AIML - CS 337

Lecture 3: ML Terminology

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Over the last two lectures, we reviewed probability and linear algebra. Now, we introduce some basic ML terminology.

1 An Example

This example is in continuation from Lecture 2. We have a graph $G = (V, E)$, where $V = \{1, \ldots, n\}$ is the set of vertices, and E is the set of edges. Define the value $x_i(t)$ to be the 'opinion' of node i at time t. $x_1(0), \ldots, x_n(0)$ are given. For $t \geq 0$,

$$x_i(t+1) = \frac{\sum_{j \in \mathcal{N}(i)} x_j(t)}{|\mathcal{N}(i)|} \tag{1}$$

where $\mathcal{N}(i)$ denotes the set of neighbors of node i. That is, the opinion of i at time $t+1$ is the average of the opinions of its neighbors at time t.
Consider $\boldsymbol{x}(t) = [x_1(t) \cdots x_n(t)]^T$. Define $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ such that $\boldsymbol{A}\boldsymbol{x}(t) = \boldsymbol{x}(t+1)$. From (1), $\boldsymbol{A}$ is a doubly-stochastic matrix, i.e., its entries are non-negative, and its rows and columns add up to 1.

We state the following result from [1]. A stochastic matrix $\boldsymbol{M}$ is called semi-positive if all entries of some power $\boldsymbol{M}^\alpha$ are positive.

Theorem 1.1. *If $\boldsymbol{M} \in \mathbb{R}^{n \times n}$ is a semi-positive doubly stochastic matrix, then $\lim_{t \to \infty} \boldsymbol{M}^t = \frac{1}{n}\boldsymbol{J}$, where $\boldsymbol{J} \in \mathbb{R}^{n \times n}$ with all entries 1.*

Thus, if $\boldsymbol{A}$ is semi-positive and doubly stochastic, then for all nodes i, $\lim_{t \to \infty} x_i(t) = \frac{\sum_{j=1}^n x_j(0)}{n}$

2 Image Classification Problem

3 Training and Validation Sets

References

[1] S. Baik and K. Bang. Limit theorem of the doubly stochastic matrices. *Kangweon-Kyungki Mathematics Journal*, 11(2):155–160, 2003.