Approximating demand dynamics in storable good industries

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Abstract

This paper develops a new approach that combines firm margins, market-level industry data and a static demand model to construct sets containing unbiased estimates of long-run price elasticities for storable good industries. It obviates the need to solve the consumer's value function and can be completed within a policy-making timeframe. This methodology allows for the effect of contemporaneous and inter-temporal substitution on pricing incentives to be measured by dynamic diversion ratios. Together with the margins, these are key inputs into a new price pressure test for mergers in industries with dynamic demand. This framework is applied to the UK laundry detergent industry from 2002 to 2012. I conduct two policy experiments that show how estimated sets of bias-corrected price elasticities and diversion ratios can be used to avoid misguided policy-decisions. In both cases demonstrating the efficacy of set-valued policy tools.

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JEL Classification: D12, C54, L11, L40, L67

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1 Introduction

Price elasticities derived from estimated demand models are key inputs into policy counterfactuals. Inter alia, they are used to analyze the impact of consumption taxes, import duties, and antitrust issues. Of specific interest are long-run price elasticities that measure the contemporaneous and dynamic demand responses to price movements arising from changes in the economic environment. However, when the demand model is mis-specified then the estimated price elasticities are biased, counterfactuals are wrong, and misguided policy decisions are more likely.

For storable goods, incorporating demand dynamics is crucial because their omission leads to biased estimates of long-run price elasticities.[1] However, estimation of dynamic demand models for this large class of goods is computationally challenging, resource intensive and require consumer level purchase diary data. As a result, using them to produce estimates of long-run elasticities is rarely feasible within a policy-making time horizon.

This paper develops a new approach to estimate long-run price elasticities and dynamic diversion ratios for storable goods. In contrast to full dynamic demand estimation, it is computationally straightforward to implement and can easily be completed within the timeframe of the policy making process. The reason for this is two-fold. First, it only uses empirical techniques that are often already used in policy work (i.e. standard demand estimation and root-finding). Second, it only requires firms' margins and market-level data spanning the period over which margins are calculated.[2]

The ease of implementation and wider applicability of this new approach come at the cost of a loss of point identification; estimates of long-run price elasticities and dynamic diversion ratios are set-valued. Consequently, counterfactuals and policy tools that use these as inputs produce a set of outcomes, rather than a single value. However, as illustrated in this paper's application, set-valued results of empirical policy analysis need not reduce their efficacy in a policy setting.

This new approach uses a dynamic model of demand and supply for storable good industries.[3] Consumer demand dynamics arise from two channels. One is the ability to store purchases for future consumption, which allows for inter-temporal substitution. The other is through price expectations. Informed by prices observed on previous shopping trips, consumer price expectations can influence the timing and identity of the product they purchase.

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[1] By not allowing a portion of short-run volume increases from temporary price cuts to draw down on future sales - including those of the promoted product - static demand models provide biased estimates of demand responses to non-transitory price changes. Specifically, own price elasticities are overstated and cross-price elasticities tend to be understated.

[2] This data is routinely collected directly from firms in the course of antitrust investigations. Where detailed internal margin data is not available but the manufacturer(s) periodically publish accounts, the analyst can use reported gross margin or, alternatively, employ the techniques described in De Loecker et al. (2020) to estimate them.

[3] Chen et al. (2008) show that omission of demand and supply dynamics for durable goods leads to biased estimation of demand elasticities.

In the supply model, firms are forward-looking and set current prices to maximise the net present value of expected profit flows. The firm's perception of the profitability of pricing strategies depends on their model of aggregate dynamic demand and their beliefs over pricing strategies. However, in practice, a firm's ability to compute perceived profits is arguably limited by the complexity of computing expected future profits. Especially in storable good industries with choice sets containing over 50 products. As such, the cognitive and computational challenges of processing such a large amount of information to compute optimal prices are arguably prohibitive.

Firms are assumed to solve an approximate version of their dynamic pricing problem. First, because firms do not observe consumers' inventory or consumption, they approximate aggregate dynamic demand. Second, to mitigate the computational complexity of evaluating the perceived distribution of future profits they simplify supply-side price dynamics by fixing price forecasts prior to setting prices.

The equilibrium concept used is Experience Based Equilibrium (EBE) developed by Fershtman and Pakes (2012). In line with the approximation strategies used by firms, EBE retains the Markovian structure of the more frequently used Markov Perfect Equilibrium (MPE) but relaxes some of its rationality constraints. In particular, it relaxes the MPE requirement that consumers' price forecasts, firms' beliefs of rivals' pricing strategies, and optimal pricing strategies must coincide. As a result, a EBE places a lower burden on the physical and cognitive resources needed by consumers and firms to engage in equilibrium play.

This dynamic model of the storable good industry provides an expression for the mark-up in terms of purchase quantities, prices, and static and dynamic demand derivatives. A combination of observed market data and outputs from a static demand model can be used to populate all but the dynamic components of the price setting equations. The effect of long-run demand responses on pricing incentives are measured by a matrix of reduced form parameters.

These reduced form parameters capture the relative importance of contemporaneous and inter-temporal cannibalisation of sales when firms set current prices. Further, I show they can also be used to correct biased price elasticities and diversion ratios that are estimated using the outputs of a mis-specified static demand model. The resulting dynamic diversion ratios are shown to be a key input into a new generalised upward price pressure index adjusted for demand dynamics (dGUPPI). This new empirical policy tool extends the existing GUPPI test and can be used to evaluate mergers in industries with dynamic demand.

To estimate the reduced form parameters, data on market outcomes is used with estimates of firm's profit margins. Because profit margins are costly to collect, they are measured across a group of products and over a period of time that exceeds the purchase cycle for storable goods. As such, they contain information on the effect that demand dynamics have on margins earned on sales of that group of products.

To be able to leverage the information contained in profit margins to estimate the reduced-form parameters, the expression for the mark-up derived in the dynamic model is summed over all the weeks and products it covers. Dividing through by

corresponding revenues gives an expression for the percentage margin in terms of a sequence of prices, purchase quantities, static demand derivatives and the reduced form parameters. Using the estimated margin, data on market outcomes over the period, and the output of a static demand model, this equation can be used to estimate the reduced form parameters.

Two further sets of conditions identify these parameters. The first adopts the arguably weak restriction that the absolute magnitude of the own-price elasticities bias exceeds those of the cross-price elasticities. The second imposes additional restrictions that reduce the number of parameters to be estimated. Under these two sets of conditions there are two bounded unknown parameters: one capturing own-price elasticity bias, the other the cross-price elasticity bias. With one equation and two reduced form parameters, their estimates belong to bounded sets.

To illustrate how this method can be employed in practice it is applied to the UK laundry detergent industry using Kantar Worldpanel purchase diary collected between 2002 and 2012. For the two firms, A and B, that dominate the UK laundry detergent industry, I use published accounts to estimate their margins for each calendar year. I find that own price elasticity bias is between 20 and 63 percent for firm A and 29 to 53 percent for firm B. However, the range in any one year is considerably narrower. The corresponding cross-price elasticity biases are 0 to 24 percent and 0 to 22 percent, respectively. Compared to dynamic diversion ratios that capture both static and inter-temporal substitution, I find that static diversion ratios are biased downward by 17 to 51 percent and 22 to 39 percent for firms A and B, respectively.

I conduct two experiments to show how the bias-adjusted elasticities and dynamic diversion ratios can be used in a policy setting. In the first experiment, I use a hypothetical brand acquisition to show how a set-valued dGUPPI can inform proposed changes in industry structure. In the other policy experiment, I assess whether a cartel in mainland Europe's laundry detergent industry involving the two firms that account for the vast majority of sales in the UK market was also in effect in the UK. To test this, I use the 'menu approach' (Bresnahan (1987)) to compare the market power estimates implied by alternative models of competitive interactions during and after the cartel.[4]

In both cases I find that conducting the policy analysis without accounting for market dynamics can lead to policy errors. In the first experiment the brand divestment would be incorrectly permitted. In the second, policy simulations using biased price elasticities suggest observed margins in the UK are most likely to be produced by anti-competitive conduct.[5] However, in the policy simulations using the set of bias-adjusted price elasticities there is little evidence to support the view that the

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[4]Nevo (2001), Slade (2004), Rojas (2008) and Miller and Weinberg (2017) have assessed market power or merger effects in differentiated product industries using the menu approach.

[5]Margins calculated using first order conditions using overly elastic own price elasticities from the static demand estimation understate market power. For the UK laundry detergent in the cartel period, the margins calculated assuming the cartel effected monopoly pricing lie below the observed gross margins. This empirical finding could wrongly support a policy makers' case that the cartel on mainland Europe had anti-competitive effects in the UK.

anti-competitive effects of the European laundry detergent cartel were experienced in the UK.

Related literature This paper contributes to the literature on dynamic demand estimation for storable goods. Following seminal papers by Erdem et al. (2003) and Hendel and Nevo (2006a), there have been several papers that have sought to apply and extend the frameworks they develop.[6]

However, this paper is most closely related to Hendel and Nevo (2013) and Perrone (2017). In contrast to other papers in this field, they develop models that can quantify the effect of omitting consumer dynamics on estimates of long-run price elasticities without incurring the substantial computational burden of solving for the consumer's value function.

Like Perrone (2017) the approach developed in the paper is straightforward to implement and is flexible in terms of consumer heterogeneity and price expectations. It also builds on Perrone (2017) by allowing for product differentiation and quantity discounts within a storable good industry. As a result, it can use market level data to produce product-level own and cross price elasticities for use in empirical policy analysis.

Hendel and Nevo (2013) develop a simple dynamic demand model for storable goods. Like the framework developed in this paper, their model can be used to estimate product level long-run own and cross price elasticities using market level data. However, it requires more restrictive assumptions on consumer storage technology, taste heterogeneity and price expectations to do so. Moreover, and importantly from a policy perspective, scaling up the model Hendel and Nevo (2013) beyond a handful of products is challenging. In contrast, the output of the model in this paper naturally scales to high-dimensional choice sets that are often observed in storable good industries.

The cost of the flexibility of my approach - the loss of point identification - is not shared by either Hendel and Nevo (2013) or Perrone (2017). However, this need not reduce the efficacy of policy analysis based on its outputs.

It also contributes to a literature analysing dynamic demand and supply models for durable goods - of which storable goods are a special case. The most closely related is Chen et al. (2008). They use data from a simulated dynamic demand and supply model of the used the car market to explore the biases in price elasticities and corresponding market power estimates when a misspecified static demand model is used instead of a dynamic demand model.[7]

Outline The remainder of the paper is structured as follows. Section 2 describes the supply side model of price setting when demand is inherently dynamic. Section 3

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[6]Pires (2016); Wang (2015); Osborne (2018); Crawford (2018) estimate a variety of extensions of these dynamic demand models using panel micro-data.

[7]More distantly related are Goettler and Gordon (2011) and Carranza (2010). Both estimate dynamic demand and supply models in durable good industries.

shows how to combine industry data on market outcomes, accounting margins, a static demand model, and a price forecasting model to recover parameters that adjust price elasticities to reflect consumer responses to permanent price changes when demand dynamics are present. Section 4 focuses on the application and policy experiments. Section 5 concludes.

2 Price setting with demand dynamics

This section presents a model of a differentiated storable good industry in which both consumers and firms are forward-looking. When setting current prices, firms take demand and supply dynamics into account. In each period firms set price to maximise their perception of the present value of expected profit flows.

When there are no inter-temporal links in demand or costs, the firm's optimisation problem is separable and is solved independently in each time period. However, in many industries, if not most, consumer demand is inherently dynamic. For example, if the firm discounts its product(s) as part of a temporary promotion, consumers may accelerate purchases and stock the product at home for future consumption.

When demand is dynamic, as is the case for storable goods, firms consider the impact current prices have on demand today and in the future. The effect of a price change today on current and expected future profits are affected by static and inter-temporal substitution. The ratio of inter-temporal to static diverted profits is shown to be closely linked to bias in price elasticities when estimated using a mis-specified static demand model.

When setting prices, firms form beliefs both consumer and firm dynamics. Therefore, firms must retain and process a lot of information to compute consumer demand and assess the profitability of a very large number of pricing strategies. The size of the task of determining optimal pricing strategies is exacerbated by the large number of products sold to consumers in storable good industries (i.e. often the choice set contains 50-150 products).

Faced with these large computational and cognitive challenges, firms are assumed to be able to track the evolution of this large scale system, they approximate aggregate dynamic demand and solve an approximate version of their dynamic pricing problem. These is consistent with best practice in the approximate dynamic programming literature developed across a wide range of applications. Powell (2011) states

> "It has been our repeated experience in many industrial applications that it is far more important to capture a high degree of realism in the transition function than it is to produce truly optimal decisions".

In line with this approach, the equilibrium concept used in the resulting high-dimensional dynamic pricing game is Experience Based Equilibrium (Fershtman and Pakes (2012)). This allows agents to have imperfect information about other industry participants and relaxes stringent rationality requirements. Agents' play need only maximise payoffs given

the information available to them and be consistent with outcomes in states they have previously experienced.

This section is structured as follows. First, consumer demand and firms' approximation to it is described. Second, the dynamic supply model is presented initially assuming firm's produce a single product. It is then extended to multi-product firms and the equilibrium concept consistent with the firms' approximations is discussed. The model is then recast in terms of reduced form parameters formed using the ratio of inter-temporal and static diversion ratios. These parameters are shown to capture omitted demand dynamics and are directly linked to the bias in price elasticities. Finally, the concept of a dynamic diversion ratios is introduced and their relationship to the reduced form parameters is derived. These are then used to modify price pressure tests used in antitrust policy making to account for omitted demand dynamics.

2.1 Consumer demand

Setup In each period consumers choose whether or not to purchase one product from a choice set of J differentiated products.[8] The products, or packs, contain storable goods and are sold in different sizes. The contents of the products can be stored in inventory ready for consumption at a later date.[9] Each product's price in period t is recorded in a J-vector, $p_t = [p_{1t}, \ldots, p_{Jt}]^\top$ where p_{jt} is the price of product j in period t. Consumer's take these prices as given.

The consumer's problem is a discrete-continuous dynamic optimisation problem. At the start of each time period, consumer i privately observes their existing inventories $I_{it} = [I_{i1t} \ldots, I_{iJt}]^\top$ where I_{ikt} is the number of consumption units of product k in stock at the beginning of period t. They also observe aggregate state variables: current prices, p_t, aggregate demand shocks, ξ_t, and any additional information used to forecast future prices, f_t.

Consumers then decide whether or not to make a new purchase in the current period. If they make no purchase then they consume from their existing inventory, $0 \leq C_{it} \leq I_{it}$. If, however, consumer i decides to purchase product j in period t its contents are added existing inventories

$$\bar{I}_{it} = I_{it} + Q_j$$

where Q_j is a J-vector whose j-th element contains the number of units of consumption contained within the product j, all other entries in Q_j are zero. Consumption is then drawn from the post-purchase inventory, $0 \leq C_{it} \leq \bar{I}_{it}$.

Finally, at end of each period consumer i pays a costs of carrying over the inventory costs into the next period are incurred on the remaining inventories. This cost is increasing

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[8] For expositional simplicity I abstract away from the possibility of their being variations in the number of good available for sale in each time period.

[9] The window for consuming a storable good and the related storage costs depend on their characteristics. For example, some storable goods, such as yoghurts and other dairy products, can be stored for three to four weeks in refrigerated conditions. Others, such as laundry detergent, are non-perishable but may occupy a relatively large portion of storage space.

in the number of consumption units carried over.

Demand dynamics The possibility of storing current purchases in inventory for future consumption is one source of demand dynamics. Provided the consumer has access to storage technology and can afford to use it, the ability to consume out of inventory means that purchase period and consumption period need not coincide. This creates inter-temporal demand linkages.

Storable goods are often sold at a constant price for sustained periods of time interspersed with frequent, stochastic, temporary sale periods.[10] From repeated shopping trips, consumers formulate a model of price the process (i.e. based on past prices). As a result, demand dynamics also arise from consumers' expectations over future price movements.

