Approximating demand dynamics in storable good industries

Alan Crawford[†]

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Abstract

This paper develops a new approach that combines firm margins, market-level industry data and a static demand model to construct sets containing unbiased estimates of long-run price elasticities for storable good industries. It obviates the need to solve the consumer's value function and can be completed within policy-making timeframe. This methodology allows for the effect of contemporaneous and inter-temporal substitution on pricing incentives to be measured by dynamic diversion ratios. Together with the margins, these are key inputs into a new price pressure test for unilateral effects arising from mergers in industries with dynamic demand. This framework is applied to the UK laundry detergent industry from 2002 to 2012. I conduct two policy experiments that show how estimated sets of bias-corrected price elasticities and diversion ratios avoid misguided policy-decisions. The first experiment uses the newly developed price pressure test to examine a hypothetical brand acquisition. The other assesses whether there is any evidence that anti-competitive conduct of a laundry detergent cartel in mainland Europe had any effect in the UK. Both experiments demonstrate the efficacy of set-valued policy tools.

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1 Introduction

Price elasticities derived from estimated demand models are key inputs into policy counterfactuals. Inter alia, they are used to analyze the impact of consumption taxes, import duties, and antitrust issues. Of specific interest are long-run price elasticities that measure the contemporaneous and dynamic demand responses to price movements arising from changes in the economic environment. However, when the demand model is mis-specified then the estimated price elasticities are biased, counterfactuals are wrong, and misguided policy decisions are more likely.

For storable goods, incorporating demand dynamics is crucial because their omission leads to biased estimates of long-run price elasticities.[1] However, estimation of dynamic demand models for this large class of goods is computationally challenging, resource intensive and require consumer level purchase diary data. As a result, using them to produce estimates of long-run elasticities is rarely feasible within a policy-making time horizon.

This paper develops a new approach to estimate long-run price elasticities and dynamic diversion ratios for storable goods. In contrast to full dynamic demand estimation, it is computationally straightforward to implement and can easily be completed within the timeframe of the policy making process. The reason for this is two-fold. First, it only uses empirical techniques that are often already used in policy work (i.e. standard demand estimation and root-finding). Second, it only requires firms' margins and market-level data spanning the period over which margins are calculated.[2]

The ease of implementation and wider applicability of this new approach come at the cost of a loss of point identification; estimates of long-run price elasticities and dynamic diversion ratios are set-valued. Consequently, counterfactuals and policy tools that use these as inputs produce a set of outcomes, rather than a single value. However, as illustrated in this paper's application, set-valued results of empirical policy analysis need not reduce their efficacy in a policy setting.

This new approach uses a dynamic model of demand and supply for storable good industries. Consumer demand dynamics arise from two channels. One is the ability to store purchases for future consumption, which allows for inter-temporal substitution. The other is through price expectations. Informed by prices observed on previous shopping trips, consumer price expectations can influence the timing and identity of the product they purchase.

In the supply model, firms are forward-looking and set current prices to maximise the net present value of expected profit flows. The firm's perception of the profitability of pricing

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[1]By not allowing a portion of short-run volume increases from temporary price cuts to draw down on future sales - including those of the promoted product - static demand models provide biased estimates of demand responses to non-transitory price changes. Specifically, own price elasticities are overstated and cross-price elasticities tend to be understated.

[2]This data is routinely collected directly from firms in the course of antitrust investigations. Where detailed internal margin data is not available but the manufacturer(s) periodically publish accounts, the analyst can use reported gross margin or, alternatively, employ the techniques described in De Loecker et al. (2020) to estimate them.

strategies depends on their model of aggregate dynamic demand and their beliefs over rivals' pricing strategies. However, in practice, a firm's ability to compute perceived profits is arguably limited by two factors. First, the magnitude of the information set needed to compute them. Second, the complexity of computing expected future profits - especially since choice sets in storable good industries contain between 50 to 150 products. As such, the cognitive and computational challenges of retaining and processing such a large amount of information to compute optimal prices are arguably prohibitive.

Firms therefore adopt a boundedly rational approach to price setting. To that end, they approximate dynamic aggregate demand and solve an approximate version of their dynamic pricing problem. Dynamic aggregate demand is approximated by truncating the history of prices and other state variables used to compute it. In the approximate dynamic pricing problem, the computational complexity of evaluating the perceived distribution of future profits is reduced by limiting the forecast horizon and assuming fix price forecasts prior to setting prices.

The equilibrium concept used is Restricted Experience Based Equilibrium (REBE) developed by Fershtman and Pakes (2012). In line with the boundedly rational behaviour of firms, REBE retains the Markovian structure of the more frequently used Markov Perfect Equilibrium (MPE) but relaxes some of its rationality constraints. In particular, it relaxes the MPE requirement that consumers' price forecasts, firms' beliefs of rivals' pricing strategies, and optimal pricing strategies must coincide. As a result, a REBE places a lower burden on the physical and cognitive resources needed by consumers and firms to engage in equilibrium play.

This dynamic model of the storable good industry provides an expression for the mark-up in terms of purchase quantities, prices, and static and dynamic demand derivatives. A combination of observed market data and outputs from a static demand model can be used to populate all but the dynamic components of the price setting equations. The effect of long-run demand responses on pricing incentives are captured by a matrix of reduced form parameters.

These reduced form parameters capture the relative importance of contemporaneous and inter-temporal cannibalisation of sales when firms set current prices. Further, I show they can also be used to correct biased price elasticities and diversion ratios that are estimated using the outputs of a mis-specified static demand model. The resulting dynamic diversion ratios are shown to be a key input into a new generalised upward price pressure index adjusted for demand dynamics (dGUPPI). This new empirical policy tool extends the existing GUPPI test and can be used to evaluate mergers in industries with dynamic demand.

To estimate the reduced form parameters, data on market outcomes is used with estimates of firm's profit margins. Because profit margins are costly to collect, they are measured across a group of products and over a period of time that exceeds the purchase cycle for storable goods. As such, they contain information on the effect that demand dynamics have on margins earned on sales of that group of products.

To be able to leverage the information contained in profit margins to estimate the

reduced-form parameters, the expression for the mark-up derived in the dynamic model is summed over all the weeks and products it covers. Dividing through by corresponding revenues gives an expression for the percentage margin in terms of a sequence of prices, purchase quantities, static demand derivatives and the reduced form parameters. Using the estimated margin, data on market outcomes over the period, and the output of a static demand model, this equation can be used to estimate the reduced form parameters.

Two further sets of conditions identify these parameters. The first adopts the arguably weak restriction that the absolute magnitude of the own-price elasticities bias exceeds those of the cross-price elasticities. Under these additional conditions, the reduced form parameters are shown to belong to a bounded set. The second imposes additional restrictions that reduce the number of parameters to be estimated. As a result, estimation focuses on the average bias in own and cross price elasticities across all products and time periods covered by the margin. Under these two sets of conditions there are two bounded unknown parameters: one capturing own-price elasticity bias, the other the cross-price elasticity bias. With one equation and two reduced form parameters, their estimates belong to bounded sets.

To illustrate how this method can be employed in practice it is applied to the UK laundry detergent industry using Kantar Worldpanel purchase diary collected between 2002 and 2012. For the two firms, A and B, that dominate the UK laundry detergent industry, I use published accounts to estimate their margins for each calendar year. I find that the range of own price elasticity biases over 2002 to 2012 is 13 and 35 percent for firm A and 20 to 46 percent for firm B. The corresponding cross-price elasticity biases are 0 to 17.5 percent and 0 to 22 percent, respectively.

I conduct two experiments to show how the bias-adjusted elasticities and dynamic diversion ratios can be used in a policy setting. In the first experiment, I use a hypothetical brand acquisition to show how a set-valued dGUPPI can inform proposed changes in industry structure. In the other policy experiment, I assess whether a cartel in mainland Europe's laundry detergent industry involving the two firms that account for the vast majority of sales in the UK market was also in effect in the UK. To test this, I use the 'menu approach' (Bresnahan (1987)) to compare the market power estimates implied by alternative models of competitive interactions during and after the cartel.[3]

In both cases I find that conducting the policy analysis without accounting for market dynamics can lead to policy errors. In the first experiment the brand divestment would be incorrectly permitted. In the second, policy simulations using biased price elasticities suggest observed margins in the UK are most likely to be produced by anti-competitive conduct.[4] However, in the policy simulations using the set of

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[3]Nevo (2001), Slade (2004), Rojas (2008) and Miller and Weinberg (2017) have assessed market power or merger effects in differentiated product industries using the menu approach.

[4]Margins calculated using first order conditions using overly elastic own price elasticities from the static demand estimation understate market power. For the UK laundry detergent in the cartel period, the margins calculated assuming the cartel effected monopoly pricing lie below the observed

bias-adjusted price elasticities there is little evidence to support the view that the anti-competitive effects of the European laundry detergent cartel were experienced in the UK.

Related literature This paper contributes to the literature on dynamic demand estimation for storable goods. Following seminal papers by Erdem et al. (2003) and Hendel and Nevo (2006), there have been several papers that have sought to apply and extend the frameworks they develop[5]

However, this paper is most closely related to Hendel and Nevo (2013a) and Perrone (2017). In contrast to other papers in this field, they develop models that can quantify the effect of omitting consumer dynamics on estimates of long-run price elasticities without incurring the substantial computational burden of solving for the consumer's value function.

Like Perrone (2017) the approach developed in the paper is straightforward to implement and is flexible in terms of consumer heterogeneity and price expectations. It also builds on Perrone (2017) by allowing for product differentiation and quantity discounts within a storable good industry. As a result, it can use market level data to produce product-level own and cross price elasticities for use in empirical policy analysis.

Hendel and Nevo (2013a) develop a simple dynamic demand model for storable goods. Like the framework developed in this paper, their model can be used to estimate product level long-run own and cross price elasticities using market level data. However, it requires more restrictive assumptions on consumer storage technology, taste heterogeneity and price expectations to do so. Moreover, and importantly from a policy perspective, scaling up the model Hendel and Nevo (2013a) beyond a handful of products is challenging. In contrast, the output of the model in this paper naturally scales to high-dimensional that are often observed in storable good industries.

The cost of the flexibility of my approach - the loss of point identification - is not shared by either Hendel and Nevo (2013a) or Perrone (2017). However, this need not reduce the efficacy of policy analysis based on its outputs.

Outline The remainder of the paper is structured as follows. Section 2 describes the supply side model of price setting when demand is inherently dynamic. Section 3 shows how to combine industry data on market outcomes, accounting margins, a static demand model, and a price forecasting model to recover parameters that adjust price elasticities to reflect consumer responses to permanent price changes when demand dynamics are present. Section 4.1 focuses on the application. First, I describe the UK laundry detergent industry, then estimate a static demand model and apply the methods described in Sections 2 and 3. I use the results to examine the impact of

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gross margins. This empirical finding could wrongly support a policy makers' case that the cartel on mainland Europe had anti-competitive effects in the UK.

[5]Pires (2016); Wang (2015); Osborne (2018); Crawford (2019) estimate a variety of extensions of these dynamic demand models using panel micro-data.

product innovation on the degree of bias in the elasticities produce by the static demand model. Finally, in Section 4.4 I contrast the conclusions of policy simulations using the unadjusted static demand model and the bias-adjusted version. Section 5 concludes.

2 Price setting with demand dynamics

This section presents a model of a differentiated storable good industry in both consumers and firms are forward-looking. Firms set current prices while accounting for demand and supply dynamics. In each period firms set price to maximise their perception of the present value of expected profit flows.

When there are no inter-temporal links in demand or costs, the firm's optimisation problem is separable and is solved independently in each time period. However, in many industries, if not most, consumer demand is inherently dynamic. For example, if the firm discounts its product(s) as part of a temporary promotion, consumers may accelerate purchases and stock the product at home for future consumption.

When demand is dynamic, as is the case for storable goods, firms consider the impact current prices have on demand today and in the future. The effect of a price change today on current and expected future profits are affected by static and inter-temporal substitution. The ratio of inter-temporal to static diverted profits is shown to be closely linked to bias in price elasticities when estimated using a mis-specified static demand model.

When setting prices, firms form beliefs over consumer price forecasts, rival firms' pricing strategies and the evolution of demand and supply shocks. Therefore, firms' must retain and process a lot of information to compute consumer demand and assess the profitability of a very large number of pricing strategies. The size of this task is exacerbated by the large number of products sold to consumers in storable good industries (i.e. often the choice set contains 50-150 products).

Faced with these large computational and cognitive challenges, firms are assumed to adopt a boundedly rational approach to price setting. As a result, firms estimate an approximation to aggregate dynamic demand and solve a simplified version of their profit optimisation. In with this boundedly rational approach to decision making, the equilibrium is used in the dynamic pricing game is Restricted Experience Based Equilibrium.

This section is structured as follows. First, the model is presented assuming that each firm produces a single product in an oligopoly. Second, it is extended to the multi-product firms. The model is then recast in terms of reduced form parameters formed using the ratio of inter-temporal and static diversion ratios. These parameters are shown to capture omitted demand dynamics and are directly linked to the bias in price elasticities. Finally, the concept of a dynamic diversion ratios is introduced and their relationship to the reduced form parameters is derived. These are then used to

modify price pressure tests used in antitrust policy making to account for omitted demand dynamics.

2.1 Consumer demand

In each period consumers choose whether or not purchase from a choice set of J differentiated products. The products, or packs, contain storable goods and are sold in different sizes. Each product's price in period t is recorded in a J-vector, $\mathbf{p}_t = [p_{1t}, \ldots, p_{Jt}]^\top$ where p_{jt} is the price price of product j in period t. Consumer's take these prices as given.

The contents of the products can be stored for several consumption periods.[6] Provided the consumer has access to storage technology, the ability to consume out of inventory means that purchase period and consumption period need not coincide. The possibility of storing current purchases in inventory for future consumption is one source of demand dynamics.

Demand dynamics also arise from consumers' expectations over future price movements. Since storable goods are often sold using temporary discounts, consumers' perceptions over the price process is also likely to affect the timing of purchases. For example, expecting higher prices in the future, some consumers might bring forward purchases to take advantage of a temporarily low sale price. In this case, expected purchase cost savings are weighed against increased storage costs.

With convex storage costs, the increase in storage costs will be larger for consumers that are already holding high levels of inventories relative to their consumption needs. Consumers in this position are therefore less likely to respond to temporary sale prices - especially when the sale is on large products.

The ability to use inventories for consumption and anticipate promotional prices means that consumer demand is inherently dynamic.[7] Therefore, in addition factors that affect myopic consumer demand (i.e. current prices and aggregate demand shocks), consumer's take into account their inventory holdings and price forecasts when making purchase decisions and consumption choices.

As alluded to above, consumer inventories are cumulative result of past purchases and consumption decisions. These are made in response to past prices, previous price forecasts, and the history of other payoff relevant state variables (i.e. demand shocks). Therefore, in addition to current prices, aggregate demand shocks and price forecasts, aggregate demand is also function of the their historical sequences. Let

$$q_{jt} = q_j \left(\mathbf{p}_t, \mathbf{p}_t^{\mathcal{H}}, \xi_t, \xi_t^{\mathcal{H}}, \hat{\mathbf{p}}_{t+1}, \hat{\mathbf{p}}_t \right) \tag{1}$$

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[6]The window for consuming a storable good and the related storage costs depend on their characteristics. For example, some storable goods, such as yoghurts and other dairy products, can be stored for three to four weeks in refrigerated conditions. Others, such as laundry detergent, are non-perishable but may occupy a relatively large portion of storage space.

