Estimating dynamic diversion ratios in storable good industries

Alan Crawford[†]

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This article estimates diversion ratios capturing the influence of dynamic substitution patterns on forward-looking storable goods firms' profits. These dynamic diverted value ratios are key inputs in a new dynamic upwards price pressure test, dGUPPI. This new method obviates the need to estimate consumers' dynamic demand function and is computable within a policy-making timeframe. To illustrate its practical use to policymakers, it is applied to the UK laundry detergent industry from 2002 to 2012. Estimated bounds on the dynamic diverted value ratios calculate set-valued dGUPPI that overturn standard static empirical policy tools that would incorrectly permit an anticompetitive hypothetical merger.

Keywords: dynamic demand, dynamic pricing, storable goods, price elasticities, diversion ratios, GUPPI

JEL Classification: D12, C54, L11, L40, L67

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[†]Universidad Carlos III de Madrid. Email: acrawfor@eco.uc3m.es

1. Introduction

Incorporating dynamics into demand models enables an analyst to accurately measure competitive interactions in storable goods industries. Static demand models do not allow a portion of short-run volume increases from temporary price cuts to draw down on future sales. As a result, own price elasticities are overstated and cross-price elasticities tend to be understated. The result is severely biased value and volume diversion ratios - key inputs into empirical policy analysis.

Consistent estimates of demand responses that capture both contemporaneous and inter-temporal substitution are difficult to obtain in practice. This is because estimation of sufficiently flexible dynamic demand models for storable goods is resource intensive, constrained by industry features, and often have stringent data requirements. As such, they are unlikely feasible within a policy-making time horizon.

This article develops a new computationally light method to estimate diversion ratios that reflect the dynamics inherent in storable good industries. Unlike the standard static case, it is only the value of the time-profile of diverted sales that directly linked to firm's pricing incentives. As such I focus on estimating 'dynamic diverted value ratios'. They measure the combined effect on the present value of profit of immediate and inter-temporal consumer responses to deep, temporary discounts that characterise storable good pricing.[1]

In contrast to existing dynamic demand estimation methods, estimation can be completed within the timeframe of the policy making process. The reason for this is two-fold. First, it uses empirical techniques that are often used in policy work (i.e. forecasting, standard static demand estimation and root-finding). Second, it only requires firms' margins and market data spanning the period over which margins are calculated.

The ease of implementation and wider applicability of this new approach come at the cost of a loss of point identification; estimates of dynamic diverted value ratios are set-valued. Consequently, counterfactuals and policy tools that use these as inputs produce a set of outcomes, rather than a single value. However, as illustrated in this article's application, set-valued results of empirical policy analysis need not reduce their efficacy in a policy setting.

This new approach uses a dynamic model of demand and supply for storable good industries.[2] Consumer demand dynamics arise from two channels. One is the ability to store purchases for future consumption, which allows for inter-temporal substitution. The other is through price expectations. Informed by prices observed on previous shopping trips, consumers form a data-driven statistical model of the price process. It is then used to forecast the distribution of expected prices. This in turn influences the timing and identity of the product purchased and the amount consumed.

[1] Section 2.4 provides a more detailed description of dynamic diverted value ratios and their calculation.

[2] Chen et al. (2008) show that the omission of demand and supply dynamics for durable goods leads to biased estimation of demand elasticities.

In the supply model, firms are forward-looking and set current prices to maximise the discounted present value of expected profit flows.[3] Firms' perception of the profitability of pricing strategies depends on their model of aggregate dynamic demand and their beliefs over rivals' pricing strategies.

Setting optimal prices requires that firms solve an infinite horizon high-dimensional dynamic programming problem with continuous controls and states. However, as highlighted by Rust (2019), limited by information processing capabilities and cognitive constraints firms cannot precisely solve such problems. Some form of approximation is necessary.

Noting that inter-temporal substitution predominantly draws forward purchases from the near future, in this article firms are assumed to use a finite multi-period lookahead rolling horizon procedure when setting prices. This approach in this article is aligned with the existing literature in approximate dynamic programming (Powell (2011); Bertsekas (2011)), reinforcement learning (Bertsekas (2021)), and is in line with empirical evidence on the behaviour of forward-looking firms (Che et al. (2007)). Since neither consumers nor firms have the resources to meet the unbounded rationality requirements of the default equilibrium concept – Markov Perfect Equilibrium - the equilibrium concept for the model is Experience Based Equilibrium (EBE) developed by Fershtman and Pakes (2012).

Optimal prices in the dynamic supply model are a function of marginal costs, current and future quantity demanded, forecast price derivatives, short-run demand derivatives and a matrix of reduced form parameters. In this article, the reduced form parameters are dynamic correction ratios that capture the effect that demand dynamics have on a firm's pricing incentives.[4] If known, ratios of the matrix's elements can be used to translate downward biased short-run estimates of diverted value ratios into dynamic diverted value ratios.

Dynamic diverted value ratios capture both contemporaneous and inter-temporal substitution patterns for storable goods and quantify the effect dynamic demand responses have on firms' pricing incentives. Further, like their static counterparts, they are shown to be a key input into a new generalised upward price pressure index adjusted for demand dynamics (dGUPPI). This new empirical policy tool extends the existing GUPPI test and can be used to evaluate mergers in industries with dynamic demand.

In the empirical application uses the firm's the optimal price equations are used to identify the bias-scaled dynamic correction ratios from data frequently available to policy analysts (i.e. market outcome data and estimates of firm's margins derived from their internal accounts). This correction is required because short-run demand derivatives estimated using a static demand model are biased estimators of the true short-run demand responses when they incorrectly omit inventory holdings.

If the effect of omitting inventory on the bias of marginal utility of income is the same for "switchers" and "brand loyal" consumers, then the bias scales the static demand derivatives by

[3] Firms are assumed not to be able to commit to future prices and retailers are assumed to be passive.

[4] As described in more detail in section 2.4, they measure the present value of future sales diverted to a good on promotion today as a proportion of the short-run profit gained of its additional sales.

the same multiplicative factor.[5] Under this arguably weak assumption, the ratio of the elements of the bias-scaled dynamic correction ratio matrix also consistently translates downward biased short-run diverted value ratios into dynamic diverted value ratios.

A three step procedure is used to estimate the bias-scaled dynamic correction ratio parameters. To implement it, the analyst is assumed to have access to high-frequency market data matching the price setting frequency (i.e. weekly) and margins measured over the same period.[6]

In the first step, the analyst specifies how price setting decisions are fed back into the internal price forecasting model. If the analyst has evidence that the firm treats price forecasts as fixed when setting prices then the expected future price derivatives are zero. With this open loop forecast assumption, no further action is required in step one. However, if the analyst believes that there is a feedback loop between price setting and forecasting, then closed-loop model of price setting and forecasting is needed.

In a closed-loop scenario a statistical model of the price process is estimated and used to construct expected price derivatives. In the application, a dynamic factor model is estimated for this purpose. The reasons are three-fold. First, it is compatible with high-dimensional forecasting. Second, it can flexibly capture observed product-specific price dynamics. Finally, estimation of high-dimensional expected price derivatives requires only basic linear algebra operations.[7]

In the second step of the estimation procedure, the analyst estimates a standard static demand model (i.e. logit, nested logit, mixed logit) using high-frequency market data matching the price setting frequency (i.e. weekly). The demand model can be estimated using aggregate market data or consumer level micro-data. The application in this article uses the latter.

The reduced form parameters are computed in step three by leveraging the information contained in profit margins. Specifically, the expression for the margin in the dynamic supply model when accumulated over multiple periods and products is equated with its empirical counterpart derived from firm's accounts. Its only unknowns are the reduced form parameters. However, with more than a handful of products, the values of these parameters (that equate the empirical and model margin) belong to sets whose bounds are likely to be too wide to be useful in a policy setting. To provide empirically useful bounds for a policy setting, further identification restrictions are imposed. These additional constraints reduce the number of parameters to be estimated to two.

[5] For example, this holds for static discrete choice demand models with homogeneous marginal utility of income (i.e. logit and nested logit).

[6] The use of margins calculated from internal accounts is common in merger screening tools. Notwithstanding the well-documented conceptual differences between the economic and accounting margins, antitrust authorities have placed evidentiary weight on empirical analysis based on firms' internal margin data (see footnote 29). See Pittman et al. (2009) on the use of accounting cost information as an input into antitrust policy. An alternative approach is to use the method proposed by De Loecker et al. (2020) to estimate mark-ups from published accounts.

[7] This approach to the closed-loop forecasting scenario may serve a sensitivity if the analyst has access to internal forecasting models.

To that end, three further parameter constraints are imposed. First, attention is restricted to average bias-adjustment parameters over the accounting period covered by the margin. Second, the overstatement of static own-price demand responses and the understatement of cross-price demand responses are constrained to be the same for all products whose sales contribute to the margin. The final constraint imposes a data-driven choice of the upper bound on the cross-price demand response adjustment parameter.

With a single equation and two unknown parameters, the final step estimation procedure searches for all values of the pair of reduced form parameters that equate the model and accounting estimate of the percentage margin. The resulting reduced form parameters are common to all products whose sales contribute to the observed margin.

To illustrate how this method can be employed in practice it is applied to the UK laundry detergent industry using Kantar Worldpanel purchase diary collected between 2002 and 2012. For the two firms, A and B, that dominate the UK laundry detergent industry, I use published accounts to estimate their margins for each calendar year.

First, I explore whether product innovation that reduced the dosage per wash affected the misspecification bias from static demand models.[8] Shrinking product sizes mean that they take up less storage space and reduce the inventory holding costs. With convex inventory costs, product compaction leads to a relative increase in the demand for large pack-sizes. The larger amount of future consumption purchased in one store visit, the larger the correlation between inventories and prices. As such, one might expect that the bias due to the omission of demand dynamics increases with the product innovation. In line with this, I find that diversion ratio biases in the UK laundry detergent industry increase during the phase of intensive product compaction.

To show how the dynamic diverted value ratios can be used in a policy setting, I use a hypothetical brand acquisition to show how a set-valued dGUPPI can inform the likelihood that harmful unilateral effects arise from changes in industry structure. I find that conducting the policy analysis without accounting for market dynamics can lead to policy errors. In the experiment without accounting for demand dynamics, the brand merger would be incorrectly permitted.

Related literature This article contributes to the nascent literature examining how firms facing solve complex high-dimensional dynamic decision problems in practice (Rust (2019); Iskhakov et al. (2020); Hortaçsu et al. (2021); Che et al. (2007)). Specifically, this article draws on techniques and insights from approximate dynamic programming (Powell (2011); Bertsekas (2011)), reinforcement learning (Bertsekas (2021)), high-dimensional forecasting (Bai and Ng (2002); Stock and Watson (2002)), and workhorse IO static demand system estimation (Berry (1994); Berry et al. (1995); Compiani (2022)) to exploit information on market power imparted using promotional pricing strategies over a prolonged period to calculate bounds on dynamic

[8]Dosage per wash refers to the physical amount of detergent needed per wash.

diverted value ratios in storable good industries.

It also contributes to the literature on dynamic demand estimation for storable goods. Following seminal articles by Erdem et al. (2003) and Hendel and Nevo (2006a), there have been several articles that have sought to apply and extend the frameworks they develop. For example, Pires (2016); Wang (2015); Osborne (2018); Crawford (2018) estimate a variety of extensions of these dynamic demand models using panel micro-data). It is also related to Hendel and Nevo (2013) and Perrone (2017). In contrast to other articles in this field, they develop models that can quantify the effect of omitting consumer dynamics on estimates of long-run price elasticities without incurring the substantial computational burden of solving for the consumer's value function.

Like Perrone (2017) the approach developed in this article is straightforward to implement and is flexible in terms of consumer heterogeneity and price expectations. It also builds on Perrone (2017) by allowing for product differentiation and quantity discounts within a storable good industry.

Hendel and Nevo (2013) develop a simple dynamic demand model for storable goods. Like the framework developed in this article, their model can be estimated using market level data. However, it requires more restrictive assumptions on consumer storage technology, taste heterogeneity and price expectations to do so. Moreover, and importantly from a policy perspective, it is challenging to scale up the model Hendel and Nevo (2013) propose beyond a handful of products. In contrast, the output of the model in this article naturally scale to high-dimensional choice sets that are often observed in storable good industries.

The cost of the flexibility of the approach in this article - the loss of point identification - is not shared by other approaches. However, this need not reduce the efficacy of policy analysis based on its outputs. This is demonstrated using a new dynamic version of the price pressure test, dGUPPI, developed in this article. This new price pressure test extends the set of tools available to antitrust practitioners assessing mergers exhibiting price dynamics.

Finally, it also contributes to a literature analysing dynamic demand and supply models for durable goods - of which storable goods are a special case. The most closely related are Chen et al. (2008). They use data from a simulated dynamic demand and supply model of the used car market to explore the biases in price elasticities and corresponding market power estimates when a misspecified static demand model is used instead of a dynamic demand model.[9]

Outline The remainder of the article is structured as follows. Section 2 describes the dynamic demand and supply side model of storable good industries. Section 3 shows how to combine industry data on market outcomes, estimated margins, a static demand model, and a price forecasting model to recover parameters to estimate dynamic diverted value ratios. Section 4 focuses on the application and policy experiments. Section 5 concludes.

[9] More distantly related are Goettler and Gordon (2011) and Carranza (2010). Both estimate dynamic demand and supply models in durable good industries.

2. Price setting with demand dynamics

This section presents a model of a differentiated storable good industry in which both consumers and firms are forward-looking. When setting current prices, firms take demand and supply dynamics into account. In each period firms set price to maximise their perception of the present value of expected profit flows.

When there are no inter-temporal links in demand or costs, the firm's optimisation problem is separable and is solved independently in each time period. However, in many industries, if not most, consumer demand is inherently dynamic. For example, if the firm discounts its product(s) as part of a temporary promotion, consumers may accelerate purchases and stock the product at home for future consumption. When demand is dynamic, as is the case for storable goods, firms consider the effect that current prices have on demand today and in the future. The effect of a price change today on current and expected future profits are affected by static and inter-temporal substitution.

When setting prices, firms form beliefs about both consumer and firm dynamics. Therefore, firms must retain and process a lot of information to compute consumer demand and assess the profitability of a very large number of pricing strategies. The size of the task of determining optimal pricing strategies is exacerbated by the large number of products sold to consumers in storable good industries (i.e. often the choice set contains 50-150 products).

Faced with these large computational and cognitive challenges, firms are only able to solve an approximate version of their dynamic pricing problem. In line with this approach, the equilibrium concept used in the resulting high-dimensional dynamic pricing game is Experience Based Equilibrium (Fershtman and Pakes (2012)). This allows agents to have imperfect information about other industry participants and relaxes stringent rationality requirements. Agents' play need only maximise payoffs given the information available to them and be consistent with outcomes in states they have previously experienced.

This section is structured as follows. First, consumer demand and firms' empirical demand function is described. Second, the dynamic supply model is presented and the equilibrium concept consistent with the firm's approximations is discussed. The model is then recast in terms of parameters - dynamic correction ratios - formed by dividing *inter-temporal* diverted value ratios by *static* diverted value ratios. These parameters capture omitted demand dynamics and can be used to construct the present value of contemporaneous and inter-temporal diverted sales. Finally, the concept of a 'dynamic diverted value ratio' is formal introduced and their relationship to the dynamic correction ratios is described. These are then used to modify price pressure tests used in antitrust policy making to account for omitted demand dynamics when elasticities are estimated using static demand models.

Hereafter, bold symbols indicate vectors and matrices.

2.1. Consumer demand

Setup In each period consumers choose whether or not to purchase one product from a choice set of J differentiated products.[10] The products (or packs) contain storable goods and are sold in different sizes. The contents of the products can be stored in inventory ready for consumption at a later date.[11] Each product's price in period t is recorded in a J-vector, $\boldsymbol{p}_t = [p_{1t}, \ldots, p_{Jt}]^\top$ where p_{jt} is the price of product j in period t. Consumers take these prices as given.

The consumer's problem is a discrete-continuous dynamic optimisation problem. At the start of each time period, consumer i privately observes their existing inventories $\boldsymbol{I}_{it} = [I_{i1t} \ldots, I_{iJt}]^\top$ where I_{ikt} is the number of consumption units of product k in stock at the beginning of period t. They also observe vectors of current prices, $\boldsymbol{p}_t$, purchase demand shocks, $\boldsymbol{\xi}_t$, seasonal demand shifters, $\boldsymbol{y}_t$, and any additional information used to forecast future prices, $\boldsymbol{f}_t$.

Given this private and market-wide information, consumers then decide whether or not to make a new purchase in the current period. If they make no purchase then they consume from their existing inventory, $\mathbf{0} \leq \boldsymbol{C}_{it} \leq \boldsymbol{I}_{it}$. If, however, consumer i decides to purchase product j in period t its contents are added to existing inventories

$$\bar{\boldsymbol{I}}_{it} = \boldsymbol{I}_{it} + \boldsymbol{Q}_j$$

where $\boldsymbol{Q}_j$ is a J-vector whose j-th element contains the number of units of consumption contained within the product j, all other entries in $\boldsymbol{Q}_j$ are zero. Consumption is then drawn from the post-purchase inventory, $\mathbf{0} \leq \boldsymbol{C}_{it} \leq \bar{\boldsymbol{I}}_{it}$. Finally, at the end of each period consumer i pays a cost to carrying over the remaining inventory into the next period. This cost is increasing in the number of consumption units carried over.

Consumer demand dynamics The possibility of storing current purchases in inventory for future consumption is one source of demand dynamics. Provided the consumer has access to storage technology and can afford to use it, the ability to consume out of inventory means that purchase period and consumption period need not coincide. This creates inter-temporal demand linkages.

Storable goods are often sold at a 'base' or 'regular' price for sustained periods of time interspersed with frequent, stochastic, temporary sale periods.[12] From repeated shopping trips, consumers formulate a model of the price process (i.e. based on past prices).

[10]For expositional simplicity I abstract away from the possibility of there being variations in the number of good available for sale in each time period.

[11]The window for consuming a storable good and the related storage costs depend on their characteristics. For example, some storable goods, such as yoghurts and other dairy products, can be stored for three to four weeks in refrigerated conditions. Others, such as laundry detergent, are non-perishable but may occupy a relatively large portion of storage space.

[12]Erdem et al. (2003); Hendel and Nevo (2006a,b); Nevo and Hendel (2012); Osborne (2013); Wang (2015) report that prices in the industries they study also exhibit these features.

To make purchases consumers go to the store, observe products' sales status, their prices and then forecast future prices. These price forecasts are taken into account when making purchase and consumption decisions.

For example, expecting higher prices in the future, some consumers might bring forward purchases to take advantage of a temporarily low sale price. In this case, expected savings are weighed against increased storage costs and can affect the timing of purchases.

