

MATH 545 Intro to Time Series Final Project

ALAN AN

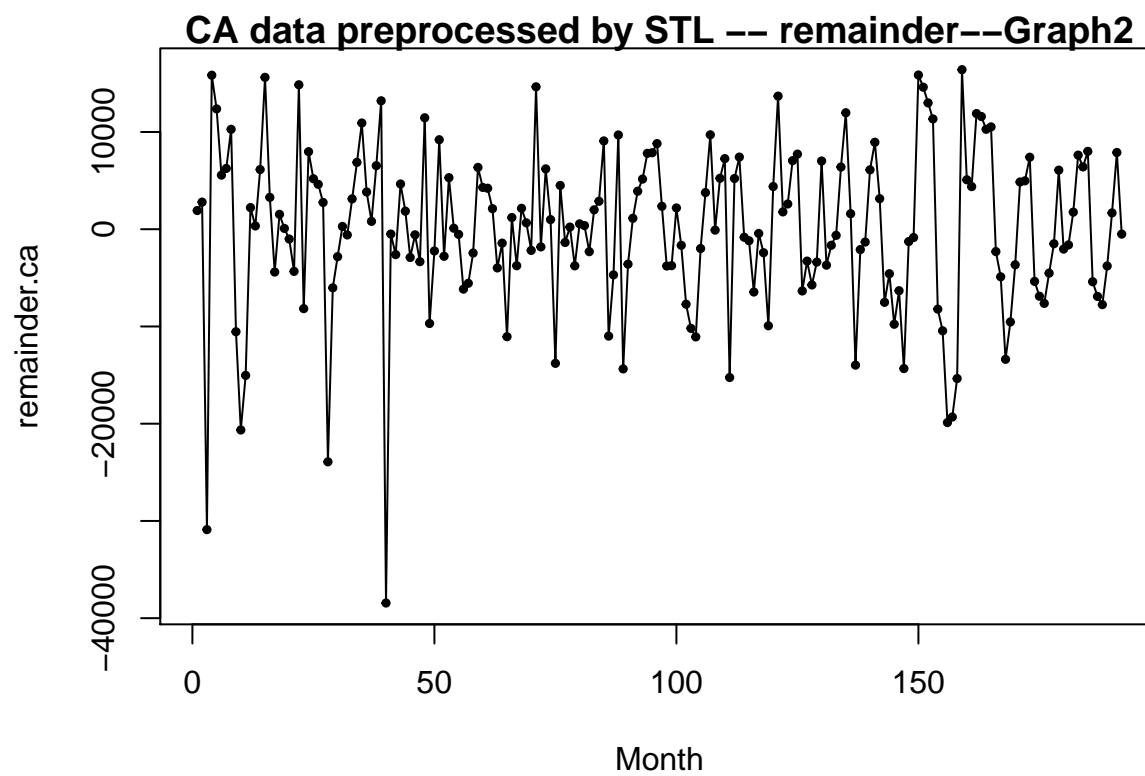
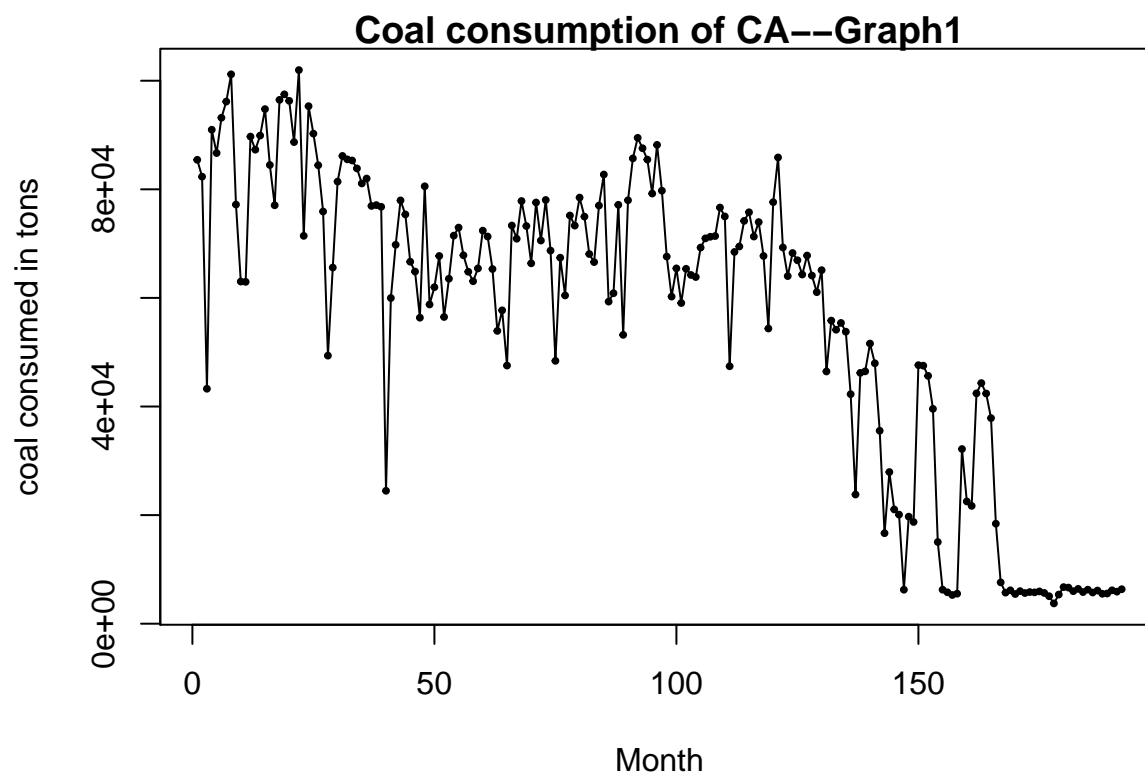
Background

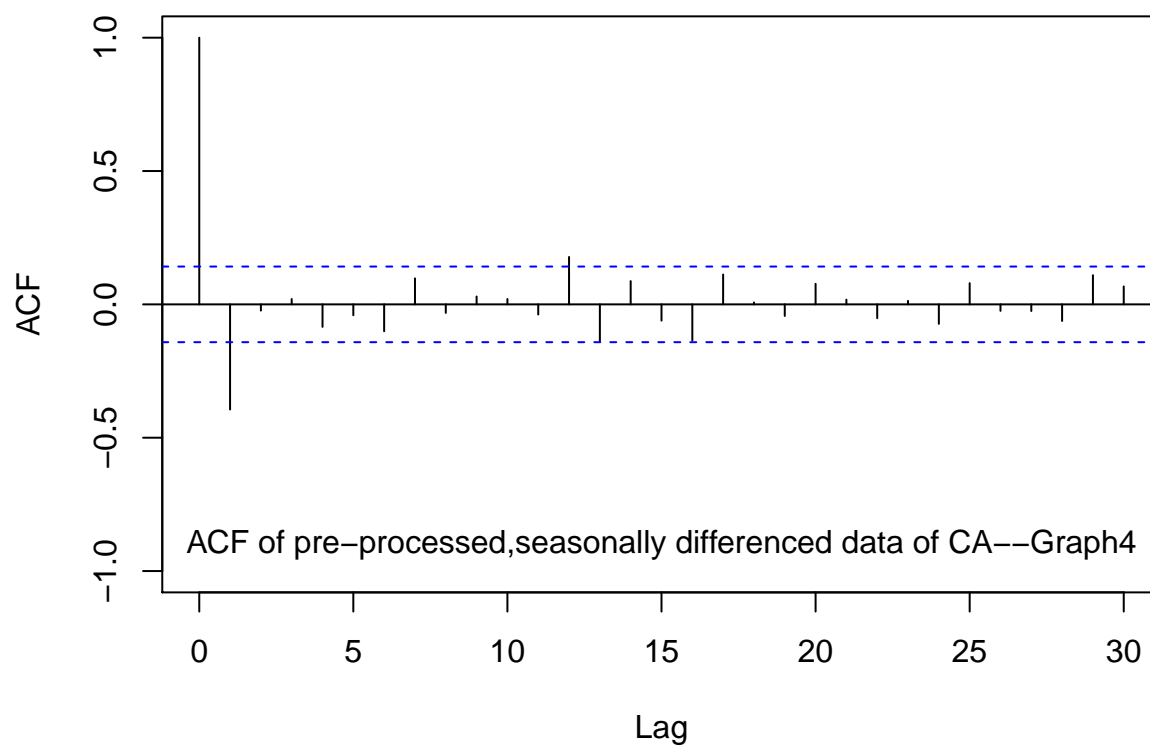
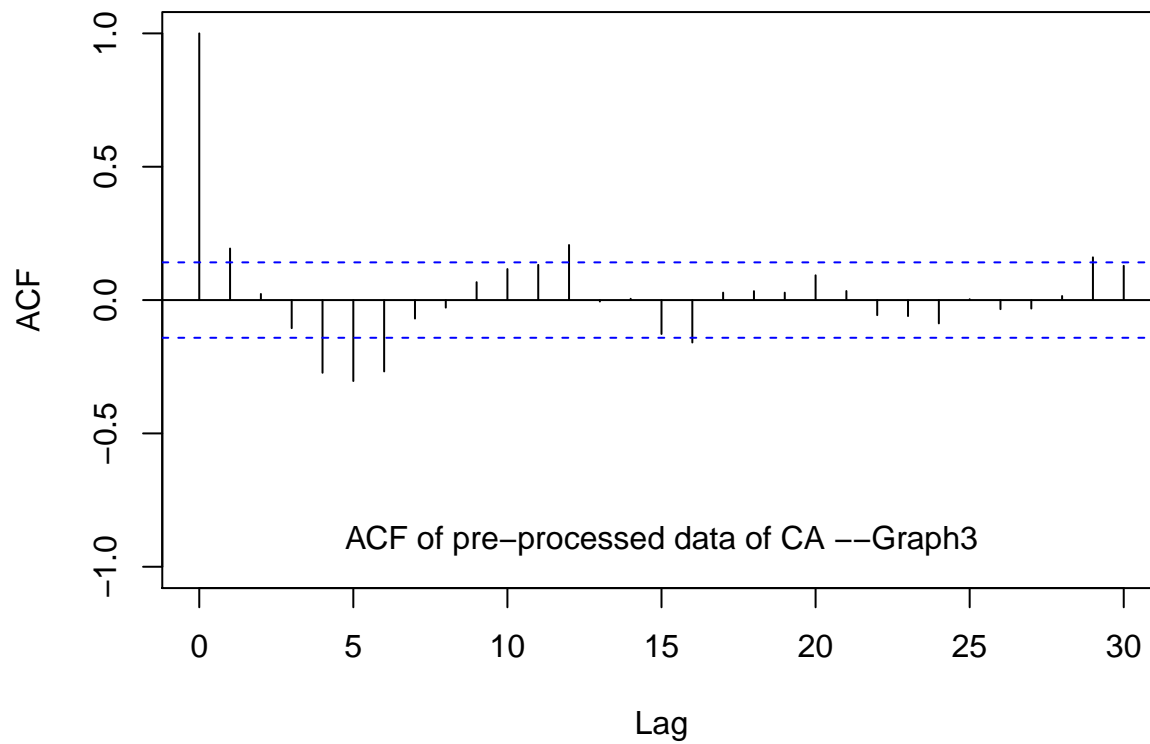
The data in the file USCoal-2016.csv contains monthly observations from January 2001 to December 2016 (192 observations) of the amount (in tons) of coal consumed in energy production in each of five US states (California, Florida, Illinois, North Carolina and Pennsylvania). Write a report on the analysis of these data sets, commenting on the following aspects:

