non-stationary signal to a local stationary signal. The usual F.T. is used when we are not intresting in time evolution. For example denoising.

3. **(5 points) Fourier filters**
Explain the difference between a low-pass filter and a high-pass filter in Fourier analysis.
List the main steps you would take to detect edges in images using a Fourier analysis in python.

- Low-pass filter takes away the high frequencies of a signal while high-pass filter takes away the low frequencies of a signal.

Detects edges:

1) Perform a 2D-fourier transform on the 2D-image, and shift it to put low frequencies on the center.
2) Create a mask (binary) to avoid low frequencies. This should be an image with a "hole" in the center.

3) Apply the mask in the 2D-transformed array by multiplying both.
4) Unshift the 2D-transformed array and perform an 2D-inverse F.T.
5) Check the result and adjust the mask if it is needed.

4. **(5 points) Noise filtering**
Imagine you are given a 2D grayscale image with background noise. Design and sketch the python workflow that you would follow to remove the noise using a Fourier-based algorithm in python.