With convex storage costs, the increase in storage costs will be larger for consumers that are already holding high levels of inventories relative to their consumption needs. Consumers in this position are therefore less likely to respond to temporary sale prices - especially when the sale is on large products.

Demand dynamics can also arise through consumption decisions. For example, consumers holding lower levels of inventory may choose to reduce consumption rather than purchase when prices are currently high and only expected to fall at some point in the future. In this case, reduced utility from lower consumption today is more than offset by the option value associated with lengthening the period of time where consumption can draw on existing inventories and consumers can wait for lower prices.

The ability to use inventories for consumption and to anticipate promotional prices means that consumer demand is inherently dynamic. Therefore, in addition to factors that affect myopic consumer demand (i.e. current prices and aggregate demand shocks), consumers take into account their inventory holdings and price forecasts when making purchase decisions and consumption choices.

Purchases and consumption As is the case for static demand models, a consumer's characteristics and taste preferences, $\boldsymbol{\eta}_i$, affect demand for storable goods. In the context of storable goods, the elements of $\boldsymbol{\eta}_i$ might include household composition, net income, dwelling size, urban/rural location and taste preferences. Consumer preferences might also be influenced by experiences of recent purchases and consumption. For example, consumers might incur switching costs, have time-varying brand loyalty, and/or exhibit habit formation.[13]

Together with inventory, current prices and the non-price aggregate state variables, consumer i's characteristics affect the purchase decisions. The probability that consumer i purchases product j in period t is[14]

$$\Pr\left(d_{it} = j\right) = s_j\left(\boldsymbol{I}_{it}, \boldsymbol{p}_t, \boldsymbol{x}_t, \boldsymbol{\eta}_i\right) \tag{1}$$

[13]Myśliwski et al. (2020) and MacKay and Remer (2019) analyse dynamic price games for firms selling storable goods to myopic consumers with state-dependent demand.

[14]Each consumer is assumed to have the same information set and use the same price forecasting model. Expanding the demand model to allow for differential information sets and, therefore, heterogeneous forecasting models is beyond the scope of this article. Amongst other things, the links between information acquisition, inventories, store choice, and shopping frequency would have to be added to the model.

where $\boldsymbol{x}_t := [\boldsymbol{\xi}_t, \boldsymbol{y}_t, \boldsymbol{f}_t]$ where $\boldsymbol{\xi}_t$ are aggregate purchase demand shocks, $\boldsymbol{y}_t$ are seasonal demand shifters, and $\boldsymbol{f}_t$ any contains other market-level information used to forecast prices. This information is revealed at the beginning of the period before firms choose prices that affect consumers' purchase decisions.

Once purchases have been made and stored at home, consumer i consumes $C_{ijt} = c_j\left(\bar{\boldsymbol{I}}_{it}, \boldsymbol{y}_t, \boldsymbol{f}_t, \boldsymbol{\eta_i}\right)$ units from post-purchase inventory of product j at time t, $\bar{I}_{ijt}$. As highlighted by the discussion in the preceding subsection, once purchases are made and inventory is updated only price expectations (and potentially seasonal demand factors), not current prices, influence consumption decisions.

Aggregate demand The empirical aggregate dynamic demand functions are assumed to be known to firms. Nair (2019) notes,

> "Generally speaking the empirical (IO and marketing) literature assumes that firms know the true model generating demand. This assumption may not be unreasonable in data-rich situations in stable markets where firms have access to rich historical information".

In line with this perspective, storable good firms tend to have extensive industry experience, have access to detailed historical information on market outcomes, and intensively research to study consumer preferences. Moreover, the industries are often mature, stable and exhibit low levels of product turnover.[15]

Product j's share of market demand in period t is therefore assumed to be known to firms. In the model above, it is obtained by integrating individual purchase decision functions over the joint distribution of household inventories and characteristics, $F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i\right)$,

$$s_j\left(\boldsymbol{p}_t, \boldsymbol{x}_t, F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i\right)\right) = \int s_j\left(\boldsymbol{I}_{it}, \boldsymbol{x}_t, \boldsymbol{f}_t, \boldsymbol{\eta}_i\right) dF\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i\right) \tag{2}$$

Given a market size M_t, the empirical the aggregate dynamic demand function for product j in period t is

$$q_j\left(\boldsymbol{p}_t, \boldsymbol{x}_t, F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i\right)\right) = M_t s_j\left(\boldsymbol{p}_t, \boldsymbol{x}_t, F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i\right)\right) \tag{3}$$

If firms could observe $F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i\right)$ in addition to M_t and $\boldsymbol{x}_t$, then they can use equation (3) to empirically evaluate aggregate demand at any price vector, $\mathbf{p}_t$. However, as noted in the model description above, realisations of consumption and inventories are private to consumers. As a result, $F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i\right)$ is not observable and firms cannot directly evaluate $q_j\left(\boldsymbol{p}_t, \boldsymbol{x}_t, F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i\right)\right)$ for use in internal planning and price setting.

[15] Even where new storable goods are introduced, they are often similar to the existing product set (i.e. they are a new combination of existing product characteristics). Moreover, they often undergo intensive market research investigation prior to their formal introduction. As such, their demand and their effect on existing products demand is likely to be well anticipated by firms.

However, firms do possess the information set to empirically evaluate an equivalent representation of equation (2)

$$q_j\left(\boldsymbol{p}_t, \boldsymbol{\omega}_t\right) = M_t s_j\left(\boldsymbol{p}_t, \boldsymbol{x}_t, \boldsymbol{p}_t^{\mathcal{H}}, \boldsymbol{x}_t^{\mathcal{H}}\right) \tag{4}$$

where $\boldsymbol{\omega}_t := [\boldsymbol{x}_t, \boldsymbol{p}_t^{\mathcal{H}}, \boldsymbol{x}_t^{\mathcal{H}}, M_t]$. This aggregate dynamic demand function can be evaluated at any vector of prices having observed the market size, non-price state variables, their histories, and past prices. In the subsequent supply model, firms are assumed to know and use this empirical aggregate dynamic demand function when setting prices.

Discussion The description of the empirical aggregate dynamic demand above is consistent with existing dynamic storable good demand models estimated in the empirical IO and marketing literature. Specifically, they can be derived from the model above by imposing the additional restrictions added to those models to help mitigate estimation challenges (i.e. curse of dimensionality, data limitations, etc.).[16]

One set of statistics excluded from the arguments of the empirical dynamic demand function are the historical realisations of aggregate market shares, $\boldsymbol{s}_t^{\mathcal{H}}$. However, once $\boldsymbol{p}_t^{\mathcal{H}}$ and $\boldsymbol{x}_t^{\mathcal{H}}$ are conditioned on the inclusion of $\boldsymbol{s}_t^{\mathcal{H}}$ in the state vector does not provide additional information about aggregate demand.[17] That is, $F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i | \boldsymbol{p}_t^{\mathcal{H}}, \boldsymbol{x}_t^{\mathcal{H}}, \boldsymbol{s}_t^{\mathcal{H}}\right) = F\left(\boldsymbol{I}_{it}, \boldsymbol{\eta}_i | \boldsymbol{p}_t^{\mathcal{H}}, \boldsymbol{x}_t^{\mathcal{H}}\right)$.

Given the availability of lagged market shares and purchase diary data from market research companies, one might wonder whether $\boldsymbol{s}_t^{\mathcal{H}}$ can be used in lieu of some of the components of $\boldsymbol{x}_t^{\mathcal{H}}$. In general, this substitution will result in an approximation of unknown quality to the true aggregate dynamic demand function for storable goods.[18]

2.2. Firms

This section describes the supply side of the storable good industry. Storable goods tend to be mature industries with the majority of sales occurring in a few large, national supermarket/specialist retail chains. It is also common for their manufacturing industry

[16] Most existing models have focused on the case where household-level purchase diary data is available. In this case, so called 'full-solution' methods are used and inventories are integrated out of a dynamic demand model using observed household's purchase history and observed prices (i.e. Erdem et al. (2003); Hendel and Nevo (2006a); Sun (2005); Wang (2015); Pires (2016); Osborne (2018); Crawford (2018)). Outside of the the 'full-solution' approach, Hendel and Nevo (2013) develop a dynamic demand model for storable goods impose more restrictive consumption, price forecasting, and storage assumptions. These additional restrictions simplify the model enough to facilitate estimation of a low-dimensional dynamic demand system with market-level data.

[17] However, $\mathbf{s}_t^{\mathcal{H}}$ can be constructed by aggregating purchase sequences across all households.

[18] Only in special cases would the resulting empirical aggregate demand model be exact. For example, Myśliwski et al. (2020) show that if consumers have a static, state-dependent aggregate demand function, then last period's market shares are needed in the state vector in a dynamic discrete promotion setting game.

structure to be oligopolistic. Manufacturers offer a large number of products. The set of products tends to evolve slowly over time. As such, the model abstracts away from entry and exit of firms and products.

Without direct information on vertical arrangements in my application, I assume manufacturers set retail prices. In the model below, the retailer's mark-up is implicitly included in a composite marginal cost for a price-setting manufacturer.[19] This represents a simplification of the complex negotiations between manufacturer and retailer that occur in practice. Extensions considering richer vertical contracting arrangements is left for future study with richer data sources.

Setup Let $n = 1, \ldots, N$ index firms in the industry sharing a common discount factor, $\delta \in (0, 1)$. The expected discounted sum of per-period profits over an infinite horizon,

$$\pi_{nt}^{NPV} = \mathbb{E} \sum_{h=t}^{\infty} \sum_{j \in \mathcal{J}_n} \left(p_{jh} - mc_{jh}\right) q_j \left(\boldsymbol{p}_h, \boldsymbol{\omega}_h\right) \tag{5}$$

where set of J_n products manufactured by firm n is denoted $\mathcal{J}_n$ and mc_{jt} is the marginal cost of production for product j in period t. Hereafter , let $\boldsymbol{m}_t := \boldsymbol{p}_t - \boldsymbol{mc}_t$ be the J-vector of mark-ups.

Firm n maximises π_{nt}^{NPV} by setting a sequence of a prices, $\{\boldsymbol{p}_{nh}\}_{h=t}^{\infty}$. To set prices firm n solves a high-dimensional dynamic programming problem (DP) for each of the products it sells. In each case it chooses a vector of continuous controls as a function of a high-dimensional and continuous industry state vector. This is a cognitively challenging, resource intensive task for firms - especially given the high frequency of price setting and limited resources they have at their disposal.

Rust (2019) discusses the existing results from the computational science literature linking the problem solving resources of agents and their ability to find optimal solutions to dynamic programs as a function of their complexity, size, and nature. He highlights Chow and Tsitsiklis' (1989) result that agents with finite cognitive and computational ability cannot compute the solution to large, complex realistic dynamic problems - like the storable good firm's dynamic pricing problem.

Rust (2019) also reviews the nascent literature of case studies investigating the optimality of choices made by firms facing dynamic profit optimisation problems and finds that this appears to be the case across many industries. Among others, these include airlines (Williams (2022)),

[19] This is the approach taken in much of the empirical IO and marketing literature analysing fast-moving consumer goods industries. Selected examples relevant to storable goods include Hendel and Nevo (2013); Pavlidis and Ellickson (2017); Myśliwski et al. (2020); Nevo (1998, 2001); Slade (2004). Moreover, the assumption that manufacturer set prices with a 'passive' retailer has also been adopted in antitrust cases involving storable goods. For example, after a detailed review of internal documents, the DG COMP adopt this assumption in COMP/M.56858 - Unilever/Sara Lee - a merger of storable personal care products ("Technical Annex: Demand estimation and merger simulation").

hotels (Cho et al. (2018)), and car rental companies (Cho and Rust (2008)). In these case studies firms' strategic choices (i.e. prices, asset replacements) are compared to those that solve the model of the firms' dynamic profit optimisation problems. Against the benchmark solutions firms' strategic choices are 'nearly optimal', but may leave room for improvement. Hortaçsu et al. (2021) also document how organisational and pricing inefficiencies in a large US airline can lead to internal procedures result that may result in "second-best" pricing policies.

The consistent message from these case studies is that firms facing a large, complex dynamic profit optimisation problems are boundedly rational. That is, they 'satisfice' and use approximations to optimal solutions in practice.

ADP Solutions to complex dynamic programs can be approximated in many ways. Approximate dynamic programming (ADP) encompasses a wide variety of techniques from a collection of disparate fields that have developed specific approaches to approximate solutions to complex dynamic programs they encounter.[20] Powell (2011) highlights that they have been successfully employed in many industries including transportation, energy, health and finance.

Instead of attempting to solve intractable, high-dimensional infinite horizon problems Powell (2011) notes that "a natural approximation is to try to solve the problem over a shorter horizon". Under this ADP approach, boundedly rational firms lookahead a short number of periods ahead, H, then solve for optimal policies given this optimisation horizon. They then implement the optimal policy for period t and move forward to period $t+1$. In the next period, they repeat this rolling horizon procedure looking ahead H periods into the future.[21]

Storable goods firms have a natural horizon linked to consumer purchase cycles that drives demand dynamics (i.e. purchase, store, consume, purchase). Because consumers respond to promotions by accelerating purchases from the near future, inter-temporal substitution takes place over a short time window. Recognising this, storable goods firms may choose prices by solving a rolling horizon approximation to their original DP where the H-period lookahead window is linked to consumers' inter-purchase durations. Provided that the majority of inter-temporal substitution has taken place once H-periods have passed, the loss of accuracy in the resulting dynamic pricing functions is likely to be limited.

The presence and length of a fixed-window rolling horizon procedure is tested by Che et al. (2007). They find that firms in the US Cereal industry use a short, fixed horizon when setting prices (i.e. not more than two periods ahead). They repeat the analysis for ketchup - a storable good. Once again they find evidence that firms in the US ketchup industry adopt boundedly rational short fixed-window rolling horizon procedure when setting prices.

In line with discussion above, hereafter firm n is assumed to be boundedly rational and

[20] See Powell (2011); Bertsekas (2011) for detailed overview of ADP.

[21] This is also known as the receding horizon procedure or, in engineering, model predictive control. It also describes online components of many reinforcement learning algorithms successfully applied to Backgammon, Chess and Go. See Rust (2019), Igami (2020) and Iskhakov et al. (2020) for a discussion of these methods.

approximates their computationally infeasible DP using a H-period lookahead rolling horizon procedure.

FOC To succinctly describe the set of first order conditions firm n satisfies when solving the approximate DP I introduce some additional notation. First, let $\boldsymbol{\Delta}_h^q$ denote the matrices of the current and inter-temporal total derivatives of (expected) demand with respect to changes in current prices. Further, let $\boldsymbol{\Delta}_h^p$ be the corresponding current and inter-temporal total derivatives of price forecasts. The (j,k)-th elements of these matrices are given by equations (6) and (7),

$$\Delta_{jkh}^q := \frac{dq_{kh}}{dp_{jt}} \,\forall\, t \leq h \leq H \tag{6}$$

$$\Delta_{jkh}^p := \frac{dp_{kh}}{dp_{jt}} \,\forall\, t < h \leq H \tag{7}$$

Further, let $\boldsymbol{\Omega}_n$ the ownership matrix for firm n whose (j,k)-th entry is

$$\Omega_{jkn} = \begin{cases} 1 & \text{if } j,k \in \mathcal{J}_n \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

and its diagonal, $\overrightarrow{\boldsymbol{\Omega}}_n$, is a J-vector encoding the set of products sold by firm n.
Using this notation, the system of first order conditions for firm n's approximation to its DP in matrix form is

$$\boldsymbol{q}_{nt} + \boldsymbol{\Delta}_{nt}^q \boldsymbol{m}_t + \mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{nh}^q \boldsymbol{m}_h + \mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{nh}^p \boldsymbol{q}_h = \mathbf{0} \tag{9}$$

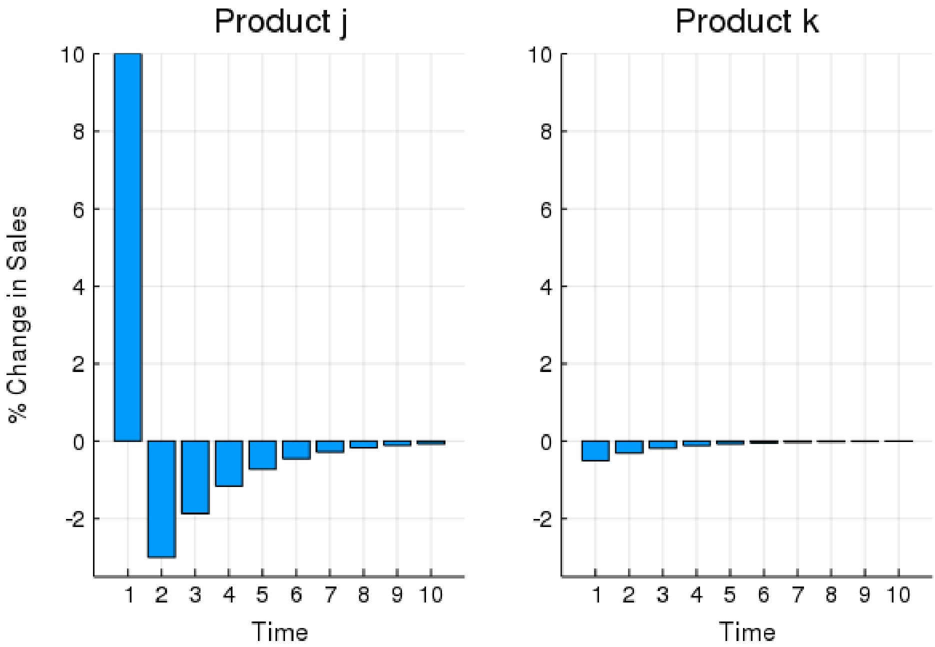
where $\boldsymbol{\Delta}_{nh}^i := \boldsymbol{\Omega}_n \odot \boldsymbol{\Delta}_h^i$ for $i \in \{p,q\}$, $\boldsymbol{q}_{nt} := \overrightarrow{\boldsymbol{\Omega}}_n \odot \boldsymbol{q}_t$, and '$\odot$' denotes the Hadamard product. The first and the second terms in equation (9) correspond to the terms that enter the first order conditions when demand is static and firms are myopic. The additional terms contain total derivatives capturing the effect that price changes today have on future quantities and prices, respectively.[22] Demand and price dynamics central to storable good competition and promotional pricing strategies are discussed next in more detail.

Aggregate demand dynamics The second and third terms in equation (9) capture the effect of contemporaneous and inter-temporal substitution on firm n's pricing incentives, respectively. Figure 1 provides a stylised graphical representation of dynamic substitution patterns.[23] The

[22] See Appendix A to see how total derivatives capture both the direct and indirect effect that price changes today have on future quantities and prices.

