Universidad Yachay Tech
School of Physical Sciences and Nanotechnology

Semester II-2025
Special Topics: Complex Systems

HOME WORK I

1. A plane pendulum of mass m and length l moves in a medium with a coefficient of friction $\gamma > 0$ that produces a friction force proportional to its velocity. In addition, the pendulum is subject to a force $F = A\cos(\nu t)$, where A and ν are the amplitude and the frequency of the force, respectively.
   a) Find the equation of motion for this system. [1]
   c) Is this system autonomous? Is it conservative or dissipative? [1]

2. The competition dynamics between two populations of sizes x (rabbits) and y (sheep) can be described by the equations

$$\begin{aligned} \dot{x} &= x(3 - x - 2y) \\ \dot{y} &= y(2 - x - y), \end{aligned}$$

   a) Find the fixed points on the phase space of the system. [1]
   b) Classify the fixed points according to their stability. [1]
   c) Can both species survive? [1]

3. Consider the following dynamical system with parameters $a, b \in \mathbb{R}$,

$$\begin{aligned} \dot{x} &= y - ax \\ \dot{y} &= \frac{x^2}{1 + x^2} - by, \end{aligned}$$

   a) Find the fixed points for this system. [1]
   b) Classify the fixed points according to their stability. [1]
   c) Can this system be chaotic for some range of parameters a and b? [1]

4. The evolution of a system is described by the equation

$$\dddot{x} + r\ddot{x} + \dot{x} - |x| + 1 = 0,$$

   where $r > 0$ is a real parameter.
   a) Find the fixed points of the system. [1]
   b) Plot the asymptotic trajectory of the system on its phase space for $r = 0.6$. [1]