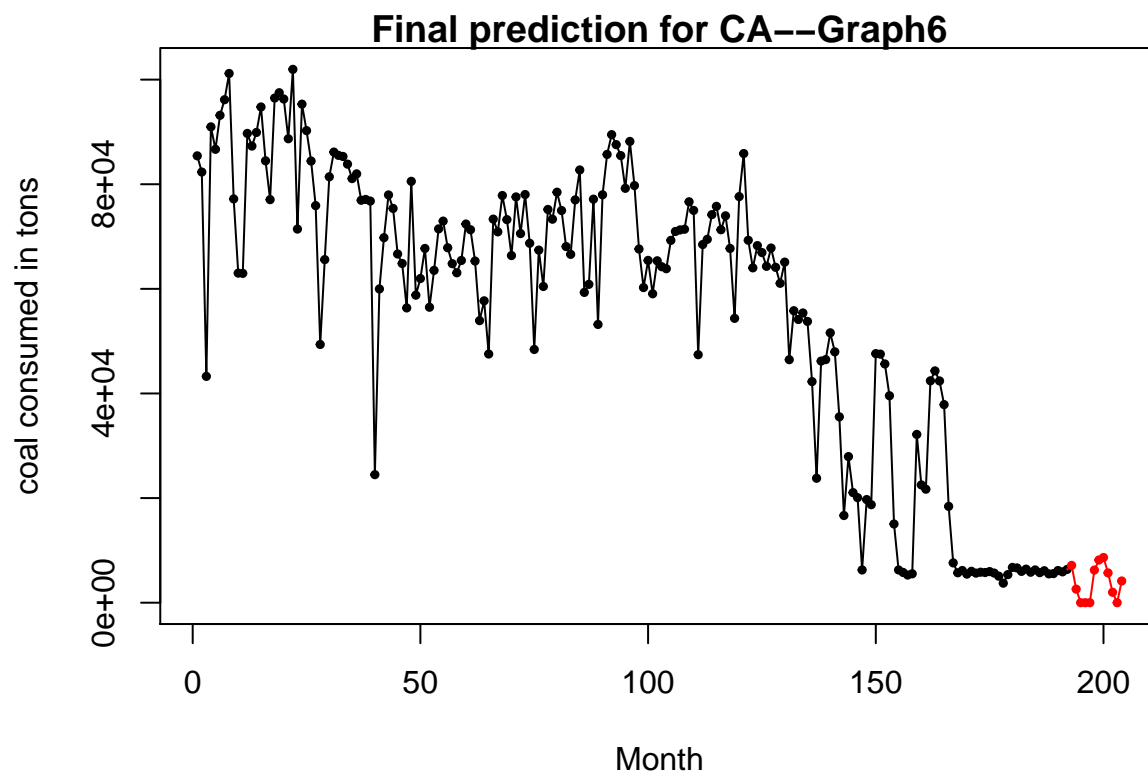
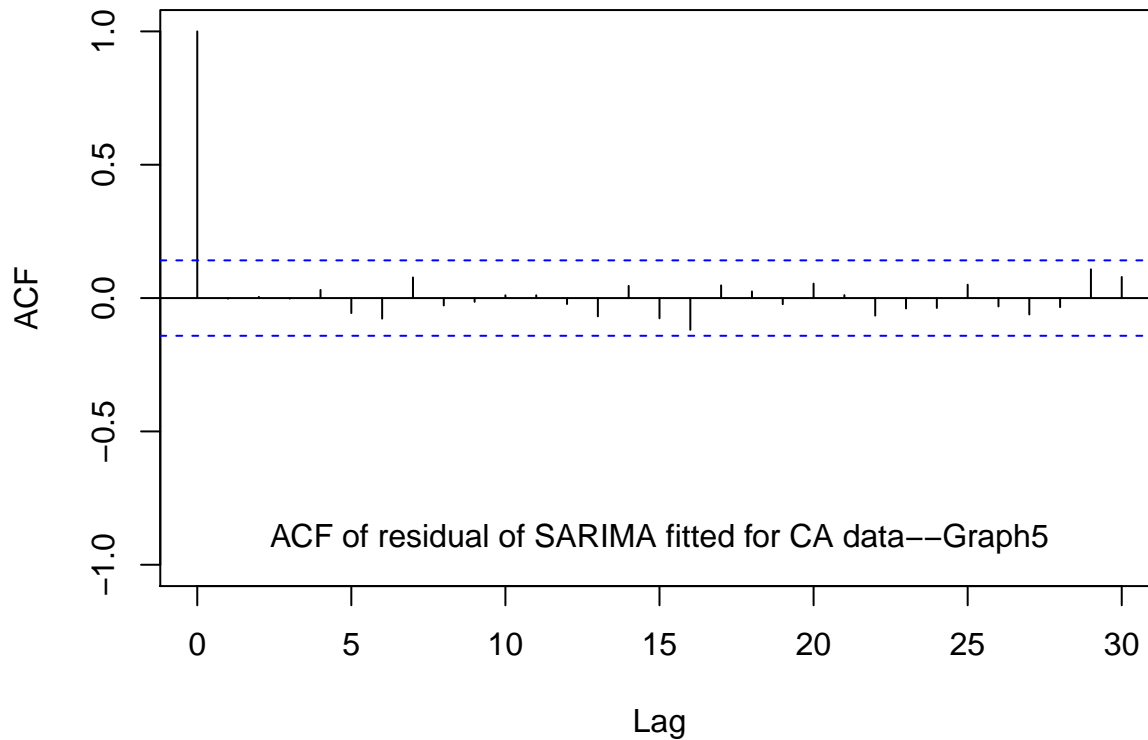
- the deterministic and stochastic time series components for each state;
- similarities and differences, in the data and proposed models, between states;
- constructed monthly forecasts for years 2017;

California

We try to analyze these data state by state. First we inspect the plot of the raw data(Graph1). We find that there is stochastically seasonal behaviour as well as a decreasing trend. Next we do the STL decomposition to remove the deterministic trend and seasonality. The remainder is shown in (Graph2). (Graph3) shows the ACF of the remainder series. We still see some seasonal component in the ACF plot (Graph3) thus we try differencing, with $d=1$ and produce (Graph4). The ACF of seasonally differenced data suggests that we can try a SARIMA model, namely with $s=12$, $d=1$, $Q=1$, (we see a spike at lag=12) $D=0$, $P=0$. We fitted different p, q from 0:7, 0:7 respectively, We found that $p, q=(4, 2)$. has the lowest AIC value. So, the final model we use is $SARIMA(4, 1, 2) \times (0, 0, 1)_{12}$. Then we inspect the residual of the fitted model. The ACF of the residuals is shown in Graph(5). We see that the model fits quite well as the residuals do seem like a white noise process. Finally, we plot the data along with the 12-month ahead predictions in (Graph 6). The predicted results is graphed in red, note that we have made an adjustment since some of the predicted value is below 0, which does not make sense in the current context, we therefore set those value to 0. Also note that we forecast the STL components separately. let $Y_t = M_t + S_t + X_t$ where M_t is the trend series, S_t is the seasonal series and X_t is the remainder series. We forecast \hat{X}_t with the SARIMA model and add it with \hat{M}_t and \hat{S}_t .



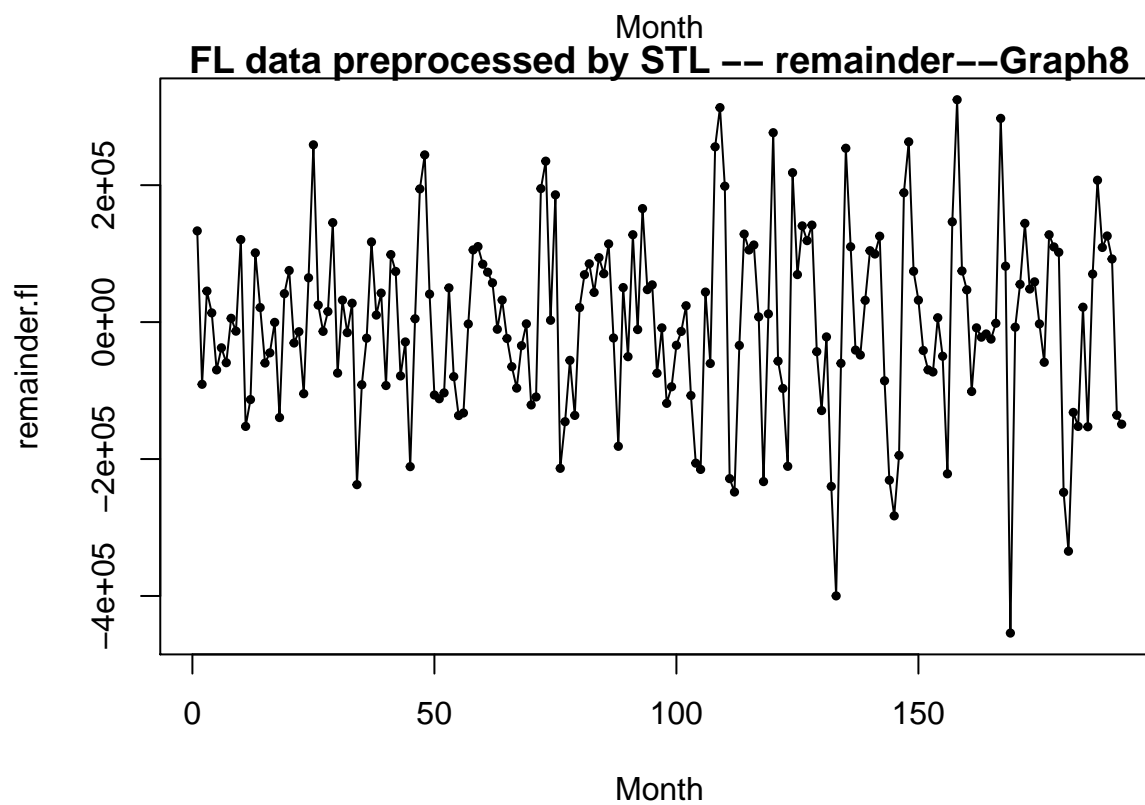
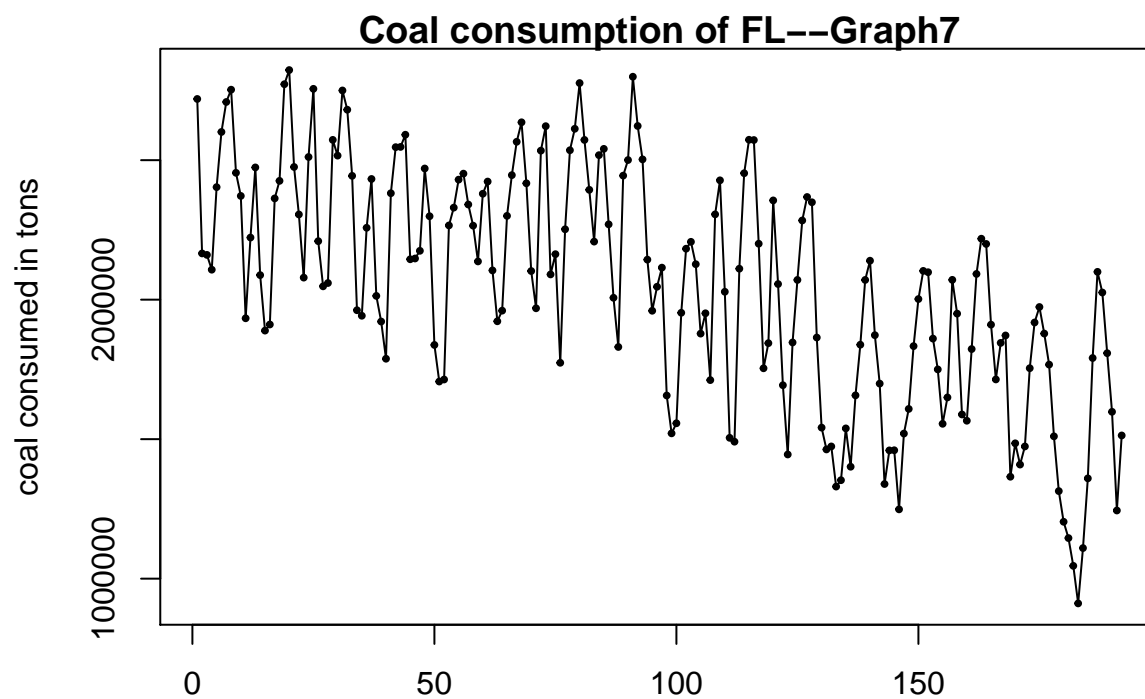