Prior to purchase, consumers update the information set they use project future prices, f_t, and make forecasts. These price forecasts are taken into account when making purchase and consumption decisions. As such, consumers' perceptions over the price process is likely to affect the timing of purchases. For example, expecting higher prices in the future, some consumers might bring forward purchases to take advantage of a temporarily low sale price. In this case, expected purchase cost savings are weighed against increased storage costs.

With convex storage costs, the increase in storage costs will be larger for consumers that are already holding high levels of inventories relative to their consumption needs. Consumers in this position are therefore less likely to respond to temporary sale prices - especially when the sale is on large products.

The ability to use inventories for consumption and anticipate promotional prices means that consumer demand is inherently dynamic. Therefore, in addition to factors that affect myopic consumer demand (i.e. current prices and aggregate demand shocks), consumer's take into account their inventory holdings and price forecasts when making purchase decisions and consumption choices.

Consumer heterogeneity As is the case for static demand models, a consumer's characteristics and taste preferences, η_{it}, affects demand for storable goods. In the context of storable goods, the elements of η_{it} might include household composition, net income, dwelling size, urban/rural location and taste preferences. Consumer preferences might also be influenced by experiences of recent purchases and consumption. For example, consumer's might incur switching costs, have time-varying brand loyalty, and/or exhibit habit formation.[11]

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[10]Erdem et al. (2003); Hendel and Nevo (2006a,b); Nevo and Hendel (2012); Osborne (2013); Wang (2015) report prices in the industries they study also exhibit these features.

[11]Myśliwski et al. (2020) and MacKay and Remer (2019) analyse dynamic price games for firms selling storable goods to myopic consumers with state-dependent demand. Because these papers estimate static state-dependent demand models, they adopt additional assumptions that specify how previous purchases affect consumer utility. Specifically, additional restrictions on state-dependence are added

Purchases Together with inventory, current prices and other aggregate state variables, consumer i's characteristics and taste preferences affect the timing of purchases, the quantity bought, and the amount consumed from post-purchase inventories. This is reflected in the probability that consumer i purchases product j in period t

$$s_{ijt} = s_j \left(I_{it}, \eta_{it}, p_t, x_t \right)$$

where $x_t := [f_t, \xi_t]$ is the vector containing all payoff relevant aggregate state variables observed by consumers prior making purchases in period t.[12]

Aggregate demand If firms know the joint distribution of pre-purchase inventories and consumer characteristics, $F(I_{it}, \eta_i)$, the size of the market, M_t, and the individual demand function, $s_j(\cdot)$, then firms can calculate aggregate demand for product j in period t by integrating out over the joint distribution of inventories and household characteristics

$$q_j \left(p_t, x_t \right) = M_t \int s_j \left(I_{it}, \eta_i, p_t, x_t \right) dF(I_{it}, \eta_i)$$

However, consumption and inventories are only observed by consumers. As a result, firms do not directly observe the joint distribution of inventories and consumer characteristics necessary to calculate aggregate demand. Instead, firms can use their industry knowledge together with histories of past prices and aggregate state variables to simulate sequence of purchase and consumption decisions.[13]

The result is a simulated approximation to the joint distribution of inventories and consumer characteristics that can be used by firms' to approximate to aggregate demand

$$q_j \left(p_t, p_t^{\mathcal{H}}, x_t, x_t^{\mathcal{H}} \right) := \frac{M_t}{R} \sum_{r=1}^{R} s_j \left(I_t^{(r)} \left(I_0^{(r)}, \eta^{(r)}, p_t^{\mathcal{H}}, x_t^{\mathcal{H}} \right), \eta^{(r)}, p_t, x_t \right)$$

where $x_t^{\mathcal{H}} := [f_t^{\mathcal{H}}, \xi_t^{\mathcal{H}}]$ and $I_0^{(r)}$ is the r-th draw from the distribution of initial inventories.[14]

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so that past aggregate market shares, together with current prices, are sufficient statistics for the evolution of Markov pricing games they analyse.

[12] Each consumer is assumed to have the same information set and use the same price forecasting model. Expanding the demand model to allow for differential information sets and, therefore, heterogeneous forecasting models is beyond the scope of this paper. Amongst other things, the links between information acquisition, inventories, store choice, and shopping frequency would have to be added to the model.

[13] Through their industry experience firms are likely to know individual demand and consumption functions, the size of the market, the distribution of initial inventories, the distribution of consumer heterogeneity, and the sequence of past prices and aggregate state variables, $p_t^{\mathcal{H}} := [p_{t-1}, \ldots, p_1]$, $f_t^{\mathcal{H}} := [f_{t-1}, \ldots, f_1]$ and $\xi_t^{\mathcal{H}} := [\xi_{t-1}, \ldots, \xi_1]$ respectively. Though, in practice, firms may approximate aggregate demand using truncated data from τ previous periods. That is, $I_{i,t-\tau} \approx I_{i0}$ and use $p_t^{\tau} := [p_{t-1}, \ldots, p_{t-\tau}]$, $x_t^{\tau} := [x_{t-1}, \ldots, x_{t-\tau}]$ in place of $p_t^{\mathcal{H}}$ and $x_t^{\mathcal{H}}$.

[14] Without placing ad hoc restrictions on the consumer demand function, and in contrast to static

This description of aggregate demand for storable goods is consistent with existing dynamic demand models.[15] However, as discussed below, the focus of the paper is on the identification and estimation of long run elasticities and a dynamic diversion ratios for use in a policy setting.[16] Both of these objects are functionals of aggregate dynamic demand and are obtained without estimation of the dynamic demand system. The additional restrictions on the demand system needed to identify them is discussed in Section 3.

2.2 Firms

Firms play the dynamic pricing game over an infinite horizon and time is discrete; $t = 0, 1, 2, \ldots, \infty$. To simplify exposition of the firm's objective function and structure of the game the model is described for a single product oligopoly. Then the model is extended to multi-product firms and the nature of the equilibrium of the dynamic pricing game is discussed.

2.2.1 Single product oligopoly

Suppose that each of the J differentiated products is produced by a single firm. In period t each firm sets the price of product j to maximise the net present value (NPV) of their perception of expected profit flows,[17]

$$V_{jt} = \mathbb{E}_t \sum_{h=t}^{\infty} \max_{p_{jh}} \delta^{h-t} \pi_j \left(p_h, c_{jh}, \omega_h \right) \tag{1}$$

where δ is a common discount factor and $\omega_t := \left[p_t^{\mathcal{H}}, x_t, x_t^{\mathcal{H}} \right]$. Firms take expectations over future demand (and/or cost) shocks and form beliefs over rivals' pricing strategies and consumers' forecasting technology.

The per-period profits earned on sales of product j in period t are

$$\pi_j \left(p_t, c_{jt}, \omega_t \right) = \left(p_{jt} - c_{jt} \right) q_j \left(p_t, p_t^{\mathcal{H}}, x_t, x_t^{\mathcal{H}} \right) \tag{2}$$

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state dependent demand for storable goods (see Myśliwski et al. (2020) and MacKay and Remer (2019)), past aggregate market shares are not sufficient statistics for aggregate demand. In addition to current values of prices and other aggregate state variables, both $p_t^{\mathcal{H}}$ and $x_t^{\mathcal{H}}$ are need to included in the state vector to compute aggregate demand. Indeed, once $p_t^{\mathcal{H}}$ and $x_t^{\mathcal{H}}$ are included, aggregate market shares provide no additional information.

[15]Most existing models have focused on the case where household-level purchase diary data is available. In this case, inventories are integrated out of a dynamic demand model using observed household's purchase history and observed prices (i.e. Erdem et al. (2003); Hendel and Nevo (2006a); Sun (2005); Wang (2015); Pires (2016); Osborne (2018); Crawford (2018)). More closely related to the objectives of this paper, Hendel and Nevo (2013) focussed on estimation of dynamic demand model for storable goods with more widely available market-level data.

[16]The concept, definition and link to demand dynamics of dynamic diversion ratios is deferred to subsection 2.3 below.

[17]Firms are assumed not to be able to commit to future prices.

where c_{jt} is the marginal cost of production for product j in period t.[18] In addition to their own price and cost of production, each firm's flow of profit depends on the aggregate demand for their product.

The optimal price for the firm manufacturing product j solves the following first-order condition

$$q_{jt} + \frac{\partial q_{jt}}{\partial p_{jt}} m_{jt} + \mathbb{E}_t \sum_{h=t+1}^{\infty} \delta^{h-t} \frac{dq_{jh}}{dp_{jt}} m_{jh} + \mathbb{E}_t \sum_{h=t+1}^{\infty} \delta^{h-t} \frac{dp_{jh}}{dp_{jt}} q_{jh} = 0 \qquad (3)$$

where $m_{jh} = p_{jh} - c_{jh}$ is the mark-up earned on sales of product j in period $h \geq t$.

The first and the second terms correspond to the terms that enter the first order conditions used by firms in a single product oligopoly to set prices when demand is static. The additional terms only contain total derivatives capturing the effect that price changes today have on future prices and quantities.

The third term is the expected change to the discounted net present value of profits earned on future sales of product j due to a change in the current price of product j. It captures the change in future profits that arises from inter-temporal substitution due to dynamic demand responses. For example, suppose a firm deeply cuts price today to promote product j. Then this term measures the profit lost due to reductions in future sales of product j because consumers accelerate their purchases to take advantage of the sale price.

Finally, the fourth term captures dynamic supply side responses. Namely, the change in expected revenues from future sales of product j that result from anticipated adjustments to future prices in response to changes in the current price of product j. To evaluate this term, firms need to compute the effect that a change in the price of their product today has on the perceived distribution of prices for the foreseeable future. These price movements are the impulse response functions consistent with firm's beliefs over rivals' future pricing strategies following a change in the price of product j.

Implicit in the fourth term is that assumption that firms' engage in 'nowcasting' - forecasts are updated using prices set in the current period. The contemporaneous evaluation of the price impulse response functions is complicated by three factors. First, the forecast horizon is infinite. Second, dynamic price strategies might be nonlinear. Third, the forecasting problem is high-dimensional when the choice set is large (i.e. for storable goods $J \approx 100$). When combined they compound the computational complexity of forecasting the future distribution of prices in response to changes in current prices.

In practice firms face non-negligible time and resource constraints. In line with this perspective, firms are assumed to solve an approximation to the first order conditions

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[18] At the cost of additional notation and structure on the supply model, the marginal cost can be specified as function observed cost shifters, z_{jt}, and privately observed i.i.d. marginal cost shocks, ν_{jt}. In this case, the marginal cost function is $c_{jt} = g(z_{jt}, v_{jt})$ and its conditional distribution given z_{jt} is $F_{g|z}(\cdot)$.

(eq. (3)) that avoids costly 'nowcasting' altogether.[19] Namely, they are assumed to make price forecasts in period t before setting prices.

A1: Firms fix price forecasts prior to setting prices: $\frac{dp_{kh}}{dp_{jt}} = 0$ for $h > t$ and $j, k \in \mathcal{J}$

Under assumption A1, the fourth term drops out of equation (3) and the first order condition becomes

$$q_{jt} + \frac{\partial q_{jt}}{\partial p_{jt}} m_{jt} + \mathbb{E}_t \sum_{h=t+1}^{\infty} \delta^{h-t} \frac{dq_{jh}}{dp_{jt}} m_{jh} = 0 \tag{4}$$

Beneficially, from the policy analysts' perspective assumption A1 eliminates the need to estimate an empirical analogue for the fourth term in equation (3). While arguably strong assumption, A1 does have the benefit of allowing the analyst to attempt to correct for main source of bias in price elasticities - the omitted demand dynamics - without incurring costly additional computation. As such it is used as the baseline specification intended for use in a policy setting.[20]

Next the model is extended to allow for multi-product firms.

2.2.2 Multi-product oligopoly

The previous section derived the first order conditions assuming that each of the J products were produced by a separate firm. However, most industries with dynamic demand $N < J$ firms manufacture multiple products in the industry. Next, the system of equations used to set prices is extended to multi-product firms.

Let f index the F firms in the industry and let $\mathcal{J}_f$ denote the set of products sold by firm f. The per-period profit function in oligopoly

$$\pi_f\left(p_t, c_{jt}, \omega_t\right) = \sum_{j \in \mathcal{J}_f} \left(p_{jt} - c_{jt}\right) q_j\left(p_t, p_t^{\mathcal{H}}, x_t, x_t^{\mathcal{H}}\right) \tag{5}$$

where the corresponding approximation to the discounted present value of profits for firm i is

$$V_{ft} = \mathbb{E}_t \sum_{t=h}^{\infty} \max_{p_{jh} \forall j \in \mathcal{J}_f} \delta^{h-t} \pi_f\left(p_t, c_{jt}, \omega_t\right) \tag{6}$$

To extend the analysis to the multi-product case, define Δ_h as a $J \times J$ matrix of demand derivatives for period h with respect to price changes in period t where $h \geq t$ that encodes the ownership structure of the products sold in the industry. If the firm produces both

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[19]Such an approach might be considered optimal for a boundedly rational firm.

[20]At the cost of a considerably more computationally intensive procedure, it is possible to weaken the restrictions on a firm's forecasting behaviour described by A1. See Annex D for a 'nowcasting' version of the model.

products j and k, the (j,k)-element of Δ_h is the inter-temporal demand derivative relevant to the pricing of firm i's products

$$\Delta^q_{(j,k),h} = \begin{cases} \frac{dq_{kh}}{dp_{jt}} & \text{if } j,k \in \mathcal{J}_f \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The first order condition of the approximate dynamic profit optimisation problem for the multi-product firms used to set p_{jt} for all $j \in \mathcal{J}_f$ in the baseline model is

$$q_{jt} + \sum_{k \in \mathcal{J}_f} \Delta^q_{(j,k),t} m_{kt} + \mathbb{E}_t \sum_{h=t+1}^{\infty} \sum_{k \in \mathcal{J}_f} \delta^{h-t} \Delta^q_{(j,k),h} m_{kh} = 0 \tag{8}$$

This system of equations serves as the basis for the empirical analysis in the remainder of this paper.

2.2.3 Equilibrium

The equilibrium concept for the dynamic pricing game is Experience Based Equilibrium (EBE) - a modification of Markov Perfect Equilibrium (MPE) developed by Fershtman and Pakes (2012).[21] A EBE retains Markovian structure of an MPE, but relaxes the restriction that agents are rational.

For agents' behaviour to meet MPE's rationality conditions in a storable good industry with forward-looking consumers and firms, there are three conditions that must hold for all possible realisations of the state vector. First, firms' beliefs over rivals' strategies coincide with their optimal price strategies. Second, they must also hold correct beliefs over the forecasting technology used by forward-looking consumers. Finally, a MPE also requires that consumers' forecasting technology is consistent with firms' equilibrium price strategies.

However, the demands that the MPE conditions place on the mental and physical resources of firms and consumers are arguably prohibitive. For example, one requirement implicit in these conditions is that all agents know the identity of the manufacturing firm for each product. While firms know the identity of manufacturers for each product, it is not obvious that consumers do. If not, the MPE condition that requires consistency between firms' equilibrium price strategies and consumers' price forecasts arguably requires more sophistication and cognitive capacity than is available to them (or at least that they would choose to allocate).

Even without these additional complications that arise from demand dynamics, firms also may not have the capacity to compute the strategies that meet MPE conditions. With a handful of products - and therefore a small state space - processing and

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[21]MPE is used by Chen et al. (2008) and Goettler and Gordon (2011) in there studies of dynamic demand and supply systems for durable goods. In contrast, in his study of the camera industry Carranza (2010) assumes that firms behave monopolistically and ignore current and future strategic interactions when optimising profits.

evaluating the value functions associated with complicated price strategies is computationally challenging. When, as is common in many storable good industries, the choice set is high-dimensional (i.e. $J \approx 100$) the state space becomes large, then the computation of strategies satisfying MPE conditions is arguably infeasible.

A EBE replaces the MPE rationality requirements with two weaker conditions. First, given the information set used by agents to make decisions, a EBE requires that agents choose actions that maximise their perceived payoffs. The second conditions requires that the perceptions held by agents are consistent with play they have observed in the past when the information set recurs.