[23] These stylised substitution patterns mirror those in Erdem et al's (2003) simulation of the quantity impulse response functions to a temporary 10 percent price cut to all Heinz products in the US Ketchup industry. Their analysis is reproduced in Appendix A.

Figure 1: Illustrative inter-temporal substitutions patterns of storable products j and k over 10 periods in response to a temporary one percent price cut of product j in $t = 1$



left panel shows the percentage change in sales of product j when the price of product j is temporarily cut by one percent in the first period. The right panel shows the corresponding percentage change in sales of product k.

The figure shows that the sales of product j rise initially in response to the temporary price cut. Also, because product k is a substitute for product j, the temporary price cut of product j diverts sales away from product k in the first period. The effect of these substitution patterns on pricing is captured by the second term in equation (9) , $\boldsymbol{\Delta}_{nt}^{q}\boldsymbol{m}_t$.

In the subsequent periods, both product j's and product k's sales are lower than those without the price cut - albeit at a diminishing rate. The loss of future sales arises because some of the demand uplift in the first period is drawn from future demand. That is, future sales of products j and k are substitutes for current sales of product j.

However, the percentage change in future sales of product k are less pronounced than product j. This indicates that future sales of product k are a more distant substitute to current sales of product j than the corresponding future sales of product j. The change in the present value of expected profits due to these inter-temporal substitution responses to price changes in the current period are captured by the third term equation (9).

Figure 1 highlights why price elasticities estimated using a mis-specified static demand model are biased for storable goods. Implicitly, a static demand model omits the change in sales from the second period onwards. As a result, future reductions in sales of product j are not netted off from the immediate demand response to the price cut. Therefore, the own-price elasticity of product j is overstated. Likewise, by not taking into account the loss of future sales of product k in response to temporary price cut of product j, the cross-price elasticity of k with respect to j is understated.

Price dynamics The fourth term in the system of first order conditions measures the expected change in the present value of revenues due to anticipated adjustments to future prices in response to changes to current prices. Hereafter, for notational convenience let

$$\boldsymbol{\Upsilon}_{nt} := \mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}^{p}_{nh} \boldsymbol{q}_h \tag{10}$$

be shorthand for the term measuring the effect that firm n's beliefs over dynamic supply side responses have on its pricing incentives. Firms' beliefs are formed from their repeated experiences of selling products in the industry. They capture product specific promotional price dynamics in each retail chain and are consistent with the observed price data.

However, tracking and forecasting a high-dimensional choice set of products' prices with retail chain specific promotional dynamics is a cognitively challenging and resource intensive task for firms.[24] Even large, well resourced firms face additional practical considerations that makes it prohibitively costly to integrate complex forecasting and dynamic pricing systems.[25] In light of these challenges, boundedly rational firms may elect to adopt tractable forecasting models and limit feedback between price forecasts and pricing decisions.

Firms facing considerable organisational frictions might adopt an open-loop forecasting model. In an open-loop model, firms takes forecasted price paths as fixed inputs into the process used to set price. The price setters do not know the underlying forecast models, nor are they given updated prices forecasts until after prices in the current period are chosen. As a result, from the price setter's perspective, future prices do not respond to current prices. Therefore, for firm n in each period t, $\boldsymbol{\Delta}^{p}_{nt} = \mathbf{0}$.

Alternatively, more sophisticated firms might employ a closed-loop price forecasting model. In a closed-loop model price setting is assumed to be fully integrated. Proposed changes in prices are fed back into the price forecasting model and the time profile of inter-temporal price derivatives updated.

In practice, the extent to which organisational structure or other frictions affect the feedback loop between prices and forecasts varies across firms. In this article's application (section 4), the baseline supply model uses a closed-loop forecast model. The open-loop model is retained for sensitivity analysis.

[24] Storable good industries with dynamic demand contain sell $J \geq 100$ products in each week sold across multiple retail outlets.

[25] For example, Hortaçsu et al. (2021) document how organisational frictions require a large US airline to make post-hoc adjustments to forecasted demand inputs so that the revenue management systems implement the desired pricing strategy.

2.3. Equilibrium

The equilibrium concept for the dynamic pricing game is Experience Based Equilibrium (EBE) - a modification of Markov Perfect Equilibrium (MPE) developed by Fershtman and Pakes (2012).[26] A EBE retains a Markovian structure of an MPE, but relaxes the restriction that agents are unboundedly rational.

For agents' behaviour to meet MPE's rationality conditions in a storable good industry with forward-looking consumers and firms, there are three conditions that must hold for all possible realisations of the state vector. First, firms' beliefs over rivals' strategies coincide with their optimal price strategies. Second, they must also hold correct beliefs over forecasting technologies used by forward-looking consumers. Finally, a MPE also requires that consumers' forecasting technology is consistent with firms' equilibrium price strategies.

However, the demands that the MPE conditions place on the cognitive and physical resources of firms and consumers are arguably prohibitive. For example, one requirement implicit in these conditions is that all agents know the identity of the manufacturing firm for each product. While firms know the identity of manufacturers for each product, it is not obvious that consumers do. If not, the MPE condition that requires consistency between firms' equilibrium price strategies and consumers' price forecasts arguably requires more sophistication and cognitive capacity than is available to them (or at least that they would optimally choose to allocate).

As highlighted above, firms also may not have the capacity to compute the strategies that meet MPE conditions. Even with a handful of products - and therefore a small state space - processing and evaluating the value functions associated with complicated price strategies is computationally challenging. When, as is common in many storable good industries, the choice set is high-dimensional (i.e. $J \approx 100$) the state space becomes large, then the computation of strategies satisfying MPE rationality conditions is arguably infeasible.

An EBE replaces the MPE rationality requirements with two weaker conditions. First, given the information set used by agents to make decisions, an EBE requires that agents choose actions that maximise their perceived payoffs. The second condition requires that the perceptions held by agents are consistent with play they have observed in the past when the information set recurs.

Both EBE conditions are met by the decisions taken by consumers and firms in the dynamic demand and price models, respectively. In accordance with the first condition, firms and consumers choose actions that maximise their perceived payoffs calculated using a restricted subset of the full information available to them (i.e. limited information of each product's manufacturer). The second condition is also met; firms' aggregate dynamic demand model, their beliefs rivals' price strategies, and consumers' beliefs over future price are all consistent

[26]MPE is used by Chen et al. (2008) and Goettler and Gordon (2011) in their studies of dynamic demand and supply systems for durable goods. In contrast, in his study of the camera industry Carranza (2010) assumes that firms behave monopolistically and ignore current and future strategic interactions when optimising profits.

with past market outcomes.

In the context of the storable good industry model above, one benefit of using EBE as an equilibrium concept is that it relaxes the implausible MPE requirement that consumer and firm beliefs necessarily coincide. Instead, it only requires that these beliefs are consistent with observed play at recurring states of the game.

2.4. Dynamic diverted value ratios

For the purposes of empirical methods described in section 3 and their relationship to commonly used empirical antitrust policy tools used to evaluate mergers, it is useful to interpret the demand dynamics in the system of first order condition (eq. (9)) in terms of diversion ratios. A diversion ratio commonly refers to volumes displaced: $DR_{jkt} = -\Delta^q_{jkt}/\Delta^q_{jjt}$. However, it is the profit earned on diverted sales that influences pricing decisions in system of first order condition (eq. (9)).

The "value of diverted sales" from product j to product k is $\Delta^q_{jkt} m_{kt}$. Measured as a fraction of the effect of price cut for product j on its profits, the value of diverted sales for product k defines the diverted value ratio between the products j and k,

$$DR^{\pi}_{jkt} = -\frac{\Delta^q_{jkt} m_{kt}}{\Delta^q_{jjt} m_{jt}} \tag{11}$$

The higher the diverted value ratio, the more intense the competitive interaction is between the two products. When analysing myopic firms' pricing incentives, volume or profit diversion ratios can be used. For example, the gross upwards price pressure index (GUPPI) - a unitless index used as a diagnostic tool in antitrust policy to indicate the likelihood that a merger will give rise to unilateral effects - can be written as function of volume or value diversion ratios.[27] However, to measure the effect of competitive intensity on forward-looking firms' pricing incentives, it is more convenient to work with the sum of contemporaneous and inter-temporal diverted value ratios.[28]

The value of inter-temporal diversion captures the effect of dynamic substitution on pricing incentives. Specifically, the inter-temporal diverted value ratio from product j to product k is

$$ITDR^{\pi}_{jkt} = -\frac{\sum_{h=t+1}^{H} \delta^{h-t} \Delta^q_{jkh} m_{kh}}{\Delta^q_{jjt} m_{jt}} \tag{12}$$

It measures the present value of future diverted sales for product k as a fraction of the profits gained on product j in the current period when it is temporarily promoted.

[27] See section 2.5 and the online appendix for more detail of GUPPI and a new dynamic version, dGUPPI.
[28] *Ibid.*

When setting prices, firms take into account the fraction of current sales that are cannibalising profits earned on future sales of their products. The additional impact of demand dynamics on pricing incentives are captured by dynamic correction ratios. The dynamic correction ratio for a pair of products j and k measures the expected change of the present value of future profits earned on sales of product k in response to a temporary price change for product j as a fraction of the change in profits earned on current sales of product k. It is calculated by dividing the inter-temporal diverted value ratio by the short-run diverted value ratio.

$$\Psi_{jkt} := \mathbb{E}\left[\frac{ITDR^{\pi}_{jkt}}{DR^{\pi}_{jkt}}\right] \tag{13}$$

When demand dynamics are present, inter-temporal substitution effects are measured by the elements a $J \times J$ matrix of dynamic correction ratios, $\boldsymbol{\Psi}_t$. In line with inter-temporal substitution patterns shown in Figure 1, its diagonal components lie between minus one and zero. This is because a fraction of the short-run demand responses are drawn from future sales of the promoted product. Similarly, because future sales of rival products are substituted in favour of the rival promoted product, the off-diagonal elements are positive.

The system of first order conditions in equation (9) can be expressed in terms of the matrix dynamic correction ratios

$$\boldsymbol{q}_{nt} + \boldsymbol{\Delta}^{q}_{nt} \odot (1 + \boldsymbol{\Psi}_t)\, \boldsymbol{m}_t + \boldsymbol{\Upsilon}_{nt} = \boldsymbol{0} \tag{14}$$

The dynamic correction ratios convert the myopic value of diverted sales into the present value of the time-profile of diverted sales for a forward-looking firm. However, if there are no demand dynamics, then $\boldsymbol{ITDR}^{\pi}_t = \boldsymbol{0}$ and therefore $\boldsymbol{\Psi}_t = \boldsymbol{0}$. Similarly, if firms are myopic, or use an open-loop forecast model, then $\boldsymbol{\Delta}^{p}_{nh} = \boldsymbol{0}$ for all $h > t$. Without demand and price dynamics, equation (14) collapses to the first order condition of the workhorse static differentiated Bertrand-Nash model of price competition.

Equation (14) serves as the basis for the empirical work recovering dynamic versions of diverted value ratios for storable goods. Before turning to the empirical methodology, I describe how the system of dynamic price setting equations can be used to extend price pressure tests to measure the strength of competitive constraints between two storable goods.

2.5. Dynamic GUPPI

The 2010 US Horizontal Merger Guidelines introduced upwards price pressure measures: UPP (Farrell and Shapiro (2010)) and the gross upwards price pressure index (GUPPI) (Moresi (2010)). Price pressure tests have been used to screen mergers and assess closeness of competition - particularly for differentiated products.[29]

[29] GUPPI and other price pressure tests have been used in many recent merger cases. For example, in the UK, these include Sainsbury's/Asda (2019), Tesco/Booker (2017), Ladbrokes/Coral (2016) and

The GUPPI is a measure of the strength of the closeness of substitution between two products, j and k. It can be expressed using volume or profit diversion ratios. Using the volume measure, the diverted sales from product j to k multiplies the margin on product k, μ_k, and the ratio of the two prices. Alternatively, GUPPI is the margin earned on sales of product j, μ_j, multiplied by the diverted value ratio from product j to k.

$$GUPPI_{jk} := DR_{jk}\mu_k \frac{p_k}{p_j} = DR^{\pi}_{jk}\mu_j \tag{15}$$

To calculate GUPPI in a policy setting requires estimates of diversion ratios, margins and prices. In practice, the market power is proxied by margins measured or derived from a firm's (internal) accounting records.[30] As such, it is typically measured over a period of time (i.e. one year). Therefore when demand and supply dynamics are present, the margin implicitly reflects the effects of both immediate and inter-temporal substitution.

Where data permits, diversion ratios are estimated using the output of a demand model. As noted above, for storable goods the volume and profit diversion ratios are downward biased when inter-temporal substitution is omitted from the procedure used to estimate demand.[31] In this case, the diversion ratios are biased and are not consistent with the competitive dynamics that affect firms' market power. To remedy this a dynamic diverted value ratio is required.

The dynamic diverted value ratio in equation (16) measures the reduction in the present value of expected profits earned on current and future sales of product k as a fraction of the present value of expected profits earned on current and future sales of product j in response to a temporary cut in the price of j. It can be expressed as a function of the dynamic correction ratios and its static counterpart

$$DDR^{\pi}_{jk} := -\frac{\mathbb{E}\sum_{h=0}^{H}\delta^h \Delta^q_{jkh} m_{kh}}{\mathbb{E}\sum_{h=0}^{H}\delta^h \Delta^q_{jjh} m_{jh}} = \frac{1+\Psi_{jk}}{1+\Psi_{jj}} DR^{\pi}_{jk} \tag{16}$$

Replacing the biased static diverted value ratios in the GUPPI formula (equation (15)) with its long-run counterpart defines a dynamic generalised upward pricing pressure index, dGUPPI.[32]

$$dGUPPI_{jk} := DDR^{\pi}_{jk}\mu_j \tag{17}$$

Cineworld/Showcase Cinema (2013). Examples in the EU and USA include Austria / Orange Austria (2012) and Dollar Tree/Family Dollar (2015), respectively.

[30] Moresi (2010) advocates the use of gross margins. Similar accounting measures derived from internal documents have been used in the cases listed in footnote 29.

[31] The potential pitfalls of assuming static demand estimation applied to time and product aggregated data is discussed at greater length in Appendix B.

[32] GUPPI is often derived from static first order conditions of two merging products. The online appendix derives dynamic GUPPI using the first order conditions from the dynamic demand and supply model from section 2.

Given that estimation of a sufficiently flexible dynamic demand model is unlikely to be feasible within the policy making time horizon, and simulation of the post-merger outcomes made even more complex by the presence of multiple equilibria, it is hoped this new policy tool will be useful for antitrust analysis of storable good industries.

The remainder of the article focuses on a new computationally light empirical methodology identify and estimate bounds on dynamic diverted value ratios. This can, in turn, facilitate the estimation of the new dGUPPI measure to evaluate the likely strength of competitive interactions relaxed by a proposed merger in storable goods industries.

3. Empirical methodology

This section describes how the first order conditions from the model (eq. (14)) are combined with data on market outcomes and margins to identify and estimate bounds on dynamic diverted value ratios and dGUPPI.

The remainder of this section describes a computationally light three-step estimation procedure. In the first step, an approximation to the dynamic price process is estimated and used to construct price forecasts. The second step uses the data on market outcomes to estimate a static demand model. It provides estimates of quantity demanded in each period and the corresponding matrix of short-run demand derivatives. In the final step, bounds on the elements of the 'reduced-form' matrix of parameters capturing demand dynamics are estimated ready for use in empirical policy analysis.

Before discussing each step, the data assumed to be available to the analyst is outlined. Finally, I describe how the output of the estimation procedure can be used in a policy setting to compute bounds on dynamic diverted value ratios and dGUPPI.

3.1. Required Data

The analyst is assumed to have an estimate of the margin for firm n and data on market outcomes over the same period. In a policy setting, the estimated margin might be calculated directly from internal management accounts - often made available at the request of antitrust authorities. Alternatively, margins might be sourced from published accounts (i.e. reported gross margins, estimated using methods described by De Loecker et al. (2020)).

Data on market outcomes contains information on the products sold in each period, their observed prices and consumer choices. The application in this article uses household level purchase diary data from Kantar WorldPanel. Aggregate level market data observed at a frequency consistent with price setting behaviour also suffices.

3.2. Step 1: Price Forecasts

In the application in Section 4, I consider two polar empirical models of the relationship between price forecasts and prices set in each period: open and closed-loop.

In an open-loop model, a firm's price setting teams treat prices forecasts as given and do not respond to current prices. Therefore, for firm n in each period t, $\boldsymbol{\Delta}^p_{nt} = \mathbf{0}$. In contrast, in the closed-loop model the price setting teams have access to the price forecasting model and the time profile of inter-temporal price derivatives are updated as they make decisions.

In the event that the firm's price forecasting model is not known the analyst, for a closed-loop approach they need to specify a statistical model to approximate it. From the analyst's perspective it is desirable if the approximation to the firms' price forecasting model: (i) can capture the key features of observed high-dimensional promotional price dynamics both across products and over time; (ii) be estimable using sparsely populated historical price data within the policy making time-horizon; and (iii) be straightforward to use to generate the time profile of dynamic price derivatives up to H-periods ahead in each period t.

One possibility is to use a dynamic factor model (DFM). A DFM provides a flexible, data-driven price forecast model that is easy to implement, is well suited to high-dimensional applications, and can be implemented with missing data.[33] Under this model each products' price process is approximated as a weighted sum of a low dimensional set of time-varying price factors that follow a κ-order Markov process. The state space representation of the dynamic factor model of the price process is

$$\boldsymbol{p}_t = \boldsymbol{\Lambda}\boldsymbol{F}_t + \boldsymbol{\epsilon}_t \tag{18}$$

$$\boldsymbol{F}_{t+1} = \boldsymbol{A}\boldsymbol{F}_t + \boldsymbol{U}_{t+1} \tag{19}$$

where $\boldsymbol{F}_t = [\boldsymbol{f}_t, \dots, \boldsymbol{f}_{t-\kappa}]^\top$ is a κR vector comprising of a low dimensional set of R-vector, $\boldsymbol{f}_t$, capturing underlying price trends or 'factors'. Also, $\boldsymbol{\Lambda} = [\boldsymbol{L}, \mathbf{0}, \dots, \mathbf{0}]$ is a $J \times \kappa R$ matrix and $\boldsymbol{L} = \left[\boldsymbol{\lambda}_1^\top, \dots, \boldsymbol{\lambda}_J^\top\right]^\top$ be a $J \times R$ matrix of factor loadings of factor loadings. Standard normalisations on factors and their loadings are imposed.[34] Finally, $\boldsymbol{\epsilon}_t$ is J-vector of price shocks and $\boldsymbol{U}_{t+1} = [\boldsymbol{u}_{t+1}, 0 \dots, 0]^\top$ is a κR vector containing innovations to $\boldsymbol{F}_{t+1}$ where

$$\begin{bmatrix} \boldsymbol{\epsilon}_t \\ \boldsymbol{u}_{t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{pp} & 0 \\ 0 & \boldsymbol{\Sigma}_{ff} \end{bmatrix}\right) \tag{20}$$

where $\boldsymbol{\Sigma}_{pp}$ and $\boldsymbol{\Sigma}_{ff}$ are the covariance matrices for equations (18) and (19), respectively. For a given R, the price factors can be non-parametrically estimated applying a truncated or 'thin'

[33] Another possibility is to use a linear-in-prices VAR(κ) approach to model the price process. While simple to use, it is ill suited to high-dimensional applications. The online appendix discusses this issue in more depth and provides more detail on the DFM.

