Inconsistency on Multimember Courts

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Abstract

Appellate courts sometimes issue inconsistent decisions. Individual judges are sometimes inconsistent too. We argue that making judges more consistent could exacerbate the problem of inconsistent courts. We do so through a variant of Arrow's model of preference aggregation in which preferences are complete, but need not be transitive. We introduce an ordinal rationality measure to compare preference relations. Using this measure, we introduce a new axiom, monotonicity in rationality, which requires the collective preference to become more rational when the individual preferences become more rational. We show that no collective choice rule satisfies monotonicity in rationality and the standard Arrovian assumptions: unrestricted domain, weak Pareto, independence of irrelevant alternatives, and non-dictatorship.

JEL Codes: D60; D70; D71; K40.

1 Introduction

Appellate courts sometimes issue inconsistent decisions. One reason, of course, is that judges are inconsistent. It should not be surprising that a court with inconsistent judges will, at times, yield inconsistent decisions. This suggests a path forward: find

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a way to make individual judges more consistent, and it must follow, as the night the day, that courts will become more consistent.

The purpose of this paper is to demonstrate that this logic, while appealing, is incorrect. It is not true that the consistency of multimember courts follows from that of its judges. This is not true regardless of whether the court uses majority rule to make collective decisions. We prove that there is no reasonable procedure that multimember courts can use to ensure that more consistent judges will not lead to less consistent verdicts.

We will make these arguments precise later in the paper. First, however, we need to explain the sense in which we use the term "consistency." Following Easterbrook (1982), we identify judicial consistency with the economists' notion of rationality of a preference relation.[1] We envision this as a model of external consistency, in that it requires sets of judgments to be consistent in relation to each other.[2]

The cost (or difficulty) of improving the consistency of individual judges depends on the reason for the inconsistency. For example, judges may be inconsistent for reasons akin to behavioral explanations of failures of rationality: judges fail to be consistent because limitations on cognition lead them to rely on heuristics that result in biased decision making.[3] It may be possible to reduce these inconsistencies at low cost by informing the judge of the inconsistency and letting them correct their behavior, or through "nudges." Some such inconsistencies, however, may be more costly to address. If it is possible to affect the consistency of individual judges, it is important to understand the implications of doing so.

An alternate explanation stems from the theory of law as a multi-criterial choice process, pioneered by Spitzer (1979). If finding the correct legal decision requires the balancing of several hard-to-compare values, then inconsistency of judicial decisions is an implication of the Arrow Impossibility Theorem (Arrow, 1963). Inconsistencies that arise from the problem of multi-criterial choice cannot be remedied except through more fundamental changes to the nature of law.[4]

To provide context to our argument, it will be helpful to envision the set of

[1]We use the term "preference" purely in the technical sense as a binary relation on a set of alternatives. By "rationality" we refer to the extent to which a preference relation satisfies transitivity or other "coherence" conditions such as quasi-transitivity, acyclicity, semi-transitivity, and the interval order. By "rational" we refer to a preference relation that satisfies transitivity. For more on coherence, see Bossert and Suzumura (2007).

[2]A decision can also be internally inconsistent, in that its conclusions contradict its preferences. Kornhauser and Sager (1986) provide a separate framework to study the internal consistency of court decisions. Their notion is largely separable from the concept of external consistency characterized by rationality, and we do not consider it in this work.

[3]As a note, the behavioral literature suggests that the cognitive biases will themselves be consistent, while the resulting judgments will not be.

[4]See also Katz and Sandroni (2017). In the related context of loopholes, Katz (2010) argues that inconsistencies arising from the nature of multi-criterial choice should not be viewed as normatively undesirable. It is harder to make a similar argument for inconsistencies that arise from cognitive biases of judges.

judicial opinions that can be issued in a particular case. Judges need not agree as to the correct opinion; furthermore, judges may lack the skill to write an opinion that perfectly encapsulates their understanding of the law. Nonetheless, judges should be able to compare these opinions in terms of *how well they reflect the law*. The judge who writes the majority opinion will generally prefer it to a dissent; that same judge may prefer the dissent to the concurrence, or vice versa.[5] We use the term "preference" to describe a set of comparisons.[6]

Throughout the paper, we will assume that these preferences are complete. This means that, when faced with two possible opinions, judges will always be able to make a comparison. For example, when comparing the majority and the dissenting opinions, each judge will believe either that (a) the majority reflects the law better than does the dissent, (b) the dissent reflects the law better than does the majority, or (c) the majority and dissent reflect the law equally well.[7]

Judicial consistency requires the preferences to be transitive. This means that if a judge believes that the majority opinion reflects the law at least as well as the dissent, and that the dissent reflects the law at least as well as the concurrence, then that judge must also believe that the majority opinion reflects the law at least as well as does the concurrence. A judge cannot simultaneously believe that the majority opinion reflects the law better than does the dissent, that the dissent reflects the law better than does the concurrence, and that the concurrence reflects the law better than does the majority opinion. We use the terms consistent and transitive interchangeably.

The canonical model of preference aggregation was introduced in Arrow (1963), which studies the aggregation of transitive preferences. In our interpretation of this model, each judge in a multimember court has a consistent preferences over judicial opinions; the model examines the methods through which these individually consistent preferences can be aggregated to form the consistent preferences of the court. Arrow (1963) shows that no reasonable method of aggregating these preferences exists, where "reasonable" is defined as satisfying several well defined axioms. Easterbrook (1982) interprets this result as saying that multimember courts may behave in an inconsistent manner even if all judges are individually consistent. Judicial inconsistency is, at times, an unavoidable consequence of multimember courts.