Both EBE conditions are met by the decisions taken by firms and consumers in the dynamic price model. In accordance with the first condition, firms and consumers choose actions that maximise their perceived payoffs calculated using a restricted subset of the full information available to them (i.e. limited information of each product's manufacturer). The second condition is also met; firms' aggregate dynamic demand model, the model they use to forecast rivals' price strategies, and consumers' price forecast model are all consistent with past market outcomes.

In the context of the storable good industry model above, one benefit of using EBE as an equilibrium concept is that it relaxes the implausible MPE requirement that consumer and firm beliefs necessarily coincide. Instead, it only requires that these beliefs are consistent with observed play at recurring states of the game.

2.3 Dynamic diversion ratios

This section expresses the dynamic component of the optimal pricing equations in terms of diversion ratios. Diversion ratios measure the closeness of substitution between two products. They are calculated as the proportion of product j's sales lost from product k as a result of a increase in product j's price

$$DR_{(j,k),t} := -\frac{\Delta^q_{(j,k),t}}{\Delta^q_{(j,j),t}} \tag{9}$$

When the diversion ratio is multiplied by the relative mark-ups of the two products it forms the diverted profit ratio. This measures effect that sales diverted to product j from product k has on product k's profits

$$DR^\pi_{(j,k),t} := -\frac{\Delta^q_{(j,k),t} m_{kt}}{\Delta^q_{(j,j),t} m_{jt}} \tag{10}$$

In the context of a static pricing game, a diverted profit ratio captures the strength of the competitive constraint internalised when a manufacturer produces substitutable products j and k. As discussed at length in subsection 2.4, the diverted profit ratio is a key component of generalised upward price pressure indices (GUPPI) used in antitrust analysis to assess unilateral merger effects.

However, when equations (9) and (10) are calculated using high frequency market data, these diversion ratios only capture the effect of contemporaneous substitution. As a result, they are not suitable to measure diversion between storable goods. This is because the inter-temporal substitution synonymous with dynamic demand for storable goods are not accounted for. By not including diversion from future sales, the numerator tends to be understated and the absolute value of the denominator overstated. These biases reinforce one another and can combine to severely bias diversion ratios downwards.

To capture the inter-temporal substitution patterns in in a diversion ratio, equation (9) is adjusted to only include the change in future sales of product k in the numerator

$$ITDR_{(j,k),t} := -\frac{\sum_{h=t+1}^{\infty} \Delta^q_{(j,k),h}}{\Delta^q_{(j,j),t}} \tag{11}$$

Similarly, the effect of inter-temporal substitution on firm profits is measured by modifying equation (10). In place of the change in current profits of product k, the new numerator is the change in NPV of its future profits in response to the temporary price cut for product j. The result is an inter-temporal diverted profit ratio between current purchases of product j and future purchases of product k.

$$ITDR^{\pi}_{(j,k),t} := -\frac{\sum_{h=t+1}^{\infty} \delta^{h-t} \Delta^q_{(j,k),h} m_{kh}}{\Delta^q_{(j,j),t} m_{jt}} \tag{12}$$

As is the case for static price setting games, the inter-temporal diverted profit ratios are a key components in dynamic pricing. Specifically, the ratio of the equation (12) to its static counterpart, equation (10), is closely connected to the bias that results from omitting demand dynamics for storable goods.

This relative dynamic diverted profit ratio for a pair of products j and k in period t is

$$\Psi_{(j,k),t} := \frac{ITDR^{\pi}_{(j,k),t}}{DR^{\pi}_{(j,k),t}} \tag{13}$$

It measures the change of the net present value of future profits earned on sales of product k in response to a temporary price change for product j as a fraction of the change in profits earned on current sales of product k. It captures the relative importance of contemporaneous and inter-temporal cannibalisation when setting current prices.

Let $\bar{\Psi}_t := \mathbb{E}_t \Psi_t$ be a $J \times J$ matrix of is the expected dynamic diverted profit ratios for all pairs of products $j, k \in \mathcal{J}$ in period t and substitute it into the dynamic price setting equations. Written in matrix form the resulting system of first order conditions is

$$q_t + \left(\Delta^q_t \odot \left(1 + \bar{\Psi}_t \right) \right) m_t = 0 \tag{14}$$

where q_t is a J-vector of aggregate quantity demanded in period t, m_t is a J-vector of mark-ups and $\odot$ denotes the Hadamard product.

If all elements of $\bar{\bar{\Psi}}_t$ are zero, then equation (14) reverts to the static differentiated Bertrand Nash optimality conditions. However, when demand dynamics are present, they provide a reduced form expression of omitted inter-temporal substitution. Moreover, the elements of $\bar{\bar{\Psi}}_t$ convert biased short-run price elasticities to their long-run counterparts that measure both contemporaneous and inter-temporal substitution.

The remainder of the paper treats the elements of $\bar{\bar{\Psi}}_t$ as parameters to be estimated from data on market outcomes and margins earned on sales over the same period. Before turning to identification and empirical analysis, I describe how the system of price setting equations (14) can be used in existing antitrust policy tools often applied to industries in which demand dynamics are common.

2.4 Antitrust policy: price pressure tests

The 2010 US Horizontal Merger Guidelines introduced upwards price pressure measures UPP (Farrell and Shapiro (2010)) and the gross upwards price pressure index (GUPPI) (Moresi (2010)). Since then, these price pressure tests have been used to screen mergers, aid market definition and assess closeness of competition - particularly for differentiated products.[22] This section shows how $\bar{\bar{\Psi}}$ can be used to adjust diversion ratios used in upward price pressure tests to reflect market dynamics in a modified GUPPI formula.

The GUPPI is a measure of the strength of the competitive constraint between two products, j and k. Formally, in response to a price rise of product j, the percentage margin earned on sales of product j multiplied by the diverted profit ratio. That is,

$$GUPPI_{(j,k)} := -\frac{\Delta^q_{(j,k)} m_k}{\Delta^q_{(j,j)} m_j} \mu_j = DR^\pi_{(j,k)} \mu_j \tag{15}$$

where μ_j is the percentage mark-up on product j.[23]

Implicit in the GUPPI calculation is that the diversion profit ratio reflects the competitive constraints that limit market power. In practice, the market power proxied by margin, is based on firm's accounts. As such, is typically measured over a prolonged period of time (i.e. one year). Therefore when demand and supply are dynamic, the margins implicitly reflect effect of both immediate and inter-temporal substitution.

For storable goods the diverted profit ratio is biased downward due the omission of inter-temporal substitution. Problematically, these biased diversion ratios are not consistent with the competitive dynamics that effect firms' market power. To remedy

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[22]GUPPI and other price pressure have been used in many recent merger cases. For example, in the UK, these include Sainsbury's/Asda (2019), Tesco/Booker (2017), Ladbrokes/Coral (2016) and Cineworld/Showcase Cinema (2013). Examples in the EU and USA include Austria / Orange Austria (2012) and Dollar Tree/Family Dollar (2015), respectively.

[23]To reflect the fact the GUPPI is usually measured with respect using data collected over multiple periods, time subscripts are temporarily suppressed.

this a dynamic diverted profit ratio (DDR) is required.

$$DDR^{\pi}_{(j,k)} := -\frac{\mathbb{E}\sum_{h=0}^{\infty}\delta^h\Delta^q_{(j,k),h}m_{kh}}{\mathbb{E}\sum_{h=0}^{\infty}\delta^h\Delta^q_{(j,j),h}m_{jh}} \tag{16}$$

The dynamic diverted profit ratio measures the reduction in the present value of expected profits earned on current and future sales of product k as a fraction of the present value of expected profits earned on current and future sales of product j in response to a temporary cut in price of j.

Replacing biased static diverted profit ratio in the GUPPI formula (equation (15)) with its dynamic counterpart defines a new GUPPI adjusted for omitted inter-temporal substitution dynamics.

$$dGUPPI_{(j,k)} := DDR^{\pi}_{(j,k)}\mu_j \tag{17}$$

Problematically for use in a policy setting, the dynamic diverted profit ratio as stated above in equation (16) is hard to quantify - even when a dynamic demand model is available to simulate future demand derivatives. Therefore, it is useful to be able to express it in terms of more readily obtainable components.

The dynamic diverted profit ratio is linked to its short-run counterpart by elements of $\bar{\Psi}$. Specifically, for product k in response to a temporary increase in the price of product j's dynamic diverted profit ratio is[24]

$$DDR^{\pi}_{(j,k)} := \frac{1 + \bar{\Psi}_{(j,k)}}{1 + \bar{\Psi}_{(j,j)}}DR^{\pi}_{(j,k)} \tag{18}$$

Using equation (18) in equation (17) it is straightforward to show dGUPPI simplifies to the product of a term containing elements of $\bar{\Psi}$ and the static GUPPI. Therefore, after estimating $\bar{\Psi}$, the construction of this new dynamic price pressure index is straightforward in a policy setting.

$$dGUPPI_{(j,k)} = \frac{1 + \bar{\Psi}_{(j,k)}}{1 + \bar{\Psi}_{(j,j)}}GUPPI_{(j,k)} \tag{19}$$

To illustrate how the dGUPPI captures demand dynamics, consider a firm assessing the impact on profits of its promotional pricing decisions on two substitutable products j and k over a one month planning horizon.

Suppose the firm producing product j is considering temporarily cutting its price from £15 to £10 for the current week. If it does so, it expects to generate an additional 100 sales as a result. However, the firm expects that 20 of those sales would occur at the

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[24]See Appendix A.1 for a derivation of $DDR^{\pi}_{(j,k)}$ in terms of $\bar{\Psi}$ and its static counterpart

regular higher price of £15 within the subsequent weeks over the next month without the price cut.

Assuming a marginal cost of £5 in all periods, 40% of short-run profits made on sales of good j immediately following the price cut come are drawn from expected profits from future profit flows. That is, the (j, j)-th element of $\bar{\Psi}$ is -0.4.

Further suppose, the effect of the price cut of product j on product k is the loss of 10 sales today and a further 3 sales over the next month. Assuming its price and cost remain unchanged over the period and its sales yield a £10 margin, the firm anticipates that consumer demand dynamics will lead to an additional 30% of losses on top of short-run profits lost on product k over the next month. Equivalently the (j, k)-th entry of Ψ is 0.3.

In this simple example, only 20% of the firm's profits are the result of sales diverted away from product k in the period of the price change and the GUPPI from k to j is 0.1. Recall, that the (j, j)-th and the (j, k)-th entry of Ψ are -0.4 and 0.3, respectively. Therefore, $DDR_{(j,k)} = 0.43$ and the dGUPPI that factors in anticipated demand dynamics is 0.217 - more than double the GUPPI calibrated with contemporaneous profit movements alone. This higher dynamic diversion ratio demonstrates that products j and k are much closer substitutes for non-transitory price changes than implied by a static analysis.

Remarks The benefit of constructing dGUPPI to assess the strength of competitive constraints in the context of storable goods is threefold. First, it does not require estimation of a computationally intensive dynamic demand model to provide measures of diversion ratios. Second, the analyst need not solve for a new equilibrium of the underlying dynamic pricing game to be able to evaluate the unilateral effects concerns arising from a merger. Finally, calculation of dGUPPI only requires inputs and empirical techniques commonly used in antitrust analysis. Namely, product margins, high frequency market data (i.e weekly or monthly), static demand estimation and manipulation of the system of linear price setting equations. As discussed in the next section, the main drawback is that the dGUPPI will be set valued because the elements of Ψ_t are partially-identified. However, as shown in policy application in sections (4.3) and (4.4), this does not preclude its efficacy in a policy setting.

3 Identification of $\bar{\Psi}_t$

This section describes how data used in demand estimation and estimates of economic margins are combined with the model in Section 2 to set identify the elements of $\bar{\Psi}_t$. It begins with a description of the data assumed to be available to the policy analyst is described.

Data For the purposes of identifying the elements of $\bar{\Psi}$ we assume that the analyst has an estimate of the margin for a collection of one or more products that covers the

same period as the data used to estimate the demand model. For example, the analyst might have access to company accounts that provides estimates of the margin earned on sales of all of its products in a particular calendar year. To estimate the elements of $\bar{\Psi}$ its is assumed that they also have access to high frequency data (i.e. weekly) on market outcomes in that same year.

Identification   Using data on market outcomes and margins measured over the same period, the elements of Ψ_t are not point identified by the combination of the price setting model described in Section 2 and the demand derivatives of a static demand model. They are only set identified. However, without further restriction, the identified sets are likely to be too wide for use in a policy setting.

To remedy this, two further restrictions on the nature of inter-temporal substitution patterns omitted from the model are added. They both capture important aspects of the nature of the missing demand dynamics.

A2: Inter-temporal substitution: Demand for product j is decreasing in its own-price in the current period and is increasing in own-price in future periods. A cut in the price for product j weakly decreases contemporaneous and demand over future periods for all other goods.

A3: $\left|\bar{\Psi}_{(j,j),t}\right| \geq \bar{\Psi}_{(j,k),t}$ $j \neq k$, $j \in \mathcal{J}_f$. Inter-temporal demand responses are strongest with respect to the product's own past prices.

A2 formalises that purchases of goods in the industries are substitutes both in the period in which they are purchased and in future periods. A3 reflects the fact that inventories built up in response to a promotion have the largest impact on the demand of the product promoted - rather than on the demand of imperfectly substitutable alternatives.[25]

Adding these two restrictions to the model from Section 2, as stated in Proposition 1, the elements of Ψ_t can be shown to be bounded above and below.[26]

Proposition 1: Assume A2 and A3 hold and that firms set prices using equation (14). Provided that promotional margins are non-negative and future sales are expected to at least cover variable costs, then the resulting prices are above marginal costs. In this case, all of the elements of Ψ belong to compact sets

$$-1 \leq \bar{\Psi}_{(j,j),t} \leq 0 \tag{20}$$

$$0 \leq \bar{\Psi}_{(j,k),t} \leq \left|\bar{\Psi}_{(j,j),t}\right| \quad \forall j \neq k \tag{21}$$

for $j, k \in \mathcal{J}_f$.

Even though the parameters can be bounded, the number of reduced form parameters in $\{\bar{\Psi}_t\}$ for $t = 1, \ldots, T$ in the system of price setting equation (14), for products in group n is $T J_n^2$.

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[25]This is consistent the findings of the existing literature on dynamic demand estimates in storable goods markets (Erdem et al. (2003); Hendel and Nevo (2006a)). Namely, the biases to own-price elasticities are larger in magnitude than biases to cross-price elasticities.

[26]See Appendix A.2 for the proof of Proposition 1.

To aid empirical work further restrictions on $\bar{\Psi}_t$ are imposed to reduce the number of parameters to be estimated. To that end, three further assumptions are added. First, assumption A4 assumes that $\bar{\Psi}_t$ stationary. This reduces the number of estimated parameters by a factor of T.

A4. $\bar{\Psi}_t = \bar{\Psi}$ for $t = 1, \ldots, T$

Second, without variation of product margins within the n groups of products, the available data does not allow the analyst to draw inference about individual $\bar{\Psi}_{(j,j)}$ for each of the products in group n. As such, a further assumption is introduced that constrains the diagonal elements of $\bar{\Psi}$ for products in group n to be the same.

A5: $\bar{\Psi}_{(j,j)} = \psi_n^{own}$ for all $j \in \mathcal{J}_n$

Third, because the margins have no additional information on covariances between mark-ups and demand derivatives, the off-diagonal elements are not identified. Therefore, I approximate the off-diagonal elements of the matrix of dynamic diversion ratios by assuming that they are the same for any two products, $k \neq j$ and $k' \neq j$.

A6: $\bar{\Psi}_{(j,k)} = \psi_n^{cross}$ for all $j \neq k, j \in \mathcal{J}_n$

Adding assumptions A5 and A6 further reduces the number of parameters from J_n^2 to 2 parameters for each group n.