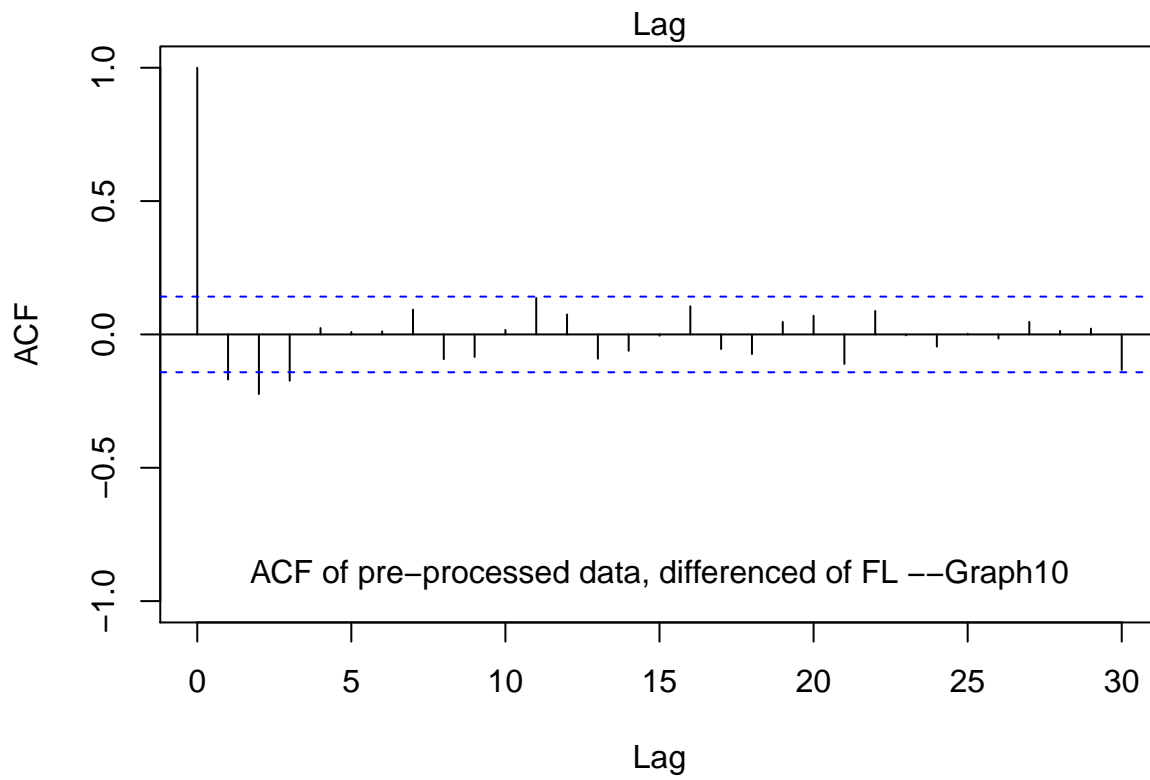
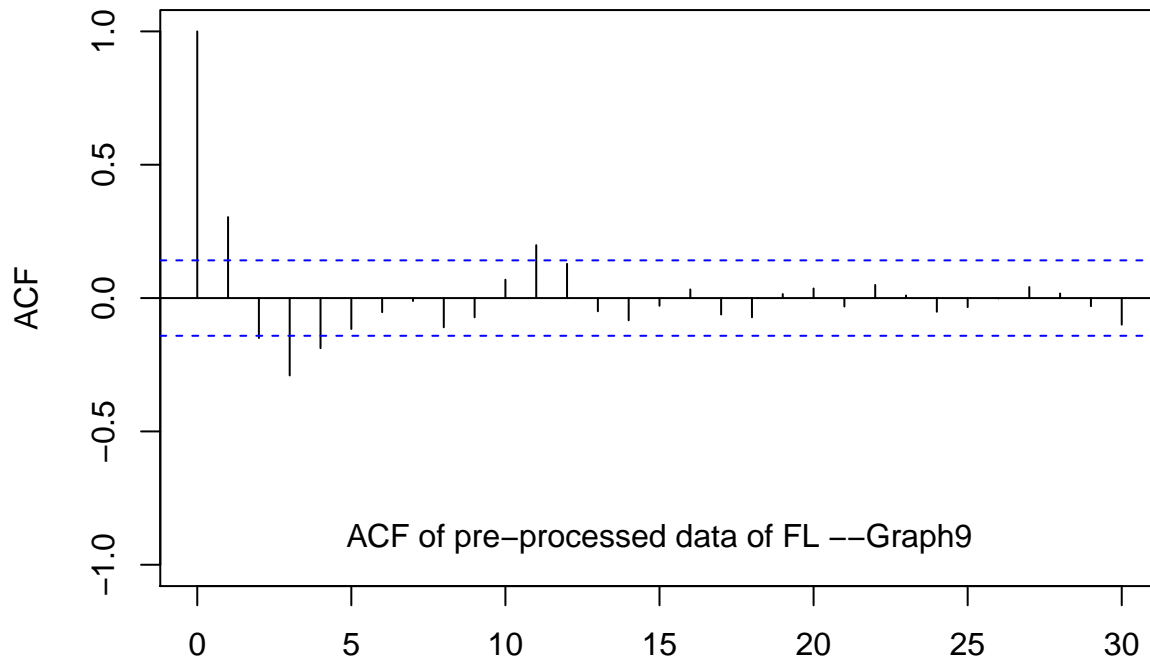


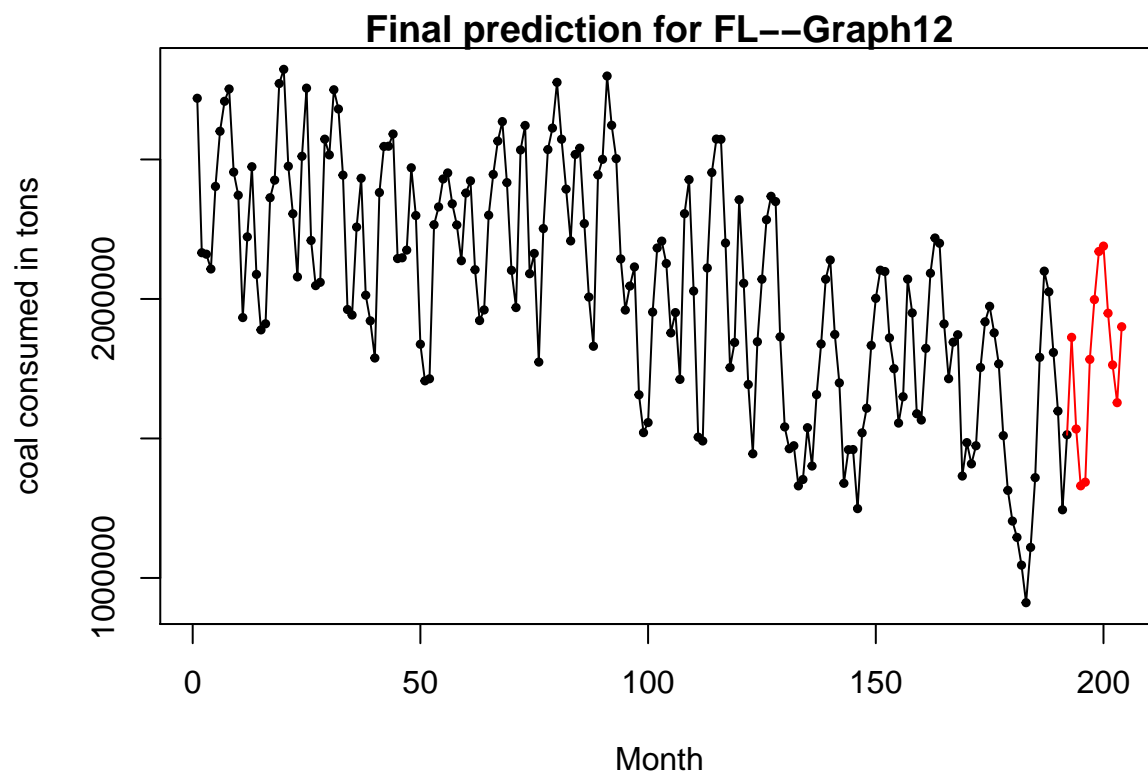
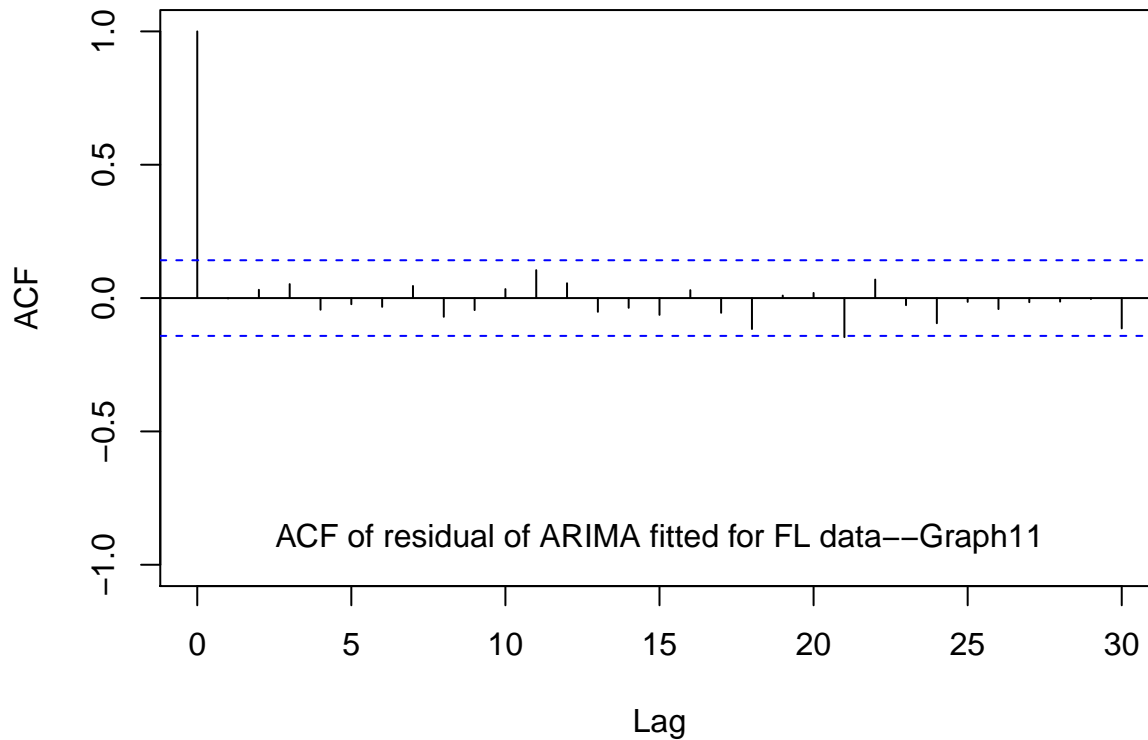
Florida

We follow a similar procedure. First we inspect Florida's raw data(Graph7). We find a decreasing trend as well as some seasonal behavior, we will need to check some ACF plots to determine whether there is stochastic seasonality or not. Next we use STL to do decomposition and plot the remainder series (Graph8) and the

ACF of the remainder series(Graph9). We see that there seems to be an increase in variance which indicates non-stationarity, We try to difference the remainder series with $d=1$ and plot the ACF of the differenced data(Graph10). We don't see any spike at lag=12. So we believe that stochastic seasonality is removed by differencing and thus an ARIMA model will suffice. We try different (p,q) vaules from 0:7 and 0:7 and find that (1,5) has the lowest AIC. We proceed to fit the model with ARIMA(1, 1, 5). We checked the ACF of the residuals(Graph 11). It looks like a white noise process. Finally we produce the 2017 Forecast for FL in (Graph 12), using the same method as we did for CA.



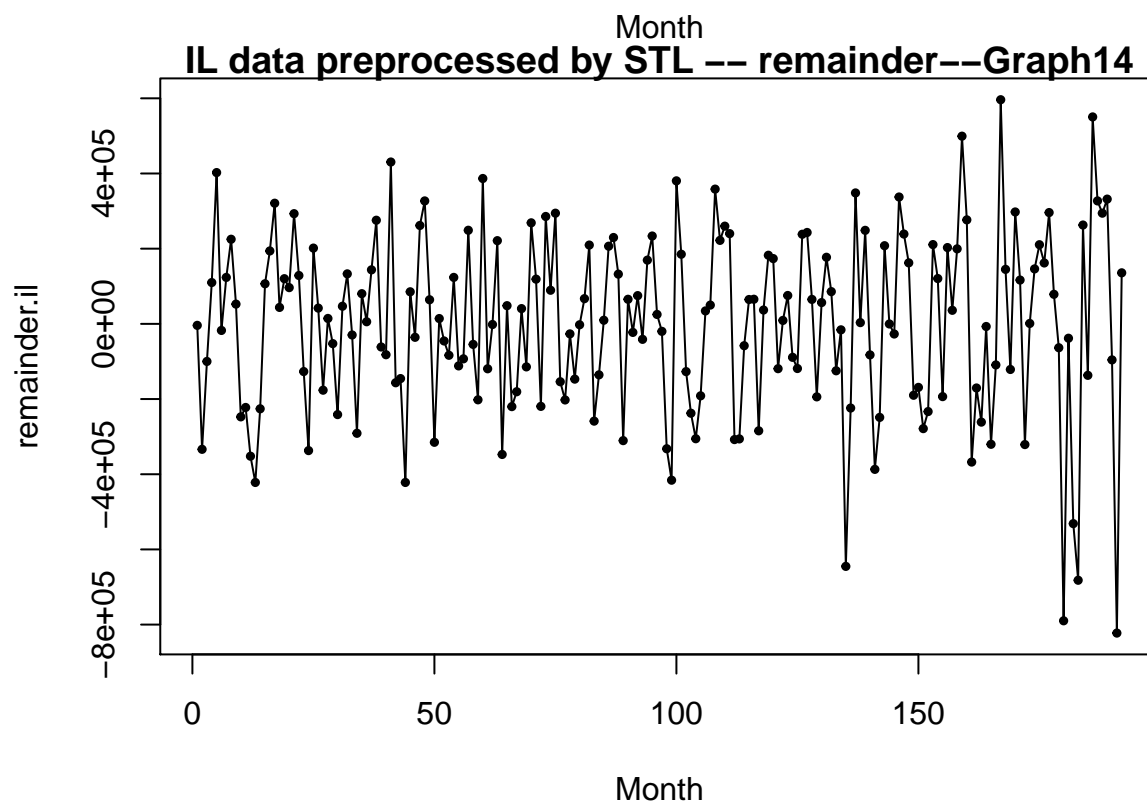
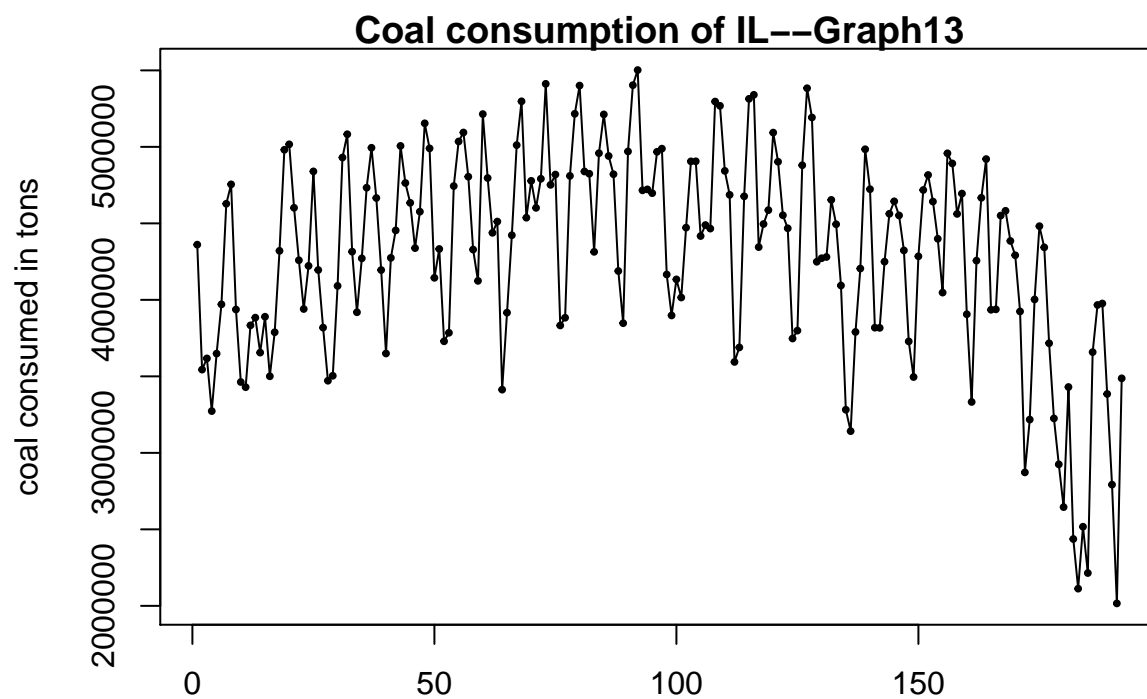