[7]At least for consumers whose discount factors are greater than zero.

be the aggregate demand for product j in period t where $\xi_t = [\xi_{1t}, \ldots, \xi_{Jt}]^\top$ and $\hat{\mathbf{p}}_{t+1} = [\hat{p}_{1,t+1}, \ldots, \hat{p}_{J,t+1}]^\top$ are J-vectors of product-specific demand shocks and product price forecasts, respectively. Further, $\mathbf{p}_t^{\mathcal{H}} := [\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_{t-1}]$, $\hat{\mathbf{p}}_t^{\mathcal{H}} := [\hat{\mathbf{p}}_2, \ldots, \hat{\mathbf{p}}_t]$, and $\xi_t^{\mathcal{H}} := [\xi_1, \xi_2, \ldots, \xi_{t-1}]$ are matrices containing price history, price forecast history, and history of aggregate demand shock until period t.

From the consumer's perspective, the depth of discounts, their timing, and the length of the promotions are uncertain. To forecast the distribution of future prices, consumers are assumed to use the history of prices observed on recent shopping trips to estimate a statistical model of the evolution of prices

$$\hat{\mathbf{p}}_{t+1} \sim G\left(\mathbf{p}_t^\kappa, \mathbf{p}_t\right) \tag{2}$$

where $\mathbf{p}_t^\kappa = [\mathbf{p}_{t-\kappa}, \ldots, \mathbf{p}_{t-1}]$ is the matrix of prices in the previous κ periods.[8]

Incorporating the price forecasting model used by consumers in equation (2), the model of aggregate dynamic demand can be expressed as a function current prices, past prices and the all other non-price market level demand determinants, $\omega_t := \left[\xi_t, \xi_t^{\mathcal{H}}\right]$.[9]

$$q_{jt} = q_j\left(\mathbf{p}_t, \mathbf{p}_t^{\mathcal{H}}, \omega_t\right) \tag{3}$$

Remarks This description of aggregate demand for storable goods is consistent with existing dynamic demand models.[10] However, as discussed below, the focus of the paper is on the identification and estimation of medium to long elasticities and a dynamic diversion ratios for use in a policy setting.[11] Both of these objects are functionals of aggregate dynamic demand and are obtained without estimation of the dynamic demand system. Therefore, in contrast to the existing dynamic demand estimation literature for storable goods, further restrictions on the demand system are not required to facilitate estimation.

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[8]Each consumer is assumed to have the same information set and use the same price forecasting model. Expanding the demand model to allow for differential information sets and, therefore, heterogeneous forecasting models is beyond the scope of this paper. Amongst other things, the links between information acquisition, inventories, store choice, and shopping frequency would have to be added to the model.

[9]Substituting in equation (2) into equation (1) gives $q_{jt} = q_j\left(\mathbf{p}_t, \mathbf{p}_t^{\mathcal{H}}, \xi_t^{\mathcal{H}}, \xi_t, G\left(\mathbf{p}_t^\kappa, \mathbf{p}_t\right), \left\{G\left(\mathbf{p}_{t-k}^\kappa, \mathbf{p}_{t-k}\right)\right\}_{k=0,1,\ldots,t}\right)$. Because price forecasts are made using current and past prices we can re-express aggregate demand as a function as $q_{jt} = q_j\left(\mathbf{p}_t, \mathbf{p}_t^{\mathcal{H}}, \xi_t, \xi_t^{\mathcal{H}}\right)$. Finally, using the definition of ω_t we get equation (3).

[10]Most existing models have focused on the case where household-level purchase diary data is available. In this case, inventories are integrated out of a dynamic demand model using observed household's purchase history and observed prices (i.e. Sun (2005); Erdem et al. (2003); Hendel and Nevo (2006); Wang (2015); Pires (2016); Osborne (2018); Crawford (2019)). More closely related to the objectives of this paper, Hendel and Nevo (2013b) focussed on estimation of dynamic demand model for storable goods with more widely available aggregate market data.

[11]The concept, definition and link to demand dynamics of dynamic diversion ratios is deferred to subsection 2.3 below.

2.2 Firms

Firms' play the dynamic pricing game over an infinite horizon and time is discrete; $t = 1, 2, \ldots, \infty$. To simplify exposition of the firm's objective function and structure of the game the model is described for a single product oligopoly. Then the model is extended to multi-product firms and the nature of the equilibrium of the dynamic pricing game is discussed.

2.2.1 Single product oligopoly

Suppose that each of the J differentiated products is produced by a single firm. In period t each firm sets the price of product j to maximise the net present value (NPV) of their perception of expected profit flows,[12]

$$V_j\left(\mathbf{x}_t\right) = \mathbb{E}_t \sum_{h=t}^{\infty} \max_{p_{jh}|\mathbf{p}_{-jh}} \delta^{h-t} \pi_j\left(p_{jh}, \mathbf{p}_{-jh}, \mathbf{x}_h\right) \tag{4}$$

where $\mathbf{x}_t := \left[\mathbf{p}_t^{\mathcal{H}}, \omega_t\right]$ is the state of the game in period t and ∇ is a common discount factor. Firms take expectations over future demand (and/or cost) shocks and form beliefs over rivals' pricing strategies and consumer's forecasting technology.

The per-period profits earned on sales of product j in period t are

$$\pi_j\left(p_{jt}, \mathbf{p}_{-jt}, \mathbf{x}_t\right) = \left(p_{jt} - c_{jt}\right) q_j\left(p_{jt}, \mathbf{p}_{-jt}, \mathbf{x}_t\right) \tag{5}$$

where $c_{jt} = g\left(z_{jt}, v_{jt}\right)$ is the marginal cost of production for product j in period t.[13] In addition to their own price and cost of production, each firms' flow of profit depends on the aggregate demand for their product.

Approximating demand Aggregate demand as stated in equation (3) requires that firms store and use the history of prices (and other state variables, ω_t). However, the cognitive and computational challenge of retaining and processing such a large amount information is arguably prohibitive. Recognising these limitations, firms are assumed to approximate the dynamic aggregate demand function using shorter price history consisting of τ-periods $\mathbf{p}_{t-1}^\tau$.

The number of periods used when approximating aggregate demand, τ, depends on the distribution of the time that elapses between purchases. In general, purchase cycles are influenced by the nature of products available for purchase (i.e. range of pack sizes, brand loyalty, price discounting) and consumer specific factors (i.e. storage costs, and consumption rates).

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[12]Firm's are assumed not to be able to commit to future prices.

[13]At the cost of additional notation and structure on the supply model, the marginal cost can be specified as function observed cost shifters, z_{jt}, and privately observed i.i.d. marginal cost shocks, ν_{jt}. In this case, the marginal cost function is $c_{jt} = g\left(z_{jt}, v_{jt}\right)$ and its conditional distribution given z_{jt} is $F_{g|z}(\cdot)$.

Assuming that the price history used to approximate demand is at least as long as the price history that firms' believe consumers use forecast future prices, $\mathbf{p}_t^\kappa \subseteq \mathbf{p}_t^\tau$, the approximate aggregate dynamic demand is

$$q_{jt} \approx q_j\left(\mathbf{p}_t, \mathbf{x}_t\right) \tag{6}$$

where, with a little abuse of notation, the state of the game is $\mathbf{x}_t := [\mathbf{p}_t^\tau, \xi_t, \xi_t^\tau]$. Prior to setting prices, firms approximate aggregate demand using the observed history of market outcomes (i.e. past prices, demand shocks, and quantities sold).

Given information contained in the state of the game and their beliefs over rivals' pricing behaviour, firms use the estimated demand to evaluate the effect that different price strategies have of profit flows. The optimal price for the firm manufacturing product j solves the following first-order condition

$$q_{jt} + \frac{\partial q_{jt}}{\partial p_{jt}} m_{jt} + \mathbb{E}_t \sum_{h=t+1}^\infty \delta^{h-t} \frac{dq_{jh}}{dp_{jt}} m_{jh} + \mathbb{E}_t \sum_{h=t+1}^\infty \delta^{h-t} \frac{dp_{jh}}{dp_{jt}} q_{jh} = 0 \tag{7}$$

where $m_{jh} = p_{jh} - c_{jh}$ is the mark-up earned on sales of product j in period $h \geq t$.

The first and the second terms correspond to the terms that enter the first order conditions used by firms in a single product oligopoly to set prices when demand is static. The additional terms only occur due to the presence of price and demand dynamics.

The third term is the expected change to the discounted net present value of profits earned on future sales of product j due to a change in the current price of product j. It captures the change in future profits that arises from inter-temporal substitution due to dynamic demand responses. For example, suppose a firm deeply cuts price today to promote product j. Then this term measures the profit lost due to reduction in future sales of product j because consumers accelerate their purchases to take advantage of the sale price.

Finally, the fourth term measures the change in expected revenues from future sales of product j that result from anticipated adjustments to future prices in response to changes in the current price of product j. To evaluate this term, firm's need to compute the effect that a change in the price of their product today has on the perceived distribution of prices for the foreseeable future. These price movements are the impulse response functions consistent with firm's beliefs over rivals' future pricing strategies following a change in the price in product j.

The evaluation of the price impulse response functions is complicated by three factors. First, the forecast horizon is infinite. Second is the possibility of non-linear dynamic price strategies. When combined they compound the computational complexity of forecasting the future distribution of prices. Third, the forecasting problem is high-dimensional when the choice set is large (i.e. for storable goods $J \approx 100$).

Approximate dynamic pricing To help mitigate the these information processing costs firms solve an approximation to their dynamic pricing program. In the modified dynamic

optimisation problem, firms approximate their expected discounted present value of profit flows by splitting it into two components.

$$V_j\left(\mathbf{x}_t\right) \approx \mathbb{E}_t \sum_{h=t}^{H} \max_{p_{jt}\mid\mathbf{P_{-jt}}} \delta^{h-t}\pi\left(p_{jh}, \mathbf{p}_{-jh}, \mathbf{x}_h\right) + \bar{V}_{jt}^{H+1} \tag{8}$$

The first is term measures the firms' perceived discounted NPV of profits over the next H-periods. The second, $\bar{V}_{jt}^{H+1}$, is an approximation to expected discounted NPV profits after period H and is assumed to be independent of current prices. Firms choose prices to maximise approximate NPV profit over a rolling window.

Even with this strategy, the forecasting problem is still high dimensional and (potentially) non-linear. To further simplify the problem, firms may restrict price strategies to be linear and use the history of market outcomes to estimate a κ-order Markov process to model the evolution of prices.

$$\mathbf{p}_t = \mathbf{A}_0 + \sum_{s=1}^{\kappa} \mathbf{A}_s\mathbf{p}_{t-s} + \mathbf{u}_t \tag{9}$$

where $\mathbf{A}_0$ is a J-vector, each $\mathbf{A}_s$ is a $J \times J$ matrix of price coefficients for $s = 1,\ldots,\kappa$ and $\mathbf{u}_t$ is a J-vector of price shocks.

In this case, the price impulse response functions entering the first order conditions of the approximate dynamic program is only a function of selected parameters in the matrices $\mathbf{A}_s$ for $s = 1,\ldots,\kappa$. Therefore, imposing this a linear function form restriction simplifies the calculation of term four in equation (7) by reducing it to taking the expectation of aggregate demand for product j.

Implicit in above analysis firms' engage in 'nowcasting' - forecasts are updated using prices set in the current period. However, given the high-dimensional nature of the forecasting problem, even this is likely to be a time consuming and resource intensive task. As such, in practice, firm's may seek to adopt a boundedly rational approach and reduce price optimisation costs by evaluating the price impulse response functions prior to setting prices in period t.

A1: Firms fix price forecasts prior to setting prices and $\frac{dp_{jh}}{dp_{jt}} = 0$ for all $j = 1,\ldots,J$ and $h = t+1,\ldots,H$

Under assumption A1, the fourth term drops out of equation (7). The first condition of the approximate dynamic profit optimisation becomes,

$$q_{jt} + \frac{\partial q_{jt}}{\partial p_{jt}}m_{jt} + \mathbb{E}_t \sum_{h=t+1}^{H} \delta^{h-t}\frac{dq_{jh}}{dp_{jt}}m_{jh} = 0 \tag{10}$$

In line with perspective that firms face non-negligible time and resource constraints, the baseline model used in the remainder of this paper assumes that firms hold forecasts fixed when setting prices.

Remarks Assuming firms used fix price forecasts prior to setting prices yields practical benefits in a policy setting. Specifically, it eliminates the need to estimate an empirical analogue for term four in equation (7) for all products and time periods in the data.

To explore why this is beneficial in a policy setting, suppose the analyst would like to relax this assumption and estimate an empirical analogue. This requires the price forecast model in equation (9) to be estimated from market data. However, most industries with dynamic demand contain $J \geq 100$ products in each week. In this case, the forecast model has $J(1 + \kappa J)$ parameters; if $J = 100$ and $\kappa = 1$ then there are over 10,000 parameters to be estimated. With so many parameters the price forecasting model will be challenging to estimate - even in industries with dynamic demand containing only a handful of products.[14]

In light of the above discussion, the baseline model intended for use in a policy setting assumes that firms adopt approximations to high dimensional demand and supply dynamics. To further simplify the resource intensive process of optimising the resulting approximation to expected NPV profit, firms are also assumed make price forecasts before setting prices. Such a process might be considered optimal for boundedly-rational firms. Next the model is extended to allow for multi-product firms.

2.2.2 Multi-product oligopoly

The previous section derived the first order conditions assuming that each of the J products were produced by a separate firms. However, most industries with dynamic demand $N < J$ firms manufacture multiple products in the industry. Next, the system of equations used to set prices is extended to multi-product firms.

Let f index the F firms in the industry and let $\mathcal{J}_f$ denote the set of products sold by firm f. The per-period profit function in oligopoly

$$\pi_f \left(\mathbf{p}_t, \mathbf{x}_t \right) = \sum_{j \in \mathcal{J}_f} \left(p_{jt} - c_{jt} \right) q_j \left(\mathbf{p}_{jt}, \mathbf{x}_t \right) \tag{11}$$

where the corresponding approximation to the discounted present value of profits for firm i is

$$V_f \left(\mathbf{x}_t \right) = \mathbb{E}_t \sum_{t=h}^{H} \max_{p_{jh} \forall j \in \mathcal{J}_f} \delta^{h-t} \pi_f \left(\mathbf{p}_t, \mathbf{x}_t \right) + \bar{V}_{ft}^{H+1} \tag{12}$$

where $\bar{V}_{ft}^{H+1}$, is an approximation to expected discounted NPV profits for firm f after period H.

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[14]To circumvent this issue a feasible alternative model of pricing strategies tailored to deal with high-dimensional forecasting problem is required. In addition to reducing the dimension of the problem, it must maintain the linearity κ-order Markov process in equation (9) that was used to simplify calculation of $\frac{dp_{jh}}{dp_{jt}} \forall h \in \{t + 1, \ldots, H\}$ in term four of equation (7). One possible alternative is to use the dynamic factor model for price forecasts suggested by Crawford (2019).