Proposition 2: Under assumptions A1-A6, the mark-up in each period can be expressed as a function of aggregate demand, prices, short-run demand derivatives and $\bar{\Psi}$.

$$\mu_n = -\frac{\sum_{t=1}^{\bar{T}} q_t^{n,\top} \left(\Delta_t^q \odot \left(1 + \bar{\Psi}_n\right)\right)^{-1} q_t}{\sum_{t=1}^{\bar{T}} q_t^{n,\top} p_t} \tag{22}$$

$$-1 \leq \psi_n^{own} \leq 0 \tag{23}$$

$$0 \leq \psi_n^{cross} \leq |\psi_n^{own}| \tag{24}$$

where q_t^n is a vector of quantities purchased whose j-th entry is q_{jt} if $j \in \mathcal{J}_n$ and 0 otherwise and $\left[\Delta_t^q \odot \left(1 + \bar{\Psi}\right)\right]$ is an invertible matrix of bias-adjusted demand derivatives.[27]

Proposition 2 provides an equation for the percentage margin earned on sales of products in group n.[28] Combining an estimated margin, data on market outcomes and the output of a static demand model, the analyst can find values for the two elements of $\bar{\Psi}$ that solve equations (22), (23) and (24). When $J_n > 1$ with only one equation the parameters in $\bar{\Psi}$ are partially, rather than point-identified.

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[27]The non-singularity of this matrix requires the arguably weak condition that the aggregate long-run diversion ratios for products in group n produced by firm f differ.

[28]See Appendix A.3 for the proof of Proposition 2.

4 Application: UK laundry detergent

In this section, the model described in Sections 2 and 3 is applied to the UK laundry detergent sector. After providing a brief overview of the industry the elements of $\bar{\Psi}$ are estimated in two steps.

First, I specify a static demand model and estimate it using household purchase diary data from Kantar Worldpanel for a large UK supermarket from 2002 to 2012.[29] Second combining margins estimated from firm accounts with market-level data (i.e. prices) and the output of the static demand model (i.e purchase probabilities and static demand derivatives), the set of elements of $\bar{\Psi}$ that solve first order conditions are estimated using the system of equations (22), (23) and (24). These sets of estimates for $\bar{\Psi}$ are used to quantify the bias in (absolute) own and cross price elasticities.

Finally, the elements of $\bar{\Psi}$ are used to conduct two policy experiments. First, I show how the dynamically adjusted GUPPI formula, dGUPPI, can be used to assess the unilateral effects that might arise following a divestment of a brand from one firm to another. Second, I test whether there is evidence that a laundry detergent cartel in mainland Europe between 2002 and 2005 affected the UK detergent market.

Both experiments highlight that the omission of demand dynamics has the potential to lead to misguided policy conclusions. Further, they demonstrate that the methods developed in the paper have the potential to prevent such errors and aid policy decision making.

4.1 The UK laundry detergent industry

The analysis of the UK laundry detergent industry is based on individual household purchase data from 1st January 2002 until 31st October 2012. Households that take part in the survey scan the barcode of the items they purchase. Using the scanned barcode, the survey records the price and number of packs bought together with the characteristics of the product purchased. In addition, the purchase date and store in which the product was bought is also recorded. The purchase data is supplemented by annually updated household demographics.

To avoid complexities related to store choice, the analysis in the remainder of this paper is conditioned on detergent purchased from a single leading UK supermarket. Prices charged by supermarkets of the same fascia are the same stock keeping unit (SKU or product) in different stores in whose footprint is greater than 280 sqft.[30] As such, purchases across different stores within a particular supermarket fascia in the UK can be pooled for the purposes of empirical analysis.

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[29]Kantar Worldpanel data is a large data set that has been widely used in research into firm and consumer behaviour. Recent examples include Griffith et al. (2018); Dubois et al. (2017); Thomassen et al. (2017); Lewbel and Nesheim (2019); Myśliwski et al. (2020).

[30]This pricing policy follows an undertaking in 2001 following a market investigation by the UK Competition Commission into the groceries industry in 2000.

Two firms account for around 75 to 85 percent of households' annual purchases of laundry detergents in each year between 2002 and 2012. Hereafter, they are referred to as firm A and firm B. Firm A's annual share of sales is between 44 and 54 percent and firm B between 29 and 36 percent. Outside of these two major producers of branded products, the retailer's private label products commands the largest share - although its share has declined from 25 percent in 2002 to 12 percent in 2012. A fringe of small niche brands account for the remainder of products sold.

Laundry detergent is sold in a diverse array of brands, formats, and pack sizes. Each pack, or product, contains a single type of detergent. In general, a detergent is defined by its format, its brand and the chemical properties of the enzymes it contains (i.e. non-bio/bio, stain removal properties, scent etc).

In addition to the supermarket's private label products and the fringe of 'other' brands, the two largest firms sell six major brands between them. These six brands are labelled alphabetically as brand A through to brand F and are differentiated by their perceived quality.[31] Using the average purchased price per wash as a indicator for consumer perception of quality, brand D is classified as a premium brand (20p per wash). Brands A, C and E are mid-range brands (17-18p per wash) and brands C and F are 'standard' brands whose price is similar to the supermarket's private label detergents (13-14p per wash).

Laundry detergent brands are sold in six formats: powder, tablets, liquids, liquid capsules, super concentrated liquid, and gel.

Formats have different dosage metrics: liquid and gel are measured in millilitres per wash, whereas powder and tablets are measured in grams per wash. Further, following a series of industry-wide initiatives that sought to reduce the environmental impact of detergent production and the development of new products, dosages tend to decline over time within formats. To avoid complications arising from correcting for changing dosages, the number of washes is used to measure pack size.

Laundry detergents can be purchased in many different pack sizes.[32] As dosages have declined over time, so has the physical storage space occupied by material needed per wash. In response, firms produce larger pack sizes over time: the average number of washes per purchased pack have increased by around 50 percent from 17 washes in 2002 to 26 washes in 2012.

In the remainder of this paper, a product is defined as a unique combination of brand, format and pack size. Table 1 provides summary statistics for the resulting 668 products together with consumer characteristics (including household size and a proxy for household income - i.e. average weekly grocery spend).

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[31]Brands are not attributed to firms for confidentiality reasons.

[32]The UK laundry detergent industry differs in this respect from the one studied by Hendel and Nevo (2006a). They restrict attention to powder products and examine brand choice conditional on size choice from a small number of discrete sizes: 16oz, 32oz, 64oz, 96oz, and 128oz. Erdem et al. (2003) also focus on only five different weight choices in the US Ketchup market in their dynamic demand estimation.

Table 1: Summary of Kantar Worldpanel data

	Mean	Median	Std dev	Min	Max
Purchase Characteristics					
Price in (£)	3.58	3.20	1.81	0.13	24.52
Price per wash (£)	0.17	0.16	0.06	0.01	0.77
Purchased Washes	23	20	12	3	100
Dosage in Grams	87.7	80	16.1	18.3	139.1
Dosage in Millilitres	52.6	45	20.8	12	125
Number of Packs	1.07	1	0.26	1	2
Household Characteristics					
Number of Equivalent Adults	2.28	2.20	0.79	1.00	8.10
Av. weekly grocery spend (£)	67.05	63.78	27.71	3.70	423.45

Source: Kantar Worldpanel

4.2 Static demand estimation for UK laundry detergent

This section describes a static discrete choice demand model of UK laundry detergent. Given the policy focus of this paper, a demand model often employed in antitrust investigations - the nested logit model - is estimated.

In contrast to many applications in a policy setting, the nested logit model is estimated using the purchase diary data from Kantar Worldpanel described in Section 4.1. The availability of micro-data enables the use of observed household characteristics to estimate a rich demand model without the additional computational costs associated with estimation of a random coefficient model.

Where the policy analyst only has aggregate market data, Berry (1994) and Berry et al. (1995) can be used to estimate conditional, nested, or random coefficient logit demand models. The approach to estimating of the elements of $\bar{\Psi}$ is as described in Section 4.3.

4.2.1 Static demand model

Detergents are sold in one of $j = 1, \ldots, J$ stock keeping units (products). A product is defined by the detergent it contains and the number of washes it contains. The products sold in market t, $\mathcal{J}_t$, are manufactured by $f = 1, \ldots, F$ firms. The outside good is denoted by $j = 0$ and represents the decision not to purchase in market t.

Household i elects to purchase good j from a market t to maximise conditional indirect utility

$$U_{ijt} = x_j^\top \beta_i + \alpha_i p_{jt} + \epsilon_{ijt} \tag{25}$$

where p_{jt} is the price of product j in market t and x_j is a K-vector of observed product attributes. Because the model is estimated using consumer micro-data, the parameters α_i and β_i depend on an L-vector of household characteristics, y_i.

The observed product attributes include the number of washes, dosage, and brand-format dummies. Household i's valuation of these attributes is captured by taste parameters

$$\beta_i = \beta + \beta_y y_i \qquad (26)$$

where y_i is an L-vector of household characteristics. To capture heterogeneous valuations of these attributes β_i has two components. The first is common to all households and is measured by a K-vector of parameters, β. The second captures the effect of household characteristics and is measured by an L-vector of parameters β_y.

Prices of product j in market t, p_{jt}, enter linearly into the indirect utility function. The marginal utility of income for each household is

$$\alpha_i = \alpha + \alpha_y y_i \qquad (27)$$

In addition to a common component, α, the effect of different household characteristics on the marginal utility of income is measure by an L-vector of parameters, α_y.

The effect of factors observed by the household, but not the econometrician, are captured by

$$\epsilon_{ijt} = \xi_{jt} + \varepsilon_{ijt}$$

It contains two terms: ξ_{jt} is an unobserved product-market component of utility common to all consumers and ε_{ijt} is a private household-specific utility shock. The household utility shocks are identically and independently distributed and follow the Generalised Extreme Value distribution. The unobserved product-market components are observed by all firms prior to setting prices. As a result, prices are likely to be correlated with ξ_{jt} and are not independent from the unobserved component of demand, $\epsilon_{ijt} \not\perp p_{jt}$. This gives rise to endogeneity concerns.

There are three channels that lead to correlation between unobserved product-market characteristics and prices. First, any additional costs incurred in the production of a positively valued unobserved characteristic will likely be passed onto households in the form of higher prices.

Second, to the extent that the unobserved attribute shifts demand outwards, equilibrium markups will reflect any additional market power this confers on the firm. In both cases, prices are likely to be positively correlated to unobserved attributes. If the endogeneity is left untreated, these correlations positively bias the marginal utility of income.

Another potential source of correlation captured by ξ_{jt} is non-price advertising and promotional activities. If unobserved non-price advertising activity - such as funding prominent store positioning - increases during periods of promotional prices, ξ_{jt} is negatively correlated with prices. Alternatively, if prices contemporaneously increase to fund non-price advertising, then ξ_{jt} is positively correlated with prices.

Because both positive and negative correlation might result, the direction of the bias due to non-price advertising is ambiguous. However, given the importance of promotional activity and unobserved factors, such as placement within the store, it is quite plausible (even likely) that non-price advertising components of ξ_{jt} are negatively correlated with prices. In this case, marginal utility of income would be biased upward.

4.2.2 Estimation and identification

Data   The demand model is estimated using a random sample of five purchases in each week from the Kantar Worldpanel purchase diary data described in Section 4.1, resulting in 2,810 choice occasions.

The number of products in choice sets ranges from 87 to 187 with a median of 120 products.[33] The set of products sold in each week is partitioned into four nests based on the number of washes contained in each product: small (S), medium (M), large (L), and extra large (XL). The size boundaries of these groups correspond to the 25th, 50th, and 75th quantile of distribution of washes in each calendar year.

Estimation and identification   In this particular application, 12 percent of product-week market shares are zero. As a result, it is not possible to use Berry (1994) to deal with endogeneity concerns. Therefore, the control function approach proposed by Petrin and Train (2010) is used to address endogeneity concerns.

The model is estimated in two steps. In the first step, the control function is estimated using prices from observed purchases.[34]

$$p_{jt} = W\left(x_j, z_{jt}; \gamma\right) + \upsilon_{jt} \tag{28}$$

The auxiliary price equation contains two components. The first, $W\left(x_j, z_{jt}; \gamma\right)$, is a function of observable product characteristics, x_j, and instruments excluded from the choice model, z_{jt}. These instruments include cost shifters and BLP instruments that shift mark-ups. Consequently, they are correlated with prices, but not unobserved demand factors. The other component is a product-market specific unobservable, υ_{jt}.

The estimated residuals are used to construct the control function, $CF\left(\hat{v}_{jt}; \rho\right)$. In the second step, the control function is added to the choice model and it is estimated by maximum likelihood.[35]   Finally, because the control function is estimated, standard errors are calculated by bootstrapping the two step estimation procedure.[36]

When controlling for endogeneity, the control function for product j in period t is linear in the residuals from the estimated auxiliary equation (equation (56)),

$$CF\left(\hat{v}_{jt}; \rho\right) = \rho\hat{v}_{jt} \tag{29}$$

The specification of the auxiliary equation contain the same product characteristics as the choice model, year fixed effects and instruments. The BLP instruments that capture the congestion of product space for each product in each consumer's choice set. In this application, the instruments are based on the number of similar products sold in the

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[33]See Appendix B.1 for a more detailed description of the construction of choice sets from the purchase diary data.

[34]When more than one price is observed per product in each week, its average weekly price is used.

[35]See Train (2009), chapter 4 for details of maximum likelihood estimation of the nested logit model.

[36]See Appendix B.3 for a more detailed description of the bootstrap algorithm.

market.[37] There are two instruments: a count of the firm's similar products and a count of rival's similar products.[38]

The static demand model parameters, α, β and ρ are common to all consumers. They are identified by variations prices, product characteristics and fitted control function residuals within consumer's choice sets, respectively. The parameters that interact consumer with price or product characteristics are identified from variation in choices across different types of consumers. The nesting parameter, σ, is identified by variations in the ex-ante expected utility of purchasing a product from a give nest for each consumer.

4.2.3 Results

The specification of conditional indirect utility include interactions between price and a proxy for household income.[39] Product characteristics include the size of the product purchased and the dosage - the amount of material (recommended) for use in a single wash. To control for household size, I also include the amount of washes purchased per equivalent adult in the household.[40] Detergent fixed effects are also included.

The results of the nested logit estimation are displayed in Table 2. It contains the parameters and standard errors of the nested logit model with and without the control function.

In both nested logit models the price coefficient is negative and households with higher income have a lower marginal utility of income. When the control function is included, the price coefficient (scaled by the nesting parameter) approximately doubles from -0.523 to -1.054.

This is consistent with correlation between unobserved product demand factors being positive. It is also reflected by the control function parameter, ρ, being positive and statistically significantly different from zero.

Table 2 also shows that households positively value larger pack-sizes. However, for smaller households this is less pronounced. This is consistent with smaller households living in smaller accommodation and facing a higher cost of storage.

The amount of material needed to do a single wash (i.e. dosage) is negatively valued, especially for 'solid' detergents. This is also consistent with the fact that households value storage space. When the dosage is lower, households can store more washes without necessarily occupying more storage space. Indeed, as noted in Section 4.1, this is one of the driving factors behind the success of the new super-concentrated and gel detergent products. By itself, this suggest the presence of inter-temporal demand links through inventories - a source of mis-specification for this static demand model.

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[37]A product is regarded as being similar if it belongs to the same size category and has the same format. Further, and in line with the construction of the choice sets, similar products must be sold in the same calendar month as their comparator.

[38]See Appendix B.2 for estimates of the auxiliary equation used to construct the control function.

[39]The proxy for household income is annual average weekly expenditure on all groceries.

[40]To calculate equivalent adults, I use the OECD-modified equivalence scale.

Table 2: Choice models: parameter estimates

| Variables | Nested Logit | | | |
	θ	se	θ	se
Price Params:				
Price	-0.275	0.021	-0.581	0.136
Price x Income	0.179	0.039	0.184	0.029
Characteristics:				
Washes	0.118	0.163	4.484	1.949
Washes per Eq. Ad.	-0.337	0.095	-0.371	0.771
Dosage	-1.222	0.108	-1.058	0.284
Dosage x Liquids	0.279	0.067	0.575	0.381
Other Params:				
λ	0.526	0.035	0.551	0.049
ρ			0.297	0.126
Detergent Fixed Effects	Yes		Yes	
N	2,810		2,810	
Log-Likelihood	-12,203		-12,199	

The nesting parameter, λ, is 0.551 and is statistically significantly different from 1. This indicates that that there are some unobserved correlations in the utility between detergents of similar sizes and rejects the independence of irrelevant alternatives imposed by a conditional logit.