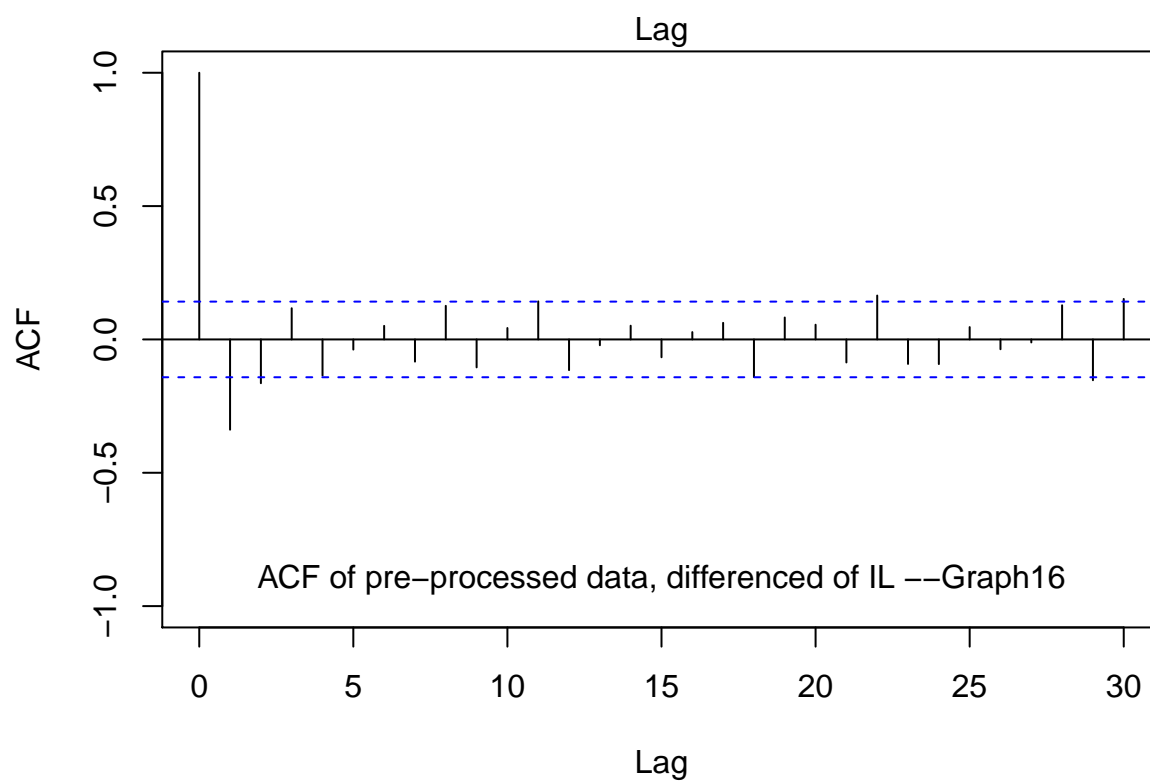
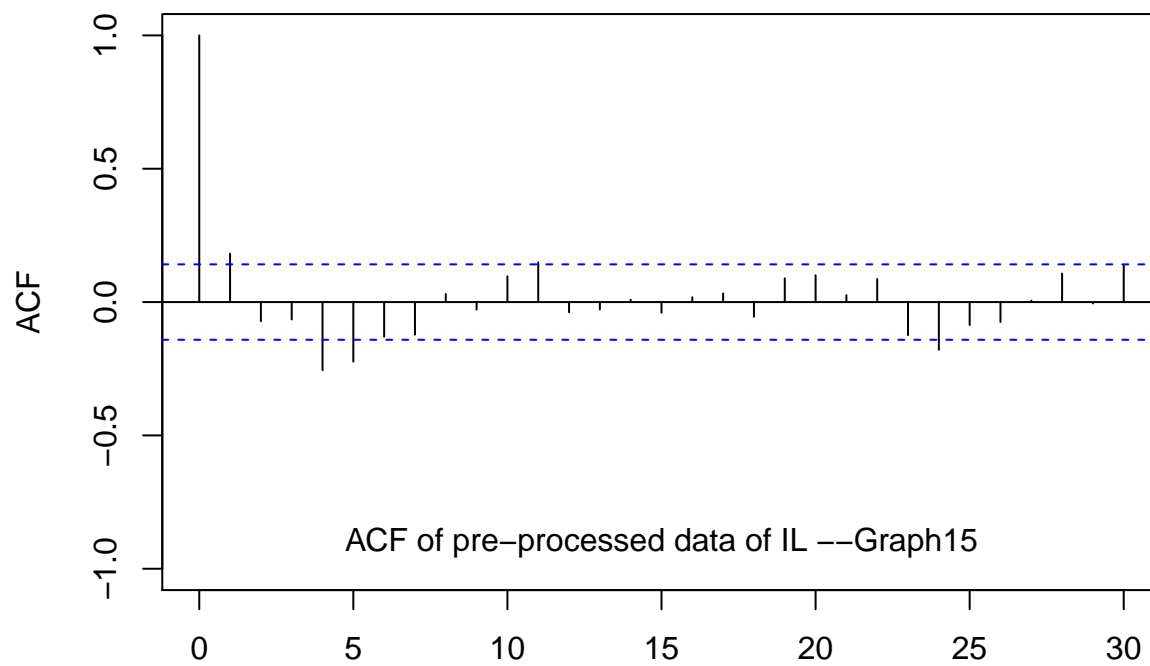


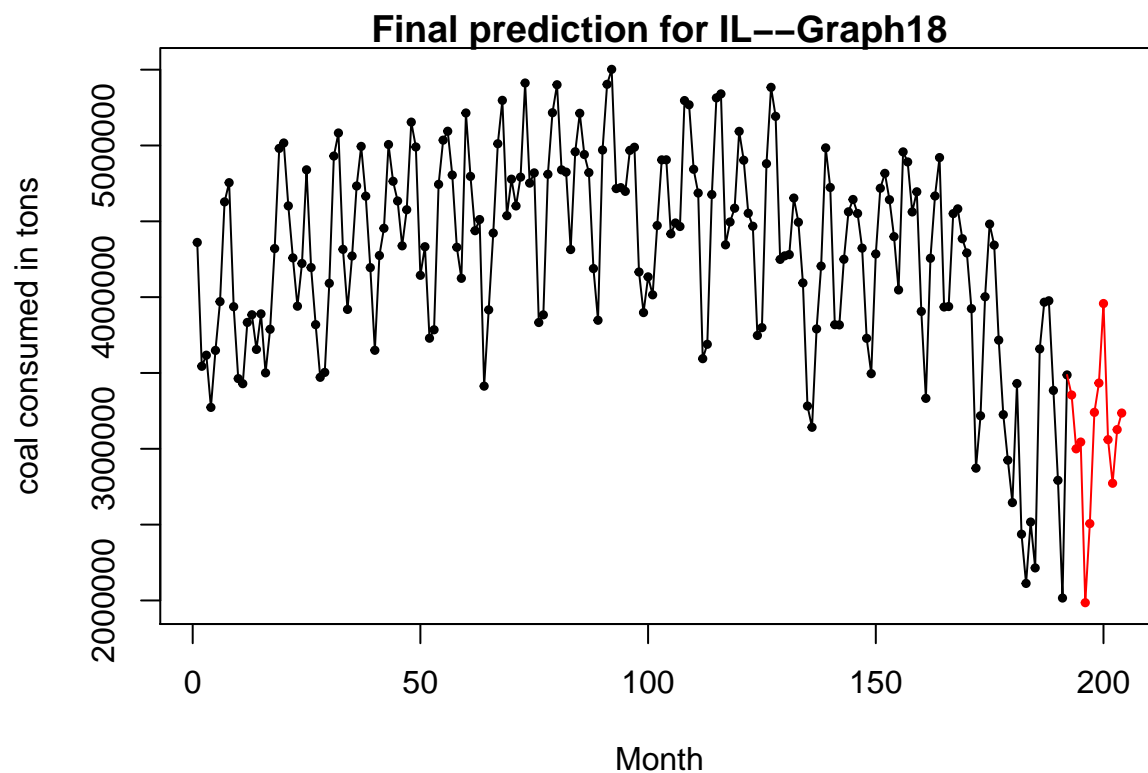
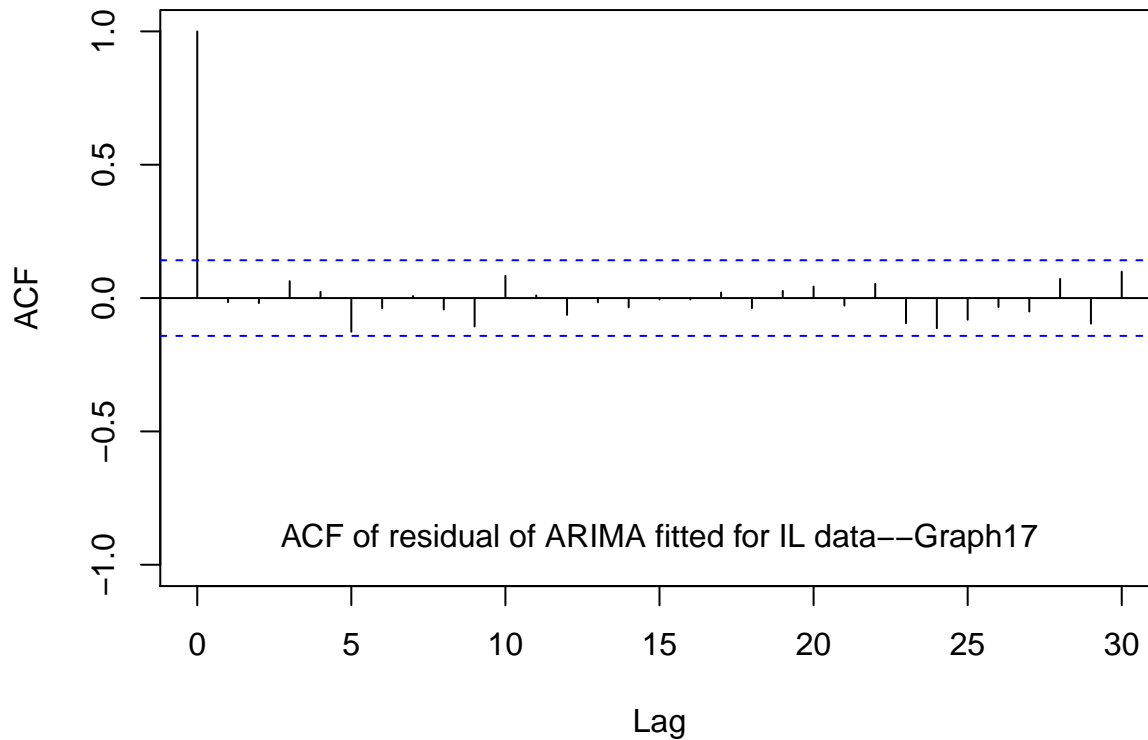
Illinois

First we inspect Illinois's raw data(Graph13). We find a decreasing trend as well as some seasonal behavior, we will need to check some ACF plots to determine whether there is stochastic seasonality or not. Next we use STL to do decomposition and plot the remainder series (Graph14) and the ACF of the remainder

series(Graph15). The data does not seem to have constant variance, thus we try to difference the remainder series with $d=1$, and plot the ACF of the differenced data(Graph16). After differencing, we see the stochastic seasonality seems to be removed. We decide to fit the Illinois data with an ARIMA model starting with $d=1$. We try different (p,q) vaules from 0:7 and 0:7 and find that $(4,5)$ has the lowest AIC. We proceed to fit the model with ARIMA(4, 1, 5). We check the ACF of the residuals(Graph 17). It looks like a white noise process. Finally we produce the 2017 Forecast for IL in (Graph 18), using the same method as we did for CA.