To extend the analysis to the multi-product case, define ∇_h as a $J \times J$ matrix of demand derivatives for period h with respect to price changes in period t where $h \geq t$ that encodes the ownership structure of the products sold in the industry. If the firm produces both products j and k, the (j, k)-element of ∇_h is the inter-temporal demand derivative relevant to the pricing of firm i's products,

$$\nabla_{(j,k),h} = \begin{cases} \frac{dq_{kh}}{dp_{jt}} & \text{if } j, k \in \mathcal{J}_f \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

The first order condition of the approximate dynamic profit optimisation problem for the multi-product firms used to set p_{jt} for all $j \in \mathcal{J}_f$ in the baseline model is,

$$q_{jt} + \sum_{k \in \mathcal{J}_f} \nabla_{(j,k),t} m_{kt} + \mathbb{E}_t \sum_{h=t+1}^{H} \sum_{k \in \mathcal{J}_f} \delta^{h-t} \nabla_{(j,k),h} m_{kh} = 0 \tag{14}$$

This system of equations serves as the basis for the empirical analysis in the remainder of this paper.

2.2.3 Equilibrium

The equilibrium concept for the dynamic pricing game is Restricted Experience Based Equilibrium (REBE) - a modification of Markov Perfect Equilibrium (MPE) developed by Fershtman and Pakes (2012). A REBE retains Markovian structure of an MPE, but relaxes the restriction that agents are rational.

For agent's behaviour meet MPE's rationality conditions in a storable good industry with both forward-looking consumers and firms, there are three conditions that must hold for all possible realisations of the state vector. First, firms' beliefs over rivals' strategies coincides with their optimal price strategies. Second, they must also hold correct beliefs' over the forecasting technology used by forward-looking consumers. Finally, a MPE also requires that consumers' forecasting technology is consistent with firms' equilibrium price strategies.

However, the demands that the MPE conditions place on the mental and physical resources of firms and consumers are arguably prohibitive. For example, one requirement implicit in these conditions is that all agents know the identity of the manufacturing firm for each product. While firm's know the identity of manufacturers for each product, it is not obvious that consumers do. If not, the MPE condition that requires consistency between firms' equilibrium price strategies and consumers' price forecasts arguably requires more sophistication and cognitive capacity than is available to them (or at least that they would choose to allocate).

Even without these additional complications that arise from demand dynamics, firms' may also not have the capacity to compute the strategies that meet MPE conditions. With a handful of products - and therefore a small state space - processing and evaluating the value functions associated complicated price strategies is

computationally challenging. When, as is common in many storable good industries, the choice set is high-dimensional (i.e. $J \approx 100$) the state space becomes large, the computation of strategies satisfying MPE conditions is arguably infeasible.

A REBE replaces the MPE rationality requirements with two weaker conditions. First, given the information set used by agents to make decisions, a REBE requires that agents choose actions that maximise their perceived payoffs. The second conditions requires the perceptions held by agents are consistent with play they have observed in the past when the information set recurs.

Both REBE conditions are met by the decisions taken by firms and consumers in the dynamic price model. In accordance with the first condition, firms and consumers are choose actions that maximise their perceived payoffs calculated using a restricted subset of the full information available to them (i.e. truncated price histories). The second condition is also met; firms' aggregate dynamic demand model, the model they use to forecast rivals' price strategies, and consumers' price forecast model are all consistent with past market outcomes.

In the context of the storable good industry model above, one benefit of using REBE as an equilibrium concept is is relaxes the implausible MPE requirement that consumer and firm beliefs necessarily coincide. Instead, it only requires that these beliefs are consistent with observed play at recurring states of the game.

2.3 Dynamic diversion ratios

This section expresses the dynamic component of the optimal pricing equations in terms of diversion ratios. Diversion ratios measure of the closeness of substitution between two products. They are calculated as the proportion of product j's sales gained from product k as a result of a decrease in product j's price

$$DR_{(j,k),t} := -\frac{\nabla_{(j,k),t}}{\nabla_{(j,j),t}} \tag{15}$$

When the diversion ratio is multiplied by the relative mark-ups of the two products it forms the diverted profit ratio. This measures effect that sales diverted to product j from product k has on product k's profits

$$DR^{\pi}_{(j,k),t} := -\frac{\nabla_{(j,k),t} m_{kt}}{\nabla_{(j,j),t} m_{jt}} \tag{16}$$

In the context of a static pricing game, a diverted profit ratio captures of the strength of the competitive constraint internalised when a manufacturer produces substitutable products j and k. As discussed at greater length below in subsection 2.4, the diverted profit ratio is a key component of generalised upward price pressure index (GUPPI) used in antitrust analysis to assess unilateral merger effects.

However, when equations (15) and (16) are calculated using high frequency market data, these diversion ratios only capture the effect of contemporaneous substitution.

Figure 1: Inter-temporal substitutions patterns of product j and k over 10 periods in response to a temporary 1% price cut of product j at $t = 1$
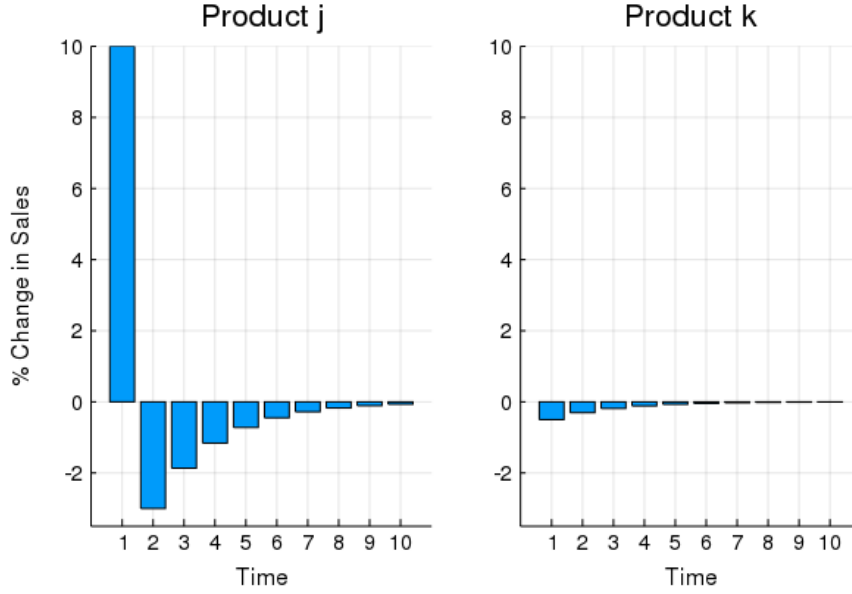
As a result they are not suitable to measure diversion between storable goods. This is because the inter-temporal substitution synonymous with dynamic demand are not accounted for. By not including diversion from future sales, the numerator tends to be understated and the absolute value of the denominator overstated. These biases reinforce one another and can combine to severely bias diversion ratios downwards.

Figure 1 provides a graphical representation of contemporaneous and inter-temporal substitution patterns for two substitutable storable goods, j and k. The left panel shows the percentage change in sales of product j when the price of product j is temporarily cut by one percent in the first period. The right panel shows the corresponding percentage change in sales of product k.

The figure shows that the sales of product j rise initially in response to the temporary price cut. Also, because product k is a substitute for product j, the temporary price cut of product j diverts sales away from product k in the first period. In the subsequent periods, both product j and product k's sales are lower than those without the price cut - albeit at a diminishing rate.

The loss of future sales arises because some of the demand uplift in the first period is drawn from future demand. That is, future sales of products j and k are substitutes for current sales of product j. However, the percentage change in future sales of product k are less pronounced than product j. This indicates that future sales of product k are a more distant substitute to current sales of product j than future the corresponding sales of product j.

To capture the inter-temporal substitution patterns in Figure 1 in a diversion ratio, equation (15) is adjusted to only include the change in future sales of product k in the

numerator,

$$\Delta DR_{(j,k),t} := -\frac{\sum_{h=t+1}^{H} \nabla_{(j,k),h}}{\nabla_{(j,j),t}} \tag{17}$$

Similarly, the effect of inter-temporal substitution on firm profits is measured by modifying equation (16). In place of the change in current profits of product k, the new numerator is the change in expected NPV of its future profits in response to the temporary price cut for product j. The result is an inter-temporal diverted profit ratio between current purchases of product j and future purchases of product k.

$$\Delta DR_{(j,k),t}^{\pi} := -\frac{\mathbb{E}_t \sum_{h=t+1}^{H} \delta^{h-t} \nabla_{(j,k),h} m_{kh}}{\nabla_{(j,j),t} m_{jt}} \tag{18}$$

As is the case for static price setting games, the inter-temporal diverted profit ratios are a key components dynamic pricing. Specifically, the ratio of the equation (18) to its static counterpart, equation (16) is closely connected to the bias that results from omitting demand dynamics for storable goods.

This relative dynamic diverted profit ratio for a pair of products j and k in period t is,

$$\Psi_{(j,k),t} := \frac{\Delta DR_{(j,k),t}^{\pi}}{DR_{(j,k),t}^{\pi}} \tag{19}$$

It measures the expected change of the net present value of future profits earned on sales of product k in response to a temporary price change for product j as a fraction of the change in profits earned on current sales of product k. It captures the relative importance of contemporaneous and inter-temporal cannibalisation when setting current prices.

Let $\boldsymbol{\Psi}_t$ be a $J \times J$ matrix for all pairs of products $j, k \in \mathcal{J}$ and substitute it into the dynamic price setting equations. Written in matrix form the resulting system of first order conditions is

$$\mathbf{q}_t + \left(\nabla_t \odot \left(1 + \boldsymbol{\Psi}_t \right) \right) \mathbf{m}_t = 0 \tag{20}$$

where $\mathbf{q}_t$ is a J-vector of aggregate quantity demanded in period t, $\mathbf{m}_t$ is a J-vector of mark-ups and $\odot$ denotes the Hadamard product.

If all elements of $\boldsymbol{\Psi}_t$ are zero, then equation (20) reverts to the static differentiated Bertrand Nash optimality conditions. However, when demand dynamics are present, they provide a reduced form expression of omitted inter-temporal substitution. Moreover, the elements of Ψ_t convert biased short-run price elasticities to their long-run counterparts that measure both contemporaneous and inter-temporal substitution.

The remainder of the paper treats the elements of $\boldsymbol{\Psi}_t$ as reduced form parameters to be estimated from data on market outcomes and margin's earned on sales over the same

period. Before turning to identification and empirical analysis, I describe how the system of price setting equations (20) can be used in existing antitrust policy tools often applied to industries in which demand dynamics are common.

2.4 Antitrust policy: price pressure tests

The 2010 US Horizontal Merger Guidelines introduced upwards price pressure measures UPP (Farrell and Shapiro (2010)) and the gross upwards price pressure index (GUPPI) (Moresi (2010)). Since then, these price pressure tests have been used to screen mergers, aid market definition and assess closeness of competition - particularly for differentiated products.[15] This section shows how Ψ can be used to adjust diversion ratios used in upward price pressure tests to reflect market dynamics in a modified GUPPI formula.

The GUPPI is a measure of the strength of the competitive constraint between two products, j and k. Formally, in response to a price rise of product j, the percentage margin earned on sales of product j multiplied by the diverted profit ratio. That is,

$$GUPPI_{(j,k)} := -\frac{\nabla_{(j,k),t} m_{kt}}{\nabla_{(j,j),t} m_{jt}} \mu_{jt} = DR^{\pi}_{(j,k),t} \mu_{jt} \tag{21}$$

where μ_{jt} is the percentage mark-up on product j in period t.

As discussed above, for storable goods the diverted profit ratio is biased downward due the omission of inter-temporal substitution. To capture the effect of inter-temporal substitution in response to a temporary change in the price for product j, a dynamic diverted profit ratio is required.

$$DDR^{\pi}_{(j,k),t} := -\frac{\mathbb{E}_t \sum_{h=t}^{H} \delta^{h-t} \nabla_{(j,k),h} m_{kh}}{\mathbb{E}_t \sum_{h=t}^{H} \delta^{h-t} \nabla_{(j,j),h} m_{jh}} \tag{22}$$

The dynamic diverted profit ratio measures the reduction in the present value of expected profits earned on current and future sales of product k as a fraction of the present value of expected profits earned on current and future sales of product j in response to a temporary cut in price of j.

Replacing biased static diverted profit ratio in the GUPPI formula (equation (21)) with its dynamic counterpart defines a new GUPPI adjusted for omitted inter-temporal substitution dynamics is created.

$$dGUPPI_{(j,k)} := DDR^{\pi}_{(j,k),t} \mu_{jt} \tag{23}$$

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[15]GUPPI and other price pressure have been used in many recent merger cases. For example, in the UK, these include Sainsbury's/Asda (2019), Tesco/Booker (2017), Ladbrokes/Coral (2016) and Cineworld/Showcase Cinema (2013). Examples in the EU and USA include Austria / Orange Austria (2012) and Dollar Tree/Family Dollar (2015), respectively.

Problematically for use in a policy setting, the dynamic diverted profit ratio as stated above in equation (22) is hard to quantify - even when a dynamic demand model is available to simulate future demand derivatives. Therefore, it is useful to be able to express it in terms of more readily obtainable components.

Lemma 1: The dynamic diverted profit ratio is linked to its short-run counterpart by elements of $\mathbf{\Psi}_t$. Specifically, for product k in response to a temporary increase in the price of product j in period t using

$$DDR_{(j,k),t}^{\pi} := -\frac{1 + \Psi_{(j,k),t}}{1 + \Psi_{(j,j),t}} DR_{(j,k),t}^{\pi} \tag{24}$$

Using equation (24) in equation (23) it is straightforward to show dGUPPI simplifies to the product of term containing elements of Ψ_t and the static GUPPI.[16] Therefore, after estimating $\mathbf{\Psi}_t$, the construction of this new dynamic price pressure index is straightforward in a policy setting.

$$dGUPPI_{(j,k)} = -\frac{1 + \Psi_{(j,k),t}}{1 + \Psi_{(j,j),t}} GUPPI_{(j,k)} \tag{25}$$

To illustrate how the dGUPPI captures demand dynamics, consider a firm assessing the impact on profits of its promotional pricing decisions on two substitutable products j and k over a one month planning horizon.

Suppose the firm producing product j is considering temporarily cutting its price from £15 to £10 for the current week. If it does so, it expects to generate an additional 100 sales as a result. However, the firm expects that 20 of those sales would occur at the regular higher price of £15 within the subsequent week over the next month without the price cut.

Assuming a marginal cost of £5 in all periods, 40% of short-run profits made on sales of good j immediately following the price cut come are drawn from expected profits from future profit flows. That is, the (j, j)-th element of $\bar{\Psi}$ is -0.4.

Further suppose, the effect of the price cut of product j on product k is the loss of 10 sales today and a further 3 sales over the next month. Assuming its price and cost remain unchanged over the period and its sales yield a £10 margin, the firm anticipates that consumer demand dynamics will lead to an additional 30% of losses on top of short-run profits lost on product k over the next month. Equivalently the (j, k)-th entry of $\bar{\Psi}$ is 0.3.

In this simple example, only 20% of the firm's profits are the result of sales diverted away from product k in the period of the price change and the GUPPI from k to j is 0.1. Recall, that the (j, j)-th and the (j, k)-th entry of $\bar{\Psi}$ are -0.4 and 0.3, respectively. Therefore, $\gamma_{(j,k)} = 0.43$ and the dGUPPI that factors in anticipated demand dynamics is 0.217 - more than double the GUPPI calibrated with contemporaneous profit movements alone. This higher dynamic diversion ratio demonstrates that products j and k are much closer substitutes for non-transitory price changes than implied by a static analysis.