4.3 Solving for $\bar{\bar{\Psi}}$

Next, the output of the estimated static demand model is combined with market outcome data to find values of $\bar{\bar{\Psi}}$ that solve equations (22), (23), and (24) for firms A and B in each year from 2002 until 2012.

In the context of an antitrust investigation brand level margins may be available over several years. However, in my case, I only have access to published annual accounts. Therefore, the published gross margins in firm A's and firm B's annual accounts from 2002 to 2012 are assumed to be good approximations to the economic margins earned on sales of their laundry detergent portfolio in the UK.[41]

Plugging in the margins, using the Kantar Worldpanel purchase data in each year and combining with output of the demand model, the analyst searches for all pairs of parameters $(\psi_n^{own}, \psi_n^{cross})$ where $n \in \{f, y\}$, $f \in \{A, B\}$ and $y = 2002, \ldots, 2012$.[42]

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[41]The financial year for firm B starts midway through the year. As such, the annual report margins are adjusted to match calendar years in the data. Figures are omitted for confidentiality reasons.

[42]For the purpose of this section, I assume that the cartel in mainland Europe had no impact on the UK laundry detergent sector, and they priced detergent according to differentiated Bertrand competition

Figure 1 shows the value of these parameters for both firms in each year in the data when the demand model outputs are evaluated at mean parameter values reported in Table 2.

It shows the pairs of $(\psi_n^{own}, \psi_n^{cross})$ that satisfy equations (22), (23), and (24) in each year for firm A and firm B. For each firm, the absolute value of the own and cross price elasticity biases are negatively correlated with one another and are close to parallel in each year. The slope measured at the extremum points of the sets of $(\psi_n^{own}, \psi_n^{cross})$ for firm A range from 0.221 to 0.269 over 2002 to 2012. For firm B they range from 0.137 to 0.149 - approximately half that of firm A.

Figure 2 shows how the sets of $(\psi_n^{own}, \psi_n^{cross})$ for each firm-year combination translate into estimates of the bias in the aggregate own and cross price elasticities for firms A (the left panel) and firm B (the right panel).[43] In both panels, the blue and orange ribbons show the bias of own and cross price elasticities, respectively.

In line with much of the dynamic demand estimation literature, Figure 2 shows that the static demand model produces overly elastic own price elasticities and cross price elasticities are biased towards zero.

For firm A, the own price elasticity bias increases over the period 2002 to 2012. In 2002 the the long-run price elasticity is overstated by 20 to 26 percent. Then in 2008, prior to the increased rollout of new compacted products with, this bias increases to 32 to 42 percent. As new more compacted products are rolled out from 2009 to 2012, the degree to which the own-price elasticity are overstated steadily increases to 47 to 63 percent by 2012. For firm A the cross price elasticities are understated by 0 to 24 percent.

Like its rival, firm B also rolled new compacted products with lower storage costs after 2007. By 2009, these larger pack sizes, firm B's own price elasticity bias is 41 to 50 percent after fluctuating between 31 and 46 percent from 2002 to 2007. However, from 2010 firm B offers deeper discounts on smaller pack sizes and the expected pack size purchased size falls - particularly for its aggressively priced budget brand. By 2011 and 2012, firm B's own price elasticity bias lies between 28 and 34. Despite these fluctuations, the width of the set of own price elasticity bias interval is relatively narrow and stable - its average width is 7.5 percentage points across all years with a standard deviation of 1.25.[44] For firm B the cross price elasticities are understated by 0 to 22 percent.

The above findings are consistent with the own price elasticity bias increasing as inter-temporal substitution due to demand dynamics intensifies. Specifically, own price elasticity bias appears to increases when firms offer new compacted detergents with lower storage costs in larger pack sizes.

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between 2002 and 2005. I revisit this issue in Section 4.4.

[43]Appendix C.2 contains the aggregate own and cross price elasticities by brand from 2002 to 2012 in the UK laundry detergent industry.

[44]For firm A the average width of the own-price elasticity bias interval across years is 10.7 percentage points with a standard deviation of 3.8.

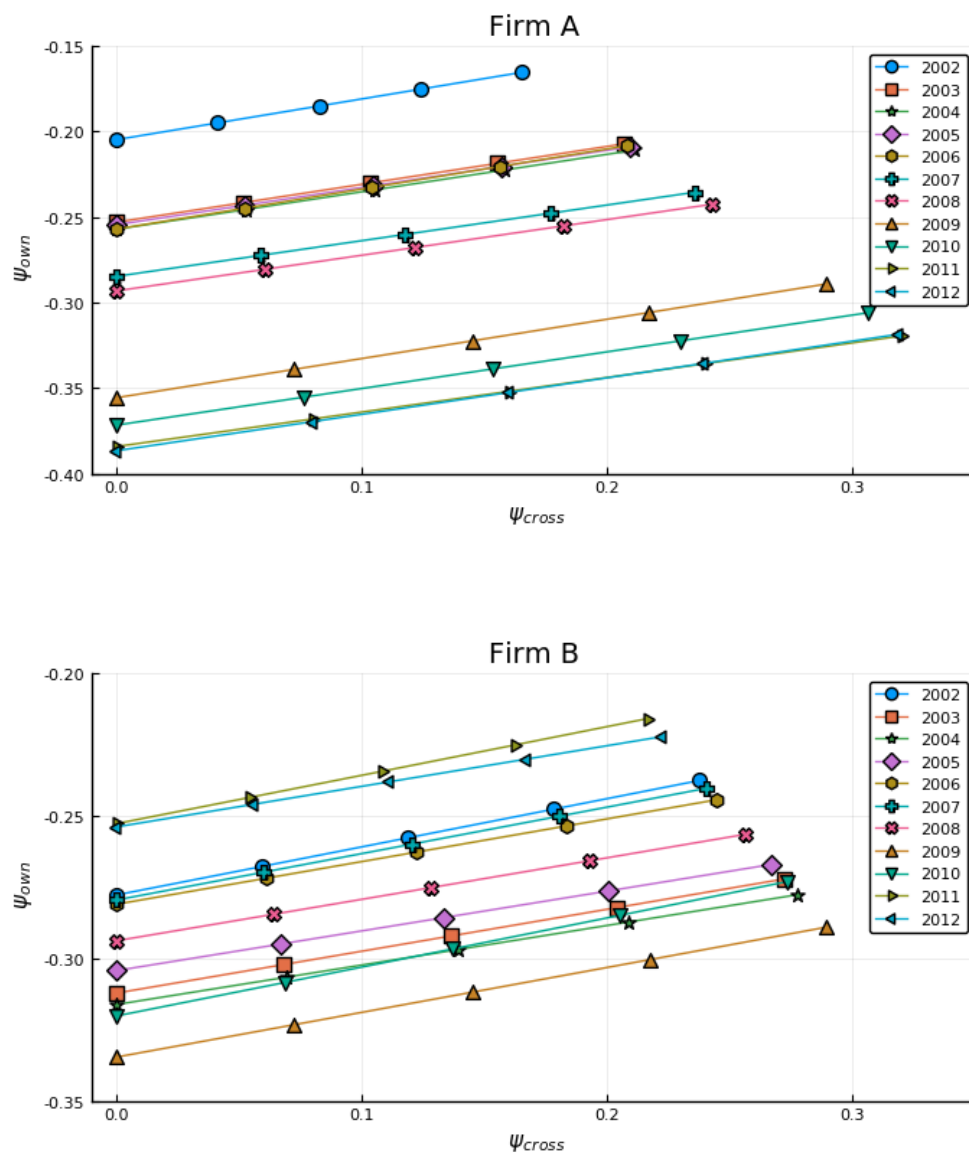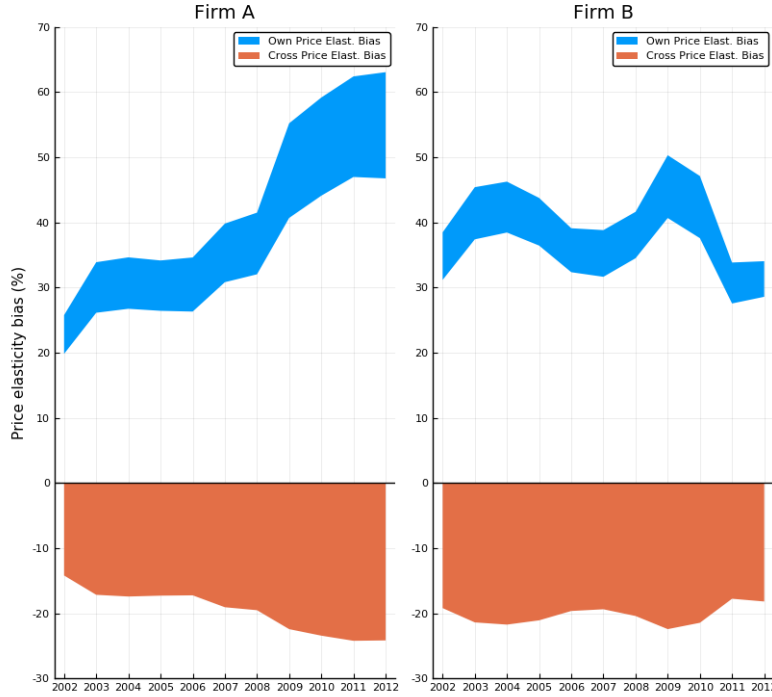Figure 1: $\left(\psi_n^{own}, \psi_n^{cross}\right)$ for Firm A and Firm B in each year from 2002 to 2012

Figure 2: Bias in price elasticities

4.4 Policy experiments

This section contains two hypothetical policy experiments using the UK laundry detergent industry. It shows how set-valued estimates of the elements in $\bar{\Psi}$ can provide evidence in support of policy decisions.

The first experiment assesses the competitive effects from a hypothetical divestment of a brand from firm B to firm A using dGUPPI. The second experiment explores whether a cartel in mainland Europe's laundry detergent industry involving the two largest firm A and B between 2002 and 2005 was in effect in the UK. In both cases I will show that set-valued empirical tools can help avoid policy making errors.

dGUPPI To illustrate how the estimates of bias-adjusted diversion ratio can be used in a policy setting, suppose brand A and brand E are produced by different firms and the firm producing A is buying brand E from the rival firm at the end of 2012. For the purpose of this exercise assume brand A is produced by firm A and brand E by firm B.[45]

To assess whether the proposed acquisition is likely to lead to a significant lessening of competition, the analyst can calculate the bias-adjusted GUPPI measure introduced in section 2.4, dGUPPI. Allowing for some tolerance in potential efficiencies arising through joint production, it is assumed that the policy analyst views a dGUPPI below

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[45]For confidentiality reason, brands cannot be explicitly assigned to firms.

five percent as sufficiently low to allow the brand divestment to occur without giving rise to anti-competitive concerns.[46]

The dGUPPI is calculated in each year to gather evidence on the likelihood of prices rising by examining strength of the incentive to increase the prices in previous years. Two upward price pressure indices are computed; one for the price increase of brand A and another for the price increase of brand E.

As discussed at in section 2.4, the dGUPPI is product of a static diversion ratio bias-adjustment term and the standard GUPPI measure. Under A5 and A6, dGUPPI for a price increase in brand A in each year is

$$dGUPPI_{(A,E)} = \frac{1 + \psi_A^{cross}}{1 + \psi_A^{own}} DR_{(A,E)} \frac{\mu_E \bar{p}_E}{\bar{p}_A} \tag{30}$$

where $DR_{(A,E)}$ is the static diversion ratio from brand A to brand E, μ_E is the percentage margin and $\bar{p}_b$ is the average price for brand $b \in \{A, E\}$.[47]

Because there is a set of pairs of $(\psi_n^{own}, \psi_n^{cross})$ that solve equations (22), (23) and (24, the dynamic diversion ratio and dGUPPI is set-valued. The corresponding sets containing the bias of the static diversion ratios for brands across all years is plotted in Figure 3.

Compared to dynamic diversion ratios that capture both static and inter-temporal substitution, static diversion ratios are biased downward by 17 to 53 percent for firm A and by 22 to 39 percent for firm B. Moreover, the combination of bias in both components of the diversion ratio creates comparatively wider identified sets of elasticity bias than for own-price elasticities. The width of the set containing the diversion bias ranges from 15 to 22 percentage points for firm A and 17 to 19 percentage points for firm B.

To compute the dGUPPI, the standard (static) GUPPI is computed using estimated margins, sales-weighted annual average prices for the brands, and brand-level diversion ratios from the demand model. The resulting static GUPPI for brand A and brand E is shown by the red lines in the left and right panels of Figure 4, respectively.

Between 2002 and 2007 the GUPPI when brand A's price is changed is relatively stable and averages 6.4 percent. After 2008, it first drops below the five percent threshold: starting from 6.4 percent in 2007 it declines to 3.4 percent in 2012. This drop in the GUPPI is driven by a decline in the diversion ratio. By 2012, the diversion ratio had fallen from 13.1 percent in 2007 to 8.4 percent by 2012. Detergent level analysis indicates

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[46]Five percent (or higher) might be used because it is often used as the floor for a significant non-transitory increase in price for market definition purposes. However, for goods that comprise a large fraction of consumption, competition authorities have used more stringent GUPPI thresholds. For example in the investigation into the proposed Asda/Sainsbury's merger the UK Competition Markets Authority a 2.75 percent GUPPI threshold; a 1.5 percent baseline with a 1.25 percent efficiencies allowance to assess local market competition. See Competition Markets Authority (2019) "Anticipated merger between J Sainsbury PLC and Asda Group Ltd: Final Report".

[47]Diversion ratio and average prices are volume weighted following Domencich and McFadden (1975). See Appendix C.1 for further details. To calculate the dGUPPI for a price increase in brand E, the analyst need only exchange the indices in equation (30).

Figure 3: Estimated set for diversion ratio bias for brand A and brand E, 2002 to 2012
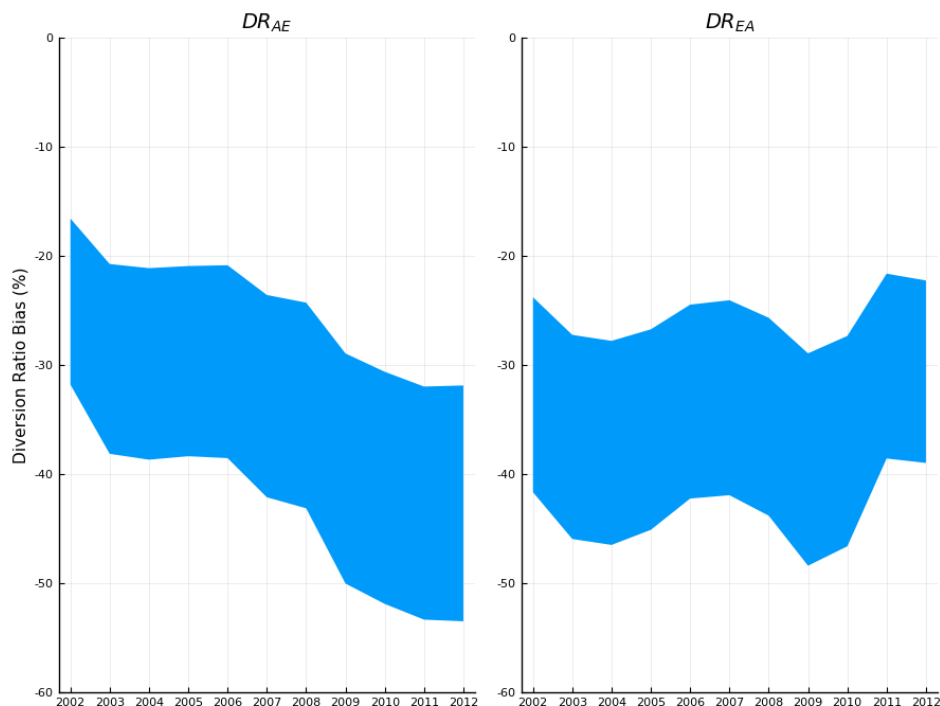
that drop occurs because brand A became a leading brand for the new gel formats rolled out from 2008 onwards. Prior to that brand E's powder and tablets were perceived to be closer substitutes by brand A customers.