[34] The normalisations imposed are $\frac{\boldsymbol{f}\boldsymbol{f}^\top}{T} = \boldsymbol{I}_R$ and $\boldsymbol{L}^\top\boldsymbol{L}$ is diagonal where $\boldsymbol{f} := [\boldsymbol{f}_1, \dots, \boldsymbol{f}_T]$ is the $R \times T$ vector of price factors.

singular value decomposition to standardised price data. The coefficients of the Markov process are estimated by using a VAR(κ) on the estimated factors.[35]

The h-periods ahead forecasted price derivatives for a price change in period t are easy to compute functions of its parameters.[36]

$$\widehat{\boldsymbol{\Delta}}^{p}_{t+h} := \boldsymbol{\Lambda}\boldsymbol{A}^{h}\boldsymbol{F}_t\boldsymbol{F}_t^{\top}\boldsymbol{\Lambda}^{\top}\left[\boldsymbol{\Lambda}\boldsymbol{F}_t\boldsymbol{F}_t^{\top}\boldsymbol{\Lambda}+\boldsymbol{\Sigma}_{pp}\right]^{-1} \tag{21}$$

They are then used to construct and empirical version of the price dynamics term in the first order condition (eq. (14)). To that end, the h-step ahead forecasts of product k's price derivatives are assumed to be uncorrelated with product k's h-step ahead demand forecast.

A1: $cov\left(\boldsymbol{\Delta}^{p}_{h}, \boldsymbol{q}_h\right) = \mathbf{0}$ for $h = 1, \ldots, H$

Under assumption A1, the expectation of the present value of revenues due to the temporary price change in period t is product of the expected price forecast derivative and the expected future quantity demanded.[37] Given H and a discount factor, the empirical analogue to this term is

$$\boldsymbol{\Upsilon}_{nt} = \mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\boldsymbol{\Delta}^{p}_{nh}\boldsymbol{q}_h = \boldsymbol{\Gamma}_{nt}\bar{\boldsymbol{q}} \tag{22}$$

where $\boldsymbol{\Gamma}_{nt} = \sum_{h=t+1}^{H}\delta^{h-t}\widehat{\boldsymbol{\Delta}}^{p}_{nh}$ and $\bar{\boldsymbol{q}} := \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{q}\left(\boldsymbol{p}_t\right)$ is a J-vector of average quantity demanded over T periods estimated using the static demand model from step two of the estimation procedure.

3.3. Step 2: Demand Estimation

Without limits on time, computation, or data resources, policy analysts can estimate a dynamic demand model. Once estimated, a dynamic demand model can be used to simulate the path of quantity responses to changes in the price process.[38] In practice, however, policy analysts are unlikely to be in a position to estimate dynamic demand models within the policy setting horizon. Instead, in practice a static differentiated product demand model can be estimated using observed weekly market outcome data.

As discussed above, price elasticities and diversion ratios computed from short-run demand derivatives are biased estimators of their long run counterparts. This is because they exclude

[35] The choice of R and κ are based on information criteria. This is discussed further in Appendix B.

[36] See the online appendix for a derivation of price derivatives.

[37] The validity of this assumption can be empirically supported by examining the covariances of $\widehat{\Delta}^{p}_{jkt+h}$ and $q_{k,t+h}$ for $h = 1, \ldots, H$ over the T periods for $k \in \mathcal{J}$. If empirical covariances are not deemed to be sufficiently close to zero, the empirical covariance for each (j, k, t, h) can be added to equation (22).

[38] Hendel and Nevo (2006a) use their dynamic demand model to simulate long-run quantity responses to permanent changes to US laundry detergent product prices. Erdem et al. (2003) simulate long-run quantity responses to transitory and permanent changes to brand level prices using their dynamic demand model of the US Ketchup industry (see Appendix A).

the possibility of inter-temporal substitution. However, short-run demand derivatives estimated using the static model, $\widehat{\mathbf{\Delta}}^q_t$, are also biased and inconsistent estimates of the contemporaneous demand responses to a temporary price cut. This is because inventory and price expectations are omitted from the static demand model.

In static discrete demand models with a common marginal utility of income for all consumers, the bias from omitting inventory and price expectations scales demand derivatives through its impact on the price coefficient. Therefore the absolute magnitude of the bias is the same for all own and cross-price demand derivatives: $B_{jk} = B_{j'k'} = B$ for all $j, k, j', k' \in \mathcal{J}$ where $B_{jk} := (\widehat{\Delta}^q_{jkt} - \Delta^q_{jkt})/\Delta^q_{jkt}$.

This restriction approximately holds for more flexible static demand models when switchers and brand-loyal consumers have a similar distribution of marginal utility of income. Hendel and Nevo (2006a) find that the derivatives computed from a static demand model omitting inventory and price forecasts overstate demand responses: $B > 0$. In turn, magnifying their existing bias as estimators of long-run demand derivatives.[39]

3.4. Step 3: Approximating demand dynamics

Demand dynamics in the first order condition (eq. (9)) are a function of the element-wise product of the short-run demand response to a temporary price cut and dynamic correction ratios, $\mathbf{\Psi}_t$. Without a dynamic demand model, the analyst cannot simulate consistent estimates of short-run demand responses to a temporary price cut in period t, $\mathbf{\Delta}^q_t$.

Alternatively, the analyst can replace them with estimates from the static demand model estimated in step two, $\widehat{\mathbf{\Delta}}^q_{nt}$. However, as noted above, static demand derivatives from the misspecified static demand model are biased estimators of the true short-run demand responses. Specifically, they contain bias due to omission of inventories and price expectations from the static demand model, $\widehat{\mathbf{\Delta}}^q_{nt} = \mathbf{\Delta}^q_{nt}(1 + B)$. If so, the dynamic demand term is the element-wise product of estimated short-run demand derivatives and a matrix of reduced-form parameters, $\boldsymbol{\theta}_t$, whose elements are a function of bias-scaled dynamic correction ratios.

$$\mathbf{\Delta}^q_{nt} \odot (1 + \mathbf{\Psi}_t) = \widehat{\mathbf{\Delta}}^q_{nt} \odot \boldsymbol{\theta}_t \tag{23}$$

where $\boldsymbol{\theta}_t := (1 + \mathbf{\Psi}_t) / (1 + B)$.

In the final step of the estimation procedure, the outputs from the previous two steps are combined with price data and firm n's margins estimated from their internal accounts to

[39] When households observe a temporary sale, those with sufficiently low existing inventory may accelerate purchases and take advantage of discounted prices. As a result, inventories tend to be positively correlated with high prices that follow the end of the temporary sale periods. Therefore, when prices exhibit positive serial correlation (i.e. they are low today, but expected to be high in the next period), omitting inventory negatively biases the static demand's price coefficient. In turn, fortifying the bias due to the exclusion of demand dynamics from the static demand model.

estimate these reduced form parameters.[40]

Since accounts are costly to compile, financial statistics tend to be produced periodically (i.e. firm-level annual gross margins). The margins calculated from firm n's internal accounts, $\widehat{\mu}_n$, reflect the cumulative effect of the sequence of pricing decisions across multiple products, cost shocks, and corresponding quantity responses spanning several time periods. As a result, they contain information on the effect that demand and price dynamics have on firms' ability to impart market power.

To leverage the information contained in the profit margin estimated directly from internal accounts covering T periods, the expression for the mark-up from the firm's first order condition is summed over all the periods and products it covers. Dividing through by corresponding revenues gives an expression for the margin in terms of a sequence of purchase quantities, estimated short-run demand derivatives, the firm's price forecasts, and $\boldsymbol{\theta}_n = \{\Omega_n \odot \boldsymbol{\theta}_t\}_{t=1}^{T}$. The resulting expression in the model for the margin earned over T periods by firm n in the model is

$$\mu_n(\boldsymbol{\theta}_n) = -\frac{\sum_{t=1}^{T} \boldsymbol{q}_{nt}^{\top} \left(\boldsymbol{\Delta}_{nt}^{q} \odot \boldsymbol{\theta}_t\right)^{-1} \left(\boldsymbol{q}_t + \boldsymbol{\Upsilon}_{nt}\right)}{\sum_{t=1}^{T} \boldsymbol{q}_{nt}^{\top} \boldsymbol{p}_t} \tag{24}$$

The final step of this procedure solves for elements in $\boldsymbol{\theta}_n$ that set the firm n's margin derived from internal accounts equal to its counterpart from the model, $\widehat{\mu}_n = \mu_n(\boldsymbol{\theta}_n)$. However, with a single margin equation covering multiple products' sales, the TJ_n^2 non-zero parameters affected by firm n's pricing decisions are not point-identified.

Even for a single product firm, there are T parameters to estimate. While the reported margin contains information on how demand and supply dynamics affects market power levied across multiple purchase cycles, only an aggregate measure of bias across the whole period can be recovered.[41] Accepting this practical limitation, assumption A2 restricts the dynamic correction factors to be mean stationary over the accounting period.[42]

A2: $\boldsymbol{\Psi} = \mathbb{E}\left[\boldsymbol{ITDR}_t^{\pi} \oslash \boldsymbol{DR}_t^{\pi}\right] \ \forall \ t = 1, \ldots, T$

Under this assumption A2, the number of parameters in $\boldsymbol{\theta}_n$ is reduced by a factor of T. For a single product firm, under A2, $\widehat{\mu}_n = \mu_n(\theta_n)$ point-identifies θ_n. However, for firms with even a handful of products the elements of $\boldsymbol{\theta}_n$ are not point identified. Moreover, given that storable good firms often sell tens of products, the bound on J_n^2 parameters is likely to be too wide to be useful in a policy setting. In practice, for use in a policy setting further restrictions are needed.[43]

[40] For expositional purposes I will focus on firm n's margin. More generally, the empirical approach described in this section can be applied to margins earned on sales of any group of products (i.e. brand level margins).

[41] Any subset of $T-1$ parameters can be set to 0 and the remaining free element can be set to solve the equation.

[42] "$\oslash$" denotes element-wise division.

[43] See footnote 41. A similar logic can be applied across products, albeit within theoretical parameter bounds.

From the model above and the existing literature, the direction and magnitude of bias using a static demand model is expected to differ by promotion status. If the good is promoted, static demand responses overstate total volume responses. For substitutes to the promoted good, total demand responses are understated by the output of static demand models. In line with theoretical and empirical findings, two different aggregate measures of bias adjustment are retained in assumption A3. One for own-price demand derivatives and another for cross-price demand derivatives.

For promoted products, the diagonal elements of $\boldsymbol{\theta}_n$ contributing to firm n's margin are constrained to be the same. This constrains the fraction of profits earned on the promoted good during the sale period that are pulled forward from its future sales to be the same for all firm n's products. Similarly, all non-promoted products are assumed to have the same fraction of current profits lost over future periods to the promoted good in the sale period. This constrains all off-diagonal elements in the non-zero rows of $\boldsymbol{\theta}_n$ to be the same.[44] Asymmetric overall substitution for individual products is inherited from the static demand estimation.

A3: $\theta_{jj} = \theta_n^{own} \in (0, 1]$ and $\theta_{jk} = \theta_n^{cross} > 1$ for all $j \neq k, j \in \mathcal{J}_n$, $k \in \mathcal{J}$

Under assumptions A2 and A3, the number of parameters in $\boldsymbol{\theta}_n$ reduces from TJ_n^2 to two parameters for each firm n. The set of values for firm n are defined by the set

$$\Theta_n^\star := \left\{ (\theta_n^{own}, \theta_n^{cross}) \in \left[\underline{\theta}_n^{own}, \overline{\theta}_n^{own}\right] \times \left[1, \overline{\theta}_n^{cross}\right] \mid \widehat{\mu}_n = \mu_n\left(\theta_n^{own}, \theta_n^{cross}\right) \right\} \tag{25}$$

where $\underline{\theta}_n^{own}$ and $\overline{\theta}_n^{own}$ are the minimum and maximum values of θ_n^{own} when θ_n^{cross}.

The upper bound, $\overline{\theta}_n^{cross}$, is application specific and chosen by the policy analyst. For example, as highlighted by the simulation in Erdem et al. (2003) in Appendix A, inter-temporal substitution declines rapidly over time and is largely complete before the median household's inter-purchase duration. Imposing assumptions on the nature of the decline in inter-temporal substitution over time (i.e. the fraction of Δ_{jkt}^q foregone exponentially falls) and the percentage of substitution completed $N \leq H$ weeks after the price cut can be used to give plausible values for $\overline{\theta}_n^{cross}$.

Having chosen $\overline{\theta}_n^{cross}$, the analyst can fix a grid of values for $\theta_n^{cross} \in \left[1, \overline{\theta}_n^{cross}\right]$ and use root-finding algorithm to find the value θ_n^{own} given θ_n^{cross} that sets the margin estimated from internal accounts equal to its counterpart in the model.

3.5. Empirical policy tools

The set of parameters, $\Theta_n^\star$, can be used to modify inputs into empirical policy tools used to assess competitive interactions. The inputs, and therefore empirical policy tools, are set-valued.

[44] For empirical support for this assumption see Figure 4 in Appendix A replicating Erdem et al's (2003) simulation of dynamic substitution patterns in the US Ketchup industry. The relative dynamic responses of non-promoted goods appear approximately symmetric over time relative to the contemporaneous response.

Section 2.5 introduced a new price pressure test applicable to the dynamic pricing model, $dGUPPI_{jk}$. Using the estimate of firm n's margin from its internal accounts to approximate product j's margin, the only missing component from the dGUPPI calculation is the equation (17) the dynamic diverted value ratio, DDR^{π}_{jk}.[45]

Dynamic diverted value ratios Recall from equation (16), the dynamic diverted value ratio is the multiplicative product of a function of dynamic correction ratios and the static diverted value ratio. Since the latter can be directly estimated using the static demand model, only the function containing elements of the matrix of dynamic correction ratios is required to compute $dGUPPI_{jk}$.

Under assumptions A2 and A3, the elements of $\Theta^{\star}_n$ can be used to compute

$$\frac{1+\Psi_{jk}}{1+\Psi_{jj}} = \frac{1+\theta^{cross}_n}{1+\theta^{own}_n} \tag{26}$$

for $j \neq k$ where $j \in \mathcal{J}_n$ and $k \in \mathcal{J}$. In turn, this facilitates consistent estimation of the bounds on dynamic diverted value ratios, DDR^{π}_{jk}. Specifically, the bounds are given by

$$\underline{DDR}^{\pi}_{jk} := \min\left\{\frac{1+\theta^{cross}_n}{1+\theta^{own}_n}\ s.t.\ (\theta^{own}_n, \theta^{cross}_n) \in \Theta^{\star}_n\right\} DR^{\pi}_{jk} \tag{27}$$

$$\overline{DDR}^{\pi}_{jk} := \max\left\{\frac{1+\theta^{cross}_n}{1+\theta^{own}_n}\ s.t.\ (\theta^{own}_n, \theta^{cross}_n) \in \Theta^{\star}_n\right\} DR^{\pi}_{jk} \tag{28}$$

where the short-run diverted value ratios is calculated using the static demand model's outputs, margins estimated from firms' accounts, and observed prices over T periods.[46]

dGUPPI These upper and lower bounds on DDR^{π}_{jk} can be then plugged into equation (17) to compute bounds on $dGUPPI_{jk}$.

$$dGUPPI_{jk} \in \left[\underline{DDR}^{\pi}_{jk}, \overline{DDR}^{\pi}_{jk}\right] \widehat{\mu}_j \tag{29}$$

The main drawback is that point-identification of dGUPPI is lost. However, as shown in the policy applications in sections (4.2), dGUPPI being set-valued does not necessarily preclude its efficacy in a policy setting.

[45]Long-run price elasticities can also be approximated using the output of the estimation procedure. See Appendix A.

[46]See the online appendix. Diversion ratios enumerated using volume weights as described Domencich and McFadden (1975).

4. Application: UK laundry detergent

The model described in section 2 and corresponding empirical methodology in section 3 is applied to the UK laundry detergent industry. The analysis of the industry uses Kantar WorldPanel individual household purchase diary data from 1st January 2002 until 31st October 2012 focusing on purchases at a leading UK supermarket.[47]

For brevity, both the estimation of the closed-loop price forecasting model and the workhorse static nested logit demand model contained in Appendix B. It also contains the results of the sensitivity analysis to the baseline model estimates of $\boldsymbol{\theta}_n$. The remainder of the section focuses on the baseline model results obtained in step three of the estimation procedure.

To place the empirical analysis into context, this section begins by outlining the key characteristics of the UK laundry industry. To illustrate how the omission of demand dynamics has the potential to lead to misguided policy conclusions, the outputs of estimation procedure are used to conduct a policy experiment. It demonstrates the methods and empirical policy tools developed in this article have the potential to prevent errors and aid policy decision making.

4.1. Industry Overview

Two firms account for around 75 to 85 percent of households' annual purchases of UK laundry detergents in each year between 2002 and 2012. Hereafter, they are referred to as firm A and firm B. Firm A's annual share of sales is between 44 and 54 percent and firm B is between 29 and 36 percent. Outside of these two major producers of branded products, the retailer's private label products commands the largest share - although its share has declined from 25 percent in 2002 to 12 percent in 2012. A fringe of small niche brands account for the remainder of products sold.

Laundry detergent is sold in a diverse array of brands, formats, and pack sizes. Each pack, or product, contains a single type of detergent. In general, a detergent is defined by its format, its brand and the chemical properties of the enzymes it contains (i.e. non-bio/bio, stain removal properties, scent etc).

In addition to the retailer's private label products and the fringe of 'other' brands, the two largest firms sell six major brands between them. These six brands are labelled alphabetically as brand A through to brand F and are differentiated by their perceived quality.[48] Using the average purchased price per wash as an indicator for consumer perception of quality, brand D is classified as a premium brand (20p per wash). Brands A, C and E are mid-range brands (17-18p per wash) and brands C and F are 'standard' brands whose price is similar to the supermarket's private label detergents (13-14p per wash).

