# Quiz 1 - Computational Physics II

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**Date:** Thursday 27 February 2025 **Duration:** 45 minutes

**Credits:** 20 points (4 questions) **Type of evaluation:** LAB

**Provide short and concise answers to the following items:**

1. **(5 points) Integration methods for Ordinary Differential Equations (ODEs)**
   (a) Explain the difference between explicit and implicit Euler integrators for ODEs.
   (b) What are the error sources when you integrate ODEs numerically in a computer?

a) Explicit Euler integrators only depends on the last state to compute the following state while implicit Euler integrators depend also in its own state to compute the actual state.

Explicit: $S(t_{j+1}) = S_j + h\,F(t_j, S(t_j))$ ; Implicit: $S(t_{j+1}) = S_j + h\,F(t_{j+1}, S(t_{j+1}))$

$S(t_{j+1}) \to$ ~~actual~~ current state ; $S(t_j) \to$ past state.

? Why systematic ? −0.5

b) The error sources are the systematic error which comes from the behaviour of the method with respect to the problem ?, and the machine error related with computer capacity of our pcs.

2. **(5 points) Runge-Kutta methods for ODEs**
   (a) Explain how Runge-Kutta (RK) methods work.
   (b) How do RK methods improve upon simpler integration methods like the Euler methods?
   (c) Design your own third-order RK method, and write down the slopes and integrator.

a) The RK methods comes from truncated Taylor series and it works by calculating different slopes within $t_j$ and $t_{j+1}$ to then made a weigthed average from ~~this~~ them with weights.

b) The advantage of RK-methods is that this method does not need calculating higher derivatives for increasing the accurancy of the solution instead it calculates the state on intermediates points. Then it could be cheaper, computational talking, for some problems. Another advantage is that this method could be very personalizable since the weigths (e.g. $c_1, c_2, p, q$) can be adapted to the problem to improve the precission, efficiency and accuracy. The stability also is higher tha Euler methods since take points within $t_j$ and $t_{j+1}$.

c) $$S(t_{j+1}) = S(t_j) + \frac{1}{4}\left(c_1 k_1 + 2c_2 k_2 + c_3 k_3\right) \to \text{RK method.}$$

The slopes are: −0.5

$$k_1 = h\,F\left(t_j + \frac{p}{4}h,\ S\left(t_j + \frac{q}{4}h\right)\right), \quad k_2 = h\,F\left(t_j + \frac{p}{2}h,\ S\left(t_j + \frac{q}{2}k_1\right)\right)$$

$$k_3 = \cancel{h}\,F\left(t_j + \cancel{\frac{p}{4}}h,\ S\left(t_j + \frac{q}{4}k_2\right)\right)$$

take full step

3. **(5 points) ODE order reduction**

Consider an object with mass, $m$, that falls from rest under the influence of gravity (i.e., along the Y axis). The object is also subjected to a drag force that arises from friction with air molecules, so its equation of motion reads:

$$m\frac{d^2y}{dt^2} = -m\,g + b\frac{dy}{dt}$$

where $g$ is the acceleration of gravity and $b$ is a friction constant.

(a) Reduce the order of this ODE to first order and write down the resulting matrix-form equation.

(b) Identify the slope function.

(c) Briefly explain what the advantages of carrying out order reduction are.

a) State vector : $S(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix} \Rightarrow F(t, S(t)) = \frac{dS(t)}{dt} = \begin{bmatrix} \dot{y}(t) \\ \ddot{y}(t) \end{bmatrix}$

Operating the ODE:

$$\frac{d^2y}{dt^2} = -g + \frac{b}{m}\frac{dy}{dt} \Rightarrow \frac{dS(t)}{dt} = \begin{bmatrix} \dot{y}(t) \\ -g + \frac{b}{m}\dot{y}(t) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 1 \\ -g/y(t) & +b/m \end{bmatrix} \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix}$$

b) The slope function F is:

$$F(t, S(t)) = \begin{bmatrix} 0 & 1 \\ -g/y(t) & b/m \end{bmatrix} S(t)$$

c) The advantage of order reduction are that we can change from a $n^{th}$-order ODE to n first-order ODEs since more of the methods are optimized to solve $1^{st}$-order ODEs, which also makes the works more efficient since calculating higher order derivatives is not needed, this would be also computationally cheaper.

4. **(5 points) Shooting method for ODEs**

(a) Explain how the shooting method works and what it is used for.

(b) Sketch an algorithm workflow to implement the shooting method for an ODE in Python.

a) The shooting method is made to solve boundary value problems (BVPs) for ODEs, and it changes a BVP to a initial value problem (IVP) which is ~~more easy~~ easier to solve since from an initial point can be calulated all the solution. It mainly consists on three steps: aim step, shooting step and iterative step (optimization).

b)