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[16]See annex A.2 for a derivation of the result in Lemma 1.

Remarks The benefit of constructing dGUPPI to assess the strength competitive constraints in the context of storable goods is threefold. First, it does not require estimation of a computationally intensive dynamic demand model to provide accurate measures of diversion ratios. Second, the analyst need not solve for a new equilibrium of the underlying dynamic pricing game to be able evaluate the unilateral effects concerns arising from a merger. Finally, calculation of dGUPPI only requires inputs and empirical techniques commonly used in antitrust analysis. Namely, product margins, high frequency market data (i.e weekly, monthly), static demand estimation and manipulation of the system of linear price setting equations. As discussed in the next section, the main drawback is that the dGUPPI will be set valued because the elements of Ψ are set-valued. However, as shown in policy application in sections (4.3) and (4.4), this does not preclude its efficacy in a policy setting.

3 Identification of Ψ

Given data on market outcomes and margins measured over the same period, the elements of Ψ_t are not identified by the combination of the model price setting model described in Section 2 and demand derivatives of a static demand model. To remedy this two further restrictions on the nature of inter-temporal substitution patterns omitted from the model are added. They both capture important aspects of the nature of the missing demand dynamics described in Figure 1.

A2: Inter-temporal substitution: Demand for product j is decreasing in own-price in the current period and is increasing in own-price in future periods. The demand for all other products is such that a cut in the price for product j weakly decreases contemporaneous and future demand over future periods for all other goods.

A3: $\left|\Psi_{(j,j),t}\right| \geq \Psi_{(j,k),t}$ $j \neq k$, $j \in \mathcal{J}_f$. Inter-temporal demand responses are strongest with respect to the product's own past prices.

A2 formalises that purchases of goods in the industries are substitutes both in the period in which they are purchased and in future periods. A3 it reflects the fact that inventories built up in response to a promotion have the largest impact on the demand of the product promoted - rather than on the demand of imperfectly substitutable alternatives.[17]

Adding these two restrictions to the model from Section 2, as stated in Proposition 1, the elements of Ψ_t can be shown to be bounded above and below.

Proposition 1: Assume A2 and A3 hold and that firms set prices using equation (20). Provided that promotional margins are non-negative and future sales are expected to at least cover variable costs, then the resulting prices are above marginal costs, then all of

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[17]This is consistent the findings of the existing literature on dynamic demand estimates in storable goods markets that that the biases to own-price elasticities are larger in magnitude than biases to cross-price elasticities. See Erdem et al. (2003); Hendel and Nevo (2006).

the elements of Ψ belong to compact sets

$$-1 \leq \Psi_{(j,j),t} \leq 0 \tag{26}$$
$$0 \leq \Psi_{(j,k),t} \leq \left|\Psi_{(j,j),t}\right| \quad \forall\, j \neq k \tag{27}$$

for $j, k \in \mathcal{J}_f$.[18]

Even though the parameters can be bounded, the number of reduced form parameters in $\{\Psi_t\}$ for $t = 1, \ldots, T$ in the system of price setting equation (20), for products in group n is $T J_n^2$.

To aid empirical work further restrictions on Ψ_t are imposed to reduce the number of parameters to be estimated. To that end, three further assumptions are added to reduce the number of parameters to be estimated for products in one of the groups, n.

First, the bias adjustment parameters are assumed to be the same in all periods over which the margin is measured. This reduces the number of estimated parameters by a factor T.

A4. $\Psi_t = \bar{\Psi}$ for $t = 1, \ldots, T$

Second, without variation of product margins within the n groups of products the available data does not allow the analyst to draw inference about individual $\bar{\Psi}_{(j,j)}$ for each the j products in group n. As such, a further assumption is introduced that constrains the diagonal elements of $\bar{\Psi}$ for products in group n to be the same.

A5: $\bar{\Psi}_{(j,j)} = \psi_n^{own}$ for all $j \in \mathcal{J}_n$

Third, because the margins have no additional information on covariances between mark-ups and demand derivatives, the off-diagonal elements are not identified. Therefore, I approximate the off-diagonal elements of matrix of dynamic diversion ratios by assuming that they are the same for any two products, $k \neq j$ and $k' \neq j$.

A6: $\bar{\Psi}_{(j,k)} = \psi_n^{cross}$ for all $j \neq k, j \in \mathcal{J}_n$

Adding assumptions A5 and A6 further reduces the number of parameters from J_n^2 to 2 parameters for each group n.

Lemma 2: Under assumptions A1-A6, the mark-up in each period can be expressed as a function of aggregate demand, prices, short-run demand derivatives and $\bar{\Psi}$.

$$\mu_n = -\frac{\sum_{t=1}^{\bar{T}} \mathbf{q}_t^{n,\top} \left(\nabla_t \odot \left(1 + \bar{\Psi}\right)\right)^{-1} \mathbf{q}_t}{\sum_{t=1}^{\bar{T}} \mathbf{q}_t^{n,\top} \mathbf{p}_t} \tag{28}$$

$$-1 \leq \psi_n^{own} \leq 0 \tag{29}$$
$$0 \leq \psi_n^{cross} \leq |\psi_n^{own}| \tag{30}$$

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[18]See Annex A.1 for the proof of Proposition 1.

where $\mathbf{q}_t^n$ is vector of quantities purchase whose j-th entry is q_{jt} if $j \in \mathcal{J}_n$ and 0 otherwise and $\left[\nabla_t \circ \left(1 + \bar{\mathbf{\Psi}} \right) \right]$ is an invertible matrix of bias-adjusted demand derivatives.[19]

Lemma 2 provides with an equation for the percentage margin earned on sales of products in group n. Combining an estimated margin, data on market outcomes and the output of a static demand model, the analyst can find values for the 2 elements of $\bar{\Psi}$ that solve equations (28), (29) and (30). When $J_n > 1$ with only one equation the parameters in $\bar{\Psi}$ are set, rather than point-identified.

4 Application: UK laundry detergent

In this section, the model described in Sections 2 and 3 is applied to the UK laundry detergent sector.

After providing a brief overview of the industry in which I highlight how technological innovations have shrunk products, reduced storage costs and led to changes in pricing behaviour, the elements of $\bar{\Psi}$ are estimated in two steps.

First, I specify a static demand model and estimate it using household purchase diary data from Kantar WorldPanel for a large UK supermarket from 2002 to 2012. Second combing margins estimated from accounting reports with market data and the output of the static demand model, the set of values for elements of $\bar{\Psi}$ that solve first order conditions are calculated. Combining the demand model with sets of estimates for $\bar{\Psi}$, I quantify the bias in (absolute) own and cross price elasticities.

Finally, the set-valued estimates of elements of Ψ are used to conduct two policy experiments. First, I show how the dynamically adjusted GUPPI formula, dGUPPI, can be used to assess the unilateral effects that might arise following a divestment of a brand from one firm to another. Second, I test whether there is evidence that a laundry detergent cartel in mainland Europe between 2002 and 2005 affected the UK detergent market.

Both experiments highlight that the omission of demand dynamics has the potential to lead to misguided policy conclusions. Further, they demonstrate that the methods developed in the paper have the potential prevent such errors and aid policy decision making.

4.1 The UK laundry detergent industry

The analysis of the UK laundry detergent industry is based on individual household purchase data from 1st January 2002 until 31st October 2012. Households that take part in the survey scan the barcode of the items they purchase. Using the scanned barcode, the survey records the price and number of packs bought together with the characteristics of the product purchased. In addition, the purchase date and store in

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[19]The non-singularity of this matrix requires that the aggregate long-run diversion ratios for products in group n produced by firm f differ.

which the product was bought is also recorded. The purchase data is supplemented by annually updated household demographics.

In the remainder of the paper, to avoid complexities related to store choice, the analysis is conditioned on detergent purchased from a single leading UK's supermarket. Prices charged by supermarkets the same fascia are the same stock keeping unit (SKU or product) in different stores in excess of 280 sq. ft. in size.[20] As such, purchases across different stores within a particular supermarket fascia in the UK can be pooled for the purposes of empirical analysis.

The UK laundry detergent industry is dominated by two firms accounting for around 75 to 85 percent of households' annual purchases of laundry detergents in each year between 2002 and 2012. Hereafter, they are referred to as firm A and firm B. Firm A's annual share of sales is between 44 and 54 percent and firm B between 29 and 36 percent. Outside of these two major producers of branded products, the retailer's private label products commands the largest share - although its share has declined from 25 percent in 2002 to 12 percent in 2012. A fringe of small niche brands account for the remainder of products sold.

Laundry detergent industry is sold in a diverse array of brands, formats, and pack sizes. Each pack, or product, contains a single type of detergent. In general, a detergent is defined by its format and brand and the chemical properties of the enzymes it contains (i.e. non-bio/bio, stain removal properties, scent etc). In the remainder of this paper, the analysis focuses on the major distinguishing features of a product - its brand, format and pack size.

In addition to the supermarket's private label products and the fringe of 'other' brands, the two largest firms sell six major brands between them. These six brands are labelled alphabetically as brand A through to brand F and are differentiated by their perceived quality.[21] Using the average purchased price per wash as a indicator for consumer perception of quality, brand D is classified as a premium brand (20p per wash). Brands A, C and E are mid-range brands (17-18p per wash) and brands C and F are 'standard' brands whose price is similar to the supermarket's private label detergents (13-14p per wash).

Laundry detergent brands are sold in six formats: powder, tablets, liquids, liquid capsules, super concentrated liquid, and gel.

Formats have different dosage metrics: liquid and gel are measured in millilitres per wash, powder and tablets are measured in grams per wash. Further, following a series of industry-wide initiatives that sought to reduce the environmental impact of detergent production and the development of new products, dosages tend to decline over time within formats. To avoid complications arising from correcting for changing dosages, the number of washes is used to measure pack size.

Laundry detergents can be purchased in many different pack sizes.[22] As dosages have

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[20]This pricing policy follows an undertaking in 2001 following a market investigation by the UK Competition Commission into the groceries industry in 2000.

[21]Brands are not attributed to firms for confidentiality reasons.

[22]The UK laundry detergent industry differs in this respect from the one studied by Hendel and Nevo

declined over time and so has the physical storage space occupied by material needed per wash. In response, firms produce larger pack sizes over time: the average number of washes per purchased pack have increased by around 50 percent from 17 washes in 2002 to 26 washes in 2012.[23]

As noted above, the product is defined as a unique combination of brand, format and pack size. Table 1 provides summary statistics for resulting 668 products together with consumer characteristics including household size and a proxy for household income - the average weekly grocery spend.

Table 1: Summary of Kantar WorldPanel data

	Mean	Median	Std dev	Min	Max
Purchase Characteristics					
Price in (£)	3.58	3.20	1.81	0.13	24.52
Price per wash (£)	0.17	0.16	0.06	0.01	0.77
Purchased Washes	23	20	12	3	100
Dosage in Grams	87.7	80	16.1	18.3	139.1
Dosage in Millilitres	52.6	45	20.8	12	125
Number of Packs	1.07	1	0.26	1	2
Household Characteristics					
Number of Equivalent Adults	2.28	2.20	0.79	1.00	8.10
Av. weekly grocery spend (£)	67.05	63.78	27.71	3.70	423.45

Source: Kantar WorldPanel

4.2 Demand for UK laundry detergent

This section describes static discrete choice demand model of UK laundry detergent. Given the policy focus of this paper, a demand model often employed in antitrust investigations - the nested logit model - is estimated.

In contrast to many applications in a policy setting, the nested logit model is estimated using the purchase diary data from Kantar WorldPanel data described in Section 4.1. The availability of micro-data enables the use of observed household characteristics to estimate a rich demand model without the additional computational costs associated with estimation of a random coefficient model.

(2006). They restrict attention to powder products and examine brand choice conditional on size choice from a small number of discrete sizes: 16oz, 32oz, 64oz, 96oz, and 128oz. Erdem et al. (2003) also focus on only five different weight choices in the US Ketchup market in their dynamic demand estimation.

[23]There is heterogeneity in the evolution of pack-sizes across brands. Purchased pack sizes increase by 20 to 30 percent for brands A, B, and D from 2002 to 2012. For brand C purchased pack sizes increase by around 70 percent and approximately double for brands E and F.

Where the policy analyst only has aggregate market data, Berry (1994) and Berry et al. (1995) can be used to estimate conditional, nested, or random coefficient logit demand models. The estimating of the elements of $\bar{\Psi}$ is unchanged described in Section 4.3 is unchanged.

4.2.1 Static demand model

Detergents are sold in one of $j = 1, \dots, J$ stock keeping units (products). A product is defined by the detergent it contains and the number of washes it contains. The products sold in market t, $\mathcal{J}_t$, are manufactured by $f = 1, \dots, F$ firms. The outside good is denoted by $j = 0$ and represents the decision not to purchase in market t.

Household i elects to purchase good j from a market t to maximise conditional indirect utility,

$$U_{ijt} = x_j^\top \beta_i + \alpha_i p_{jt} + \epsilon_{ijt} \tag{31}$$

where p_{jt} is the price of product j in market and x_j is a K-vector of observed product attributes. Since the model is estimated using consumer micro-data, the parameters α_i and β_i depend on an L-vector of household characteristics, y_i.

The observed product attributes include the number of washes, dosage, and brand-format dummies. Household i's valuation of these attributes is captured by taste parameters,

$$\beta_i \;\; = \;\; \beta + \beta_y y_i \tag{32}$$

where y_i is an L-vector of household characteristics. To capture heterogeneous valuations of these attributes β_i has two components. The first is common to all households and is measured by a K-vector of parameters, β. The second captures the effect of household characteristics and is measured by an L-vector of parameters β_y.

Prices of product j in market t, p_{jt}, enter linearly into the indirect utility function. The marginal utility of income for each household is

$$\alpha_i \;\; = \;\; \alpha + \alpha_y y_i \tag{33}$$

In addition to a common component, α, the effect of different household characteristics on the marginal utility of income is measure by an L-vector of parameters, α_y.

The effect of factors observed by the household, but not the econometrician, are captured by

$$\epsilon_{ijt} = \xi_{jt} + \varepsilon_{ijt}$$

It contains two terms: ξ_{jt} is an unobserved product-market component of utility common to all consumers and ε_{ijt} is a private household-specific utility shock. The household utility shocks are identically and independently distributed and follow the Generalised Extreme Value distribution. The unobserved product-market components are observed by all firms prior to setting prices. As a result, prices are likely to be correlated with

ξ_{jt} and are not independent from the unobserved component of demand, $\epsilon_{ijt} \not\perp p_{jt}$. This gives rise to endogeneity concerns.

There are three channels that lead to correlation between unobserved product-market characteristics and prices. First, any additional costs incurred in the production of a positively valued unobserved characteristic will likely be passed onto households in the form of higher prices.

Second, to the extent that the unobserved attribute shifts demand outwards, equilibrium markups will reflect any additional market power this confers on the firm. In both cases, prices are likely to be positively correlated to unobserved attributes. If the endogeneity is left untreated, these correlations positively bias the marginal utility of income.

Another potential source of correlation captured by ξ_{jt} is non-price advertising and promotional activities. If unobserved non-price advertising activity - such as funding prominent store positioning - increases during periods of promotional prices, ξ_{jt} is negatively correlated with prices. If, on the other hand, prices contemporaneously increase to fund non-price advertising, then ξ_{jt} is positively correlated with prices.