[47] A more detailed description of the data is provided in Appendix B.

[48] Brands are not attributed to firms for confidentiality reasons.

Laundry detergent brands are sold in six formats: powder, tablets, liquids, liquid capsules, super concentrated liquid, and gel.

Formats have different dosage metrics: liquid and gel are measured in milliliters per wash, whereas powder and tablets are measured in grams per wash. Further, following a series of industry-wide initiatives that sought to reduce the environmental impact of detergent production and the development of new products, dosages tend to decline over time within formats. To avoid complications arising from correcting for changing dosages, the number of washes is used to measure pack size.

Laundry detergents can be purchased in many different pack sizes.[49] As dosages have declined over time, so has the physical storage space occupied by material needed per wash. In response, firms produce larger pack sizes over time: the average number of washes per purchased pack have increased by around 50 percent from 17 washes in 2002 to 26 washes in 2012.

In this application a product is defined as a unique combination of brand, format and pack size. Table 1 provides summary statistics for the resulting 668 products together with consumer characteristics (including household size and a proxy for household income - i.e. average weekly grocery spend).

Table 1: Summary of Kantar Worldpanel data

	Mean	Median	Std dev	Min	Max
Purchase Characteristics					
Price in (£)	3.58	3.20	1.81	0.13	24.52
Price per wash (£)	0.17	0.16	0.06	0.01	0.77
Purchased Washes	23	20	12	3	100
Dosage in Grams	87.7	80	16.1	18.3	139.1
Dosage in Millilitres	52.6	45	20.8	12	125
Number of Packs	1.07	1	0.26	1	2
Household Characteristics					
Number of Equivalent Adults	2.28	2.20	0.79	1.00	8.10
Av. weekly grocery spend (£)	67.05	63.78	27.71	3.70	423.45

Source: Kantar Worldpanel

[49] The UK laundry detergent industry differs in this respect from the one studied by Hendel and Nevo (2006a). They restrict attention to powder products and examine brand choice conditional on size choice from a small number of discrete sizes: 16oz, 32oz, 64oz, 96oz, and 128oz. Erdem et al. (2003) also focus on only five different weight choices in the US Ketchup market in their dynamic demand estimation.

Table 2: Inter-purchase duration in weeks

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Median	3	4	4	4	4	4	4	5	5	5	5
Mean	5	6	7	7	7	7	8	9	9	10	10

4.2. Solving for $\boldsymbol{\theta}_n$

As is often the case in policy analysis, having reviewed the available evidence there a number of inputs chosen by the analyst. In this case, these include modifications to reflect key idiosyncratic industry features and parameter calibrations. Specifically, to estimate $\boldsymbol{\theta}_n$ the analyst specifies the optimisation window (H), the firm's discount factor, the price forecast model, and $\overline{\theta}_n^{cross}$ for the baseline model. Sensitivity analysis evaluates the robustness of conclusions drawn to input assumptions. Next, I describe these choices for a baseline model of the UK laundry detergent industry over the sample period.

Baseline Model In the baseline model, the optimisation horizon is set equal to the median inter-purchase duration observed in the data in each year.

Table 2 presents the median and mean inter-purchase durations for households buying UK laundry detergent in the Kantar Worldpanel data from 2002 to 2012. As a result, H grows from three weeks in 2002, to four weeks from 2003 to 2008, and five weeks from 2009. This increase in inter-purchase duration over time is consistent with the ability of consumer to take advantage of product innovation reducing storage cost per wash and buy larger products.

Table 2 shows that the mean inter-purchase duration is, as expected, larger the median - doubling from 5 weeks in 2002 to 10 weeks by 2011. Unlike the median, the mean is affected by a handful of households with long gaps between purchases - to whom price dynamics are unlikely to be a first order purchase determinant. For this reason, an analysis with H equal to mean inter-purchase duration is retained as a scenario to evaluate the sensitivity of the baseline model with an alternative closed-loop forecast scenario with a longer optimisation window.

The firm's weekly discount factor reflects the cost of capital obtainable at the time from financial markets in the sample. For this purpose, the sample is split pre and post the 2008 financial crisis. Reflecting underlying movement in UK base rates, the firm's weekly discount factor is $\delta = 0.998$ for the period 2002 to 2008 and $\delta = 0.999$ thereafter.

Having specified H and δ, the analyst also needs to choose a price forecasting model to calculate the $\boldsymbol{\Gamma}_{nt}\bar{\boldsymbol{q}}$ for $t = 1, \ldots, T$ in $\mu_n(\boldsymbol{\theta}_n)$. In the baseline model, prices are forecast using the DFM estimated in the first step of the estimation procedure. The open loop forecasting model, in which $\boldsymbol{\Gamma}_{nt}\bar{\boldsymbol{q}} = 0$, is retained as another scenario to evaluate the robustness of the key conclusions to the closed-loop assumption in the baseline model.

Table 3: $\overline{\theta}_n^{cross}$ by optimisation horizon

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
H = median	1.36	1.59	1.59	1.59	1.59	1.59	1.59	1.82	1.82	1.82	1.82
H = mean	1.83	2.06	2.30	2.30	2.30	2.30	2.55	2.80	2.80	3.04	3.04

$\overline{\theta}_n^{cross}$ is chosen assuming that most inter-temporal substitution are purchases that would have occurred in the near future. Specifically, the fraction of switchers is assumed to exponentially decline over the chosen optimisation window.

$$\overline{\theta}_n^{cross} = \sum_{h=0}^{H} \delta^h \exp(-rh) \tag{30}$$

Table 3 shows the values for $\overline{\theta}_n^{cross}$ under the mean and median inter-purchase duration. In the baseline model, the parameter controlling the rate of exponential decline, r, is calibrated so that inter-temporal substitution is largely complete by period H.[50] In the application, it is assumed that two percent of contemporaneous switchers would have purchased a rival product in H periods time. The resulting values for $\overline{\theta}_n^{cross}$ are 1.36 in 2002, 1.59 between 2003 to 2008, and 1.84 from 2009 onwards. Setting the optimisation window to the mean, rather than the median, inter-purchase duration provides higher values for $\overline{\theta}_n^{cross}$. The sensitivity of the baseline model to $\overline{\theta}_n^{cross}$ is part of the scenario with a longer optimisation window.[51]

Using inter-purchase duration to compute $\overline{\theta}_n^{cross}$ and the forecast horizon links the estimates of $(\theta_n^{own}, \theta_n^{cross})$ to the changes in the size of products sold. With more washes in inventory, inter-purchase durations increase and inter-temporal substitution is a larger component of overall substitution. All else equal, omitting demand dynamics might be expected to increase bounds on θ_n^{own} and θ_n^{cross} when products contain more washes.

Finally, laundry detergents are subject to VAT in the UK. The first order conditions are updated accordingly. For simplicity, changes in VAT rates are assumed not to be anticipated by firms and computations reflect the VAT prevailing in the UK at the time.[52]

[50] In Erdem et al's (2003) simulation, inter-temporal substitution is compared to the contemporaneous demand response from a dynamic demand model. As noted above, the contemporaneous demand response is likely to be overstated by a static demand model. Therefore, $\overline{\theta}_n^{cross}$ might be interpreted as a conservative estimate.

[51] Table 3 shows the values of $\overline{\theta}_n^{cross}$ under the median and mean inter-purchase duration. Appendix B shows how results are affected when the scenario when the optimisation window equals the mean inter-purchase duration.

[52] From the beginning of the sample until 30th November 2008 UK VAT was 17.5%. From 1st December 2008 to 30th December 2009 it was cut to 15%. It was returned to 17.5% from 1st January 2010 until 3rd January 2011. Since 4th January 2011 UK VAT has been 20%.

The results of the subsequent analysis under the different scenarios noted above are presented in Appendix B. This sensitivity analysis shows that while some magnitudes change, the key conclusions of the policy results are qualitatively unchanged.

Estimation In the final step of the estimation procedure, the baseline model is combined with market outcome data and estimated margins to find values of $\boldsymbol{\theta}_n$ that solve equation (24) for firms A and B in each year from 2002 to 2012.

In the context of antitrust investigations, brand level margins may be available over several years. However, in my case, I only have access to published annual accounts. Therefore, the published gross margins in firm A's and firm B's annual accounts from 2002 to 2012 are assumed to be good approximations to the economic margins earned on sales of their laundry detergent portfolio in the UK.[53]

To following procedure is used to compute the elements of $\Theta_n^\star$. First, fix a value for θ_n^{cross} from the interval $\left[1, \overline{\theta}_n^{cross}\right]$. Next, plug in firm n's margin, observed prices, and other parameterised components of the baseline model. Finally, solve for θ_n^{own} so that $\widehat{\mu}_n = \mu_n\left(\theta_n^{own} \mid \theta_n^{cross}\right)$. Repeating the procedure for a grid of points spanning the set of possible values for $\theta_n^{cross} \in \left[1, \overline{\theta}_n^{cross}\right]$ and interpolating the results defines the set of solution pairs: $(\theta_n^{own}, \theta_n^{cross}) \in \Theta_n^\star$.[54]

Results When $\theta_n^{own} = \theta_n^{cross} = 1$ the static demand model dictates substitution patterns and the model's percentage margin calculation over T periods contains no adjustment for demand dynamics. If, as anticipated, the static demand model overstates own price elasticities and understates cross price elasticities, the model generates too little market power and cannot match observed margins.

Figure 2 shows $(\theta_n^{own}, \theta_n^{cross}) \in \Theta_n^\star$ in each year for firm A (the left panel) and for firm B (the right panel). They trace out the combination of asymmetric inter-temporal substitution adjustments for the promoted and rival products that equate the model's margin to that estimated from accounting data. In line with the findings in dynamic demand literature, as expected, $(\theta_n^{own}, \theta_n^{cross}) \in \Theta_n^\star$ are positively correlated.

This correlation arises because both own and cross product inter-temporal substitution increase market power. The strongest inter-temporal substitution for the promoted good, $\theta_n^{own} = \underline{\theta}_n^{own}$, occurs when there are no demand dynamics with rival products. However, as θ_n^{cross} increases and approaches its upper bound, the firm accrues market power from recaptured future sales of its substitute products. As a result, the model can match the margin when future sales of the promoted good are perceived by consumers as more distant substitutes for current sales.

[53] The financial year for firm B starts midway through the year. As such, the annual report margins are adjusted to match calendar years in the data. Figures are omitted for confidentiality reasons.

[54] The first two periods are dropped in 2002 because two lags are required to produce estimates $\boldsymbol{\Gamma}_{nt}\bar{\boldsymbol{q}}$.

The size of the adjustment and evolution over time differs for firms A and B. Between 2002 and 2006, 20 to 30 percent of firm A's of its uplift in profits from the promotion is drawn from its future profits. From 2007 onwards, the bias due to the omission of inter-temporal substitution increases. By 2011, a larger adjustment to the static demand model's derivatives is required to match margins. In 2011 and 2012, over 50 percent of A's promotional profits draw down on the present value of its promotional products' future sales.

The bias due to the omission of inter-temporal substitution increases because products contain more consumption units following storage product compaction innovations in 2007 and 2008 (see Figure 6 in Appendix B). Because firm A hold the purchased price per wash broadly constant (see Figure 5 in Appendix B), increasing the number of washes leads to proportionately higher prices. In the static nested logit demand model estimated, higher prices lead to more elastic demand responses and therefore even lower margins in the unadjusted supply model. This effect is compounded by the fact that product innovation allows firm A to reconfigure their portfolio to contain more distant substitutes over time.[55]

The right panel of Figure 2 shows the sets $(\theta_B^{own}, \theta_B^{cross}) \in \Theta_B^\star$ are less dispersed over time than for firm A. From 2002 to 2008, between 31 and 39 percent of firm $B's$ promotional profits are drawn from its promoted products' future sales.

After 2008 - as product compaction is applied to more of its products - these figures increase to between 39 and 45 percent in 2009 and 2010. However, after 2011 only 31 to 35 percent of the promotional profit uplift are accelerated future purchases. At first glance this is puzzling - like firm A, firm B's compaction innovation also led to increased product sizes and an increasingly different product range.[56] However, the trend can be explained by differences in pricing strategy and an observed reduction in market power after 2010.

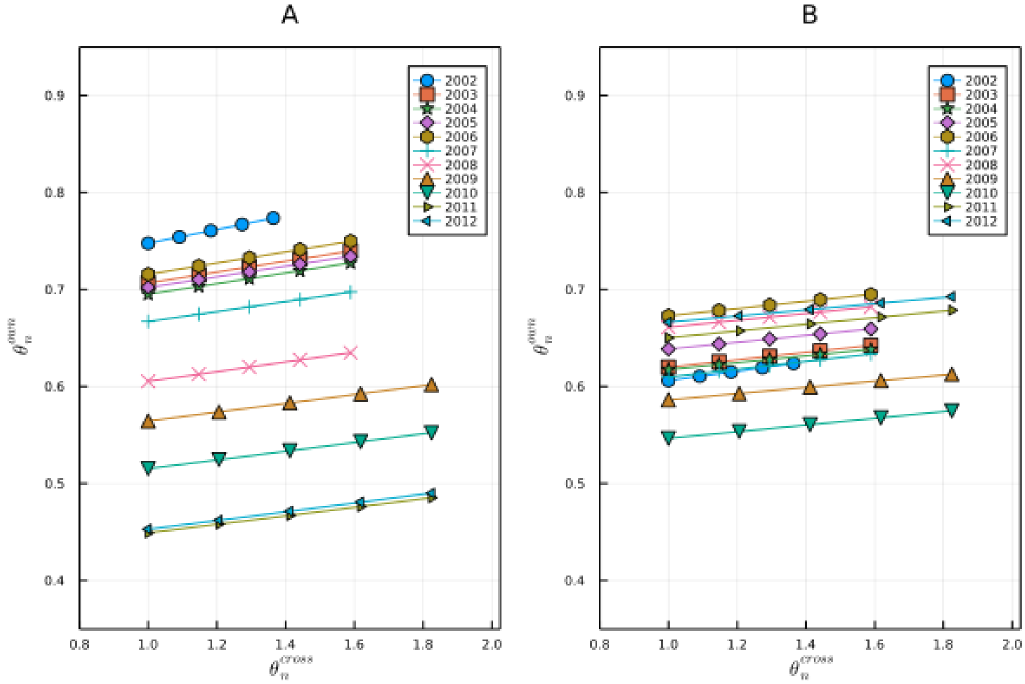
Unlike firm A, Figure 5 shows that firm B chose to hold posted price per wash stable, but offer increasingly deep discounts from 2007 onwards. This leads to a lower purchased price per wash. Even though product sizes increase after 2007 as product are compacted, firm B's prices increase to a lesser degree than firm A and the demand response is only slightly more elastic. This mitigates the need for increasingly large adjustments to match observed margin from 2008 to 2010.

The subsequent increase bounds on θ_B^{own} can be attributed to the combination of a 10 percentage point reduction in the observed margin in 2011 and 2012 and increasingly deep discounting. Together, the effect they have on market power are large enough to more than offset the increased elasticity due to continued growth in product prices.

[55] The ratio of θ_n^{cross} to θ_n^{own} is increasing when products in $\mathcal{J}_n$ are perceived to be closer substitutes. The ratio of θ_A^{cross} to θ_A^{own} falls 38 percent from 0.071 in 2002 to 0.044 by 2011.

[56] The ratio of θ_B^{cross} to θ_B^{own} for $(\theta_B^{own}, \theta_B^{cross}) \in \Theta_B^\star$ falls 35 percent from 0.049 in 2002 to 0.032 by 2012.

Figure 2: $(\theta_n^{own}, \theta_n^{cross})$ for Firm A and Firm B in each year from 2002 to 2012



4.3. Antitrust policy experiment

To illustrate how the estimates of $(\theta_n^{own}, \theta_n^{cross}) \in \Theta_n^{\star}$ can be used in a policy setting, suppose brand A and brand E are produced by different firms. Further assume that the firm producing A is buying brand E from the rival firm at the end of 2012. For the purpose of this exercise assume brand A is produced by firm A and brand E by firm B.[57]

Further, for the purpose of this hypothetical acquisition assume that brand A is the closest substitute for brand E. Therefore, if empirical policy analysis can show that brands A and E are sufficiently distant competitors, then the acquisition is more likely to be permitted.

To assess whether the proposed acquisition is likely to lead to a significant lessening of competition, the analyst can calculate dGUPPI. Allowing for some tolerance in potential efficiencies arising through joint production, it is assumed that the policy analyst views a dGUPPI below five percent as sufficiently low to allow the brand merger to occur without giving rise to anti-competitive concerns.[58]

[57] For confidentiality reason, brands cannot be explicitly assigned to firms.

[58] Five percent (or higher) might be used because it is often used as the floor for a significant non-transitory increase in price for market definition purposes. However, for goods that comprise a large fraction of consumption, competition authorities have used more stringent GUPPI thresholds. For example in the investigation into the proposed Asda/Sainsbury's merger the UK Competition Markets Authority used a 2.75 percent GUPPI threshold; a 1.5 percent baseline with a 1.25 percent efficiencies allowance to assess local

The dGUPPI is calculated in each year to gather evidence on the likelihood of prices rising by examining the strength of the incentive to increase the prices in previous years. Two dGUPPI indices are computed; one for the price increase of brand A and another for the price increase of brand E. In both cases they are compared to a static GUPPI. [59]

The static GUPPI for brand A and brand E is shown by the red lines in the left and right panels of Figure 3, respectively. Between 2002 and 2007 the $GUPPI_{AE}$ is relatively stable and averages 6.4 percent. After 2008, it first drops below the five percent threshold: starting from 6.1 percent in 2007 it declines to 3.8 percent in 2012. This drop in the $GUPPI_{AE}$ is driven by a decline in the diverted value ratio - it fell from 11.9 percent in 2007 to 7.6 percent in 2012. Detergent level analysis indicates that drop occurs because brand A became a leading brand for the new gel formats rolled out from 2008 onwards. Prior to that brand E's powder and tablets were perceived to be closer substitutes by brand A customers.

$GUPPI_{EA}$ is below the five percent threshold in each year over the period - an average of 4.2 percent across the period. Therefore, relying solely on a static GUPPI measures, an analyst might conclude that although the brands were closer competitors in the past, innovation in product formats and product compaction since 2007/8 led to the brands being sufficiently distant competitors by 2012 to permit the acquisition for both brand A's and brand E's customers.

However, once the diversion ratios are adjusted for bias due to the omission of demand dynamics, the analyst's conclusions are reversed. The resulting range of possible values of the dGUPPI is shown by the blue bands in Figure 3.