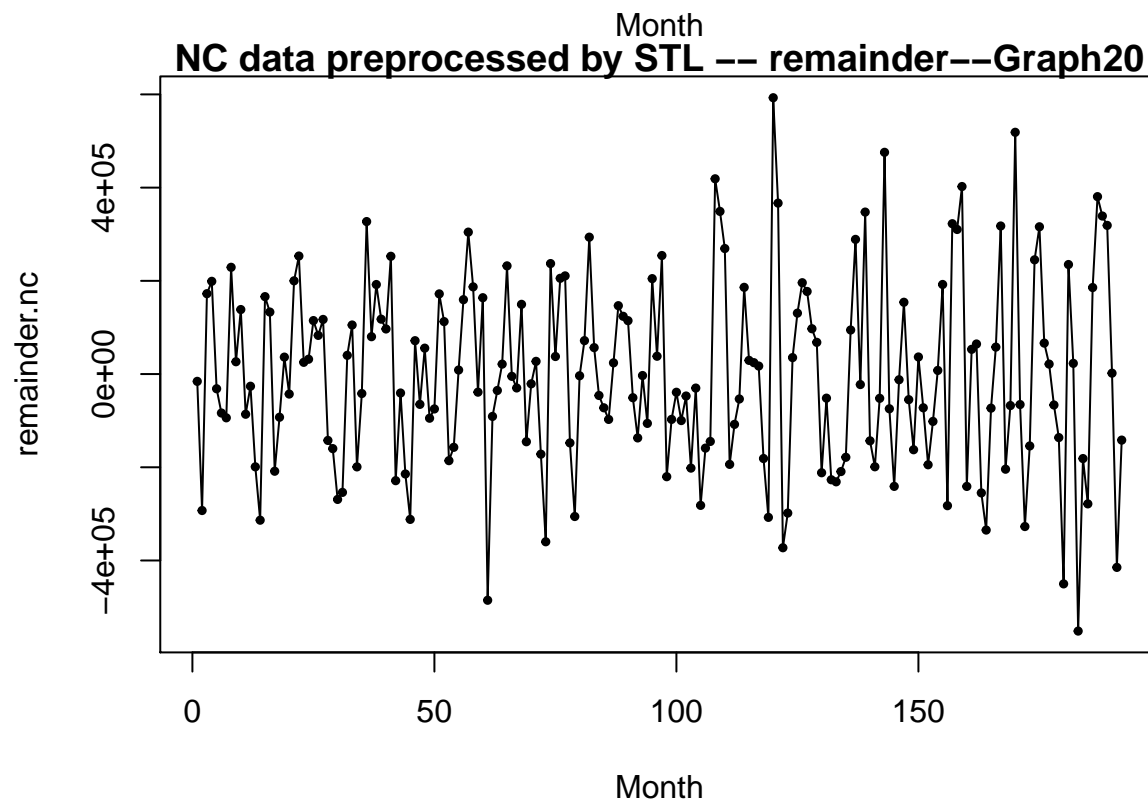
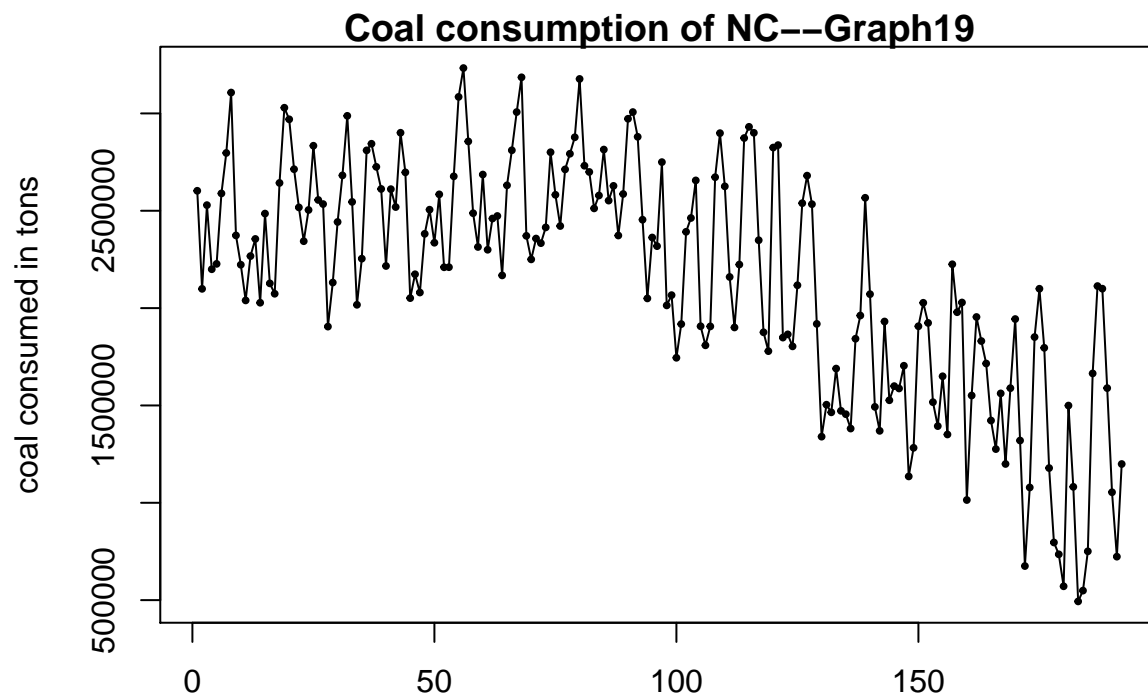


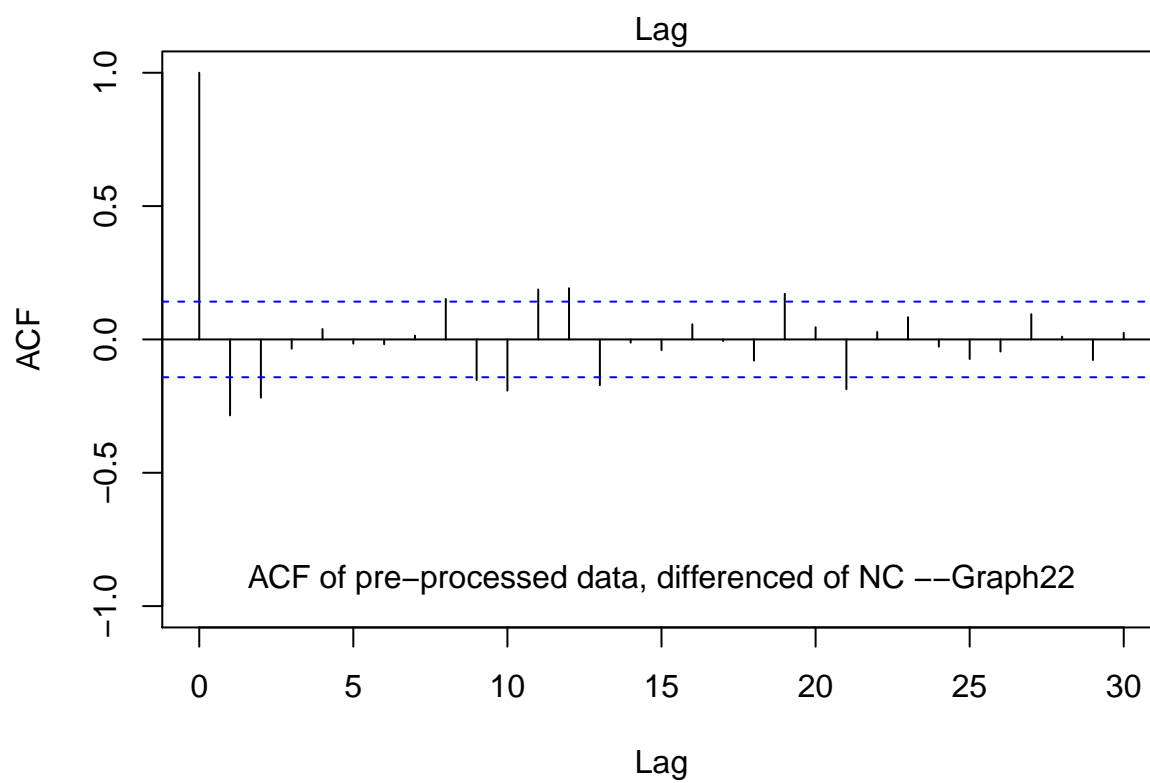
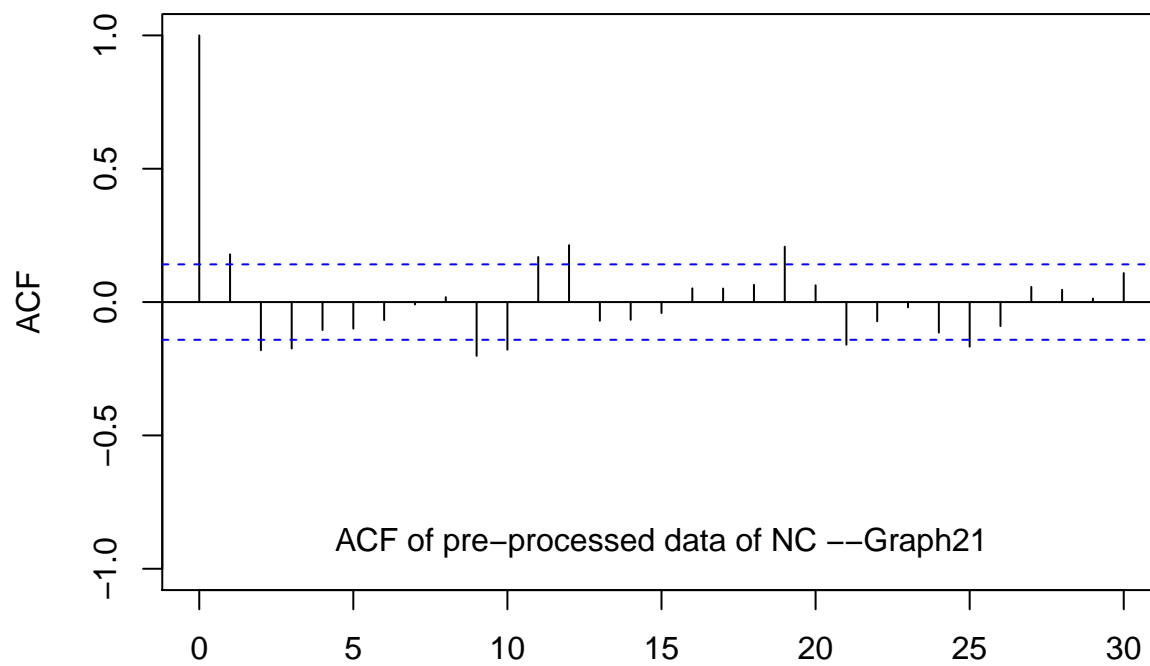