Since the both positive and negative correlation might result, the direction of the bias due to non-price advertising is ambiguous. However, given the importance of promotional activity and unobserved factors, such as placement within the store, it is quite plausible (even likely) that non-price advertising components of ξ_{jt} are negatively correlated with prices. In this case, marginal utility of income would be biased upward.

4.2.2 Estimation

The demand model is estimated using a random sample of 5 purchases in each week from the Kantar WorldPanel purchase diary data described in Section 4.1, resulting in 2,810 choice occasions.

The number of products in choice sets ranges from 87 to 187 with a median of 120 products.[24] The set of products sold in each week is partitioned into four nests based on the number of washes contained in each product: small (S), medium (M), large (L), and extra large (XL). The size boundaries of these groups correspond to the 25th, 50th, and 75th quantile of distribution of washes in each calendar year.

In this particular application, because 12 percent of product-week market shares are zero. As a result, it is not possible to use the workhorse Berry et al. (1995) approach to deal with endogeneity concerns. Therefore, the control function approach proposed by Petrin and Train (2010) is used to address endogeneity concerns.

The model is estimated in two steps. In the first step, the control function is estimated using prices from observed purchases.[25]

$$p_{jt} = W\left(x_j, z_{jt}; \gamma\right) + \upsilon_{jt} \tag{34}$$

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[24]See annex B.1 for a more detailed description of the construction of choice sets from the purchase diary data.

[25]When more than one price is observed per product in each week, its average weekly price is used.

The auxiliary price equation contains two components. The first, $W(x_j, z_{jt}; \gamma)$, is a function of observable product characteristics, x_j, and instruments excluded from the choice model, z_{jt}. These instruments include cost shifters and BLP instruments that shift mark-ups. Consequently, they are correlated with prices, but not unobserved demand factors. The other component is a product-market specific unobservable, υ_{jt}.

The estimated residuals are used to construct the control function, $CF(\hat{v}_{jt}; \rho)$. In the second step, the control function is added to the choice model and it is estimated by maximum likelihood.[26] Finally, because the control function is estimated, standard errors are calculated by bootstrapping the two step estimation procedure.[27]

4.2.3 Results

To estimate the nested logit model, the set of products sold in each week is partitioned into four nests based on the number of washes contained in each product: small (S), medium (M), large (L), and extra large (XL). The size boundaries of these groups correspond to the 25th, 50th, and 75th quantile of distribution of washes in each calendar year. The nesting parameter is assumed to be common to all nests.

The specification of conditional indirect utility include interactions between price and a proxy for household income.[28] Product characteristics include the size of the product purchased and the dosage - the amount of material (recommended) for use in a single wash. To control for household size, I also include the amount of washes purchased per equivalent adult in the household.[29] Detergent fixed effects are also included.

When controlling for endogeneity, the control function for product j in period t is linear in the residuals from the estimated auxiliary equation (equation (61)),

$$CF(\hat{v}_{jt}; \rho) = \rho \hat{v}_{jt} \tag{35}$$

The specification of the auxiliary equation contain the same product characteristics as the choice model, year fixed effects and instruments. The BLP instruments that capture the congestion of product space. The instruments are based on the number of similar products sold in the market.[30] There are two instruments: a count of the firm's similar products and a count of rival's similar products.[31]

The results of the nested logit estimation are displayed in Table 2. It contains the parameters and standard errors of the nested logit model with and without the control function.

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[26]See Train (2009), chapter 4 for details of maximum likelihood estimation of the nested logit model.

[27]See Annex B.3 for a more detailed description of the bootstrap algorithm.

[28]The proxy for household income is annual average weekly expenditure on all groceries.

[29]To calculate equivalent adults, I use the OECD-modified equivalence scale.

[30]A product is regarded as being similar if it belongs to the same size category and has the same format. Further, and in line with the construction of the choice sets, similar products must be sold in the same calendar month as their comparator.

[31]Estimates of the auxiliary equation used to construct the control function together with wider discussion of its application can be found in Annex B.2

Table 2: Choice models: parameter estimates

| | Nested Logit | | | |
Variables	θ	se	θ	se
Price Params:				
Price	-0.275	0.021	-0.581	0.136
Price x Income	0.179	0.039	0.184	0.029
Characteristics:				
Washes	0.118	0.163	4.484	1.949
Washes per Eq. Ad.	-0.337	0.095	-0.371	0.771
Dosage	-1.222	0.108	-1.058	0.284
Dosage x Liquids	0.279	0.067	0.575	0.381
Other Params:				
λ	0.526	0.035	0.551	0.049
ρ			0.297	0.126
Detergent Fixed Effects	Yes		Yes	
N	2,810		2,810	
Log-Likelihood	-12,203		-12,199	

In both nested logit models the price coefficient is negative and households with higher income have a lower marginal utility of income. When the control function is included, the price coefficient (scaled by the nesting parameter) approximately doubles from -0.523 to -1.054.

This is consistent with correlation between unobserved product demand factors being positive. It is also reflected by the control function parameter, ρ, being positive and statistically significantly different from zero.

Table 2 also shows that households positively value larger pack-sizes. However, for smaller households this is less pronounced. This is consistent with smaller households living in smaller accommodation and facing a higher cost of storage.

The dosage, or amount of material needed to do a single wash is negatively valued, especially for 'solid' detergents. This is also consistent with the fact that households value storage space. When the dosage is lower, households can store more washes without necessarily occupying more storage space. Indeed, as noted in Section 4.1, this is one of the driving factors behind the success of the new super-concentrated and gel detergent products. By itself, this suggest the presence of inter-temporal demand links through inventories - a source of mis-specification for this static demand model.

The nesting parameter, λ, is 0.551 and is significantly different from 1. This indicates that that there are some unobserved correlations in the utility between detergents of similar sizes and rejects the independence of irrelevant alternatives imposed by a conditional logit.

4.3 Solving for $\bar{\bar{\Psi}}$

Next, the output of the estimated static demand model is combined with market outcome data to find values of $\bar{\bar{\Psi}}$ that solve equations (28), (29), and (30) for the firm A and firm B in each year from 2002 until 2012.

In the context of an antitrust investigation brand level margins may be available over several years. However, in my case, I only have access to published annual accounts. Therefore, the published gross margins in firm A's and firm B's annual accounts from 2002 to 2012 are assumed to be good approximations to the economic margins earned on sales of their laundry detergent portfolio in the UK.[32]

Plugging in the margins, using the Kantar WorldPanel purchase data in each year and combining with output of the demand model, the analyst searches for all pairs of parameters $(\psi_n^{own}, \psi_n^{cross})$ where $n \in \{f, y\}$, $f \in \{A, B\}$ and $y = 2002, \ldots, 2012$.[33]

Figure 2 shows the value of these parameters for both firms in each year in the data when the demand model outputs are evaluated at mean parameter values reported in Table 2.

It shows the pairs of $(\psi_n^{own}, \psi_n^{cross})$ that satisfy equations (28), (29), and (30) in each year for firm A and firm B. For each firm, the absolute value of the own and cross price elasticity biases are negatively correlated with one another and are close to parallel in each year. The slope measured at the extremum points of the sets of $(\psi_n^{own}, \psi_n^{cross})$ for firm A range from 0.221 to 0.269 over 2002 to 2012. For firm B range of arc-tangents is 0.137 to 0.149 - approximately half that of firm A.

Figure 3 shows how the sets of $(\psi_n^{own}, \psi_n^{cross})$ for each firm-year combination translate into estimates of the bias in the aggregate own and cross price elasticities for firms A (the left panel) and firm B (the right panel).[34] In both panels, the blue and orange ribbons show the bias of own and cross price elasticities, respectively. The bias of the own price elasticity is stated with respect to its absolute value.

In line with the much of the dynamic demand estimation literature, Figure 3 shows that the static demand model produces overly elastic own price elasticities and cross price elasticities biased toward zero.

Specifically, for firm A's products its own price elasticity bias fluctuates over the period 2002 to 2012 and lies between 13 to 35 percent. Despite these fluctuations, the width of the set of bias estimates is relatively narrow and stable - its average width in each year is 6.3 percentage points with a standard deviation of 1.3.
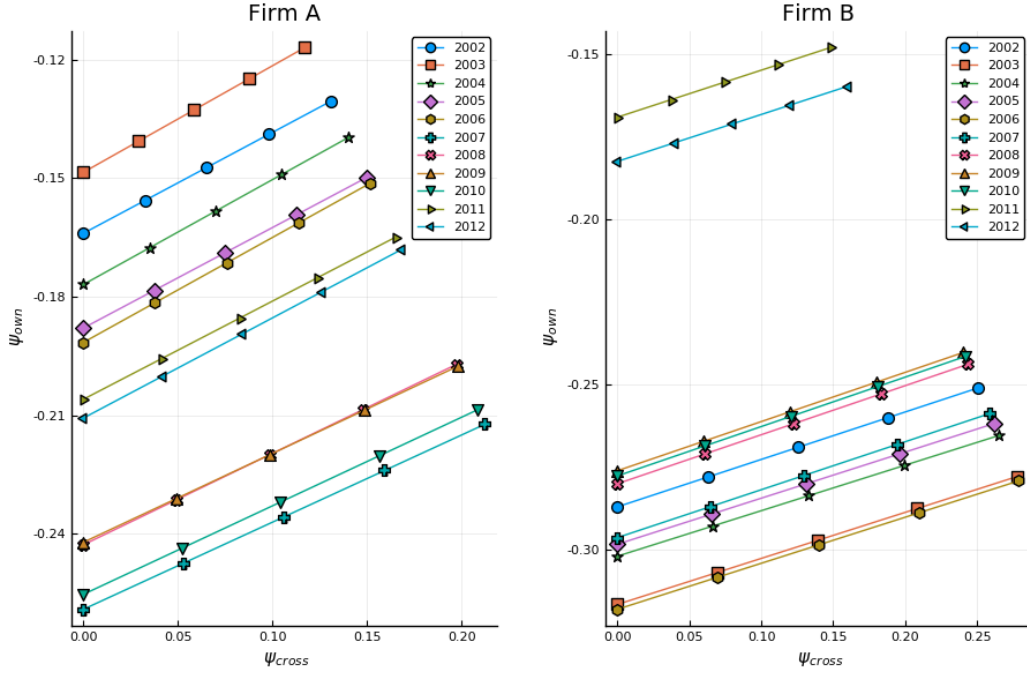
For firm B, the own price elasticity bias tends to be larger in most years. It is between 30 and 46 percent until 2010, then falls to around 20 percent. The width of this set is

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[32]The financial year for firm B starts midway through the year. As such, the annual report margins are adjusted to match calendar years in the data. Figures are omitted for confidentiality reasons.

[33]For the purpose of this section, I assume that the cartel in mainland Europe had no impact on the UK laundry detergent sector, and they priced detergent according to differentiated Bertrand competition between 2002 and 2005. I revisit this issue in Section 4.4.

[34]Annex C.2 contains the aggregate own and cross price elasticities by brand from 2002 to 2012 in the UK laundry detergent industry.

Figure 2: $(\psi_n^{own}, \psi_n^{cross})$ for Firm A and Firm B in each year from 2002 to 2012

similar to firm A's at 6.3 percentage points with a slightly higher standard deviation of 2.4.

For both firms the width of the set of cross price elasticity biases are considerably wider than the set of own price elasticity biases. For firm A the cross price bias range is 0 to 17.5 percent and for firm B is 0 to 22 percent. The reason for this is that the upper bound on ψ^{cross} imposed by restriction A3 is arguably too loose.

In a policy setting, the analyst might have access to other evidence that support adoption of a more stringent assumption. For example, they might amend restriction A3 to $\kappa\psi_n^{cross} \leq |\psi_n^{own}|$ where $\kappa > 1$. In the UK laundry detergent industry, a larger value of κ would reduce the upper bound on the set of the cross price elasticity bias estimates and narrow the set of own price elasticity biases. The result is narrower set of values for the bias in diversion ratios. As discussed below, this can be helpful for policy-making.

4.4 Policy experiments

This section contains two hypothetical policy experiments using the UK laundry detergent industry. It shows how set-valued estimates of the elements in Ψ can be provide evidence in support of policy decisions.

The first experiment assesses the competitive effects from a hypothetical divestment of a brand from firm B to firms A using dGUPPI. The second experiment explores whether a cartel in mainland Europe's laundry detergent industry involving the two largest firm

Figure 3: Bias in price elasticities
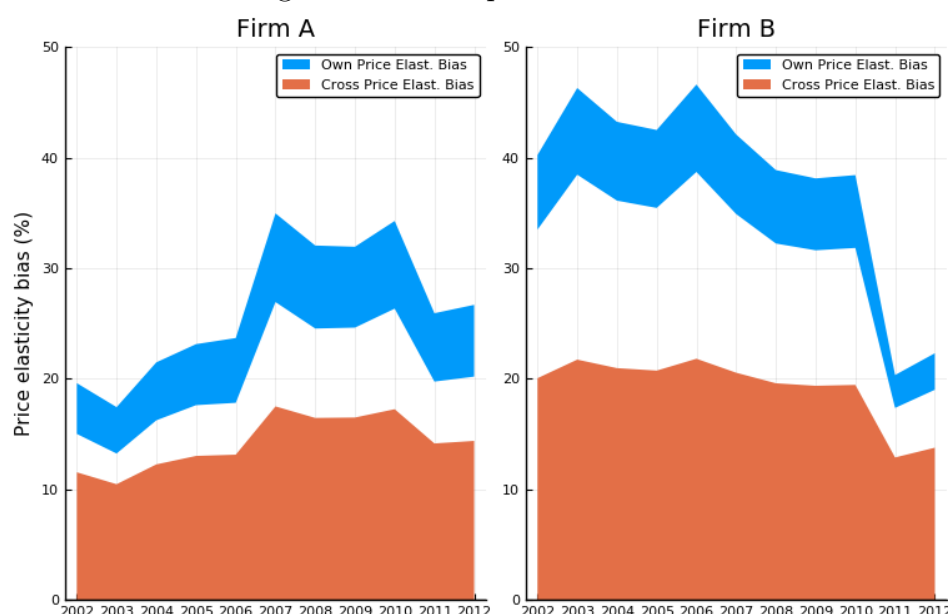
A and B between 2002 and 2005 had any effect in the UK. In both cases I will show that set-valued empirical tools can help avoid policy making errors.

dGUPPI To illustrate how the estimates of bias-adjusted diversion ratio can be used in a policy setting, suppose brand A and brand E are produced by different firms and the firm producing A is buying brand E from the rival firm at the end of 2012. For the purpose of this exercise assume brand A is produced by firm A and brand E by firm B.[35]

To assess whether the proposed acquisition is likely to lead to a significant lessening of competition, the analyst can calculate the bias-adjusted GUPPI measure introduced in section 2.4, dGUPPI. Allowing for some tolerance in potential efficiencies arising through joint production, it is assumed that the policy analyst views a dGUPPI below five percent as sufficiently low to allow the brand divestment to occur without giving rise to anti-competitive concerns.[36]

The dGUPPI is calculated in each year to gather evidence on the likelihood of prices rising by examining strength of the incentive to increase the prices in previous years. Two upward price pressure indices are computed; one for the price increase of brand A and another for the price increase of brand E.

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[35]For confidentiality reason, brands cannot be explicitly assigned to firms.