Unlike the GUPPI measure from the perspective of brand A customers, the GUPPI for brand E customers is below the five percent threshold in each year over the period - an average of 4.3 percent across the period. Therefore, relying solely on a static GUPPI measures, an analyst might conclude that although the brands were closer competitors in the past, innovation in product formats and product compaction since 2007/8 led to the brands being sufficiently distant competitors by 2012 to permit the acquisition for both brand A's and brand E's customers.

However, once the diversion ratios are adjusted for bias due to the omission of demand dynamics, the analyst's conclusions are reversed. The resulting range of possible values of the dGUPPI is shown by the blue bands in Figure 4.

The left panel shows the range of possible values for dGUPPI when brand A's price changes. It shows that once inter-temporal substitution is accounted for, the price increase predicted by dGUPPI lies between 4.9 and 8.4 percent - largely above the five percent threshold. By itself, this finding would be sufficient to reverse the conclusions of the static GUPPI analysis.

The right panel shows the range of possible values for dGUPPI when brand E's price changes is 5 to 8.4 percent. Unlike the static GUPPI, the analyst cannot now rule out that the dGUPPI lies strictly below the five percent threshold in all years. Again, these

Figure 4: GUPPI and dGUPPI: brand A and brand E, 2002 to 2012
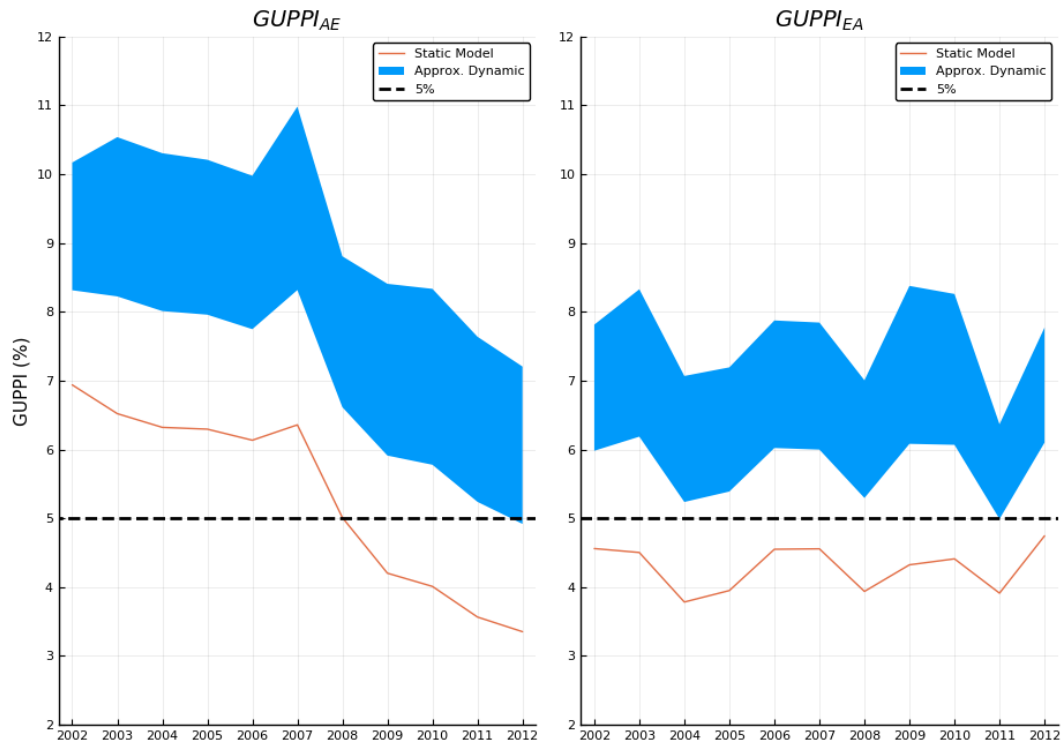
findings support prohibition of the firm producing brand A acquiring brand E.

The EU detergent cartel: UK impact? In April 2011 the European Commission (EC) found that firm A, firm B and a firm C had entered into a cartel agreement that restricted competition in the market for heavy duty laundry detergent powder. The infringement was first brought to the attention of the EC when firm C 'blew the whistle' on the cartel in exchange for immunity from prosecution and/or reduced fines. Subsequent investigation led to the finding that the cartel was effective over the period 7th January 2002 to 8th March 2005. The EC highlighted four restrictive elements of the cartel:

1. indirect price restrictions resulting from the parties agreeing not to pass on any cost savings that resulted from compaction of products

2. explicit reduction of promotional activity

3. direct price increases

4. an exchange of commercially sensitive information

According to the EC, the laundry detergent cartel (LD cartel) had anti-competitive effects in Belgium, France, Germany, Greece, Italy, Portugal, Spain and The Netherlands.[48] One notable absentee from the list of countries affected is the UK - a

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[48]European Commission Decision, 'COMP/39579 - Consumer Detergents'.

country in which the 'whistleblower', firm C, had virtually no market presence. However, the laundry detergent market in the UK is dominated by the other two firms in the cartel; firm A and firm B.

This section combines the structural demand model estimated in Section 4 together with various supply-side models to estimate market power under different types of firm conduct. By comparing the estimated margins associated with different models of firm behaviour to the observed margins for firm A and firm B from 2002 to 2005, the possibility that the collusive activities in mainland Europe were also evident in the UK is investigated.

Two policy experiments are conducted. In the first demand derivatives from the static demand model are used a inputs in to an analysis of the market power of firm A and firm B during the cartel period. In the second, this analysis is repeated with the set of demand derivatives adjusted for bias by the parameters $(\psi^{own}, \psi^{cross})$.

To test whether the firms' conduct was competitive or collusive between 2002 and 2005, the set of estimates for $(\psi^{own}, \psi^{cross})$ must be calibrated using another year where we know there was no cartel. The set of parameters for $(\psi^{own}, \psi^{cross})$ from 2006 are used because they are closest to the period in question and are less likely to be influenced by other factors (i.e. product innovations in 2007 and 2008).

In both experiments, the demand model outputs are evaluated at mean parameter values. As a result the impact of uncertainty arising from estimating the demand model's parameters is not reflected in the predicted margins in the subsequent analysis. Conducting the additional computations to construct the confidence sets that reflect uncertainty over the inputs on counterfactual simulations can make the policy conclusions more robust.[49]

To estimate the market power, assumptions on the nature of supply-side competition and the shape of the cost function are added to each demand model in each experiment. Throughout I assume that marginal cost is (locally) constant.

To examine the impact of the intensity of competition on market power, the ownership matrix is altered to reflect different ownership structures. I consider four different supply side models to analyse market power: (1) assumes that firms engage in Bertrand-Nash price competition and each product is manufactured by a single firm, (2) assumes multi-product Bertrand-Nash competition, (3) assumes that firm A and firm B collude over powder products only,[50] and (4) assumes that firm A and firm B set all prices jointly as a branded product monopolist. Scenarios (1) and (4) are intended as lower and upper bounds on the estimated market power.[51]

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[49]An additional step needed to construct confidence sets. At the end of each bootstrap iteration (see Appendix B.3) solve for $(\psi^{own}, \psi^{cross})$ and then simulate the policy counterfactual of interest. The convex hull of the set counterfactual outcomes provides a (conservative) confidence set reflecting uncertainty over demand model parameters.

[50]To reflect the cartel scenario, I assume firm B (firm A) take into account joint profits of a firm A (firm B) powder products, but do not set its price.

[51]I do not have information on the identity of producers of private label products. For the purposes of this paper I assume that firm A and firm B have no share in private label's profits. This is unlikely

Figure 5: Policy experiment with $\left(\psi_{f,y}^{own}, \psi_{f,y}^{cross}\right) = (0,0)$ for firm $f \in \{A, B\}$ and $y \in$ [2002, 2005]: gross margins and implied market power during the cartel period: 2002 - 2005
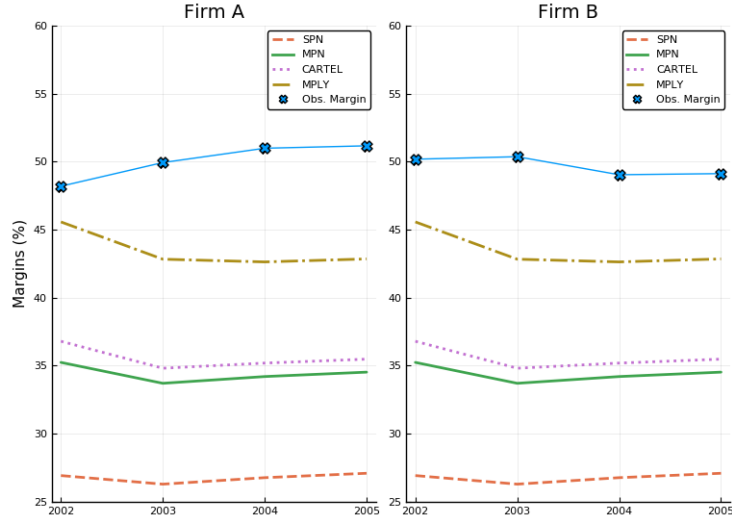
Note: SPN refers to single Product differentiated Nash-Bertrand competition, MPN refers to multi-product differentiated Nash-Bertrand competition, CARTEL refers to pricing where firms A and B collude over prices for powdered detergents and MPLY refers to firmsA and B pricing as a single entity (i.e. monopoly pricing).

The results of the first experiment are shown in Figure 5. The left panel in the figure plots the observed margins in 2002 to 2005 for firm A alongside the margins implied by the demand model and each of the four firm conduct models. The right panel mirrors this analysis for firm B.

The figure shows that the observed margins of around 50% are well in excess of the margins implied by multi-product Bertrand-Nash. Moreover, the observed margins are well in excess of the 40% margin implied by monopoly pricing.

The fact that observed margins lie above the monopoly outcome serves as a warning that the demand model is mis-specified. Especially since the known biases that arise from omitting demand dynamics in storable goods industries would understate market power in this experiment. However, based on these results from the mis-specified demand model, a (naive) policy analyst might conclude that the collusive conduct of these two dominant firms in Europe affected market outcomes in the UK laundry detergent market.

However, the fact that observed margins lie so far above the cartel outcome might serve as a warning that the demand model is mis-specified. Especially because the known biases that arise from omitting demand dynamics in storable goods industries would understate market power in this experiment. To check this hypothesis I repeat the analysis adjusting for biases due the omission of demand dynamics by setting $\left(\psi_{f,y}^{own}, \psi_{f,y}^{cross}\right) = \left(\psi_{f,2006}^{own}, \psi_{f,2006}^{cross}\right)$ for firm $f \in \{A, B\}$ and $y \in [2002, 2005]$.

Figure 6 shows the results of the same policy experiment but using the set of estimated

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to be the case. As such the level of market power in multi-product scenarios (2) and (3) are likely to understate market power compared to the 'true' ownership matrix.

Figure 6: Policy experiment with $\left(\psi_{f,y}^{own}, \psi_{f,y}^{cross}\right) = \left(\psi_{f,2006}^{own}, \psi_{f,2006}^{cross}\right)$ for firm $f \in \{A, B\}$ and $y \in [2002, 2005]$: gross margins and implied market power during the cartel period

Note: SPN refers to single Product differentiated Nash-Bertrand competition, MPN refers to multi-product differentiated Nash-Bertrand competition, CARTEL refers to pricing where firms A and B collude over prices for powdered detergents. The monopoly case is excluded from the graph.

parameters for ψ^{own} and ψ^{cross} from 2006 for both firms. The left panel of the figure plots the observed margin for firm A from 2002 to 2005 with the band of the predicted margins from the different models of conduct. Because the estimates of ψ^{own} and ψ^{cross} are set valued, the menu approach predicts a range of margins in each year. The right hand panel produces the same information for firm B. In both panels, the monopoly outcomes are omitted from the plot because they predict margins over 70%.

In contrast to the first policy experiment, the band of margins predicted under competitive conduct largely coincides with the observed margin for firm A in 2003 to 2005. In 2002, the observed margins lie below those implied by competitive conduct.

For firm B, the predicted band of margins under competitive conduct lies below the observed margins. However, the observed margins also lie just below the lower bound of margins consistent with cartel conduct. While not conclusive, taken together the findings of this policy experiment do not find compelling evidence that the collusive conduct from mainland Europe occurred in the UK.

While it should be borne in mind that this experiment is based on an approximation to observed margins, it does highlight how important it is to attempt to correct for known biases in empirical policy work.

5 Conclusion

This paper develops a new approach to estimate sets of long-run price elasticities and dynamic diversion ratios for storable goods industries. To identify the missing demand

dynamics from static demand models, the approach leverages information on market power over multiple purchase cycles contained in estimated margins.

By combining the output of a static demand model with observed prices and margins, I show how to recover parameters that capture the effect of inter-temporal substitution omitted from static models on firm's pricing incentives. These parameters are used to construct set-valued estimates of the long-run price elasticity matrix and dynamic diversion ratios - key inputs into antitrust and other policy areas.

In particular, I show dynamic diversion ratios can be combined with margins in a new generalised upward price pressure index adjusted for demand dynamics - dGUPPI. This new empirical policy tool extends the existing GUPPI test and can be used to evaluate mergers in industries with dynamic demand.

To illustrate this new framework, I apply it to the UK laundry detergent industry from 2002 to 2012. For the two firms that dominate the industry, I find that the own price elasticity bias ranges from 20 to just over 60 percent over the period. The magnitude of this bias appears to increase as demand dynamics become more prominent feature of consumer behaviour.

The bias of cross-price elasticities range from 0 to 17 for firm A and 0 and 22 percent for firm B. Together, the own and cross price elasticity biases when demand dynamics are incorrectly omitted are reinforced when computing diversion ratios. Compared to dynamic diversion ratios that capture both static and inter-temporal substitution, static diversion ratios are biased downward by 17 to 51 percent and 22 to 39 percent for firms A and B, respectively.

To show how the set-valued estimates of policy inputs can be used to aid policy-making I conduct two policy experiments. The first shows how sets of dynamic diversion ratio produce dGUPPI ranges that can help a competition authority evaluate the likelihood of unilateral effects arising from a hypothetical brand acquisition from a rival. The second uses set-valued estimates of the long-run price elasticity matrix to assess whether a cartel in mainland Europe's laundry detergent was effective in the UK. In both experiments, the use of set-valued policy inputs is central to avoiding policy errors.

The application demonstrates that this new approach is well suited to policy work. It only requires standard empirical methods and readily available data on margins and market-level industry outcomes. As a result, it can be easily implemented within the policy making time-horizon. More generally, it highlights the potential of empirical analysis producing set-valued outcomes to aid the policy-making process.

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Appendix A: Proofs and derivations

A.1 $DDR_{(j,k)}$

Derivation of the dynamic diverted profit ratio as the product of a term containing elements of $\bar{\Psi}$ and its static counterpart[52]

$$DDR_{(j,k)} := -\frac{\mathbb{E}\sum_{h=0}^{\infty}\delta^h \Delta_{(j,k),h}^q m_{kh}}{\mathbb{E}\sum_{h=0}^{\infty}\delta^h \Delta_{(j,j),h}^q m_{jh}} \tag{31}$$

$$= -\frac{\Delta_{(j,k)}^q m_k + \mathbb{E}\sum_{h=1}^{\infty}\delta^h \Delta_{(j,k),h}^q m_{kh}}{\Delta_{(j,k)}^q m_j + \mathbb{E}\sum_{h=1}^{\infty}\delta^h \Delta_{(j,j),h}^q m_{jh}} \tag{32}$$

$$= -\frac{\left(\Delta_{(j,k)}^q m_k + \mathbb{E}\sum_{h=1}^{\infty}\delta^h \Delta_{(j,k),h}^q m_{kh}\right)\frac{\Delta_{(j,k)}^q m_k}{\Delta_{(j,k)}^q m_k}}{\left(\Delta_{(j,k)}^q m_j + \mathbb{E}\sum_{h=1}^{\infty}\delta^h \Delta_{(j,j),h}^q m_{jh}\right)\frac{\Delta_{(j,j)}^q m_j}{\Delta_{(j,j)}^q m_j}} \tag{33}$$

$$= -\frac{\left(\Delta_{(j,k)}^q m_k + \mathbb{E}\sum_{h=1}^{\infty}\delta^h \Delta_{(j,k),h}^q m_{kh}\right)/\Delta_{(j,k)}^q m_k}{\left(\Delta_{(j,k)}^q m_j + \mathbb{E}\sum_{h=1}^{\infty}\delta^h \Delta_{(j,j),h}^q m_{jh}\right)/\Delta_{(j,j)}^q m_j} \times \frac{\Delta_{(j,k)}^q m_k}{\Delta_{(j,j)}^q m_j} \tag{34}$$

$$= \frac{1 + \mathbb{E}\left[\frac{ITDR_{(j,k)}}{DR_{(j,k)}}\right]}{1 + \mathbb{E}\left[\frac{ITDR_{(j,j)}}{DR_{(j,j)}}\right]} DR_{(j,k)}^{\pi} \tag{35}$$

$$= \frac{1 + \bar{\Psi}_{(j,k)}}{1 + \bar{\Psi}_{(j,j)}} DR_{(j,k)}^{\pi} \tag{36}$$

A.2 Proposition 1

Recall

$$\Psi_{(j,k),t} := \frac{\sum_{h=t+1}^{\infty}\delta^{h-t} \Delta_{(j,k),h}^q m_{kh}}{\Delta_{(j,k),t}^q m_{kt}} = \frac{ITDR_{(j,k),t}^{\pi}}{DR_{(j,k),t}^{\pi}} \tag{37}$$

for all $j, k \in \mathcal{J}_f$ is the relative dynamic profit diversion ratio.