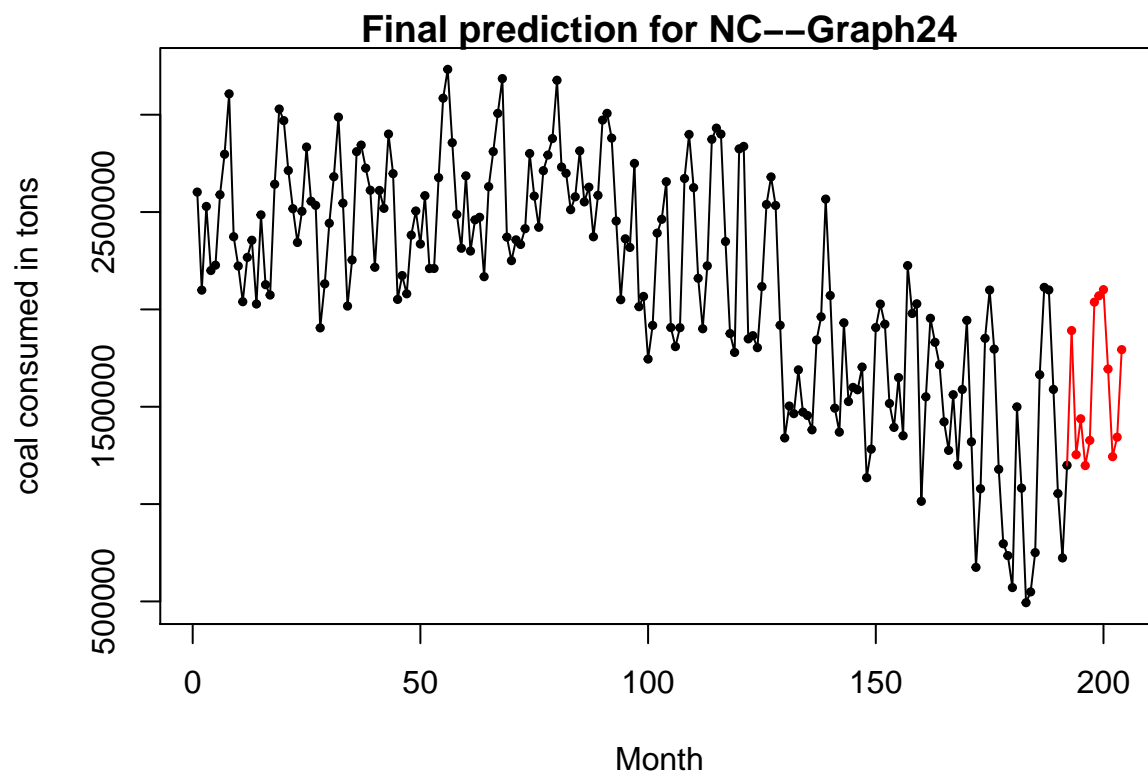
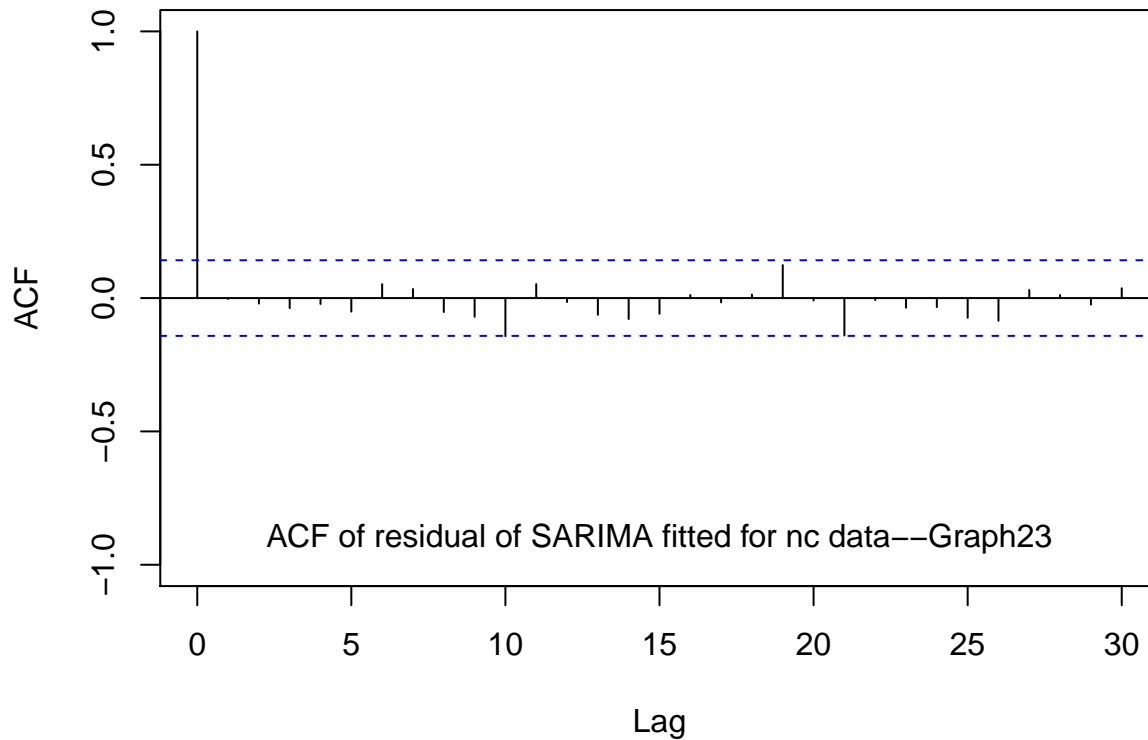
North Carolina

First we inspect North Carolina's raw data(Graph19). We find a decreasing trend as well as some seasonal behavior, we will need to check some ACF plots to determine whether there is stochastic seasonality or not. Next we use STL to do decomposition and plot the remainder series (Graph20) and the ACF of the remainder

series(Graph21). The data does not seem to have constant variance, thus we try to difference the remainder series with $d=1$ and plot the ACF of the differenced data(Graph22). The stochastic seasonality seems to be removed. The plot has a spike at lag=12, suggesting a SARIMA model with $s=12, P=0, Q=1, d=0, D=0, d=1$. We try different (p,q) vaules from 0:7 and 0:7 and find that $(7,6)$ has the lowest AIC. We proceed to fit the model with $SARIMA(7, 1, 6) \times (0, 0, 1)_{12}$. We check the ACF of the residuals(Graph 23). It looks like a white noise process. Finally we produce the 2017 Forecast for NC in (Graph 24), using the same method as we did for CA.



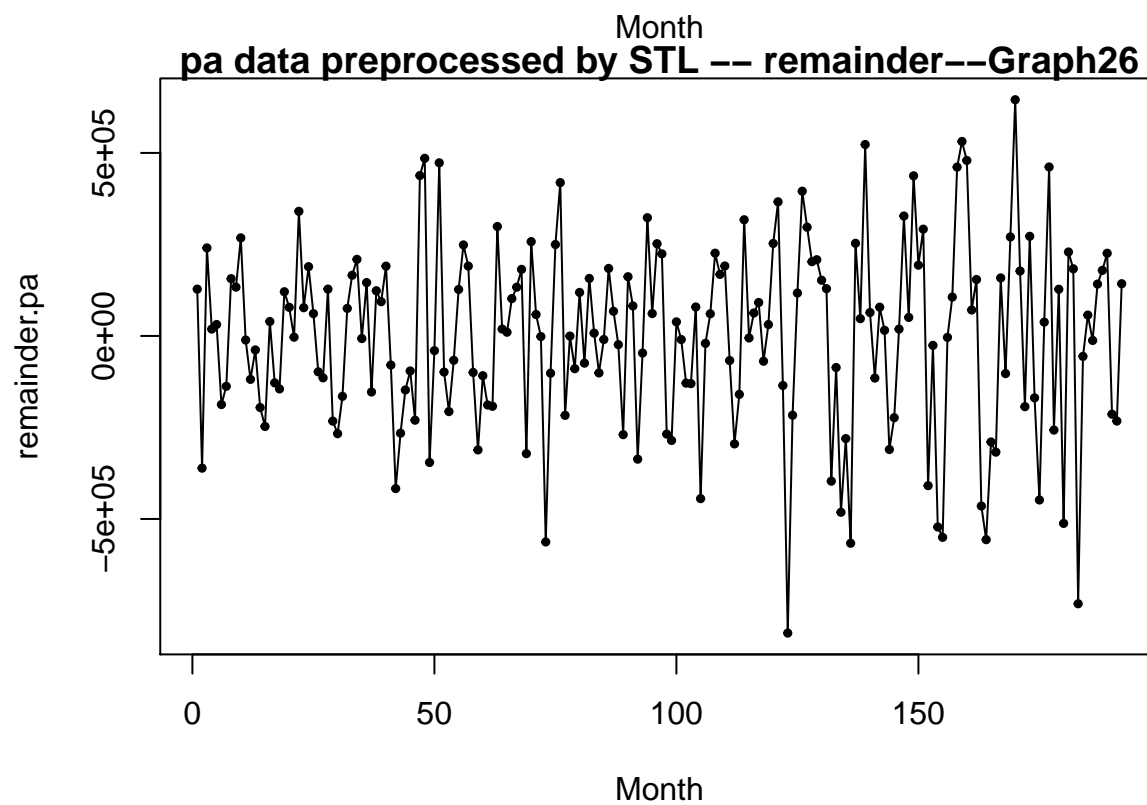
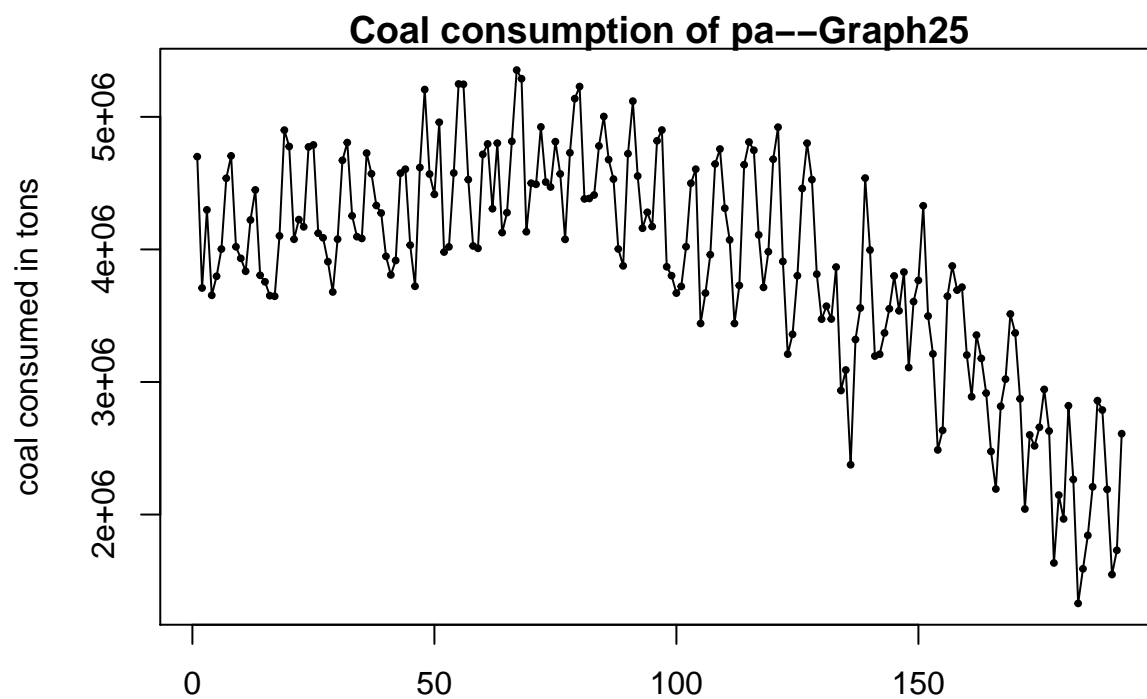