[36]Five percent (or higher) might be used because it is often used as the floor for a significant non-transitory increase in price for market definition purposes. However, for goods that comprise a large fraction of consumption, competition authorities have used more stringent GUPPI thresholds. For example in the investigation into the proposed Asda/Sainsbury's merger the UK Competition Markets Authority a 2.75 percent GUPPI threshold; a 1.5 percent baseline with a 1.25 percent efficiencies allowance to assess local market competition. See Competition Markets Authority (2019) "Anticipated merger between J Sainsbury PLC and Asda Group Ltd: Final Report".

Figure 4: Estimated set for diversion ratio bias for brand A and brand E, 2002 to 2012
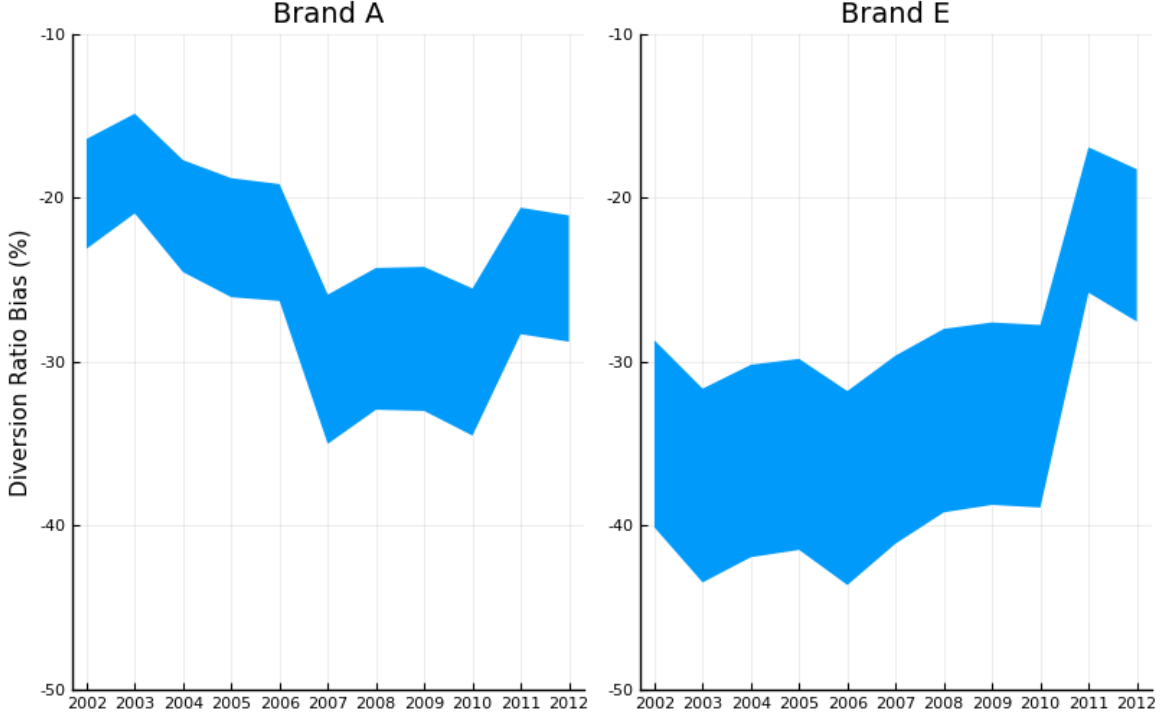
As discussed at in section 2.4, the dGUPPI is product of a static diversion ratio bias-adjustment term and the standard GUPPI measure. Under A5 and A6, dGUPPI for a price increase in brand A in each year is

$$DGUPPI_{(A,E)} = \frac{1 + \psi_A^{cross}}{1 + \psi_A^{own}} DR_{(A,E)} \frac{\mu_E \bar{p}_E}{\bar{p}_A} \tag{36}$$

where $DR_{(A,E)}$ is the static diversion ratio from brand A to brand E, μ_E is the percentage margin and $\bar{p}_b$ is the average price for brand $b \in \{A, E\}$.[37]

Since there is a set of pairs of $(\psi_n^{own}, \psi_n^{cross})$ that solve equations (28), (29) and (30, the dynamic diversion ratio and dGUPPI is set-valued. The corresponding sets containing the bias of the static diversion ratios for brands across all years is plotted in Figure 4.[38]

To compute the dGUPPI, the standard (static) GUPPI is computed using estimated margins, sales-weighted annual average prices for the brands from the market data, and brand-level diversion ratios from the demand model. The resulting static GUPPI for brand A and brand E is shown by the red lines in the left and right panels of Figure 5, respectively.

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[37]Diversion ratio and average prices are volume weighted following Domencich and McFadden (1975). See annex C.1 for further details. To calculate the DGUPPI for a price increase in brand E, the analyst need only exchange the indices in equation (36).

[38]The bias in the static diversion ratios for brand $j \in \{A, E\}$ in year $y \in \{2002, \ldots, 2012\}$ is calculated $\Gamma_{jy} - 1$.

Between 2002 and 2007 the GUPPI when brand A's price is changes is relatively stable and averages 6.7 percent over this period. However, starting from 6.8 percent in 2007 in declines to 4.3 percent in 2012 - where it first drops below the five percent threshold. This drop in the GUPPI is driven by a decline in the diversion ratio. By 2012, the diversion ratio had fallen 64 percent from 13.1 percent in 2007 to 8.4 percent by 2012. Detergent level analysis indicates that drop occurs because brand A became a leading branding for the new gel formats in 2008. Prior to that brand E's powder and tablets were perceived to be closer substitutes by brand A customers.

The GUPPI when brand E's price is changed is also relatively stable from 2002 to 2007, but at a lower average of 4.1 percent across the period. Again driven by falling diversion ratios, it falls 60 percent from 4.3 percent in 2007 to 2.6 percent in 2011. However, in 2012, the GUPPI jumps back to 3.7 percent - its 2008 level.[39] Unlike the GUPPI measure from the perspective of brand A customers, the GUPPI for brand E customers is below the five percent threshold in each year over the period.

Relying solely on a static GUPPI measures, the analyst concludes that, although the brands were closer competitors in the past, innovation in product formats and product compaction since 2007/8 led to the brands being sufficiently distant competitors by 2012 to permit the acquisition.

However, the once the diversion ratios are adjusted by the lower and upper bounds of the bias term, the analyst's conclusions are reversed. The resulting range of possible values of the DGUPPI is shown by the blue bands in Figure 5.

The left panel shows the range of possible values for dGUPPI when brand A's price changes. It shows that once inter-temporal substitution is accounted for the price increase predicted by dGUPPI lies between 5.5 and 6.1 percent - above the five percent threshold. By itself, this finding would be sufficient to reverse the conclusions of the static GUPPI analysis.

The right panel shows the range of possible values for dGUPPI when brand E's price changes. Unlike the static GUPPI, the analyst cannot now rule out that the dGUPPI lies strictly below the five percent threshold in all years, except 2011. Again, these findings support prohibition of the firm producing brand A acquiring brand E.

The EU detergent cartel: UK impact? In April 2011 the European Commission (EC) found that firm A, firm B and a firm C had entered into a cartel agreement that restricted competition in the market for heavy duty laundry detergent powder. The infringement was first brought to the attention of the EC when firm C 'blew the whistle' on the cartel in exchange for immunity from prosecution and/or reduced fines. Subsequent investigation led to the finding that the cartel was effective over the period 7th January 2002 to 8th March 2005. The EC highlighted four restrictive elements of cartel:

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[39]In 2012, brand E re-aligned its price per wash by 12 percent on the previous year to 20.7p - close to brand A's price per wash. This reversed year-on-year declines in price per wash from 2002 to 2011.

Figure 5: GUPPI: Brand A and Brand E, 2002 to 2012
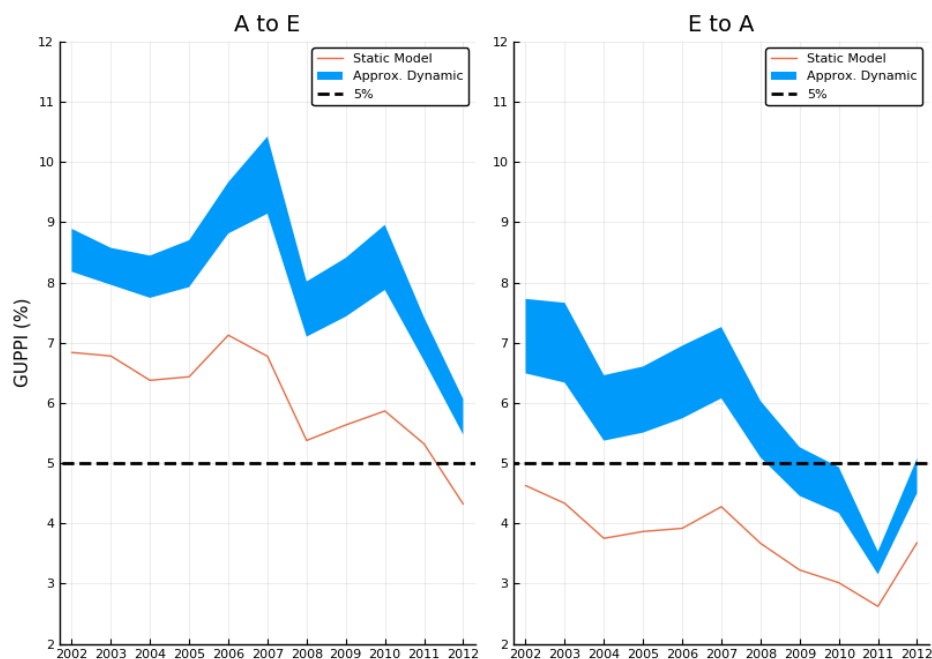
1. indirect price restrictions resulting from the parties agreeing not to pass on any cost savings that resulted from compaction of products

2. explicit reduction of promotional activity

3. direct price increases

4. an exchange of commercially sensitive information

According to the EC, the laundry detergent cartel (LD cartel) had anti-competitive effects in Belgium, France, Germany, Greece, Italy, Portugal, Spain and The Netherlands.[40] One notable absentee from the list of countries affected is the UK - a country in which the 'whistleblower', firm C, had virtually no market presence. However, the laundry detergent market in the UK is dominated by the other two firms in the cartel; firm A and firm B.

This section combines the structural demand model estimated in Section 4 together with various supply-side models to estimate market power under different types of firm conduct. By comparing the estimated margins associated with different models of firm behaviour to the observed margins for firm A and firm B from 2002 to 2005, the possibility that the collusive activities in mainland Europe were also evident in the UK is investigated.

Two policy experiments are conducted. In the first demand derivatives from the static demand model are used a inputs in to an analysis of the market power of firm A and firm B during the cartel period. In the second, this analysis is repeated with the set of demand derivatives adjusted for bias by the parameters $(\psi^{own}, \psi^{cross})$.

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[40]European Commission Decision, 'COMP/39579 - Consumer Detergents'.

To test whether the firms' conduct was competitive or collusive between 2002 and 2005, the set of estimates for $(\psi^{own}, \psi^{cross})$ must be calibrated using another year where we know there was no cartel. The set of parameters for $(\psi^{own}, \psi^{cross})$ from 2006 are used because they are closest to the period in question and are less likely to be influenced by other factors (i.e. product innovations in 2007 and 2008).

In both experiments, the demand model outputs are evaluated at mean parameter values. As a result the impact of uncertainty arising from estimation the demand model's parameters is not reflected in the predicted margins in the subsequent analysis. In a live policy setting conducting the additional computations to confidence intervals that reflect uncertainty over the inputs on counterfactual simulations would make the policy conclusions more robust.[41]

To estimate the market power, assumptions on the nature of supply-side competition and the shape of the cost function are added to each demand model in each experiment. Throughout I assume that marginal cost is (locally) constant.

To examine the impact of the intensity of competition on market power, the ownership matrix is altered to reflect different ownership structures. I consider four different supply side models to analyse market power: (1) firms engage in Bertrand-Nash price competition and each product is manufactured by a single firm, (2) assumes multi-product Bertrand-Nash competition, (3) assume firm A and firm B collude over powder products only,[42] and (4) assume that firm A and firm B set all prices jointly as a branded product monopolist. Scenarios (1) and (4) are intended as lower and upper bounds on the estimated market power.[43]

The results of the first experiment are shown in Figure 6. The left panel in the figure plots the observed margins in 2002 to 2005 for firm A alongside the margins implied by the demand model and each of the four firm conduct models. The right panel mirrors this analysis for firm B.

The figure shows that the observed margins of around 50% are well in excess of the margins implied by multi-product Bertrand-Nash and the cartel, but are similar to those implied by monopoly pricing.

Based on these results from the mis-specified demand model, the policy maker might

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[41]To implement this in the policy experiment described in this section is to bootstrap the demand model outputs over the range of parameter values in the confidence interval of interest. Using this approach, demand derivatives and purchase probabilities differ over bootstrap iterations and the first order conditions produce a corresponding set of predicted margins. However, in policy experiment 2, there is an additional step needed; within each bootstrap iteration the policy maker need to solve for $(\psi^{own}, \psi^{cross})$ in the chosen comparator market. Through this additional channel, uncertainty over demand model parameters can indirectly influence the size of the sets of predicted margins in counterfactual analysis.

[42]To reflect the cartel scenario, I assume firm B (firm A) take into account joint profits of a firm A (firm B) powder products, but do not set its price.

[43]I do not have information on the identity of producers of private label products. For the purposes of this paper I assume that firm A and firm B have no share in private label's profits. This is unlikely to be the case. As such the level of market power in multi-product scenarios (2) and (3) are likely to understate market power compared to the 'true' ownership matrix.

Figure 6: Policy experiment 1 ($\psi^{own} = \psi^{cross} = 0$): gross margins and implied market power during the cartel period: 2002 - 2005
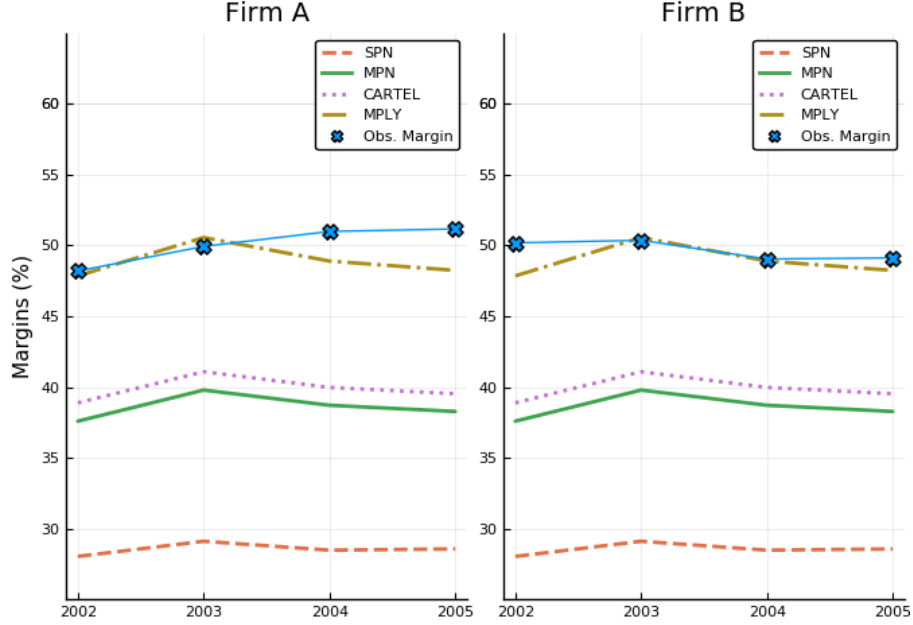
Note: SPN refers to single Product differentiated Nash-Bertrand competition, MPN refers multi-product differentiated Nash-Bertrand competition, CARTEL refers to pricing where firms A and B collude over prices for powdered detergents and MPLY refers to A and B pricing as a single entity (i.e. monopoly pricing).

conclude that the collusive conduct enabled these two dominant firms to achieve a monopoly outcome. As such, they use this as evidence to support the conclusion that the cartel in Europe affected market outcomes in the UK laundry detergent market.