The left panel shows the range of possible values for $dGUPPI_{AE}$. It shows that once intertemporal substitution is accounted for, the price increase predicted by $dGUPPI_{AE}$ in 2012 lies between 8.4 and 14.3 percent - well above the five percent threshold. By itself, this finding would be sufficient to reverse the conclusions of the static GUPPI analysis.

The right panel shows the range of possible values for $dGUPPI_{EA}$. By 2012, the range of possible values for $dGUPPI_{EA}$ is 6.2 to 10.9 percent. Unlike the static GUPPI, the analyst now cannot rule out that the dGUPPI lies strictly below the five percent threshold in all years. These findings would support a prohibition of the firm producing brand A to acquire brand E.

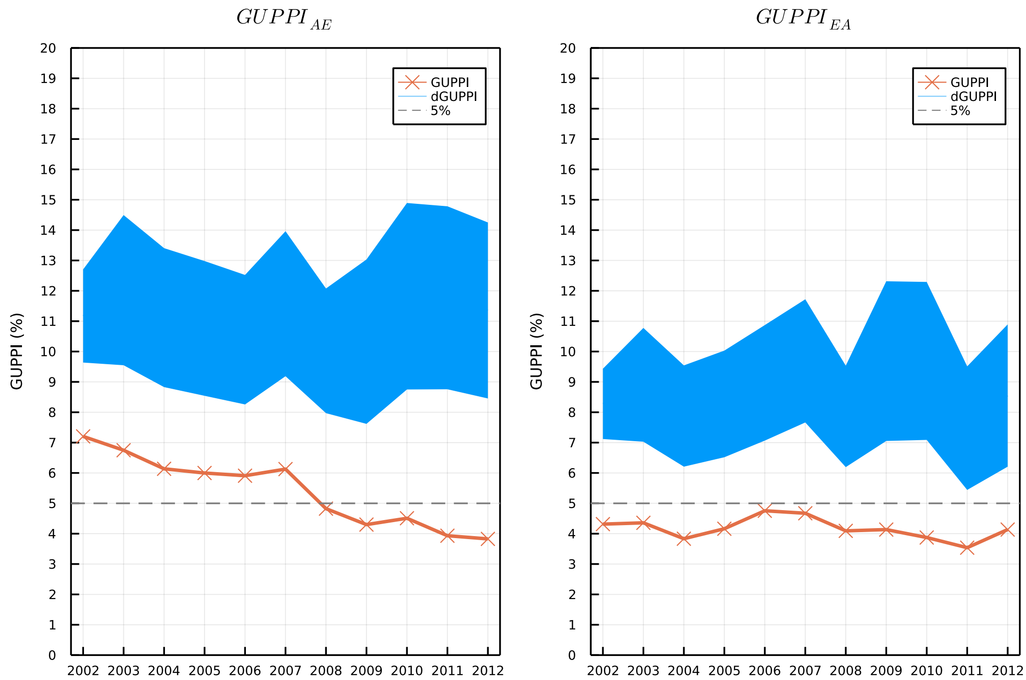
5. Conclusion

This article develops a new approach to estimate bounds on sets of dynamic diverted value ratios for storable goods industries. To identify substitution dynamics missing from static demand models, the approach leverages information on market power over multiple purchase cycles contained in margins derived from a firm's internal accounts. To estimate the bounds,

market competition. See Competition Markets Authority (2019) "Anticipated merger between J Sainsbury PLC and Asda Group Ltd: Final Report".

[59] GUPPI and dGUPPI are calculated using equations (15) and (29) , respectively.

Figure 3: GUPPI and dGUPPI: brand A and brand E, 2002 to 2012



these are equated to the margin from a dynamic demand and supply model for forward-looking storable good firms.

By combining the output of a price forecasting and static demand models with prices and margins, I recover parameters that capture the effect of inter-temporal substitution omitted from static models of firms' pricing incentives. These parameters are used to construct bounds on estimates of dynamic diverted value ratios.

I show dynamic diverted value ratios can be combined with margins in a new generalised upward price pressure index adjusted for demand dynamics - dGUPPI. This new empirical policy tool extends the existing GUPPI test and can be used to evaluate mergers in industries with dynamic demand.

To illustrate this new framework, I apply it to the UK laundry detergent industry from 2002 to 2012. I show that static diversion ratios are severely downward biased measures of their long-run counterparts. As expected, this bias grows when inter-temporal substitution becomes an increasingly large component of aggregate consumer switching in response to promotional activity.

To show how the set-valued estimates of policy inputs can be used to aid policy-making I show how sets of dynamic diverted value ratios produce dGUPPI ranges that can help a competition authority evaluate the likelihood of unilateral effects arising from a hypothetical brand acquisition from a rival. In turn, I show that the use of set-valued policy inputs is central to avoiding policy errors.

The application demonstrates that this new approach is well suited to policy work. It only requires standard empirical methods and readily available data on margins and market-level industry outcomes. As a result, it can be easily implemented within the policy making time-horizon. More generally, it highlights the potential of empirical analysis producing set-valued outcomes to aid the policy-making process.

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Appendices

A. Dynamic Storable Good Model

A.1. First Order Conditions

First consider the case of two single product firms competing in a duopoly setting prices to maximise expected discounted profits over only two periods. In this case, for firm f selling only product j the discounted profits earned over the periods are given by

$$\pi_{nt}^{NPV} = (p_{jt} - mc_{jt})\, q_j\,(\boldsymbol{p}_t, \boldsymbol{\omega}_t) + \delta \mathbb{E}_t\,(p_{j,t+1} - c_{j,t+1})\, q_j\,\left(\boldsymbol{p}_{t+1}, \boldsymbol{\omega}_{t+1}\right)$$

where mc_{jt} is the marginal cost of production for product j in period t.

Firm n's optimal price of product j in the current time period satisfies the first order condition given below

$$\begin{aligned} q_{jt} + \frac{\partial q_{jt}}{\partial p_{jt}} m_{jt} + \delta \mathbb{E}\left[\left(\frac{\partial q_{j,t+1}}{\partial p_{j,t+1}}\frac{\partial p_{j,t+1}}{\partial p_{jt}} + \frac{\partial q_{j,t+1}}{\partial p_{k,t+1}}\frac{\partial p_{k,t+1}}{\partial p_{jt}}\right) m_{j,t+1} + \frac{\partial p_{j,t+1}}{\partial p_{jt}} q_{j,t+1}\right] &= 0 \quad (31)\\ \Longrightarrow\ q_{jt} + \frac{\partial q_{jt}}{\partial p_{jt}} m_{jt} + \delta \mathbb{E}\left[\frac{d q_{j,t+1}}{d p_{jt}} m_{j,t+1} + \frac{d p_{j.t+1}}{d p_{jt}} q_{j,t+1}\right] &= 0 \quad (32)\\ \Longrightarrow\ q_{jt} + \frac{\partial q_{jt}}{\partial p_{jt}} m_{jt} + \delta \mathbb{E}\frac{d q_{j,t+1}}{d p_{jt}} m_{j,t+1} + \delta \mathbb{E}_t \frac{d p_{j,t+1}}{d p_{jt}} q_{j,t+1} &= 0 \quad (33) \end{aligned}$$

where $m_{jt} := p_{jt} - mc_{jt}$ and product k is produced by the rival firm.

The first two terms in equation (31) are the usual terms in static differentiated Bertrand duopoly equilibrium. The third term in equation (31) shows how a change in current prices affects the expected discounted profits in the second period through two channels.

First, a change in the price of product j in period t affects quantity demanded of product j in period $t+1$ indirectly through the effect its change has on both products, j and k, prices in period $t+1$. This total derivative, shown in equation (32), multiplies the mark-up for product j in period $t+1$. This term captures the expected change in future revenue due to demand dynamics arising from the price change of product j.

Second, there is an additional expected revenue effect that arises due to the effect that a change in the current price of product j has on the forecast price of product j in the next period. This derivative multiplies forecast demand of product j in period $t+1$. With only two periods, this partial derivative can be equivalently written once again as a total derivative. This substitution has been made in equations (32) and (33).

If we were to consider the problem of pricing over a horizon of three, rather two periods, the final term in equation (33) is shown in equation (34) . It includes all direct and indirect effects of changes in current prices on future prices. That is, it is product of the total derivative of the effect of current price on the price of product j in period $t+2$

$$\delta \mathbb{E} \underbrace{\left(\frac{\partial p_{j,t+2}}{\partial p_{j,t+1}} \frac{\partial p_{jt+1}}{\partial p_{jt}} + \frac{\partial p_{j,t+2}}{\partial p_{k,t+1}} \frac{\partial p_{k,t+1}}{\partial p_{jt}} + \frac{\partial p_{j,t+2}}{\partial p_{jt}} \right)}_{= \frac{dp_{j,t+2}}{dp_{jt}}} q_{j,t+2} = \delta \mathbb{E} \frac{dp_{j,t+2}}{dp_{jt}} q_{j,t+2} \tag{34}$$

Next consider the single product duopoly dynamic pricing problem to a discrete time infinite horizon. In this case the firm n maximises the expected discounted sum of per-period profits for H-periods ahead

$$\pi_{nt}^{NPV} = (p_{jt} - c_{jt})\, q_j (\boldsymbol{p}_t, \boldsymbol{\omega}_t) + \mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} (p_{jh} - c_{jh})\, q_j (\boldsymbol{p}_h, \boldsymbol{\omega}_h)$$

Further, let $\boldsymbol{\Delta}_h^q$ and $\boldsymbol{\Delta}_h^p$ be the matrices of the current and inter-temporal derivatives of expected future demand and price forecasts in period $h = t, t+1, \ldots, H$ with respect to change in current prices. The (j,k)-th elements of these matrices are given by equations (6) and (7).

Extending the two period result above, the optimal price of product j in period t of the H-period horizon problem satisfies the first order condition

$$q_{jt} + \Delta_{jjt}^q m_{jt} + \mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{jjh}^q m_{jh} + \mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{jjh}^p q_{jh} \quad = \quad 0$$

as required.

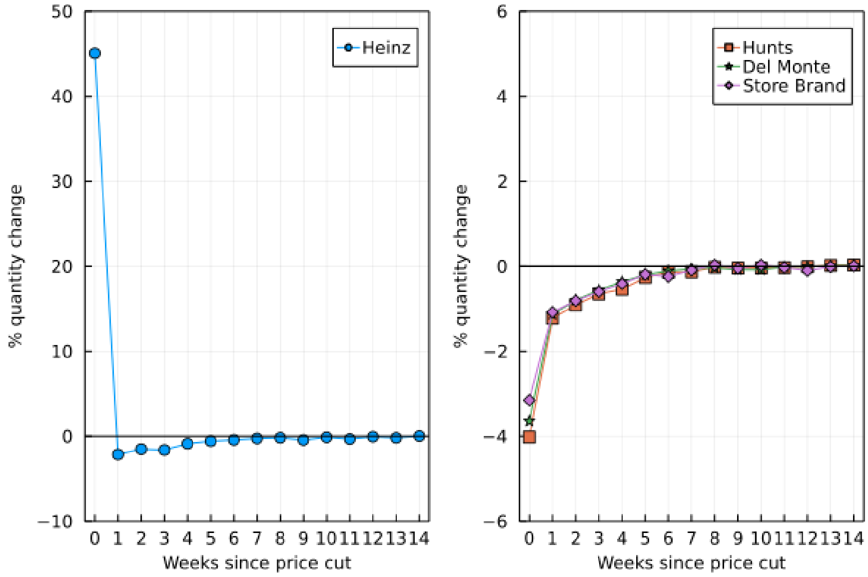
A.2. Inter-temporal substitution patterns for storable goods

Diversion ratios measure the closeness of substitution between two products. They are calculated as the proportion of product j's sales lost from product k as a result of an increase in product j's price

$$DR_{jk} := -\frac{\Delta_{jk}^q}{\Delta_{jj}^q} \tag{35}$$

However, when the components in equation (35) are calculated using short-run demand responses they are not suitable to measure diversion between storable goods. This is because the inter-temporal substitution synonymous with dynamic demand for storable goods are not accounted for. By not including diversion from future sales, the numerator tends to be understated and the absolute value of the denominator overstated. These biases reinforce one another and combine to severely bias diversion ratios downwards.

Figure 4: Erdem et al. (2003) simulated percentage changes in weekly quantities sold in response to a temporary 10 percent price cut to Heinz branded ketchup when consumer price expectations reflect the temporary price change.



To illustrate the contemporaneous and inter-temporal substitution patterns for storable goods, Figure 4 reproduces Erdem et al. (2003) policy simulation of US ketchup brands' quantity response to a temporary, one week 10 percent price cut. This price cut is assumed to be fully reflected in consumer price expectations. The left panel of Figure 4 plots the quantity impulse response functions for the promoted brand, Heinz. The right panel shows the corresponding quantity impulse responses for rival branded ketchups: Hunts, Del Monte and the store brand.

The percentage changes to quantity in week zero correspond to the short-run demand responses. Specifically, Heinz's quantity sold rises by 45.1 percent. The corresponding quantity sold of rival brands falls by 3 to 4 percent. The resulting short-run diversion ratios from Heinz to its rivals are 2.2 percent for Hunts, 1.5 percent for Del Monte and 0.5 percent for the store brand.

Inter-temporal substitution patterns are illustrated in Figure 4 by the drop in quantities sold in the weeks following the price cut. For all brands the size of the quantity response diminishes rapidly as as time passes. By week 11 - the median inter-purchase duration - inter-temporal substitution is complete.

In light of this, Heinz could set their optimisation horizon to match the median inter-purchase duration (i.e. $H = 11$) and accurately approximation to the solution to the original DP. In practice, as highlighted by Che et al. (2007), even setting $H = 1$ or $H = 2$ may produce useful practical approximate solutions.

To measure the strength of inter-temporal substitution, equation (35) is adjusted to only include the change in future sales of product k in the numerator

$$ITDR_{jk} := -\frac{\sum_{h=1}^{H} \Delta^q_{jkh}}{\Delta^q_{jj}} \tag{36}$$

This inter-temporal diversion ratio measures the fraction of j's sales uplift in the period in which its price is temporarily cut that are pulled forward from future sales of product k.

In Erdem et al. (2003) US Ketchup simulation, the inter-temporal diversion ratios are 2.1, 1.3 and 0.6 percent from Heinz to Hunts, Del Monte and the store brand respectively - almost as large as the contemporaneous responses. The inter-temporal diversion ratio for Heinz today to Heinz in the future is 20 percent - approximately 10 times larger than any rival brand. This demonstrates that the purchase acceleration effect is strongest for the promoted brand. Moreover, it shows that consumers perceive that the closest substitute to current sales of Heinz are future sales of Heinz.

The dynamic or long-run diversion ratio measures the total substitution. Following a temporary change to the current price to product j, the overall diversion from product j to product k is given by the ratio of the accumulated change in product k's sales to the net change in product j's sales over all periods.

$$DDR_{jk} = -\frac{\sum_{h=0}^{H} \Delta^q_{jkh}}{\sum_{h=0}^{H} \Delta^q_{jkh}} \tag{37}$$

In Erdem et al. (2003), the long-run dynamic diversion ratios calculated using a baseline of average weekly sales are 5.3 percent for Hunts, 3.5 percent for Del Monte and 1.4 percent for the store brand. In each case more around 2.5 times their downward biased short-run counterparts.

A.3. Dynamic diverted value ratios

Below I show that the dynamic diverted value ratio is the product of a term containing elements of $\mathbf{\Psi}_t$ and its static counterpart.

$$
\begin{aligned}
DDR^{\pi}_{jkt} &:= -\frac{\mathbb{E}\sum_{h=t}^{H}\delta^{h-t}\Delta^{q}_{jkh}m_{kh}}{\mathbb{E}\sum_{h=t}^{H}\delta^{h-t}\Delta^{q}_{jjh}m_{jh}} && (38)\\
&= -\frac{\Delta^{q}_{jkt}m_{kt} + \mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\Delta^{q}_{jkh}m_{kh}}{\Delta^{q}_{jjt}m_{jt} + \mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\Delta^{q}_{jjh}m_{jh}} && (39)\\
&= -\frac{\left(\Delta^{q}_{jkt}m_{kt} + \mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\Delta^{q}_{jkh}m_{kh}\right)\frac{\Delta^{q}_{jkt}m_{kt}}{\Delta^{q}_{jkt}m_{kt}}}{\left(\Delta^{q}_{jjt}m_{jt} + \mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\Delta^{q}_{jjh}m_{jh}\right)\frac{\Delta^{q}_{jjt}m_{jt}}{\Delta^{q}_{jjt}m_{jt}}} && (40)\\
&= -\frac{\left(\Delta^{q}_{jkt}m_{kt} + \mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\Delta^{q}_{jkh}m_{kh}\right)/\Delta^{q}_{jkt}m_{kt}}{\left(\Delta^{q}_{jjt}m_{jt} + \mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\Delta^{q}_{jjh}m_{jh}\right)/\Delta^{q}_{jjt}m_{jt}} \times \frac{\Delta^{q}_{jkt}m_{kt}}{\Delta^{q}_{jjt}m_{jt}} && (41)\\
&= \frac{1+\Psi_{jkt}}{1+\Psi_{jjt}}DR^{\pi}_{jkt} && (42)
\end{aligned}
$$

as required.

A.4. Long-run price elasticities

Price elasticities are key inputs into many empirical policy tools. Given the dynamic nature of storable good demand, the quantity responses to price changes have a cross-sectional and time dimension. The price elasticities that most closely reflect the dynamic demand responses considered by firms when choosing prices in the model corresponds the quantity impulse response function resulting from a temporary change in price.