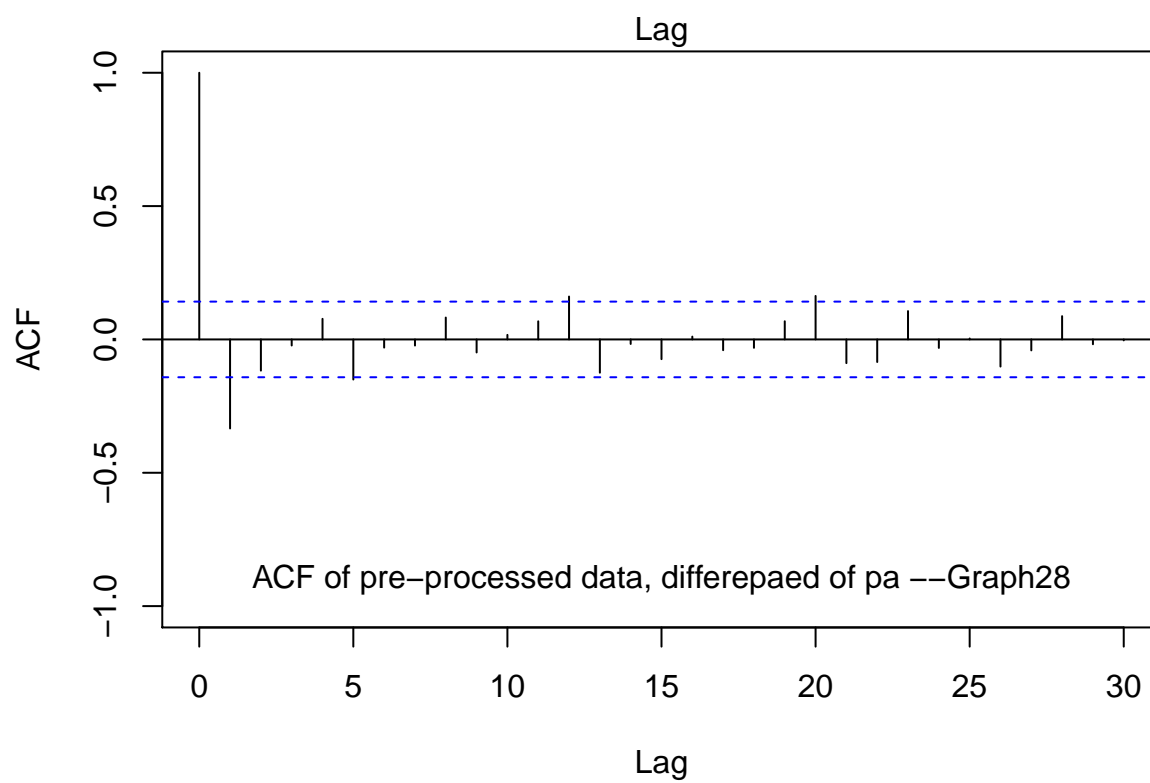
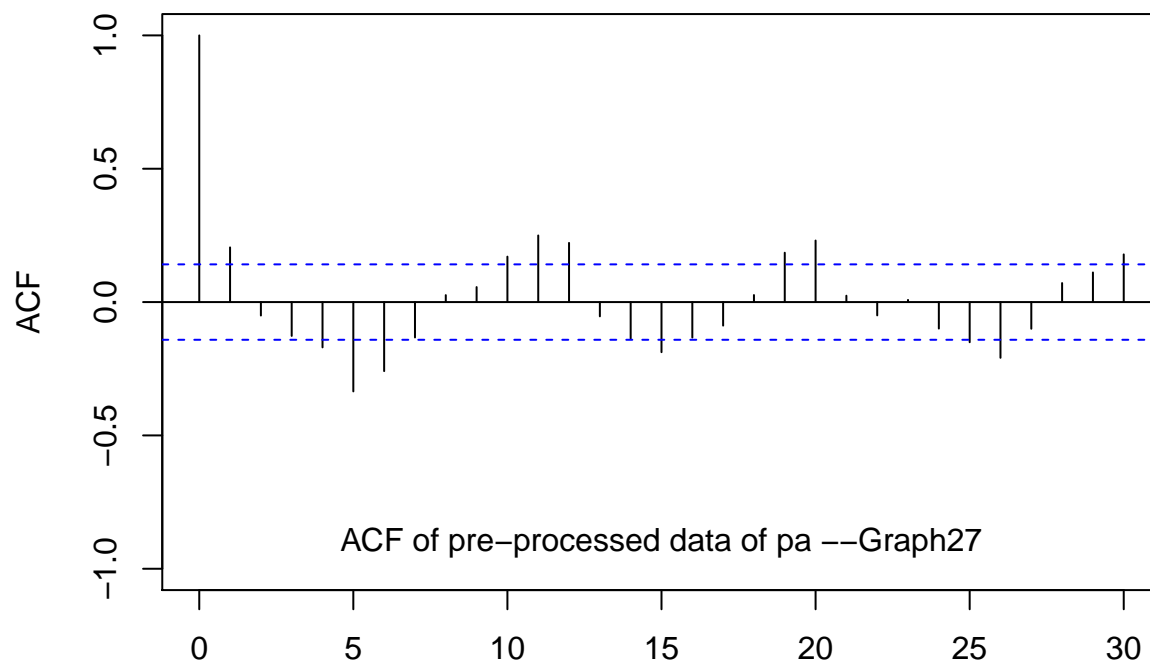


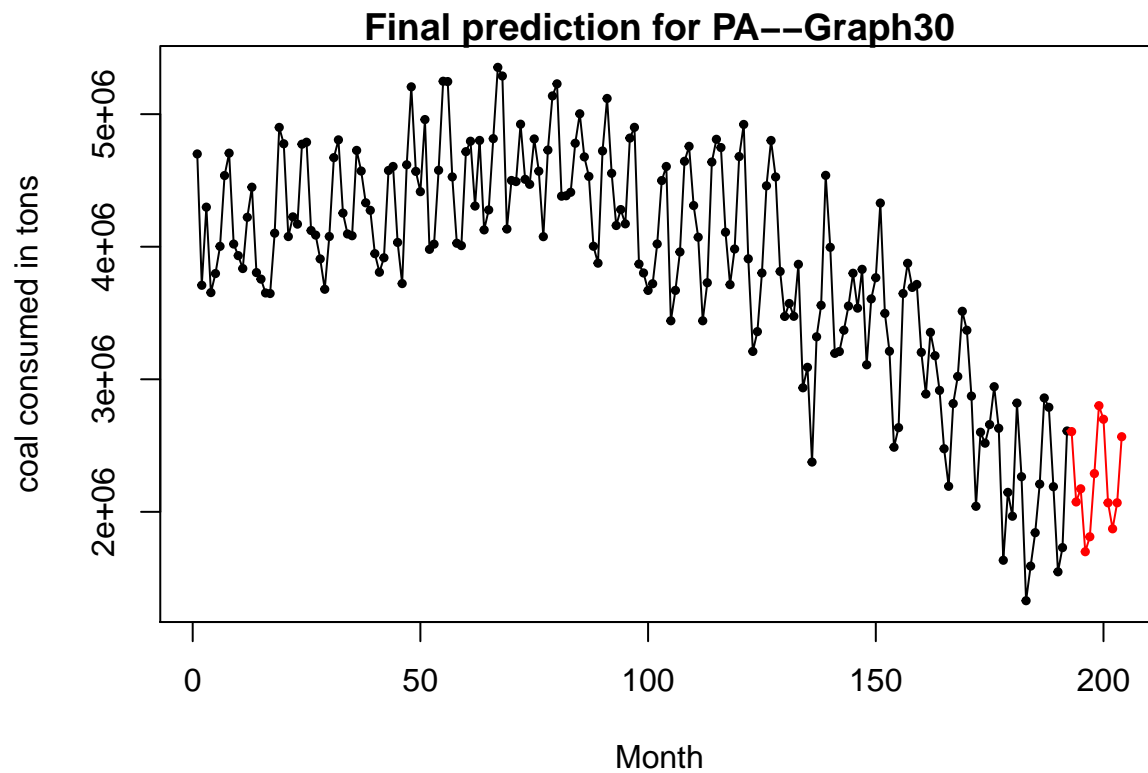
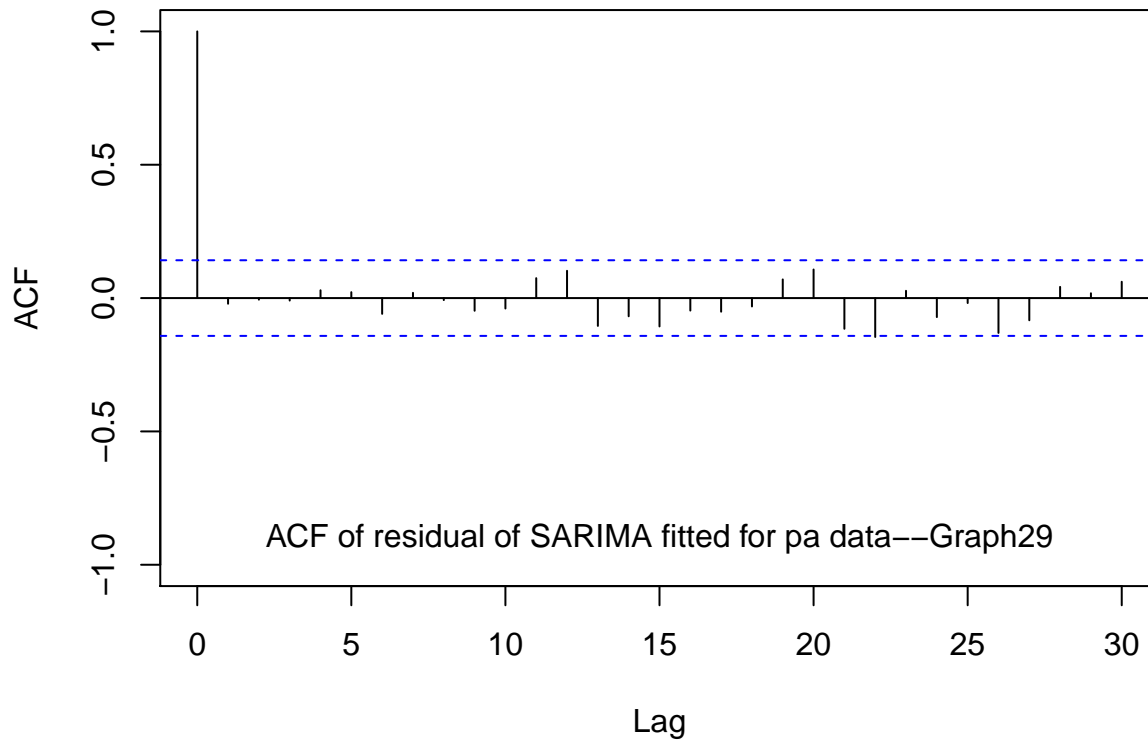
Pennsylvania

First we inspect Pennsylvania's raw data(Graph25). We find a decreasing trend as well as some seasonal behavior, we will need to check some ACF plots to determine whether there is stochastic seasonality or not. Next we use STL to do decomposition and plot the remainder series (Graph26) and the ACF of the remainder

series(Graph27). The data does not seem to have constant variance, thus we try to difference the remainder series with $d=1$ and plot the ACF of the differenced data(Graph28). The stochastic seasonality seems to be removed. The plot does not have a spike at lag=12, thus we think an ARIMA model with $d=1$ should suffice. We try different (p,q) values from 0:7 and 0:7 and find that (5,4) has the lowest AIC. We proceed to fit the model with ARIMA(5, 1, 4). We check the ACF of the residuals(Graph 29). It looks like a white noise process. Finally we produce the 2017 Forecast for PA in (Graph 30), using the same method as we did for CA.







In conclusion we have

CA: SARIMA(4, 1, 2) \times (0, 0, 1)₁₂ FL: ARIMA(1, 1, 5) IL: ARIMA(4, 1, 5)

NC: SARIMA(7, 1, 6) \times (0, 0, 1)₁₂ PA: ARIMA(5, 1, 4)

We see that all the models have integrated component d=1, that is all the stochastic seasonality can be

roughly accounted by differencing once at lag 1. In addition, we see all the data from 5 states can be modelled by relative small order of (p,q) of ARIMA/SARIMA models. However, in California and North Carolina, the coal consumption are more heavily affected by the stochastic seasonal component, thus we need to use SMA to account it. Moreover, we observe that all the data have decreasing trends. We inspected the STL graphs (not plotted in the final pdf) and find out the deterministic periodicity are quite similar among all the states. The assumption is that the coal consumption relates closely to the weather/seasons. For the trend component, we observe that California has the sharpest decreasing slope. We think this observation corresponds the fact that they have abundant solar energy in California and their advancement in solar panel technology helps them greatly to reduce the coal consumption.