However, the fact that observed margins lie so far above the cartel outcome might serve as a warning that the demand model is mis-specified. Especially since the known biases that arise from omitting demand dynamics in storable goods industries would understate market power in this experiment. As such, we proceed to check this hypothesis by adjusted for known

Figure 7 shows the results of the same policy experiment but using the set of estimated parameters for ψ^{own} and ψ^{cross} from 2006 for both firms. The left panel of the figure plots the observed margin for firm A from 2002 to 2005 with the band of the predicted margins from the different models of conduct. Because the estimates of ψ^{own} and ψ^{cross} are set valued, the menu approach predicts a range of margins in each year. The right hand panel produces the same information for firm B. In both panels, the monopoly outcomes are omitted from the plot because they predict margins over 70%.

In contrast to the first policy experiment, the observed margins also lie well below the lower bound of margins consistent with cartel conduct. In fact, the set of margins predicted by competitive conduct lie on or slightly below those implied by competitive conduct. Therefore, and in contrast to the the first policy experiment using biased demand model inputs, the findings of this policy experiment do not find compelling

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Figure 7: Policy experiment 2: gross margins and implied market power during the cartel period: 2002 - 2005
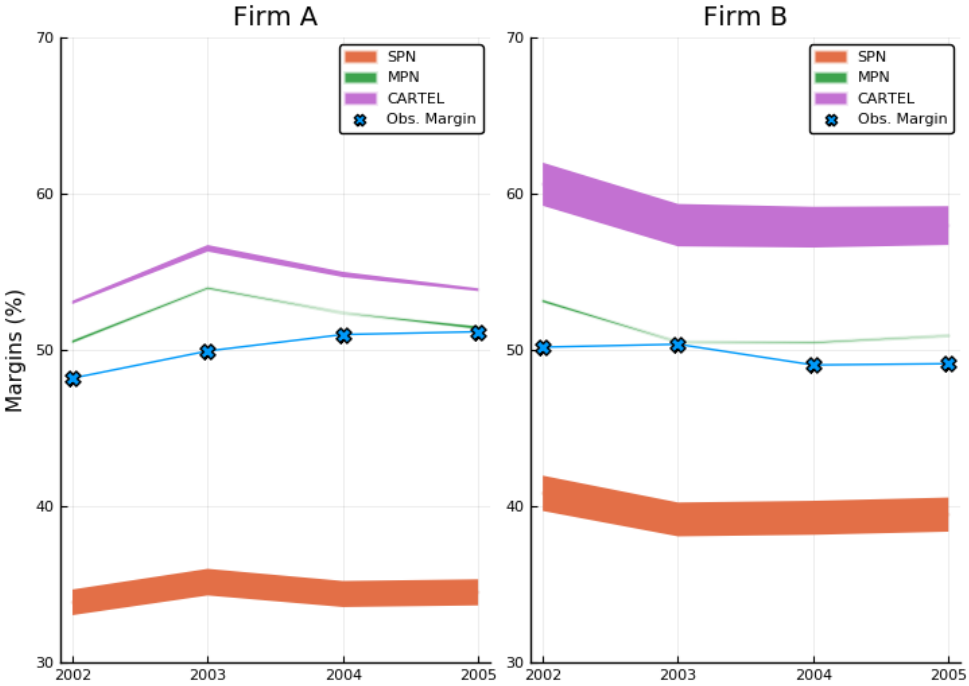
Note: SPN refers to single Product differentiated Nash-Bertrand competition, MPN refers multi-product differentiated Nash-Bertrand competition, CARTEL refers to pricing where firms A and B collude over prices for powdered detergents. The monopoly case is excluded from the graph.

evidence that the collusive conduct from mainland Europe occurred in the UK.

5 Conclusion

This paper develops a new approach to estimate sets of long-run price elasticities and dynamic diversion ratios for storable goods industries. To identify the missing demand dynamics from static demand models, the approach leverages information on market power over multiple purchase cycles contained in estimated margins.

By combining the output of a static demand model with observed prices and margins, I show how to recover parameters that capture effect of inter-temporal substitution omitted from static models on firm's pricing incentives. These parameters are used to construct set-valued estimates of the long-run price elasticity matrix and dynamic diversion ratios - key inputs into antitrust and other policy areas.

In particular, I show dynamic diversion ratios can be combined with margins in a new generalised upward price pressure index adjusted for demand dynamics - dGUPPI. This new empirical policy tool extends the existing GUPPI test and can be used to evaluate mergers in industries with dynamic demand.

To illustrate this new framework, I apply it to the UK laundry detergent industry from 2002 to 2012. For the two firms that dominate the industry, I find that the absolute own price elasticities are overstated by 13 and 35 percent and 20 to 46 percent, respectively. The upper bound on the bias of cross-price elasticities 17 and 22 percent.

To illustrate how the set-valued estimates of policy inputs can be used to aid policy-making I conduct two policy experiments. The first shows used shows how sets of dynamic diversion ratio produce dGUPPI ranges that can help a competition authority evaluate the likelihood of unilateral effects arising from a hypothetical brand acquisition from a rival. The second uses set-valued estimates of the long-run price elasticity matrix to assess whether a cartel in mainland Europe's laundry detergent had any effect in the UK. In both experiments, the use of set-valued policy inputs is central to avoiding policy errors.

The application demonstrates that this new approach is well suited to policy work. It only requires standard empirical methods and readily available data on margins and market-level industry outcomes. As a result, it can be easily implemented within the policy making time-horizon. More generally, it highlights the potential of empirical analysis producing set-valued outcomes to aid policy-making process.

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Annex A: Proofs and derivations

A.1 Proposition 1

Recall

$$
\Psi_{(j,k),t} := \frac{\mathbb{E}_t \sum_{h=t+1}^{\infty} \delta^{h-t} \nabla_{(j,k),h} m_{kh}}{\nabla_{(j,k),t} m_{kt}} = \frac{\Delta DR^{\pi}_{(j,k),t}}{DR^{\pi}_{(j,k),t}} \tag{37}
$$

for all $j, k \in \mathcal{J}_f$ is the relative dynamic profit diversion ratio.

Under condition A2 all products are substitutes, the promoted good's future sales are a substitute for current sales. A cut in the price today, will lead to a reduction in demand in the future, $\nabla_{(j,j),h} \geq 0 \; \forall j \in \mathcal{J}_f \; h > t$. Since promotional mark-ups and expected future profits are non-negative, then $\Delta DR^{\pi}_{(j,j),t} \geq 0$ and $DR^{\pi}_{(j,j),t} \leq 0$. As such, the numerator and denominator in $\Psi_{(j,j),t}$ have opposing signs, thus $\Psi_{(j,j),t} \leq 0 \; \forall \; j \in \mathcal{J}_f$.

A similar argument establishes that the off-diagonal elements of Ψ_t are non-negative. Briefly, since (i) promotional mark-ups and expected future profits are non-negative (ii) both current and future sales of rival product k are substitutes for the promoted good j, then $DR \geq 0$ and $\Delta DR^{\pi}_{(j,k),t} \geq 0 \; \forall \; j \neq k, : j, k \in \mathcal{J}_f$. Both the numerator and denominator in $\Psi_{(j,k),t} \; j \neq k$ have the same sign, hence the off-diagonal have a zero lower bound $\Psi_{(j,k),t} \geq 0 \; \forall \; j \neq k, : j, k \in \mathcal{J}_f$.

The lower bound on the diagonal elements of Ψ_t is derived from the first order conditions

for product j

$$q_{jt} + \mathbb{E}_t \sum_{h=t}^{H} \sum_{k \in \mathcal{J}_f} \delta^{h-t} \nabla_{(j,k),h} m_{kh} = 0 \tag{38}$$

$$\implies -\sum_{k \in \mathcal{J}_f} \nabla_{(j,k),t} m_{kt} - \mathbb{E}_t \sum_{h=t+1}^{H} \sum_{k \in \mathcal{J}_f} \delta^{h-t} \nabla_{(j,k),h} m_{kh} = q_{jt} \tag{39}$$

$$\implies DR^{\pi}_{(j,j),t} + \Delta DR^{\pi}_{(j,j),t} + \sum_{k \neq j} DR^{\pi}_{(j,k),t} + \sum_{k \neq j} \Delta DR^{\pi}_{(j,k),t} = \frac{q_{jt}}{\nabla_{(j,j),t} m_{jt}} \leq 0 \tag{40}$$

$$\implies DR^{\pi}_{(j,j),t} + \Delta DR^{\pi}_{(j,j),t} \leq 0 \tag{41}$$

$$\implies \Delta DR^{\pi}_{(j,j),t} \leq -DR^{\pi}_{(j,j),t} \tag{42}$$

$$\implies \frac{\Delta DR^{\pi}_{(j,j),t}}{DR^{\pi}_{(j,j),t}} \geq -1 \tag{43}$$

$$\implies \Psi_{(j,j),t} \geq -1 \tag{44}$$

where in the third line above uses the fact $\sum_{k \neq j} DR^{\pi}_{(j,k),t} \geq 0$ and $\sum_{k \neq j} \Delta DR^{\pi}_{(j,k),t} \geq 0$ and in the second last line that $DR^{\pi}_{(j,j),t} = -1 \leq 0$. Thus, from equation (44) and using $\Psi_{(j,j),t} \leq 0$, provides bounds for the diagonal elements of Ψ_t:

$$-1 \leq \Psi_{(j,j),t} \leq 0 \,\forall\, j \in \mathcal{J}_f \tag{45}$$

Finally, from condition A3, $\left|\Psi_{(j,j),t}\right| > \Psi_{(j,k),t}$ provides an upper bound on the off-diagonal elements of Ψ_t.

$$0 \leq \Psi_{(j,k),t} \leq \left|\Psi_{(j,j),t}\right| \,\forall\, j \neq k, j,k \in \mathcal{J}_f \tag{46}$$

$\blacksquare$

A.2 Lemma 1

$$DDR := -\frac{\mathbb{E}_t \sum_{h=t}^{H} \delta^{h-t} \nabla_{(j,k),h} m_{kh}}{\mathbb{E}_t \sum_{h=t}^{H} \delta^{h-t} \nabla_{(j,j),h} m_{jh}} \tag{47}$$

$$= -\frac{\nabla_{(j,k),t} m_{kt} + \mathbb{E}_t \sum_{h=t+1}^{H} \delta^{h-t} \nabla_{(j,k),h} m_{kh}}{\nabla_{(j,t),t} m_{jt} + \mathbb{E}_t \sum_{h=t+1}^{H} \delta^{h-t} \nabla_{(j,j),h} m_{jh}} \tag{48}$$

$$= -\frac{\left(\nabla_{(j,k),t} m_{kt} + \mathbb{E}_t \sum_{h=t+1}^{H} \delta^{h-t} \nabla_{(j,k),h} m_{kh} \right) \frac{\nabla_{(j,k),t} m_{kt}}{\nabla_{(j,k),t} m_{kt}}}{\left(\nabla_{(j,t),t} m_{jt} + \mathbb{E}_t \sum_{h=t+1}^{H} \delta^{h-t} \nabla_{(j,j),h} m_{jh} \right) \frac{\nabla_{(j,j),t} m_{jt}}{\nabla_{(j,j),t} m_{jt}}} \tag{49}$$

$$= -\frac{\left(\nabla_{(j,k),t} m_{kt} + \mathbb{E}_t \sum_{h=t+1}^{H} \delta^{h-t} \nabla_{(j,k),h} m_{kh} \right) \Big/ \nabla_{(j,k),t} m_{kt}}{\left(\nabla_{(j,t),t} m_{jt} + \mathbb{E}_t \sum_{h=t+1}^{H} \delta^{h-t} \nabla_{(j,j),h} m_{jh} \right) \Big/ \nabla_{(j,j),t} m_{jt}} \times \frac{\nabla_{(j,k),t} m_{kt}}{\nabla_{(j,j),t} m_{jt}} \tag{50}$$

$$= -\frac{1 + \Psi_{(j,k),t}}{1 + \Psi_{(j,j),t}} DR^{\pi}_{(j,k),t} \tag{51}$$

∎

A.3 Lemma 2

Let

$$q_{jt}^n = \begin{cases} q_{jt} & \text{if } j \in \mathcal{J}_n \\ 0 & \text{otherwise} \end{cases} \tag{52}$$

Then from the FOC we can derive an equation for the percentage margin earned on sales of products in group n over the T periods

$$\mathbf{q}_t + \left(\nabla_t \circ \left(1 + \bar{\mathbf{\Psi}}\right)\right)(\mathbf{p}_t - \mathbf{c}_t) = 0 \tag{53}$$

$$\implies \mathbf{p}_t - \mathbf{c}_t = -\left(\nabla_t \circ \left(1 + \bar{\mathbf{\Psi}}\right)\right)^{-1}\mathbf{q}_t \tag{54}$$

$$\implies \mathbf{q}_t^{n,\top}(\mathbf{p}_t - \mathbf{c}_t) = -\mathbf{q}_t^{n,\top}\left(\nabla_t \circ \left(1 + \bar{\mathbf{\Psi}}\right)\right)^{-1}\mathbf{q}_t \tag{55}$$

$$\implies T^{-1}\sum_{t=1}^{T}\mathbf{q}_t^{n,\top}(\mathbf{p}_t - \mathbf{c}_t) = -T^{-1}\sum_{t=1}^{T}\mathbf{q}_t^{n,\top}\left(\nabla_t \circ \left(1 + \bar{\mathbf{\Psi}}\right)\right)^{-1}\mathbf{q}_t \tag{56}$$

$$\implies \frac{T^{-1}\sum_{t=1}^{T}\mathbf{q}_t^{n,\top}(\mathbf{p_t} - \mathbf{c_t})}{T^{-1}\sum_{t=1}^{T}\mathbf{q}_t^{n,\top}\mathbf{p}_t} = -\frac{T^{-1}\sum_{t=1}^{T}\mathbf{q}_t^{n,\top}\left(\nabla_t \circ \left(1 + \bar{\mathbf{\Psi}}\right)\right)^{-1}\mathbf{q}_t}{T^{-1}\sum_{t=1}^{T}\mathbf{q}_t^{n,\top}\mathbf{p}_t} \tag{57}$$

$$\implies \mu_n = -\frac{\sum_{t=1}^{T}\mathbf{q}_t^{n,\top}\left(\nabla_t \circ \left(1 + \bar{\mathbf{\Psi}}\right)\right)^{-1}\mathbf{q}_t}{\sum_{t=1}^{T}\mathbf{q}_t^{n,\top}\mathbf{p}_t} \tag{58}$$

where μ_n is the percentage margin earned on sales of product in group n over the T periods.

