Linear Model For Maximal Response (Approximation)

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Linear diff.eq. model.

$\boldsymbol{e}'(t) = \boldsymbol{A}\,\boldsymbol{e}(t) + \Delta\boldsymbol{s}(t,\boldsymbol{e})$, $\boldsymbol{s}(t,\boldsymbol{e})$ is difference caused by stimulation

Need to solve matrix $\boldsymbol{A}$ from the system. This can be done from the measurements where we assume $\boldsymbol{\Delta s}$ is small and MSE optimize for a linear problem, $\boldsymbol{\Delta}\,\boldsymbol{E}/\Delta t = \boldsymbol{A}\,\boldsymbol{E}$, where $\boldsymbol{\Delta E}$ and $\boldsymbol{E}$ are measurements matrixes from the data. $\boldsymbol{A}$ is maybe time-dependant but change slowly (constant) so we keep estimating $\boldsymbol{A}(t)$ from measurements.

The variable $\boldsymbol{e}(t)$ are brain EEG measurements or other variables computed and measured from the brain. We want to predict future $\boldsymbol{e}(t_0+T)$ and minimize error

$\varepsilon(T) = \int_{t_0}^{t_0+T} \frac{1}{2}\|\boldsymbol{e}(t) - \boldsymbol{e}_{\text{target}}\|^2\,d\,t$

$\frac{d\varepsilon(T)}{d\,T} = \frac{1}{2}\|\boldsymbol{e}(t_0+T) - \boldsymbol{e}_{\text{target}}\|^2 = 0$

Assuming $\boldsymbol{\Delta s}$ is small after the initial step at t_0 we can solve for $\boldsymbol{e}(t)$.

$\boldsymbol{e}(t) = \exp(\boldsymbol{A}\,(t-t_0))\,\boldsymbol{e}(t_0)$

And we get equation

$\frac{1}{2}\|\exp(\boldsymbol{A}\,(T-t_0))\,\boldsymbol{e}(t_0) - \boldsymbol{e}_{\text{target}}\|^2 = 0$

This second order equation has easy solution when we solve for $\boldsymbol{e}(t_0) = \boldsymbol{e}_0 + \boldsymbol{\Delta s}(t_0)$

$\boldsymbol{e}_0 + \boldsymbol{\Delta s}(t_0) = \exp(\boldsymbol{A}\,(T-t_0))^{-1}\,\boldsymbol{e}_{\text{target}}$

$\boldsymbol{\Delta s}(t_0) = \exp(\boldsymbol{A}\,(T-t_0))^{-1}\,\boldsymbol{e}_{\text{target}} - \boldsymbol{e}_0$

We assume starting point is always zero $t_0 = 0$ so the final target for stimulation is:

$\boldsymbol{\Delta s}(0) = \exp(\boldsymbol{A}\,T)^{-1}\,\boldsymbol{e}_{\text{target}} - \boldsymbol{e}_0$

After solving $\Delta\boldsymbol{s}$, **approx.** optimal stimulation pictures are scanned for best stimulus that has minimal distance to target delta.

The problem is that diff.eq. model is maybe WRONG. The difference is not maybe linearly dependent on current values of the points but only from the past difference (current slope of $\boldsymbol{e}$).

This means we can maybe extend measurements $\boldsymbol{e}(t)$ with extra slope variables

$\hat{\boldsymbol{e}}(t) = [\boldsymbol{e}(t), \Delta e(t)/\Delta t]$. These slope variables are maybe linearly related to differential equation variables.