APPENDIX

```

par(mar=c(4,4,1,0))
X.ca<-as.numeric(Y.ca)
plot(X.ca,pch=19,cex=0.4,xlab='Month',ylab="coal consumed in tons");lines(X.ca);title('Coal consumption

STL.ca<-stl(Y.ca,s.window = "periodic")
remainder.ca<-as.vector(STL.ca$time.series[, "remainder"])
trend.ca<-STL.ca$time.series[, "trend"]
seasonal.ca<-STL.ca$time.series[, "seasonal"]
par(mar=c(4,4,1,0))
plot(remainder.ca,pch=19,cex=.5,xlab="Month");lines(remainder.ca);title("CA data preprocessed by STL --
#plot(trend.ca,pch=19,cex=.5,xlab="Month");lines(trend.ca);title("California graph preprocessed by STL

par(mar=c(4,4,1,0))
acf(remainder.ca,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data of CA --Graph3')

par(mar=c(4,4,1,0))
differ.ca<-diff(remainder.ca,lag=1)
#differ.ca<-diff(differ.ca,lag=12)
#par(mar=c(1,1,1,1))
acf(differ.ca,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed,seasonally differenced data of CA--Graph4')

#AIC.mat<-BIC.mat<-matrix(0,8,8)
#for(p in 0:7){
#for(q in 0:7){
#fit.ca<-arima(remainder.ca,order=c(p,1,q),seasonal=list(order=c(0,0,1),period=12),
#include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
#AIC.mat[p+1,q+1]<-AIC(fit.ca)
#BIC.mat[p+1,q+1]<-BIC(fit.ca)
#}
#}
#rownames(AIC.mat)<-paste("p=",c(0:7),sep="");colnames(AIC.mat)<-paste("q=",c(0:7),sep="")
#round(AIC.mat-min(AIC.mat),3)
fit.ca<-arima(remainder.ca,order=c(4,1,2),seasonal=list(order=c(0,0,1),period=12),
include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
par(mar=c(4,4,1,0))
acf(residuals(fit.ca),ylim=range(-1,1),lag.max=30,main='')
text(15,-.9,'ACF of residual of SARIMA fitted for CA data--Graph5')

library("forecast")
t.ca<-forecast(trend.ca,h=12)

```

```

s.ca<-forecast(seasonal.ca,h=12)
m.ca<-forecast(fit.ca,h=12)
prediction.ca<-as.numeric(m.ca$mean)+as.numeric(t.ca$mean)+as.numeric(s.ca$mean)
# this step does the correction when the predicted value is below 0
for (i in 1:12){
  if(prediction.ca[i]<0)
    prediction.ca[i]=0
}

```

```

n<-192
par(mar=c(4,4,1,0))
plot(1:n,X.ca,type='l',xlim=range(1:(n+12)),ylim=range(c(X.ca,prediction.ca)), xlab='Month',ylab="coal consumed in tons",
points(1:n,X.ca,pch=19,cex=0.5)
lines(c(n,n+1),c(X.ca[n],prediction.ca[1]),col='red')
lines((n+1):(n+12),prediction.ca,col='red')
points((n+1):(n+12),prediction.ca,pch=19,cex=0.5,col='red')

```

```

par(mar=c(4,4,1,0))
X.fl<-as.numeric(Y.fl)
plot(X.fl,pch=19,cex=0.4,xlab='Month',ylab="coal consumed in tons");lines(X.fl);title('Coal consumption in tons')
STL.fl<-stl(Y.fl,s.window = "periodic")
remainder.fl<-as.vector(STL.fl$time.series[, "remainder"])
trend.fl<-STL.fl$time.series[, "trend"]
seasonal.fl<-STL.fl$time.series[, "seasonal"]
par(mar=c(4,4,1,0))
plot(remainder.fl,pch=19,cex=.5,xlab="Month");lines(remainder.fl);title("FL data preprocessed by STL --Graph9")
par(mar=c(4,4,1,0))
acf(remainder.fl,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data of FL --Graph9')
remainder.fl<-diff(remainder.fl,lag=1)
acf(remainder.fl,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data, differenced of FL --Graph10')

```

```

#AIC.mat<-BIC.mat<-matrix(0,8,8)
#for(p in 0:7){
#for(q in 0:7){
#fit.fl<-arima(remainder.fl,order=c(p,1,q),
#include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
#AIC.mat[p+1,q+1]<-AIC(fit.fl)
#}
#}
#rownames(AIC.mat)<-paste("p=",c(0:7),sep="");colnames(AIC.mat)<-paste("q=",c(0:7),sep="")
#round(AIC.mat-min(AIC.mat),3)

```

```

fit.fl<-arima(remainder.fl,order=c(1,1,5),
include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
par(mar=c(4,4,1,0))
acf(residuals(fit.fl),ylim=range(-1,1),lag.max=30,main='')
text(15,-.9,'ACF of residual of ARIMA fitted for FL data--Graph11')

```

```

t.fl<-forecast(trend.fl,h=12)
s.fl<-forecast(seasonal.fl,h=12)
m.fl<-forecast(fit.fl,h=12)

```

```

prediction.fl<-as.numeric(m.fl$mean)+as.numeric(t.fl$mean)+as.numeric(s.fl$mean)
# this step does the correction when the predicted value is below 0
for (i in 1:12){
  if(prediction.fl[i]<0)
    prediction.fl[i]=0
}

n<-192
par(mar=c(4,4,1,0))
plot(1:n,X.fl,type='l',xlim=range(1:(n+12)),ylim=range(c(X.fl,prediction.fl)), xlab='Month',ylab="coal consumed in tons",pch=19,cex=0.5)
points(1:n,X.fl,pch=19,cex=0.5)
lines(c(n,n+1),c(X.fl[n],prediction.fl[1]),col='red')
lines((n+1):(n+12),prediction.fl,col='red')
points((n+1):(n+12),prediction.fl,pch=19,cex=0.5,col='red')

par(mar=c(4,4,1,0))
X.il<-as.numeric(Y.il)
plot(X.il,pch=19,cex=0.4,xlab='Month',ylab="coal consumed in tons");lines(X.il);title('Coal consumption in tons')
STL.il<-stl(Y.il,s.window = "periodic")

remainder.il<-as.vector(STL.il$time.series[, "remainder"])
trend.il<-STL.il$time.series[, "trend"]
seasonal.il<-STL.il$time.series[, "seasonal"]
par(mar=c(4,4,1,0))
plot(remainder.il,pch=19,cex=.5,xlab="Month");lines(remainder.il);title("IL data preprocessed by STL --Graph15")
par(mar=c(4,4,1,0))
acf(remainder.il,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data of IL --Graph15')
remainder.il<-diff(remainder.il,lag=1)
#remainder.il<-diff(remainder.il,lag=12)
acf(remainder.il,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data, differenced of IL --Graph16')

#AIC.mat<-BIC.mat<-matrix(0,8,8)
#for(p in 0:7){
#for(q in 0:7){
#fit.il<-arima(remainder.il,order=c(p,1,q),
#include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
#AIC.mat[p+1,q+1]<-AIC(fit.il)

#}
#}
#rownames(AIC.mat)<-paste("p=",c(0:7),sep="");colnames(AIC.mat)<-paste("q=",c(0:7),sep="")
#round(AIC.mat-min(AIC.mat),3)

fit.il<-arima(remainder.il,order=c(4,1,5),
include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
par(mar=c(4,4,1,0))
acf(residuals(fit.il),ylim=range(-1,1),lag.max=30,main='')
text(15,-.9,'ACF of residual of ARIMA fitted for IL data--Graph17')

t.il<-forecast(trend.il,h=12)
s.il<-forecast(seasonal.il,h=12)
m.il<-forecast(fit.il,h=12)

```

```

prediction.il<-as.numeric(m.il$mean)+as.numeric(t.il$mean)+as.numeric(s.il$mean)
# this step does the correction when the predicted value is below 0
for (i in 1:12){
  if(prediction.il[i]<0)
    prediction.il[i]=0
}

n<-192
par(mar=c(4,4,1,0))
plot(1:n,X.il,type='l',xlim=range(1:(n+12)),ylim=range(c(X.il,prediction.il)), xlab='Month',ylab="coal consumed in tons",pch=19,cex=0.5)
points(1:n,X.il,pch=19,cex=0.5)
lines(c(n,n+1),c(X.il[n],prediction.il[1]),col='red')
lines((n+1):(n+12),prediction.il,col='red')
points((n+1):(n+12),prediction.il,pch=19,cex=0.5,col='red')

par(mar=c(4,4,1,0))
X.nc<-as.numeric(Y.nc)
plot(X.nc,pch=19,cex=0.4,xlab='Month',ylab="coal consumed in tons");lines(X.nc);title('Coal consumption in tons over time')

STL.nc<-stl(Y.nc,s.window = "periodic")
remainder.nc<-as.vector(STL.nc$time.series[, "remainder"])
trend.nc<-STL.nc$time.series[, "trend"]
seasonal.nc<-STL.nc$time.series[, "seasonal"]
par(mar=c(4,4,1,0))
plot(remainder.nc,pch=19,cex=.5,xlab="Month");lines(remainder.nc);title("NC data preprocessed by STL --Graph20")
par(mar=c(4,4,1,0))
acf(remainder.nc,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data of NC --Graph21')
remainder.nc<-diff(remainder.nc,lag=1)
#remainder.nc<-diff(remainder.nc,lag=12)
acf(remainder.nc,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data, differenced of NC --Graph22')

#AIC.mat<-BIC.mat<-matrix(0,8,8)
#for(p in 0:7){
#for(q in 0:7){
#fit.nc<-arima(remainder.nc,order=c(p,1,q),seasonal=list(order=c(0,0,1),period=12),
#include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
#AIC.mat[p+1,q+1]<-AIC(fit.nc)

#}
#}
#rownames(AIC.mat)<-paste("p=",c(0:7),sep="");colnames(AIC.mat)<-paste("q=",c(0:7),sep="")
#round(AIC.mat-min(AIC.mat),3)

fit.nc<-arima(remainder.nc,order=c(7,1,6),seasonal=list(order=c(0,0,1),period=12),
include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
par(mar=c(4,4,1,0))
acf(residuals(fit.nc),ylim=range(-1,1),lag.max=30,main='')
text(15,-.9,'ACF of residual of SARIMA fitted for nc data--Graph23')

t.nc<-forecast(trend.nc,h=12)
s.nc<-forecast(seasonal.nc,h=12)
m.nc<-forecast(fit.nc,h=12)

```



```

prediction.nc<-as.numeric(m.nc$mean)+as.numeric(t.nc$mean)+as.numeric(s.nc$mean)
# this step does the correction when the predicted value is below 0
for (i in 1:12){
  if(prediction.nc[i]<0)
    prediction.nc[i]=0
}

```

```

n<-192
par(mar=c(4,4,1,0))
plot(1:n,X.nc,type='l',xlim=range(1:(n+12)),ylim=range(c(X.nc,prediction.nc)), xlab='Month',ylab="coal consumed in tons")
points(1:n,X.nc,pch=19,cex=0.5)
lines(c(n,n+1),c(X.nc[n],prediction.nc[1]),col='red')
lines((n+1):(n+12),prediction.nc,col='red')
points((n+1):(n+12),prediction.nc,pch=19,cex=0.5,col='red')

```

```

par(mar=c(4,4,1,0))
X.pa<-as.numeric(Y.pa)
plot(X.pa,pch=19,cex=0.4,xlab='Month',ylab="coal consumed in tons");lines(X.pa);title('Coal consumption in tons')
STL.pa<-stl(Y.pa,s.window = "periodic")
remainder.pa<-as.vector(STL.pa$time.series[, "remainder"])
trend.pa<-STL.pa$time.series[, "trend"]
seasonal.pa<-STL.pa$time.series[, "seasonal"]
par(mar=c(4,4,1,0))
plot(remainder.pa,pch=19,cex=.5,xlab="Month");lines(remainder.pa);title("pa data preprocessed by STL --Graph26")
par(mar=c(4,4,1,0))
acf(remainder.pa,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data of pa --Graph27')
remainder.pa<-diff(remainder.pa,lag=1)
#remainder.pa<-diff(remainder.pa,lag=12)
acf(remainder.pa,lag.max=30,ylim=range(-1,1),main='')
text(15,-0.9,'ACF of pre-processed data, differenced of pa --Graph28')

```

```

#AIC.mat<-BIC.mat<-matrix(0,8,8)
#for(p in 0:7){
#for(q in 0:7){
#fit.pa<-arima(remainder.pa,order=c(p,1,q),
#include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
#AIC.mat[p+1,q+1]<-AIC(fit.pa)
#}
#}
#rownames(AIC.mat)<-paste("p=",c(0:7),sep="");colnames(AIC.mat)<-paste("q=",c(0:7),sep="")
#round(AIC.mat-min(AIC.mat),3)

```

```

fit.pa<-arima(remainder.pa,order=c(5,1,4),
include.mean=FALSE,method='ML',optim.control=list(maxit=1000))
par(mar=c(4,4,1,0))
acf(residuals(fit.pa),ylim=range(-1,1),lag.max=30,main='')
text(15,-.9,'ACF of residual of SARIMA fitted for pa data--Graph29')

```

```

t.pa<-forecast(trend.pa,h=12)
s.pa<-forecast(seasonal.pa,h=12)
m.pa<-forecast(fit.pa,h=12)
prediction.pa<-as.numeric(m.pa$mean)+as.numeric(t.pa$mean)+as.numeric(s.pa$mean)

```

```

# this step does the correction when the predicted value is below 0
for (i in 1:12){
  if(prediction.pa[i]<0)
    prediction.pa[i]=0
}

n<-192
par(mar=c(4,4,1,0))
plot(1:n,X.pa,type='l',xlim=range(1:(n+12)),ylim=range(c(X.pa,prediction.pa)), xlab='Month',ylab="coal c
points(1:n,X.pa,pch=19,cex=0.5)
lines(c(n,n+1),c(X.pa[n],prediction.pa[1]),col='red')
lines((n+1):(n+12),prediction.pa,col='red')
points((n+1):(n+12),prediction.pa,pch=19,cex=0.5,col='red')

par(mar=c(4,4,1,0))
plot(STL.ca)
par(mar=c(4,4,1,0))
plot(STL.fl)
par(mar=c(4,4,1,0))
plot(STL.il)
par(mar=c(4,4,1,0))
plot(STL.nc)
par(mar=c(4,4,1,0))
plot(STL.pa)

```