The bounds in equations (29) and (30) follow from the application of assumptions A5 and A6 to Proposition 1. ∎

Annex B: Data and Estimation

B.1 Choice sets

The choice sets are constructed using all purchases at stores at the major UK supermarket between 1st January 2002 until 31st October 2012. The purchase dairy data on records when a product purchase. As a result, it is not easy to conclusively say which products were available at the point of purchase and can therefore enter the choice set. To address this, I assume that a product is eligible for the choice set if its purchase is observed in the same calendar month as the purchased product. This approach leads to the inclusion of products in the choice set without observed prices. To remedy this product prices need to be imputed.[44]

An interactive fixed effect model with r factors is used to impute prices:

$$\mathbf{P} = \mathbf{\Lambda}\mathbf{F}^{\top} + \varepsilon \tag{59}$$

where $\mathbf{P}$ is a $J \times T$ matrix of prices, $\mathbf{\Lambda}$ is a $J \times r$ matrix of factor loadings, $\mathbf{F}$ is a $T \times r$ factor matrix and ε a $J \times T$ matrix of errors. I return to the choice of the number of factors below.

---

[44]Approximately 7 percent of observations are imputed

A truncated single value decomposition (SVD) with tailored to sparse data is used fit the interactive fixed effect model r factors[45]

$$\mathbf{\Omega} \odot \mathbf{P} = \mathbf{\Omega} \odot \left(\mathbf{U}_r \mathbf{S}_r \mathbf{V}_r^\top \right)$$

where $\mathbf{\Omega}$ is a matrix whose (j,k)-th element is 1 if the price of product j in period t is observed and zero otherwise. The loadings and factors are given by $\mathbf{\Lambda} = \mathbf{U}_r \mathbf{S}_r$ and $\mathbf{F} = \mathbf{V}_r$.

Following Bai and Ng (2002), the number of factors are chosen to minimise penalised objective function

$$r^\star = \arg \min_{r \in \{1,2,\ldots,R_{MAX}\}} \ln(W_r) + r \frac{J+T}{JT} \ln\left(\frac{JT}{J+T}\right) \tag{60}$$

where the sum of squared residuals with r factors is

$$W_r = (JT)^{-1} \left(\mathbf{\Omega} \odot \left(\mathbf{P} - \mathbf{U}_r \mathbf{S}_r \mathbf{V}_r^\top \right) \right)^\top \left(\mathbf{\Omega} \odot \left(\mathbf{P} - \mathbf{U}_r \mathbf{S}_r \mathbf{V}_r^\top \right) \right)$$

Models with $r \in \{1, 2, \ldots, 10\}$ are fitted and $r^\star = 2$ maximises equation (60). Missing prices are imputed using

$$(\mathbf{1} - \mathbf{\Omega}) \odot \mathbf{P} = (\mathbf{1} - \mathbf{\Omega}) \odot \mathbf{\Lambda}^\star \mathbf{F}^{\star\top}$$

where $\mathbf{\Lambda}^\star = \mathbf{U}_{r^\star} \mathbf{S}_{r^\star}$ and $\mathbf{F}^\star = \mathbf{V}_{r^\star}$.

Finally, if a product is purchased in a promoted bundle of two units, the opportunity to purchase two units of product is included in the choice set. If there is no observed purchase of two units, it is assumed it is not available as a promotional bundle and the total price is twice that of a single unit of the product.

B.2 Control Function

The control function is formulated using the residuals of an auxiliary pricing equation

$$p_{jt} = W(x_j, z_{jt}; \gamma) + v_{jt} \tag{61}$$

The possible sources of endogeneity is reflected by the correlation of product-specific unobservables in the choice model, ξ_{jt}, and the unobservable component of the auxiliary price equation, v_{jt}. When observed prices are regressed on product characteristics and instruments, the estimated residuals from this regression contains the correlation between p_{jt} and ξ_{jt}. As a result, the estimated residuals, $\hat{v}_{jt} := p_{jt} - \mathbb{E}[p_{jt} | x_j, z_{jt}]$, are used to construct a control function, $CF(\hat{v}_{jt}; \rho)$ with parameters ρ.

---

[45]The Julia package TSVD.jl is used to fit the model

When this control function is added to the choice model it conditions out that part of prices correlated with ξ_{jt}. As a result, it resolves the endogeneity that arises from the unobserved product-market terms, ξ_{jt}. The unobserved component of demand is now independent of price, $\epsilon_{ijt} \perp p_{jt} \mid CF(\hat{v}_{jt}; \rho)$, and $\epsilon_{ijt} =^d \varepsilon \overset{iid}{\sim} GEV$.[46]

The results of estimating the auxiliary equation are show in Table 3.

Table 3: Control Function: parameter estimates

Variables	θ	se
Intercept	1.589	0.058
Characteristics:		
Washes	14.503	0.036
Dosage	0.678	0.102
Dosage x Liquids	0.953	0.135
Instruments:		
Num. own similar prods	-0.038	0.004
Num. rival's similar prods	-0.040	0.003
Detergent Fixed Effects	Yes	
Year Fixed Effects	Yes	
N	59,035	
R^2	0.797	

Annex C. Aggregate elasticities and diversion ratios

This annex describes the formulas used to aggregate individual elasticities and diversion ratio into their aggregate counterparts. It also shows the aggregate own and cross price elasticity estimates from the nested logit model with and without bias adjustments using elements of $\bar{\bar{\Psi}}$.

C.1 Aggregation of elasticities

Following Domencich and McFadden (1975), aggregate elasticities are calculated by weighting household's product level elasticities by their share of the total demand of the set of products being aggregated over.

To provide further details, let $z \in \{own, within, cross\}$ and E^z_{ijt} be the corresponding price elasticity of demand of product j for consumer i in period t. Further, suppose we want to measure the price elasticity of aggregate demand for brand b over $t = 1, \ldots, T$ periods. As such, let $\mathcal{J}_b$ the set of all products of brand b.

---

[46]Where $=^d$ is defined as "follows the same distribution as".

Then aggregate price elasticities can be obtained from their individual counterparts by calculating

$$E_b^z = \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{j \in \mathcal{J}_b} \omega_{ijt} E_{ijt}^z$$

where the share of total demand for brand b over T periods accounted for by consumer i's demand for product j in period t is

$$\omega_{ijt} = \frac{\theta_{it} \Pr(d_{it} = j)}{\sum_{i'=1}^{N} \sum_{t'=1}^{T} \sum_{k \in \mathcal{J}_b} \theta_{i't'} \Pr(d_{i't'} = k)}$$

and θ_{it} is sampling weight reflecting the number of type i consumers in period t. Similarly, the aggregate diversion ratio between brands b_1 and b_2 is computed as

$$DR_{(b_1, b_2)} = \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{j \in \mathcal{J}_{b_1}} \sum_{k \in \mathcal{J}_{b_2}} \omega_{ijt} DR_{(j,k),it}$$

C.2 Aggregate price elasticities

When computing elasticities the probability of purchase is adjusted for the share of the outside good. In this application the share of the outside good is informed by observed household purchase frequencies in the demand Kantar WorldPanel data. In 2002 household's purchase detergent once every three weeks on average. Following the steady increase in the number of washes per pack, this average inter-purchase duration extends to four weeks between 2003 and 2008 and five weeks thereafter. In line with this approach, the outside share is 67 percent in 2002, 75 percent between 2003 and 2008, and 80 percent between 2009 and 2012.

Figures (8), (9) and (10) show the aggregate own and cross price elasticities by brand over 2002 to 2012. The dashed orange lines are the unadjusted price elasticities and the blue ribbon are bias-adjusted price elasticities.

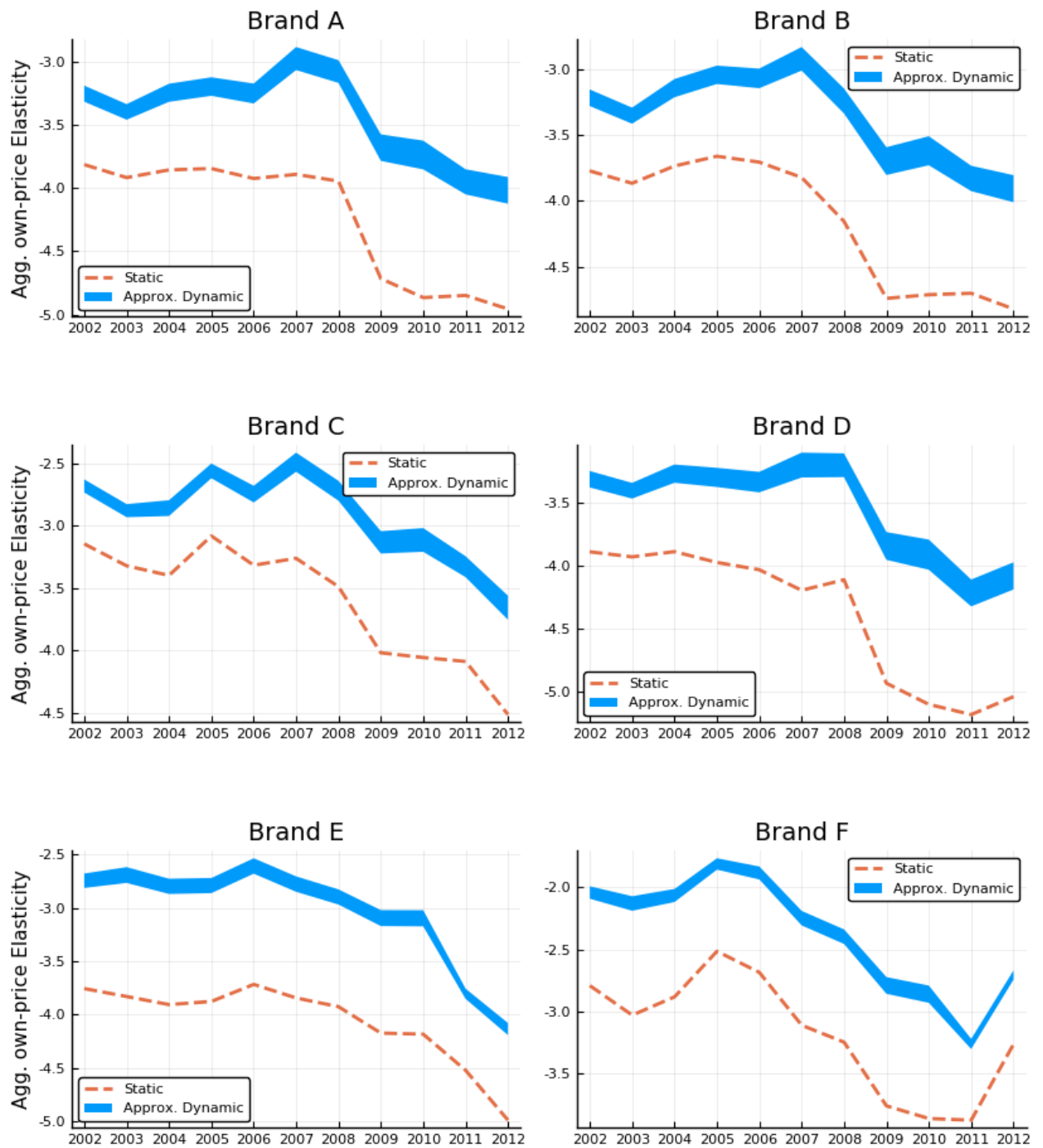Figure 8: Own price elasticities by brand: unadjusted and adjusted from 2002 to 2012

Figure 9: Within-nest cross price elasticities by brand: unadjusted and adjusted from 2002 to 2012
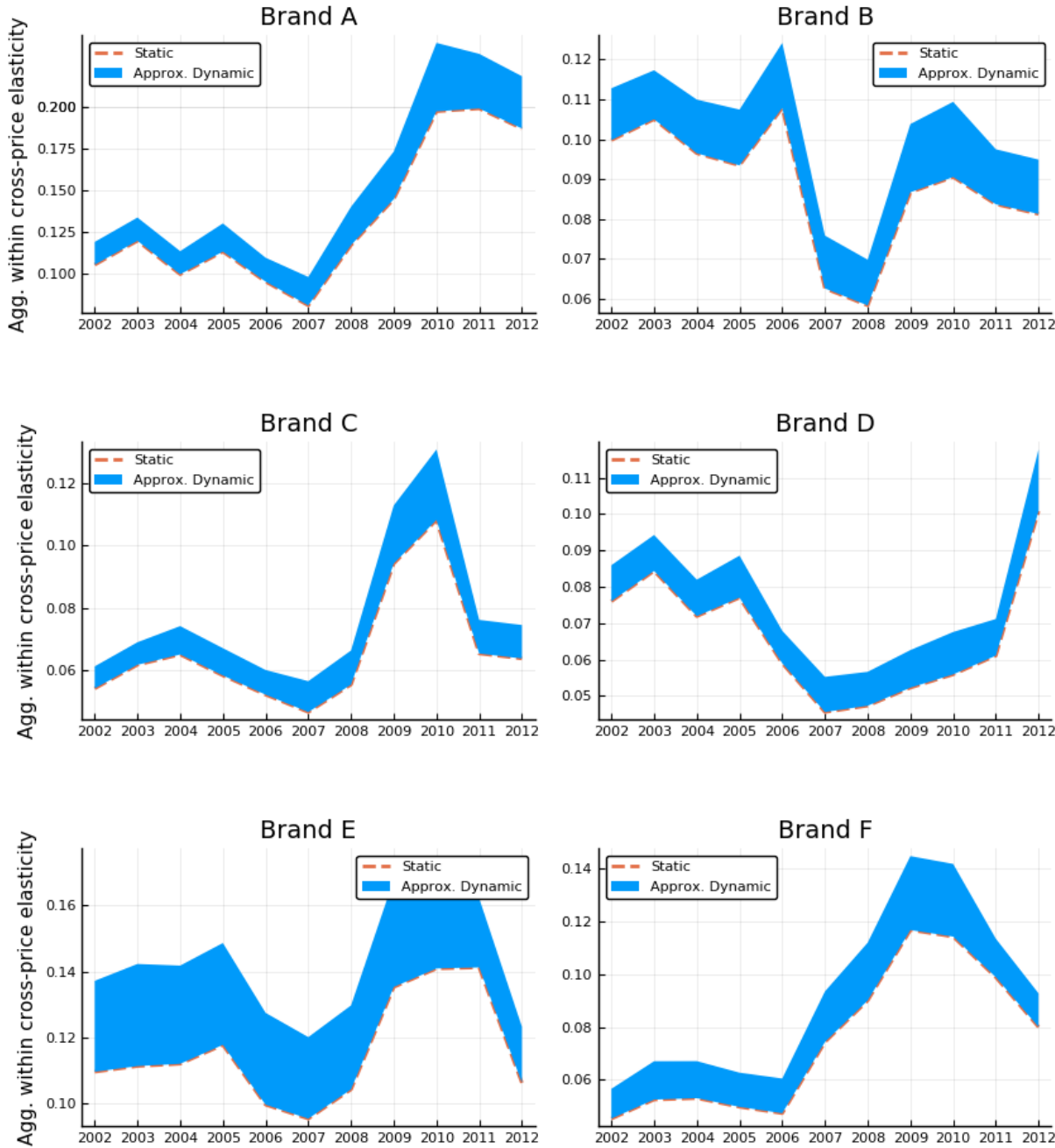
Figure 10: Across-nest price elasticities by brand: unadjusted and adjusted from 2002
to 2012