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[52]For brevity, only quantities, demand derivatives and margins in the future are have a time index.

Under condition A2 all products are substitutes, the promoted good's future sales are a substitute for current sales. A cut in the price today, will lead to a reduction in demand in the future, $\Delta^q_{(j,j),h} \geq 0 \, \forall j \in \mathcal{J}_f \, h > t$. Because promotional mark-ups and expected future profits are non-negative, then $ITDR^\pi_{(j,j),t} \geq 0$ and $DR^\pi_{(j,j),t} \leq 0$. As such, the numerator and denominator in $\Psi_{(j,j),t}$ have opposing signs, thus $\bar{\Psi}_{(j,j),t} \leq 0 \, \forall j \in \mathcal{J}_f$.

A similar argument establishes that the off-diagonal elements of Ψ_t are non-negative. Briefly, because (i) promotional mark-ups and expected future profits are non-negative (ii) both current and future sales of rival product k are substitutes for the promoted good j, then $DR_{(j,k),t} \geq 0$ and $ITDR^\pi_{(j,k),t} \geq 0 \, \forall j \neq k, : j, k \in \mathcal{J}_f$. Both the numerator and denominator in $\Psi_{(j,k),t} \, j \neq k$ have the same sign, hence the off-diagonal have a zero lower bound: $\bar{\Psi}_{(j,k),t} \geq 0 \, \forall j \neq k, : j, k \in \mathcal{J}_f$.

The lower bound on the diagonal elements of $\bar{\Psi}_t$ is derived from the first order conditions for product j

$$q_{jt} + \mathbb{E}_t \sum_{h=t}^{\infty} \sum_{k \in \mathcal{J}_f} \delta^{h-t} \Delta^q_{(j,k),h} m_{kh} = 0 \tag{38}$$

$$\implies -\sum_{k \in \mathcal{J}_f} \Delta^q_{(j,k),t} m_{kt} - \mathbb{E}_t \sum_{h=t+1}^{\infty} \sum_{k \in \mathcal{J}_f} \delta^{h-t} \Delta^q_{(j,k),h} m_{kh} = q_{jt} \tag{39}$$

$$\implies DR^\pi_{(j,j),t} + ITDR^\pi_{(j,j),t} + \sum_{k \neq j} DR^\pi_{(j,k),t} + \mathbb{E}_t \sum_{k \neq j} ITDR^\pi_{(j,k),t} = \frac{q_{jt}}{\Delta^q_{(j,j),t} m_{jt}} \leq 0 \tag{40}$$

$$\implies DR^\pi_{(j,j),t} + \mathbb{E}_t \left[ITDR^\pi_{(j,j),t} \right] \leq 0 \tag{41}$$

$$\implies \mathbb{E}_t \left[ITDR^\pi_{(j,j),t} \right] \leq -DR^\pi_{(j,j),t} \tag{42}$$

$$\implies \mathbb{E}_t \left[\frac{ITDR^\pi_{(j,j),t}}{DR^\pi_{(j,j),t}} \right] \geq -1 \tag{43}$$

$$\implies \bar{\Psi}_{(j,j),t} \geq -1 \tag{44}$$

The second line expands the second term from the line above. The third line substitutes in static and inter-temporal diverted profit ratios and uses the fact $\sum_{k \neq j} DR^\pi_{(j,k),t} \geq 0$ and $\sum_{k \neq j} ITDR^\pi_{(j,k),t} \geq 0$. After further simplifications, the second last line uses the fact that that $DR^\pi_{(j,j),t} = -1 \leq 0$ by construction. Thus, from equation (44) and using $\bar{\Psi}_{(j,j),t} \leq 0$, the final line provides bounds for the diagonal elements of $\bar{\Psi}_t$.

$$-1 \leq \bar{\Psi}_{(j,j),t} \leq 0 \, \forall j \in \mathcal{J}_f \tag{45}$$

Finally, condition A3 provides an upper bound on the off-diagonal elements of $\bar{\Psi}_t$.

$$0 \leq \bar{\Psi}_{(j,k),t} \leq \left| \bar{\Psi}_{(j,j),t} \right| \, \forall j \neq k, j, k \in \mathcal{J}_f \tag{46}$$

$\blacksquare$

A.3 Proposition 2

Let

$$q_{jt}^n = \begin{cases} q_{jt} & \text{if } j \in \mathcal{J}_n \\ 0 & \text{otherwise} \end{cases} \tag{47}$$

Then from the FOC we can derive an equation for the percentage margin earned on sales of products in group n over the T periods

$$q_t + \left(\Delta_t^q \odot \left(1 + \bar{\Psi}\right)\right) \left(p_t - c_t\right) = 0 \tag{48}$$

$$\implies p_t - c_t = -\left(\Delta_t^q \odot \left(1 + \bar{\Psi}\right)\right)^{-1} q_t \tag{49}$$

$$\implies q_t^{n,\top} \left(p_t - c_t\right) = -q_t^{n,\top} \left(\Delta_t^q \odot \left(1 + \bar{\Psi}\right)\right)^{-1} q_t \tag{50}$$

$$\implies T^{-1} \sum_{t=1}^{T} q_t^{n,\top} \left(p_t - c_t\right) = -T^{-1} \sum_{t=1}^{T} q_t^{n,\top} \left(\Delta_t^q \odot \left(1 + \bar{\Psi}\right)\right)^{-1} q_t \tag{51}$$

$$\implies \frac{T^{-1} \sum_{t=1}^{T} q_t^{n,\top} \left(p_t - c_t\right)}{T^{-1} \sum_{t=1}^{T} q_t^{n,\top} p_t} = -\frac{T^{-1} \sum_{t=1}^{T} q_t^{n,\top} \left(\Delta_t^q \odot \left(1 + \bar{\Psi}\right)\right)^{-1} q_t}{T^{-1} \sum_{t=1}^{T} q_t^{n,\top} p_t} \tag{52}$$

$$\implies \mu_n = -\frac{\sum_{t=1}^{T} q_t^{n,\top} \left(\Delta_t^q \odot \left(1 + \bar{\Psi}\right)\right)^{-1} q_t}{\sum_{t=1}^{T} q_t^{n,\top} p_t} \tag{53}$$

where μ_n is the percentage margin earned on sales of product in group n over the T periods.

The bounds in equations (23) and (24) follow from the application of assumptions A5 and A6 to Proposition 1. ∎

Appendix B: Data and Estimation

B.1 Choice sets

The choice sets are constructed using all purchases at stores at the major UK supermarket between 1st January 2002 until 31st October 2012. The purchase diary data only records when and where a product is purchased. As a result, due to sampling variation some products are not observed in the data even though are available for purchase. This makes it difficult to determine which products were available at the point of purchase. To address this, I assume that a product is eligible for the choice set if a purchase is observed in the same calendar month as the purchased product. This approach leads to the inclusion of products in the choice set without observed prices. To remedy this these product's prices need to be imputed.[53]

An interactive fixed effect model with r factors is used to impute prices:

$$P = \Lambda F^\top + \varepsilon \tag{54}$$

---

[53]Approximately 7 percent of observations are imputed

where P is a $J \times T$ matrix of prices, Λ is a $J \times r$ matrix of factor loadings, F is a $T \times r$ factor matrix and ε a $J \times T$ matrix of errors. I return to the choice of the number of factors below.

A truncated single value decomposition (SVD) with tailored to sparse data is used fit the interactive fixed effect model r factors,[54]

$$\Omega \odot P = \Omega \odot \left(U_r S_r V_r^\top \right)$$

where Ω is a matrix whose (j, k)-th element is 1 if the price of product j in period t is observed and zero otherwise. The loadings and factors are given by $\Lambda = U_r S_r$ and $F = V_r$.

Following Bai and Ng (2002), the number of factors are chosen to minimise penalised objective function

$$r^\star = \arg \min_{r \in \{1, 2, \dots, R_{MAX}\}} \ln \left(W_r \right) + r \frac{J + T}{JT} \ln \left(\frac{JT}{J + T} \right) \tag{55}$$

where the sum of squared residuals with r factors is

$$W_r = (JT)^{-1} \left(\Omega \odot \left(P - U_r S_r V_r^\top \right) \right)^\top \left(\Omega \odot \left(P - U_r S_r V_r^\top \right) \right)$$

Models with $r \in \{1, 2, \dots, 10\}$ are fitted and $r^\star = 2$ maximises the objective function in equation (55). Missing prices are imputed using

$$(1 - \Omega) \odot P = (1 - \Omega) \odot \Lambda^\star F^{\star\top}$$

where $\Lambda^\star = U_{r^\star} S_{r^\star}$ and $F^\star = V_{r^\star}$.

Finally, if a product is purchased in a promoted bundle of two units, the opportunity to purchase two units of product at is included in the choice set. If there is no observed purchase of two units, it is assumed it is not available as a promotional bundle and the total price is twice that of a single unit of the product.

B.2 Control Function

The control function is formulated using the residuals of an auxiliary pricing equation

$$p_{jt} = W \left(x_j, z_{jt}; \gamma \right) + v_{jt} \tag{56}$$

The possible source of endogeneity is reflected by the correlation of product-specific unobservables in the choice model, ξ_{jt}, and the unobservable component of the auxiliary price equation, v_{jt}. When observed prices are regressed on product characteristics and instruments, the estimated residuals from this regression contains the correlation between p_{jt} and ξ_{jt}. As a result, the estimated residuals,

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[54]The Julia package TSVD.jl is used to fit the model

$\hat{v}_{jt} := p_{jt} - \mathbb{E}[p_{jt}|x_j, z_{jt}]$, are used to construct a control function, $CF(\hat{v}_{jt}; \rho)$ with parameters ρ.

When this control function is added to the choice model it conditions out that part of prices correlated with ξ_{jt}. As a result, it resolves the endogeneity that arises from the unobserved product-market terms, ξ_{jt}. The unobserved component of demand is now independent of price, $\epsilon_{ijt} \perp p_{jt} \mid CF(\hat{v}_{jt}; \rho)$, and $\epsilon_{ijt} =^d \varepsilon \overset{iid}{\sim} GEV$.[55]

The results of estimating the auxiliary equation are shown in Table 3.

Table 3: Control Function: parameter estimates

Variables	θ	se
Intercept	1.589	0.058
Characteristics:		
Washes	14.503	0.036
Dosage	0.678	0.102
Dosage x Liquids	0.953	0.135
Instruments:		
Num. own similar prods	-0.038	0.004
Num. rival's similar prods	-0.040	0.003
Detergent Fixed Effects	Yes	
Year Fixed Effects	Yes	
N	59,035	
R^2	0.797	

B.3 Bootstrap procedure

The bootstrap procedure of Kim and Petrin (2010) is as follows:

1. The price regression form the first stage are bootstrapped and residuals calculated.
2. For each bootstrap sample the demand model is run on the combination of the original data and the bootstrapped residuals.
3. The variance of the parameters from 500 bootstrap samples is calculated (see Cameron and Trivedi (2005)).
4. The bootstrapped standard errors are the square root of the sum of the bootstrapped parameter variance and the unadjusted parameter variance.

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[55]Where $=^d$ is defined as "follows the same distribution as".

Appendix C. Aggregate elasticities and diversion ratios

This appendix describes the formulas used to aggregate individual elasticities and diversion ratio into their aggregate counterparts. It also shows the aggregate own and cross price elasticity estimates from the nested logit model with and without bias adjustments using elements of $\bar{\Psi}$.

C.1 Aggregation of elasticities

Following Domencich and McFadden (1975), aggregate elasticities are calculated by weighting household's product level elasticities by their share of the total demand of the set of products being aggregated over.

To provide further details, let $z \in \{own, within, cross\}$ and E^z_{ijt} be the corresponding price elasticity of demand of product j for consumer i in period t. Further, suppose we want to measure the price elasticity of aggregate demand for brand b over $t = 1, \ldots, T$ periods. As such, let $\mathcal{J}_b$ the set of all products of brand b.

Then aggregate price elasticities can be obtained from their individual counterparts by calculating

$$E^z_b = \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{j \in \mathcal{J}_b} \omega_{ijt} E^z_{ijt}$$

where the share of total demand for brand b over T periods accounted for by consumer i's demand for product j in period t is

$$\omega_{ijt} = \frac{\theta_{it} \Pr(d_{it} = j)}{\displaystyle\sum_{i'=1}^{N} \sum_{t'=1}^{T} \sum_{k \in \mathcal{J}_b} \theta_{i't'} \Pr(d_{i't'} = k)}$$

and θ_{it} is sampling weight reflecting the number of type i consumers in period t.

Similarly, the aggregate diversion ratio between brands b_1 and b_2 is computed as

$$DR_{(b_1, b_2)} = \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{j \in \mathcal{J}_{b_1}} \sum_{k \in \mathcal{J}_{b_2}} \omega_{ijt} DR_{(j,k),it}$$

C.2 Aggregate price elasticities

When computing elasticities the probability of purchase is adjusted for the share of the outside good. In this application the share of the outside good is informed by observed household purchase frequencies in the demand Kantar Worldpanel data. In

2002 household's purchase detergent once every three weeks on average. Following the steady increase in the number of washes per pack, this average inter-purchase duration extends to four weeks between 2003 and 2008 and five weeks thereafter. In line with this approach, the outside share is 67 percent in 2002, 75 percent between 2003 and 2008, and 80 percent between 2009 and 2012.

Figures (7), (8) and (9) show the aggregate own and cross price elasticities by brand over 2002 to 2012. The dashed orange lines are the unadjusted price elasticities and the blue ribbon are bias-adjusted price elasticities.

Appendix D: Nowcasting

To explore alternatives to assumption A1, suppose the analyst would like to relax this assumption and estimate an empirical analogue for equation (3). As such, the analyst needs to evaluate the expectation of the discounted sum of the product of firms' beliefs over price impulse response functions and the future quantity demanded of each product over an infinite horizon

One empirical strategy is to simplify the calculation of the expectation of the product between the impulse response functions and quantities demanded. Specifically, impose some restriction on firms' beliefs over the impulse response functions. Indeed, assumption A1 is one such, arguably strong, restriction.

An alternative is suppose firms do engage in nowcasting but believe that dynamic pricing functions are linear in prices.

A1': Firms believe that their and rivals' dynamic pricing functions are linear in prices: : $\frac{dp_{kh}}{dp_{jt}} \in \mathbb{R}$ for $h > t$ and $j, k \in \mathcal{J}$

Under linearity $\frac{dp_{kh}}{dp_{jt}}$ is constant for all $h > t$ and the expectation simplifies in to the product of two expectations

$$\mathbb{E}_t \sum_{h=t+1}^{\infty} \delta^{h-t} \frac{dp_{jh}}{dp_{jt}} q_{jh} = \sum_{h=t+1}^{\infty} \delta^{h-t} \mathbb{E}_t \left[\frac{dp_{jh}}{dp_{jt}} \right] \mathbb{E}_t [q_{jh}]$$

The main benefit of this is approach is that the impulse response functions are constants and are therefore independent of future quantities.