Dynamic substitution patterns are captured by matrices containing short-run and long-run price elasticities, respectively. Let their (j,k)-th elements are denoted by

$$\varepsilon_{jkt}^{SR} = \Delta_{jkt}^{q} \frac{p_{jt}}{q_{kt}} \tag{43}$$

$$\varepsilon_{jkt}^{LR} = \sum_{h=t}^{H} \Delta_{jkh}^{q} \frac{p_{jt}}{\bar{q}_k} \tag{44}$$

where $\bar{q}_k$ is average weekly price of product j and quantity sold for product k over the firm's optimisation horizon of H-periods. Defining $\phi_{jkh} := \frac{\Delta_{jkt+h}^{q}}{\Delta_{jkt}^{q}}$ as the ratio of the total demand derivative in h-periods to its current value, the long-run price elasticity can also be expressed as

$$\varepsilon_{jkt}^{LR} = \Delta_{jkt}^{q} \left(1 + \sum_{h=t+1}^{H} \phi_{jkh}\right) \frac{\bar{p}_j}{\bar{q}_k} \tag{45}$$

Unlike diverted values, the dynamic correction ratio cannot, in general, be used to correct the bias in the short-run price elasticity for omitted inter-temporal substitution. This is because $\phi_{jkt} \neq \Psi_{jkt}$. As equation (46) makes clear, in general, they are only equal when there is no discounting and the mark-ups are constant over time.

$$\Delta_{jkt}^{q} \left(1 + \Psi_{jkt}\right) = \Delta_{jkt}^{q} \left(1 + \sum_{h=t+1}^{H} \delta^{h-t} \phi_{jkh} \frac{m_{kh}}{m_{kt}}\right) \tag{46}$$

Therefore, when δ is close to one, and the mark-ups are roughly constant (i.e. small price changes and constant marginal costs), then one might approximate long-run price elasticities using the elements of $\mathbf{\Psi}_t$. However, for arc-elasticities that measure responses to deep discounts, or firm's with high discount rates then the quality of the approximation might degrade.

$$\varepsilon_{jkt}^{LR} \approx \Delta_{jkt}^{q} \left(1 + \Psi_{jkt}\right) \frac{p_{jt}}{\bar{q}_k} \tag{47}$$

To obtain an empirical approximation to the long-run price elasticity using the output of the estimation procedure described in section 3, the analyst can compute

$$\varepsilon_{jkt}^{LR} \approx \begin{cases} \widehat{\Delta}_{jkt}^{q} \left(1 + \theta_n^{own}\right) \frac{p_{jt}}{\bar{q}_k} & \text{for } j = k \\ \widehat{\Delta}_{jkt}^{q} \left(1 + \theta_n^{cross}\right) \frac{p_{jt}}{\bar{q}_k} & \text{otherwise} \end{cases} \tag{48}$$

B. UK Laundry detergent application

This appendix provides an overview of the application to the UK laundry detergent industry. It then details the estimation of the price process and a static nested logit demand model, respectively. Finally, the results of the sensitivity analysis to the baseline model estimating dynamic correction factors are presented.

B.1. Data

The analysis of the UK laundry detergent industry is based on individual household purchase data from 1st January 2002 until 31st October 2012. Households that take part in the survey scan the barcode of the items they purchase. Using the scanned barcode, the survey records the price and number of packs bought together with the characteristics of the product purchased. In addition, the purchase date and store in which the product was bought is also recorded. The purchase data is supplemented by annually updated household demographics.

To avoid complexities related to store choice, the analysis in the remainder of this article is conditioned on detergent purchased from a single leading UK supermarket. Prices charged by UK supermarkets are the same for all of their stores if their footprint is greater than 280 sqft.[60] As such, purchases across different stores within a particular supermarket fascia in the UK can be pooled for the purposes of empirical analysis.

B.2. Step1: Price forecasting model

The application presents results for an open-loop and closed-loop price forecast model. With access to internal documents, a policy analyst may know the forecasting model used by firms. In this case, the open and closed loop models may serve as sensitivities.

For the closed loop scenario, the high-dimensional dynamic factor model (eq. (18) and (19)) described in section 3.2 is used in this application. The dynamic factor model is estimated using average weekly prices from purchases made by households in the Kantar Worldpanel data over 10 years.

Given this long horizon, structural breaks might occur in the underlying price process. The iterative procedure described in Baltagi et al. (2021) is used to detect the presence, number, and location of structural breaks. Details of the implementation of the iterative procedure described in Baltagi et al. (2021) to detect structural breaks in the applied statistical model of the dynamic price process are described below. This includes the choice of its hyper-parameters and necessary minor modifications to enable calculation of its critical values as degrees of freedom increase.

Hyper parameters The minimum length of the regime window is 52 weeks. With 562 weeks in the sample, the corresponding trimming parameter is approximately 0.1 - in line with Baltagi et al. (2021) recommendation when serial correlation might be present. The HAC estimator of covariance matrix uses a Bartlett kernel with a bandwidth of $2 \times \lfloor T^{1/5} \rfloor$.

A maximum of eight price factors are allowed in each period. The optimal number of factors in each regimes is chosen to minimise the residual sum of squares plus the IC_2 penalty term

[60] This pricing policy follows an undertaking in 2001 following a market investigation by the UK Competition Commission into the groceries industry in 2000.

defined in Bai and Ng (2002). The optimal number of lags is chosen using the Bayes Information Criteria.

Implementation One technical problem is that prices are not observed in all weeks. To allow for missing price data, the factor model is fitted the Julia package (TSVD - Truncated Singular Value Decomposition) using a 'thin' singular value decomposition with R factor applied to the a sparse matrix containing only observed prices implementing the method developed by Larsen (1998).

A second issue is that the response surfaces in Bai and Perron (2003) appear to poorly approximate critical values when evaluated far outside of the domain of simulated critical values used to estimate them. This problem arises because because the number of regressors in a vector auto-regressions increases quadratically in R. Therefore, at even moderate values of R the quality of the approximation to critical values using the response surface can quickly degrade (i.e. they produce large negative critical values). To consider these cases, critical values are extrapolated from 'stable' tabulated critical values using a 'KingFit' procedure in the the Julia package CurveFit.

Test results Table 4 contains the result of the iterative testing procedure. The upper panel shows three tests - supF, UDmax, WDmax - to test for the presence of structural breaks under the null of no structural breaks. All tests rejects the null hypothesis at the 1 percent level.

The bottom panel shows the results of the iterative test procedure. In each case, the null of l structural breaks is tested against the alternative of $l+1$ structural breaks. The test continues until the null cannot be rejected.

The first column in Table 4 contains the number of structural breaks under the alternative hypothesis (i.e. $l+1$). The second column is the number of regressors in Bai and Perron (2003) critical values response surfaces.[61] The test statistic and critical values at the ten, five and one percent level comprise the remaining columns.

The iterative procedure's null hypothesis is not rejected for the first time when $l = 6$ and defines seven pricing regimes. The resulting breakpoints, number of price factors and the number of lags in the factor VAR in each regime are shown in Table 5.[62]

A summary of the detailed test results above are shown in Table 5. The first four columns list the regimes, the start and end weeks, and the their length in weeks. The last two columns show the number of latent factors, $R^\star$, and lags of the factor VAR, $\kappa^\star$, in each regime. The number of factors in each regime uses penalised least squares (Bai and Ng (2002)). The Bayes Information Criteria is used to select and number lags, $\kappa^\star$, in the factor VAR.

[61] Under the null of l structural breaks, $q = R_i(R_i+1)/2$ where R_i is the number latent price factors associated with the largest value of the test statistic for across $l+1$ regimes.

[62] The modulus of the eigenvalues of A in each regime is strictly less than 1 and the factor VAR in each regime are stationary.

Table 4: Structural Break Tests

Tests for at least one structural break				
		Critical Values		
Breaks	Test Stat.	10%	5%	1%
$\sup F\,(1 \mid 0)$	2.52	0.8	0.83	1.05
UDmax	2.52	1.09	1.13	1.35
WDmax: 10%	2.82	1.12	-	-
WDmax: 5%	3.00	-	1.17	-
WDmax: 1%	3.05	-	-	1.42

Iterative structural break tests					
			Critical Values		
$l+1$	q	$\sup F_{NT}\,(l+1 \mid l)$	10%	5%	1%
1	3	30.75	15.68	17.54	21.35
2	6	48.55	22.75	24.85	28.74
3	6	48.55	23.26	25.35	29.17
4	6	48.55	23.71	25.79	29.55
5	6	39.22	24.13	26.19	29.89
6	3	38.38	17.96	19.77	23.28
7[†]	36	30.21	64.70	74.40	80.47

[†] Critical values for the number of degrees of freedom lie outside tabulated values in Bai and Perron (2003). Reported critical values are extrapolated using a 'KingFit' procedure in the Julia package CurveFit.

Table 5: Price dynamics: breakpoints, $R^{\star}$, and $\kappa^{\star}$

	Start		End				
Regime	Year	Week	Year	Week	Weeks	$R^{\star}$	$\kappa^{\star}$
1	2002	1	2003	29	81	3	2
2	2003	30	2006	45	172	2	1
3	2006	46	2008	24	83	8	1
4	2008	25	2009	24	52	5	1
5	2009	25	2010	41	69	2	1
6	2010	42	2011	42	53	7	1
7	2011	43	2012	42	52	2	1

This procedure detects six structural breaks in the statistical model of price dynamics over the sample period. The first three regimes cover two-thirds of the sample - each lasting between 1.5 and 3 years. In contrast, each of the last five years contains a structural break.

Industry pricing trends To give the clustering of structural breaks towards the end of the sample in context, Figure 5 uses a series of box plots to display the distribution of price per wash in each quarter from 2002 to 2012. The red line plots the average posted price per wash and the green line plots the average price per wash of purchased products. The top panel shows the price per wash distribution for firm A and bottom panel shows this distribution for firm B.

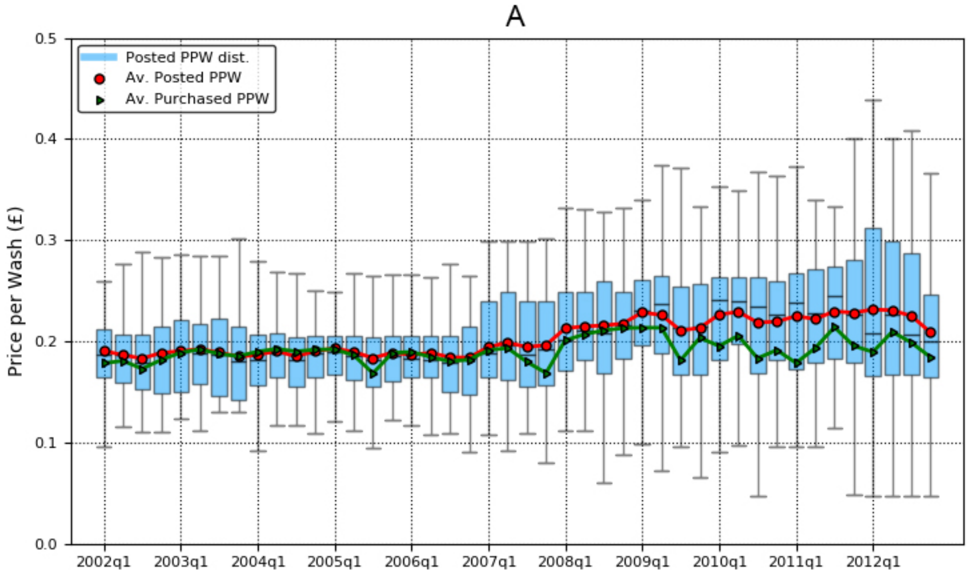
For both firm A and firm B, the whiskers and inter-quartile range of box plots from first quarter in 2002 up to the final quarter in 2006 are relatively constant. Over this period, the whiskers tend to lie between 10p and 30p per wash and the interquartile ranges lie between 15p and 23p per wash. It coincides with the first two regimes of 81 and 172 weeks, respectively.

From the first quarter in 2007 onwards, the whiskers and inter-quartile ranges of the box plot fan out for both firm A and firm B. This increased price dispersion coincides with the introduction of new product formats in 2008 for firm A and 2007 for firm B. Both innovations dramatically reduce storage space needed per wash and firms sell products containing more washes. In turn, offering consumers new detergents with a more favourable trade-off between pack size and storage costs.

To explore role that new products and containing compaction innovation had on consumer demand on pricing in turn, Figure 6 shows the share of revenue by format and the number of washes in a pack in each year over the sample period.

The top panel shows that new product formats quickly capture market share over 2007-2010 and quickly replaced similar but more bulky alternatives (i.e. tablet and non-concentrated liquids). After both new format introductions efforts to gradually reduce other formats dosage to improve their storage space-compaction tradeoff continued. Over the period 2008 to 2012,

Figure 5: Price per wash (PPW) distribution: firms A and B from 2002 to 2012.



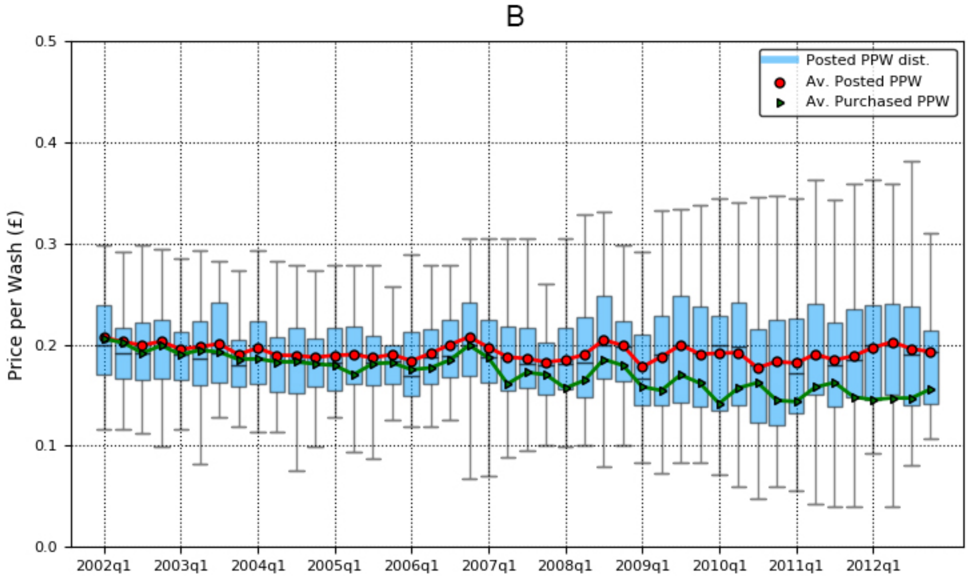
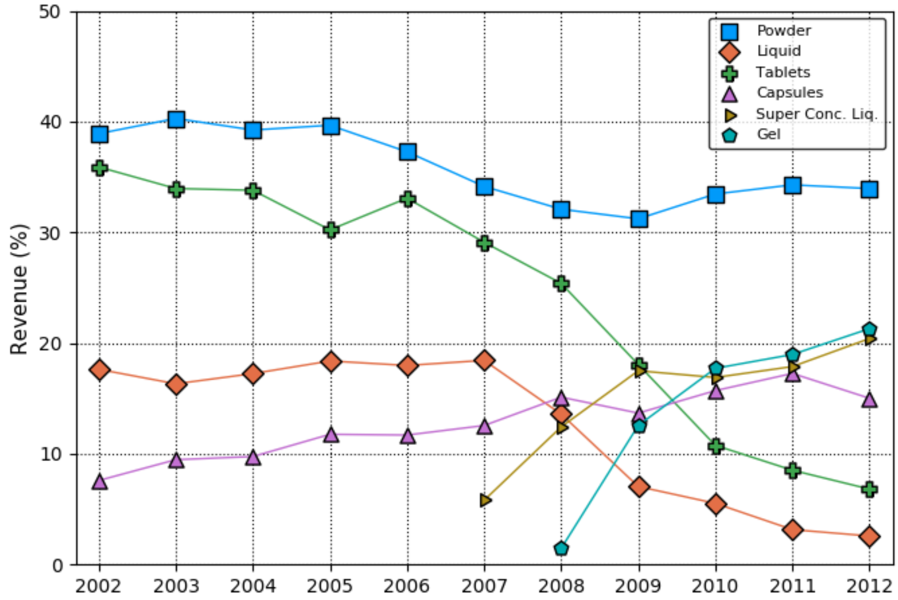
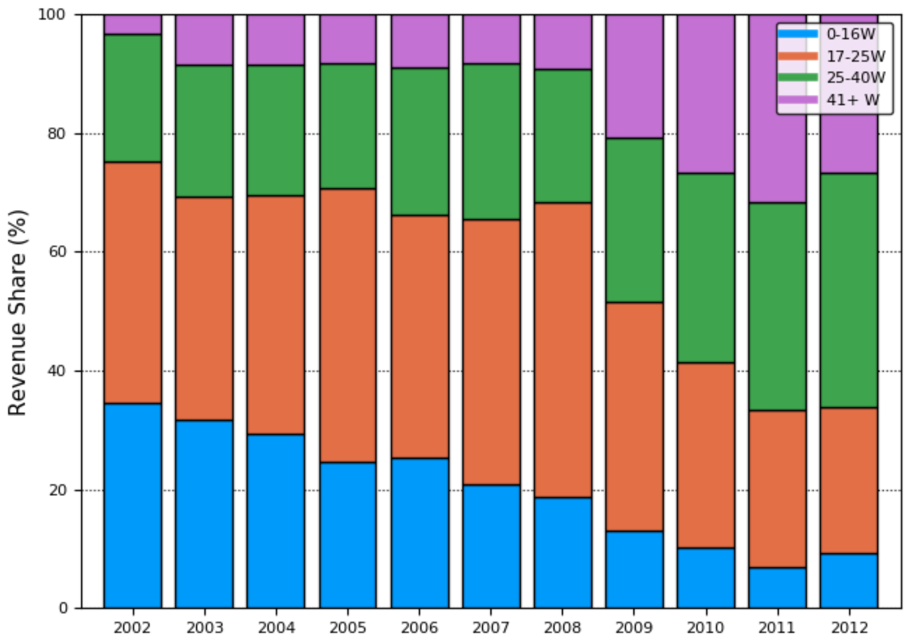


Figure 6: Revenue share by format and pack size from 2002 to 2012





the average dose per wash fell by 15 percent for powder, 9 percent for capsules and 7 percent for tablets.

The impact of new formats and smaller dosage for existing formats on the composition on product sizes is shown in the bottom panel. As expected, this storage cost savings per wash appear to have contributed to the increasing popularity of larger packs of laundry detergent. In 2002, around 75% of household spend was on SKUs with fewer than 24 washes, 10 years later this figure was less than 35 percent. Notably, as the product compaction is sped up across the firms' range of products, consumers increasingly shift to larger products. Beginning in 2008, these large annual shifts in product size purchased coincide with the the more frequent structural breaks in the price process

Taken together, the rise of new formats, more compact dosages and lower storage costs per wash from 2007/8 onwards coincide with an increase the demand for product containing more washes. As a result consumer can meet consumption needs with fewer purchases and inter-purchase durations increase.[63] In turn, faced with fewer consumer interactions, firms compete more intensely to attract consumers by engaging in deeper and more frequent discounting.

Other contributing explanations for the observed changes in pricing behaviour towards the end of sample period, inter alia, include increased price sensitivity of households due to changes in the macroeconomic climate and changes in pricing strategy resulting from firm investments from cognitive capacity and information resources.

B.3. Step 2: Estimation of the static demand model

This section describes a static discrete choice demand model of UK laundry detergent estimated in step two of the estimation procedure. Given the policy focus of this article, a demand model often employed in antitrust investigations - the nested logit model - is estimated.

