Comp Challenge #3

(a)

$$ALLD \rightarrow \overset{p_{cc}\ p_{cd}\ p_{dc}\ p_{dd}}{\{0,0,0,0\}}$$

$$TFT \rightarrow \{1,0,1,0\}$$

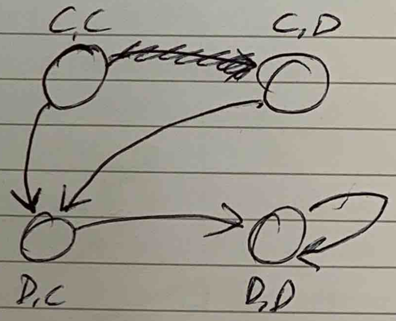
$$ALLC \rightarrow \{1,1,1,1\}.$$

Look at payoff for each combination.

ALLD – TFT, ALLD – ALLC, ALLD – ALLD,
TFT – ALLC, TFT – TFT, ALLC – ALLC.

ALLD-TFT.

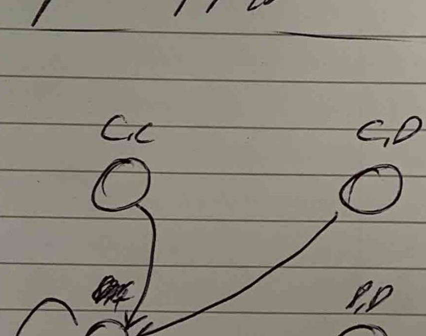
$$M_{ALLD-TFT} = \begin{array}{cccc} CC & CD & DC & DD \end{array}$$
$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



D,D is an absorbing state. The expected payoff is then Zero.

ALLD-ALLC

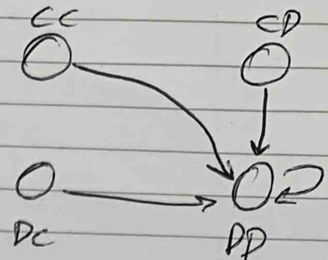
$$M_{ALLD-ALLC} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



D,C is absorbing state. Payoff is: $B/2$.

ALLD-ALLD

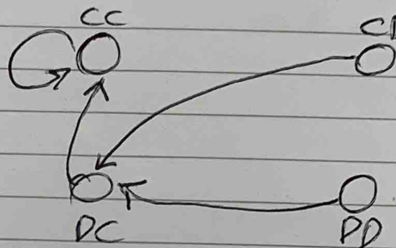
$$M_{ALLD-ALLD} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



DD is absorbing. Payoff is 0.
(zero)

TFT-ALLC

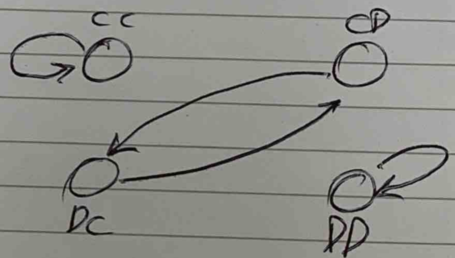
$$M_{TFT-ALLC} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



CC is absorbing. Payoff is $(B-C)$.

TFT-TFT

$$M_{TFT-TFT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



If initial is CC $\longrightarrow$ payoff is $(B-C)$.

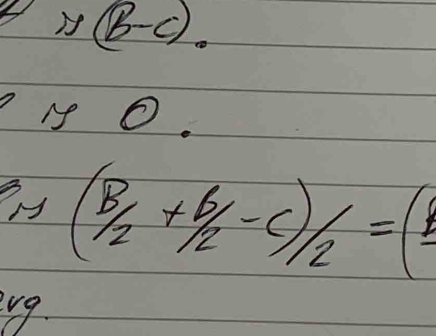
If initial is DD $\longrightarrow$ payoff is 0.

If initial is CD or DC $\longrightarrow$ payoff is $\left(\frac{B}{2}+\frac{B}{2}-C\right)/2 = \left(\frac{B-C}{2}\right)$.

on avg.