Assuming that firms believe impulse price functions are consistent with observed price movements, they can be consistently estimated from the data using high-dimensional forecasting methods.[56] The empirical analogue of average quantity sold of product j can be used to estimate the second component. Then, provided the dynamic pricing function meet stability requirements, the infinite sum of the products can be empirically evaluated. The cost of this approach is additional computation for the policy analyst.

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[56]In Crawford (2018) consumer use linear dynamic factor models to forecast prices in dynamic demand models for storable good industries.

Figure 7: Own price elasticities by brand: unadjusted and adjusted from 2002 to 2012
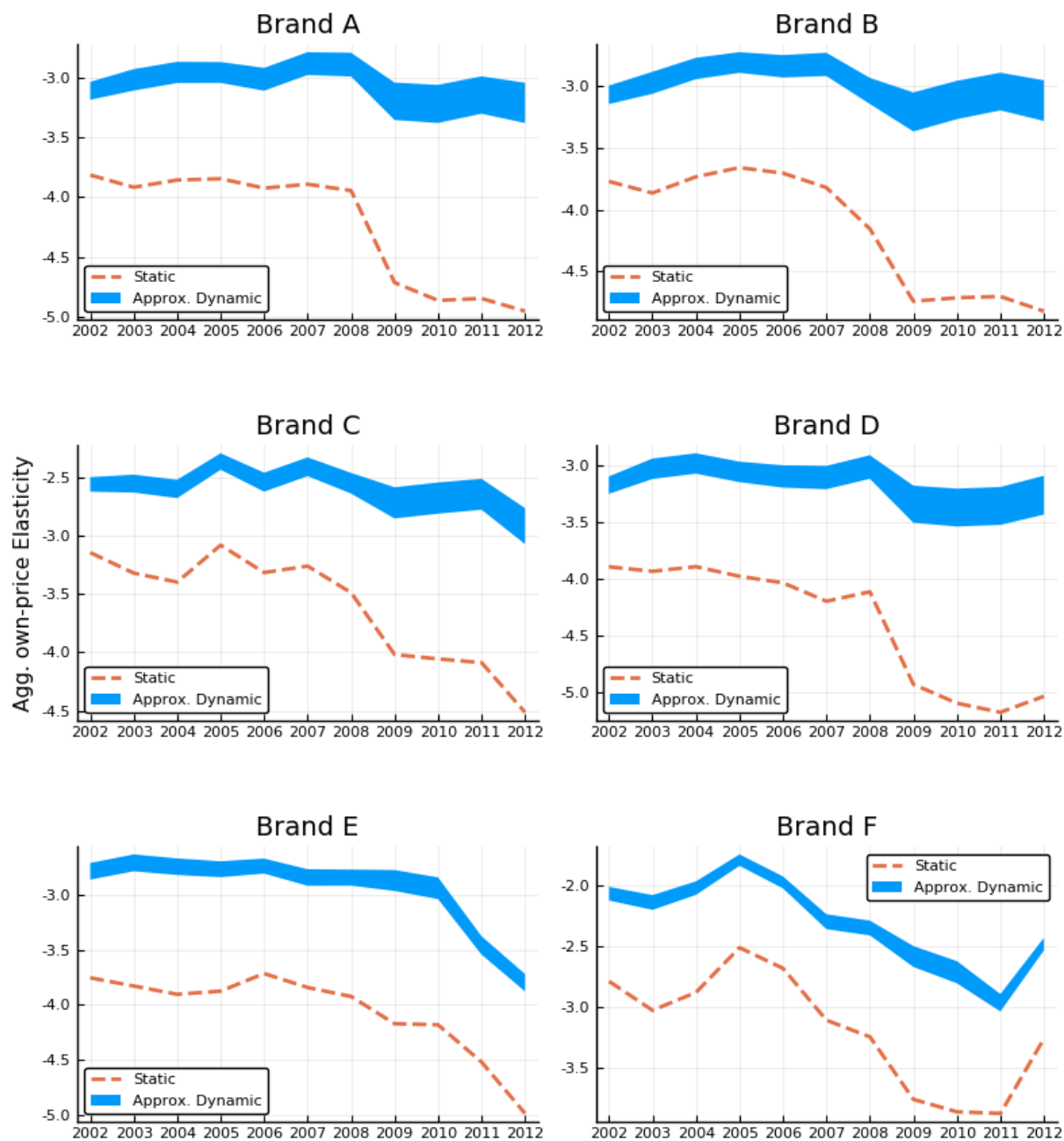
Figure 8: Within-nest cross price elasticities by brand: unadjusted and adjusted from 2002 to 2012
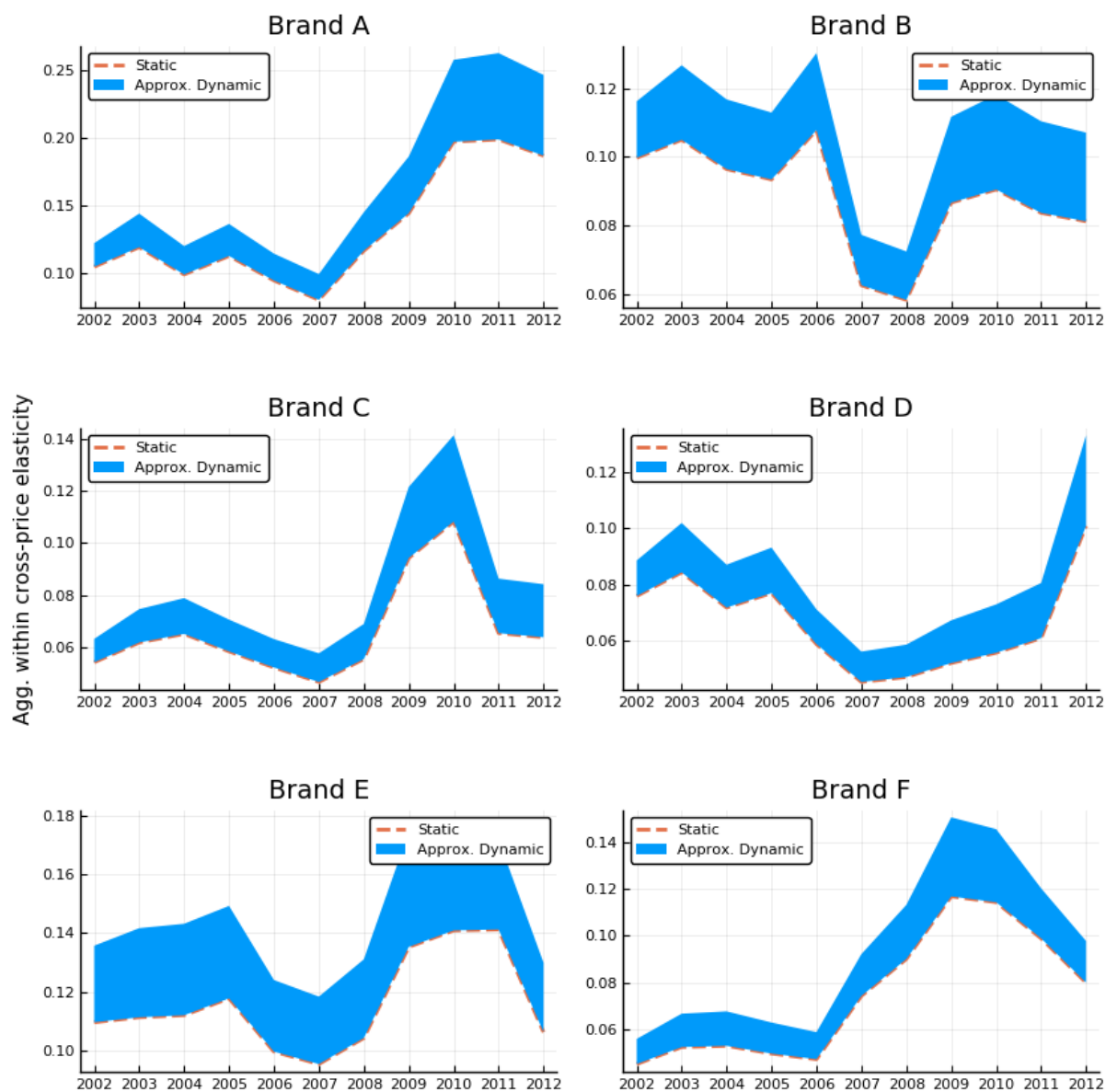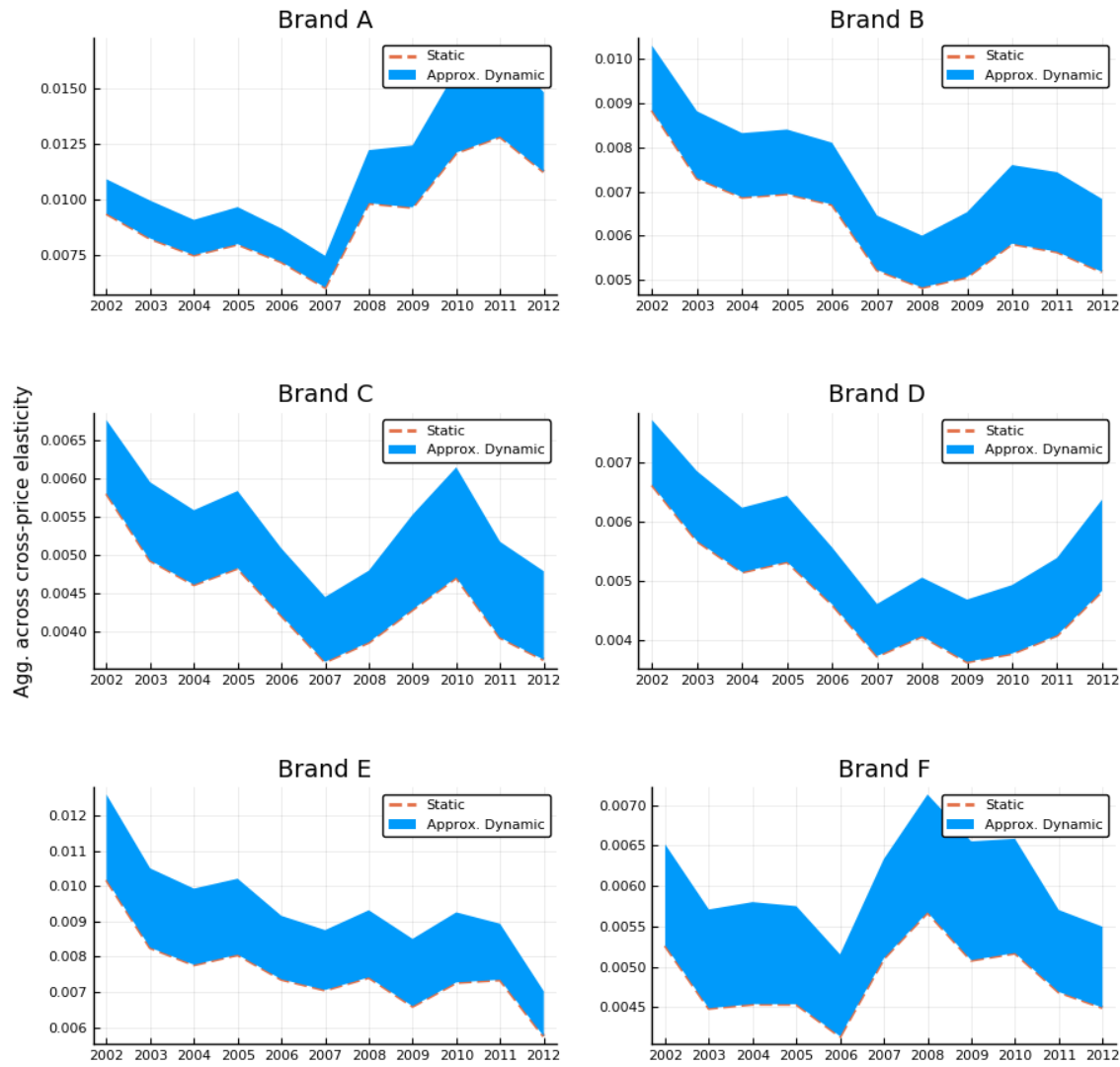
Figure 9: Across-nest price elasticities by brand: unadjusted and adjusted from 2002 to 2012

One possibility is to assumed that firms use the data on the history of market outcomes to estimate a κ-order Markov process to model the evolution of prices and use it to make price forecasts.

$$p_t = A_0 + \sum_{s=1}^{\kappa} A_s p_{t-s} + \epsilon_t \tag{57}$$

where A_0 is a J-vector, each A_k is a $J \times J$ matrix of price coefficients for $k = 1, \ldots, \kappa$ and ϵ_t is a J-vector of price shocks.

However, a noted above, most industries with dynamic demand contain $J \geq 100$ products in each week. In this case, the forecast model has $J(1 + \kappa J)$ parameters; if $J = 100$ and $\kappa = 1$ then there are over 10,000 parameters to be estimated. With so many parameters the price forecasting model will be challenging to estimate - even in industries with dynamic demand containing only a handful of products. Moreover, tracking and forecasting $J \geq 100$ product's prices is a cognitively challenging and resource intensive task for firms. Indeed, boundedly-rational firms may optimally choose a more parsimonious forecasting model.

Alternatively assume that through their repeated experiences of selling products in the industry firms are assumed to be able to relate individual products price movements to a low dimensional set of price trends that underpin observed price dynamics. If so, firms need only to track and forecast this low dimensional set of price trends. The resulting forecast model is

$$p_{jt} = \lambda_j^\top f_t + \epsilon_{jt} \tag{58}$$
$$F_{t+1} = AF_{t-k} + U_{t+1} \tag{59}$$

where f_t is an R-vector of low dimensional set of price trends or 'factors' and $F_t = [f_t, \ldots, f_{t-\tau}]^\top$ is a τR vector.

Further let $L = \left[\lambda_1^\top, \ldots, \lambda_J^\top \right]^\top$ be a $J \times R$ matrix of factor loadings and $\Lambda = [L, 0, \ldots, 0]$ is a $J \times \tau R$ matrix of time-invariant factor loadings.

In the factor VAR, the coefficient matrix is $\tau R \times \tau R$

$$A = \begin{bmatrix} A_1 & A_2 & \cdots & A_{\tau-1} & A_\tau \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & I & 0 \end{bmatrix}$$

and $U_{t+1} = [u_{t+1}, 0 \ldots, 0]^\top$ is a τR vector containing innovations to f_{t+1} where

$$\begin{bmatrix} \epsilon_t \\ u_{t+1} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{pp} & 0 \\ 0 & \Sigma_{ff} \end{bmatrix} \right) \tag{60}$$

Using this forecasting model the $J \times J$ matrix of expected changes to forecast prices h-periods ahead as a result of price changes in the current period is given by

$$\mathbb{E}_t \left[\frac{dp_{t+h}}{dp_t} \right] = \Lambda A^h F_t F_t^\top \Lambda^\top \left[\Lambda F_t F_t^\top \Lambda^\top + \Sigma_{pp} \right]^{-1}$$

Further let

$$\Delta_h^p = \Upsilon^n \odot \frac{dp_h}{dp_t}$$

where

$$\Upsilon_{(j,k)}^n = \begin{cases} 1 & \text{if } j, k \in \mathcal{J}_n \\ 0 & \text{otherwise} \end{cases}$$

is the matrix indicating which products belong to group n.

Then an empirical version of term four in the first order condition is

$$\mathbb{E}_t \left[\sum_{h=t}^{\infty} \delta^{h-t} \Delta_h^p q_h^n \right] = \Upsilon^n \odot \left(\Lambda \left(I - \delta A \right)^{-1} \Gamma_t \right) \bar{q}^n$$

where

$$\Gamma_t := F_t F_t^\top \Lambda^\top \left[\Lambda F_t F_t^\top \Lambda^\top + \Sigma_{pp} \right]^{-1}$$

and $\bar{q}^n$ is the average quantity from the model.