In practice, demand system estimation involves some element of time aggregation. To recover short-run demand responses to the temporary price changes it is arguably desirable for any time aggregation of input data matches the time interval used in practice by firms when setting prices as closely as possible. The nested logit model is estimated using the purchase diary data from Kantar Worldpanel described in above. The availability of micro-data enables the use of observed household characteristics to estimate a rich demand model.[64]

[63] See Table 2.

[64] Where the policy analyst only has aggregate market data, Berry (1994) and Berry et al. (1995) can be used to estimate conditional, nested, or random coefficient logit demand models. Alternatively, if the dimensionality of the choice is sufficiently low, then the analyst might choose to estimate an Almost Ideal Demand System (Deaton and Muellbauer (1980)) or non-parametrically estimate a static discrete demand model (Compiani (2022)). Once estimated, the approach to estimating of the elements of $\boldsymbol{\theta}$ is as described in Section 4.2.

Static demand model Household i elects to purchase good j from a market t to maximise conditional indirect utility

$$U_{ijt} = \boldsymbol{X}_j^{\top}\boldsymbol{\beta}_i + \alpha_i p_{jt} + \xi_{jt} + \varepsilon_{ijt} \tag{49}$$

where $\boldsymbol{X}_j$ is a K-vector of observed product attributes. The outside good is denoted by $j = 0$ and represents the decision not to purchase in market t.

Because the model is estimated using consumer micro-data, the parameters α_i and $\boldsymbol{\beta}_i$ depend on observed household characteristics. There are two components of utility not observed by the analyst: ξ_{jt} is an unobserved product-market component of utility common to all consumers and ε_{ijt} is a private household-specific utility shock. The household utility shocks are identically and independently distributed and follow the Generalised Extreme Value distribution.

The unobserved product-market components are observed by all firms prior to setting prices. As a result, prices are likely to be correlated with ξ_{jt} and are not independent from the unobserved component of demand. This gives rise to endogeneity concerns. There are three channels that lead to correlation between unobserved product-market characteristics and prices.

First, any additional costs incurred in the production of a positively valued unobserved characteristic will likely be passed onto households in the form of higher prices. Second, to the extent that the unobserved attribute shifts demand outwards, equilibrium markups will reflect any additional market power this confers on the firm. In both cases, prices are likely to be positively correlated to unobserved attributes. If the endogeneity is left untreated, these correlations positively bias the marginal utility of income.

Another potential source of correlation captured by ξ_{jt} is non-price advertising and promotional activities. If unobserved non-price advertising activity - such as funding prominent store positioning - increases during periods of promotional prices, ξ_{jt} is negatively correlated with prices. Alternatively, if prices contemporaneously increase to fund non-price advertising, then ξ_{jt} is positively correlated with prices.

Because both positive and negative correlation might result, the direction of the bias due to non-price advertising is ambiguous. However, given the importance of promotional activity and unobserved factors, such as placement within the store, it is quite plausible (even likely) that non-price advertising components of ξ_{jt} are negatively correlated with prices. In this case, marginal utility of income would be biased upward.

Choice Sets The demand model is estimated using a random sample of five purchases in each week from the Kantar Worldpanel purchase diary data resulting in 2,810 choice occasions.

The choice sets are constructed using all purchases at stores at the major UK supermarket between 1st January 2002 and 31st October 2012. The purchase diary data only records when and where a product is purchased. As a result, due to sampling variation some products are not observed in the data even though they are available for purchase. This makes it difficult to determine which products were available at the point of purchase. To address this, I assume that a product is eligible for the choice set if a purchase is observed in the same calendar month as the purchased product. Further, if a product is purchased in a promoted bundle of two units, the opportunity to purchase two units of product is included in the choice set. If there is no observed purchase of two units, it is assumed it is not available as a promotional bundle and the total price is twice that of a single unit of the product.

This approach leads to the inclusion of products in the choice set without observed prices. To remedy this these products' prices need to be imputed.[65] Where necessary, the DFM describe in section B.2 is used to impute prices.

The number of products in choice sets ranges from 87 to 187 with a median of 120 products. The set of products sold in each week is partitioned into four nests based on the number of washes contained in each product: small (S), medium (M), large (L), and extra large (XL). The size boundaries of these groups correspond to the 25th, 50th, and 75th quantile of distribution of washes in each calendar year.

In this particular application, 12 percent of product-week market shares are zero. As a result, it is not possible to use Berry (1994) to deal with endogeneity concerns. Therefore, the control function approach proposed by Petrin and Train (2010) is used to address endogeneity concerns.

Estimation The model is estimated in two steps. In the first step, the control function is estimated using prices from observed purchases.[66]

$$p_{jt} = W\left(\boldsymbol{X}_j, \boldsymbol{Z}_{jt}; \boldsymbol{\gamma}\right) + \upsilon_{jt} \tag{50}$$

The auxiliary price equation contains two components. The first, $W\left(\boldsymbol{X}_j, \boldsymbol{Z}_{jt}; \boldsymbol{\gamma}\right)$, is a function of observable product characteristics and instruments excluded from the choice model, $\boldsymbol{Z}_{jt}$. These instruments include cost shifters and BLP instruments that shift mark-ups. Consequently, they are correlated with prices, but not unobserved demand factors. The other component is a product-market specific unobservable, υ_{jt}.

In this application, the instruments are based on the number of similar products sold in the market.[67] There are two instruments: a count of the firm's similar products and a count of rivals' similar products. The estimated residuals are used to construct the control function, $CF\left(\hat{\upsilon}_{jt}; \rho\right)$. In the second step, the control function is added to the choice model and it is estimated by maximum likelihood.[68] Finally, because the control function is estimated, standard errors are calculated by bootstrapping the two step estimation procedure.[69]

[65] Approximately 7 percent of observations are imputed.

[66] When more than one price is observed per product in each week, its average weekly price is used.

[67] A product is regarded as being similar if it belongs to the same size category and has the same format. Further, and in line with the construction of the choice sets, similar products must be sold in the same calendar month as their comparator.

[68] See Train (2009), chapter 4 for details of maximum likelihood estimation of the nested logit model.

[69] The bootstrap procedure of Kim and Petrin (2010) is as follows. First, the price regression from the first stage are bootstrapped and residuals calculated. Next, for each bootstrap sample the demand model is run on the combination of the original data and the bootstrapped residuals. The variance of the parameters from

The possible source of endogeneity is reflected by the correlation of product-specific unobservables in the choice model, ξ_{jt}, and the unobservable component of the auxiliary price equation, v_{jt}. When observed prices are regressed on product characteristics and instruments, the estimated residuals from this regression contains the correlation between p_{jt} and ξ_{jt}. As a result, the estimated residuals, $\hat{v}_{jt} := p_{jt} - \mathbb{E}\left[p_{jt}|\boldsymbol{x}_j, \boldsymbol{z}_{jt}\right]$, are used to construct a control function $CF\left(\hat{v}_{jt}; \rho\right) = \rho\hat{v}_{jt}$.

When this control function is added to the choice model it conditions out that part of the prices correlated with ξ_{jt}. As a result, it resolves the endogeneity that arises from the unobserved product-market terms, ξ_{jt}. The unobserved component of demand is now independent of price, $\epsilon_{ijt} \perp p_{jt} \mid CF\left(\hat{v}_{jt}; \rho\right)$, and $\epsilon_{ijt} =^d \varepsilon \overset{iid}{\sim} GEV$.[70]

The results of estimating the auxiliary equation are shown in Table 6.

Table 6: Control Function: parameter estimates

Variables	θ	**se**
Intercept	1.589	0.058
Characteristics		
Washes	14.503	0.036
Dosage	0.678	0.102
Dosage x Liquids	0.953	0.135
Instruments		
Num. own similar prods	-0.038	0.004
Num. rival's similar prods	-0.040	0.003
Detergent Fixed Effects	Yes	
Year Fixed Effects	Yes	
N	59,035	
R^2	0.797	

Identification The static demand model parameters, α, β and ρ are common to all consumers. They are identified by variation in prices, product characteristics and fitted control function residuals within consumers' choice sets. The parameters that interact consumer with price or product characteristics are identified from variation in choices across different types of consumers. The nesting parameter, λ, is identified by variations in the ex-ante expected utility of purchasing a product from a give nest for each consumer.

500 bootstrap samples is calculated (see Cameron and Trivedi (2005)). Finally, the bootstrapped standard errors are the square root of the sum of the bootstrapped parameter variance and the unadjusted parameter variance.

[70] Where $=^d$ is defined as "follows the same distribution as".

Table 7: Choice models: parameter estimates

Variables	Nested Logit			
	θ	**se**	θ	**se**
Price Params				
Price	-0.275	0.021	-0.581	0.136
Price x Income	0.179	0.039	0.184	0.029
Characteristics				
Washes	0.118	0.163	4.484	1.949
Washes per Eq. Ad.	-0.337	0.095	-0.371	0.771
Dosage	-1.222	0.108	-1.058	0.284
Dosage x Liquids	0.279	0.067	0.575	0.381
Other Params				
λ	0.526	0.035	0.551	0.049
ρ			0.297	0.126
Detergent Fixed Effects	Yes		Yes	
N	2,810		2,810	
Log-Likelihood	-12,203		-12,199	

Results The results of the nested logit estimation are displayed in Table 7. It contains the parameters and standard errors of the nested logit model with and without the control function.

The specification of conditional indirect utility include interactions between price and a proxy for household income.[71] Product characteristics include the size of the product purchased and the dosage - the amount of material (recommended) for use in a single wash. To control for household size, I also include the amount of washes purchased per equivalent adult in the household.[72] Detergent fixed effects are also included.

In both nested logit models the price coefficient is negative and households with higher income have a lower marginal utility of income. When the control function is included, the price coefficient (scaled by the nesting parameter) approximately doubles from -0.523 to -1.054.

This is consistent with correlation between unobserved product demand factors being positive. It is also reflected by the control function parameter, ρ, being positive and statistically significantly different from zero.

Table 7 also shows that households positively value larger pack-sizes. However, for smaller households this is less pronounced. This is consistent with smaller households living in smaller accommodation and facing a higher cost of storage.

The amount of material needed to do a single wash (i.e. dosage) is negatively valued, especially

[71] The proxy for household income is annual average weekly expenditure on all groceries.

[72] To calculate equivalent adults, I use the OECD-modified equivalence scale.

for 'solid' detergents. This is also consistent with the fact that households value storage space. When the dosage is lower, households can store more washes without necessarily occupying more storage space. Indeed, this is one of the driving factors behind the success of the new super-concentrated and gel detergent products. By itself, this suggest the presence of inter-temporal demand links through inventories - a source of mis-specification for this static demand model.

The nesting parameter, λ, is 0.551 and is statistically significantly different from 1. This indicates that that there are some unobserved correlations in the utility between detergents of similar sizes and rejects the independence of irrelevant alternatives imposed by a conditional logit.

Remarks With the goal of measuring long-run demand responses, it may be tempting to apply a static demand model to a coarser time-partition of the market data. For example, estimating a static demand model on data aggregated to monthly frequency then using its demand derivatives to approximation to long-run price elasticities. Unfortunately, given complex promotional pricing patterns and corresponding dynamic demand responses, it is not clear how the resulting demand derivatives relate to long-run substitution patterns.

Another potential concern is that any bias may be compounded when the output of static demand model estimated on time aggregated data is used as in input into a mis-specified, time-aggregated static Nash-Bertrand supply model. The corresponding policy simulations inherit and potentially magnify the (unknown) biases in its inputs. Even if the misspecified supply side model were considered useful as an approximation to long-run industry outcomes, the correct inputs would be those derived from quantity simulations using a dynamic demand model, not those from misspecified static model applied to arbitrarily time aggregated data.

B.4. Step 3: Estimation θ in the UK laundry detergent industry

This section provides further details on the estimation of $\boldsymbol{\theta}$ under the baseline model and presents sensitivity analysis referred to in the main text.

B.4.1. Incorporating VAT

When VAT is included in the model, the net price received by the firm is the market price divided by one plus the VAT level in that period

$$\pi_{nt}^{NPV} = \mathbb{E}\sum_{h=t}^{H}\sum_{j\in\mathcal{J}_n}\delta^{h-t}\left(\frac{p_{jh}}{1+VAT_h} - mc_{jh}\right)q_j\left(\boldsymbol{p}_h, \boldsymbol{\omega}_h\right) \tag{51}$$

With VAT the first order conditions are

$$\frac{\boldsymbol{q}_{nt}}{1+VAT_t} + \boldsymbol{\Delta}^q_{nt}\widetilde{\boldsymbol{m}}_t + \mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\boldsymbol{\Delta}^q_{nh}\widetilde{\boldsymbol{m}}_h + \frac{\mathbb{E}\sum_{h=t+1}^{H}\delta^{h-t}\boldsymbol{\Delta}^p_{nh}\boldsymbol{q}_h}{1+VAT_t} = \mathbf{0} \tag{52}$$

where $\boldsymbol{w}_t = \frac{\boldsymbol{p}_t}{1+VAT_t}$ and $\widetilde{\boldsymbol{m}}_t := \boldsymbol{w}_t - \boldsymbol{mc}_t$.

The model's margin equation over T periods incorporating VAT is

$$\mu\left(\theta^{own}_n, \theta^{cross}_n\right) := -\frac{\sum_{t=1}^{T}\boldsymbol{q}^{\top}_{nt}\left(\widehat{\boldsymbol{\Delta}}^q_{nt}\odot\boldsymbol{\theta}_n\right)^{-1}\left(\boldsymbol{q}_t + \boldsymbol{\Gamma}_{nt}\bar{\boldsymbol{q}}\right)\oslash\left(1+VAT_t\right)}{\sum_{t=1}^{T}\boldsymbol{q}^{\top}_{nt}\boldsymbol{w}_t} \tag{53}$$

where "$\oslash$" denotes element-wise division.

B.4.2. Sensitivity analysis

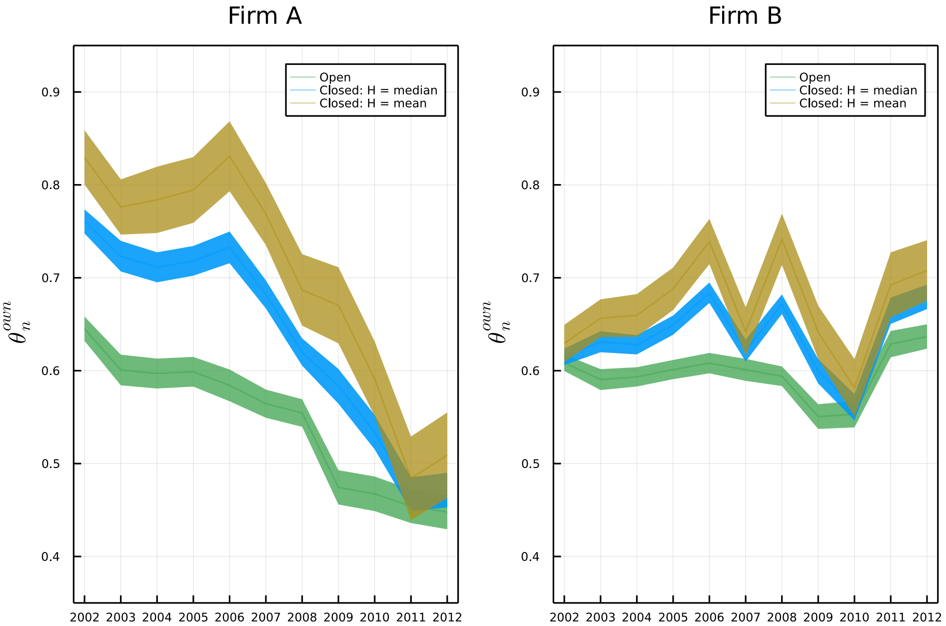
In addition to the baseline model, I consider two scenarios to test the findings of the policy analysis to alternative model calibrations. The first scenario (S1) is the same as the baseline model but an open-loop price forecasting model is used. This scenario reflects the behaviour of a boundedly rational firm with significant organisational friction that limit feedback between price setting and forecasting process. That is, $\boldsymbol{\Delta}^p_t = \mathbf{0}$.

The second scenario (S2) is the baseline model with closed-loop price forecasting model with an optimisation window equal to the mean inter-purchase duration. The values of $\bar{\theta}^{cross}_n$ under this scenario are larger than the baseline model and are shown in the last row of Table 3. It reflects a situation where a consumer's inter-temporal substitution is a larger component of overall substitution.In this scenario, households accelerate purchases in response to a promotion that would have happened further in to the future.

The results of scenarios are summarised by two charts: (i) the bounds $\left[\underline{\theta}^{own}_n, \bar{\theta}^{own}_n\right]$ under each scenario; and (ii) dGUPPI bounds under each scenario. Figure 7 shows that allowing closed-loop forecasting models with longer optimisation windows reduces the adjustment needed match margins. This is especially pronounced for firm A. This is because firm A has more products than firm B and is consistent with promotional pricing being used as an inter-temporal price discrimination tools to increase market power.

However, after 2009 the adjustments to short-run own-price demand responses overlap to a greater extent for all three scenarios and for both companies. This coincides with the period where discounts are deeper, more frequent and both firm products are aggressively compacted and sold in larger sizes. These observations are consistent with both firms being less able to use inter-temporal price discrimination strategies in this period to increase market power. In turn,

Figure 7: $\theta_n^{own} \in \Theta_n^{\star}$: Sensitivity analysis with $\overline{\theta}_n^{cross}$ calibrated with baseline model's inter-temporal substitution



suggesting that promotional pricing strategies may be more focused on consumer acquisition than inter-temporal price discrimination.

Figure 8 shows the dGUPPI ranges under the three scenarios from 2002 to 2012. The solid lines plot the lower bound of dGUPPI and the dashed lines the corresponding upper bounds.

For $dGUPPI_{AE}$, relative to the two other scenarios the baseline model produces conservative estimates for the range of dGUPPI. Without adjustments for market power arising from inter-temporal price discrimination, the open-loop forecast model has a higher lower bound than the baseline model. In contrast, both the baseline and the closed-loop model scenario with a longer optimisation window have similar lower bounds. As expected, given a higher value the upper bound on θ_n^{cross} the closed-loop sensitivity has a higher upper bound than the baseline $dGUPPI_{AE}$. These patterns are broadly similar for the $dGUPPI_{EA}$ - though less pronounced.

Overall, the qualitative conclusion of the baseline model is unchanged in all scenarios. Namely, in the past consumers perceive brands A and E to be close enough substitutes to flag the potential for harmful unilateral effects to arise if brand E is acquired by the firm producing brand A.

Figure 8: dGUPPI: Sensitivity analysis with $\overline{\theta}_n^{cross}$ calibrated with baseline model's intertemporal substitution