ALLC-ALLC

$$M_{ALLC-ALLC} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$



CC is absorbing. Payoff is $(B-C)$.

(a)

	ALLD	TFT
ALLD	0, 0	0, 0
TFT	0, 0	(B-C), (B-C)

assuming initial state is (C, C).

	ALLC	TFT
ALLC	(B-C), (B-C)	B-C, B-C
TFT	B-C, B-C	B-C, B-C

	ALLD	ALLC
ALLD	0, 0	$B/2$, $B/2 - C$
ALLC	$B/2 - C$, $B/2$	B-C, B-C.

(a) Write down the payoffs & fitness.

In general,

$$\begin{array}{ccc} & A & B \\ A & \multicolumn{2}{c}{\multirow{2}{*}{}} \\ \end{array}$$

$$\begin{array}{c|cc} & A & B \\ \hline A & a & b \\ B & c & d \end{array}$$

i individuals of A
$(N-i)$ " " " B.

Expected payoff:

to $A \rightarrow$ $F_i = \dfrac{a(i-1) + b(N-i)}{N-1}$

to $B \rightarrow$ $G_i = \dfrac{c \cdot i + d(N-i-1)}{N-1}$.

Let's look at the 6 possible combinations:

- ALLD-TFT.

Exp. payoffs
$$\begin{cases} F_{ALLD} = \dfrac{0 \cdot (i-1) + 0(N-i)}{N-1} = 0 \\ G_{TFT} = \dfrac{0 \cdot i + (B-C)(N-i-1)}{N-1} = \dfrac{(B-C)(N-i-1)}{N-1}. \end{cases}$$

fitnesses.
$$\begin{cases} f_{ALLD} = 1-\omega + \omega(0) = 1-\omega. \\ g_{TFT} = 1-\omega + \omega\left[\dfrac{(B-C)(N-i-1)}{N-1}\right] \end{cases}$$

- ALLD - ALLC

$$F_{ALLD} = \frac{0 \cdot (i-1) + \left(\frac{B}{2}\right)(N-i)}{N-1} = \frac{\frac{B}{2}(N-i)}{N-1}$$

$$G_{ALLC} = \frac{\left(\frac{B}{2}-c\right)(i) + (B-c)(N-i-1)}{N-1}$$

$$f_{ALLD} = 1 - \omega + \omega \cdot \left[\frac{\frac{B}{2}(N-i)}{N-1}\right]$$

$$g_{ALLC} = 1 - \omega + \omega \cdot \left[\frac{\left(\frac{B}{2}-c\right)i + (B-c)(N-i-1)}{N-1}\right]$$

- ALLD-ALLD

$$F_{ALLD} = 0$$

$$G_{ALLD} = 0.$$

$$f_{ALLD} = 1-\omega$$

$$g_{ALLD} = 1-\omega$$

- ALLC-TFT

$$F_{ALLC} = \frac{(B-C)(i-1) + (B-C)(N-i)}{N-1}$$

$$G_{TFT} = \frac{(B-C)i + (B-C)(N-i-1)}{N-1} = B-C$$

$$f_{ALLC} = 1 - w + w \overbrace{\left[\frac{(B-C)(i-1) + (B-C)(N-i)}{N-1}\right]}^{B-C}$$

$$g_{TFT} = 1 - w + w \overbrace{\left[\frac{(B-C)i + (B-C)(N-i-1)}{N-1}\right]}^{B-C}$$

- TFT - TFT

$$F_{TFT} = \frac{(B-C)i + (B-C)(N-i-1)}{N-1} = B-C$$

$$G_{TFT} = B-C$$

$$f_{TFT} = 1 - w + w(B-C)$$

$$g_{TFT} = 1 - w + w(B-C)$$

- ALLC - ALLC

$$\left.\begin{aligned} F_{ALLC} &= B-C \\ G_{ALLC} &= B-C \end{aligned}\right\}$$ Same cancellation as above.

$$f_{ALLC} = 1 - w + w(B-C)$$

$$g_{ALLC} = 1 - w + w(B-C)$$