We modify Arrow's framework to remove the assumption that preferences are transitive. In other words, we depart from the standard assumption that individual judges are consistent. While individual consistency may be desirable, it is also difficult to achieve. Judges are human.

We also formalize the language of "more rational" and "less rational." These terms

[5]Whether the correct party prevails in a particular case is only one factor in determining whether it reflects the law. A judge in the majority may find the legal arguments of a dissent to be more compelling than that of a concurrence in the same case.

[6]We do not assume that these comparisons reflect a personal want or desire on the part of the judges.

[7]Here, the language "reflects the law better than" represents a strict preference, while "reflects the law equally as well as" represents indifference.

have no meaning in Arrow's framework, in which a preference is either rational (that is, transitive) or is not, but cannot be somewhere in between. To talk about more and less rational, we use the concept of a rationality measure introduced by Afriat (1973). A rationality measure is a means by which preferences can be compared in terms of their coherence. Rather than pick a specific such measure, we define a broad class of such measures; our argument applies to any measure in this class.[8]

Using this concept, we define a new axiom: *monotonicity in rationality.* An aggregation method defines the preferences of the court as a function of the preferences of the individual judges. Suppose that at least one of these preferences changes, but each judge is at least as rational as before. Monotonicity in rationality requires that the court's preference must also be at least as rational as it was before. That is, an increase in rationality on the part of a judge should not make the court's preference less rational.[9]

Having removed the requirement of transitivity and introduced the monotonicity in rationality axiom, we are able to ask the question: is there a reasonable procedure that multimember courts can use to ensure that more consistent judges will not lead to less consistent verdicts?

To keep our analysis simple and clear, we retain the rest of the axioms in Arrow's framework: *weak Pareto*, *independence of irrelevant alternatives*, and *non-dictatorship*. Furthermore, following Arrow, we do not restrict the set of allowable preferences.[10] The *weak Pareto* axiom requires the aggregation method to respect unanimous opinions; that is, when all judges believe that one opinion better reflects the law than another, then the court believes that as well. The *independence of irrelevant alternatives* axiom requires that the comparison of two opinions does not depend on the existence of the third; that is, the determination of whether one concurrence reflects the law better than another is not affected by beliefs about the dissent. The *non-dictatorship* axiom requires that there is no dictator—that is, one judge whose view determines the outcome regardless of what the other judges believe.[11]

With this model, we reach our main result: no method of aggregating judges' beliefs can simultaneously satisfy monotonicity in rationality, weak Pareto, independence of irrelevant alternatives, and non-dictatorship. An increase in the consistency of an individual judge can lead to a decrease in the consistency of the court as a whole.

We illustrate this result by means of a simple example. It is clear that many

[8]Our definition is consistent with existing measures of rationality including those of Afriat (1973), Houtman and Maks (1985), and Varian (1990), as well as the money pump index (Echenique et al., 2011), the minimal swaps index (Apesteguia and Ballester, 2015), and the minimal cost index (Dean and Martin, 2016).

[9]This axiom is satisfied as long as it holds for some rationality measure; consequently, it fails only if it fails for every rationality measure.

[10]As in Arrow, the full strength of this assumption is not necessary for our result to hold.

[11]A dictatorship is approximately equivalent to a court with a single judge. This is not necessarily undesirable, but it undoes the point of a multimember court.

methods of aggregating judges' beliefs satisfy the remaining assumptions imposed by Arrow (1963): unrestricted domain, weak Pareto, independence of irrelevant alternatives, and non-dictatorship. A simple example is the method of majority decision, in which opinion x is weakly preferred to opinion y whenever the majority weakly prefers x to y. (For more on the method of majority decision, see Sen (1964, 1966).)

However, the method of majority decision has an undesirable property. Suppose that there are three judges, Alice, who prefers x to y, y to z, and x to z, Bob, who prefers z to x, x to y, and z to y, and Carol who prefers x to z, z to y, and y to x. Alice and Bob have consistent preferences; Carol does not. By the method of majority decision, x is preferred to y, z is preferred to y, and x is preferred to z, leading to a transitive and consistent collective preference. However, suppose that Carol realizes that her preferences are inconsistent and seeks to "correct" them. Upon reflection, she retains her view that opinion y better reflects the law than opinion x, but changes mind about opinion z, so that she now prefers y to z and z to x. As a consequence, the method of majority decision leads to the collective preference x to y, y to z, and z to x, and is no longer transitive. In this case, the collective preference became less consistent *because* Carol became more consistent.

This example suggests a problem with the method of majority decision. However, our theorem shows that this problem extends far beyond majority rule. Every collective choice rule which satisfies the remaining assumptions of Arrow (1963) will have this undesirable property. And it will have this property regardless of how we define rationality, as long as the measure used is within our broadly defined class. No matter what we do, we cannot remove the possibility that an increase in the consistency of individual judges will lead to a decrease in the consistency of the court as a whole. An implication is that the problem cannot be solved by abandoning majority rule.[12]

This result also suggests that we should exercise care when trying to increase the consistency of individual judges. For example, one might encourage a judge, when deciding a case, to revisit their past decisions relevant to the case at hand. This practice would enable the judge to ensure the new opinions are consistent with the old, or alternatively, to explain the older decision as a mistake. Similarly, attorneys might strategically cite older decisions of a judge to make that judge more likely to remember their past opinions. Such practices, whether intentional or not, can make it easier for litigants to "game" the decisions of a court.

[12]For this reason, we do not argue that the method of majority decision is any worse than any other method in this context. Several studies, including Sen (1966), Inada (1969), and Batra and Pattanaik (1972), examine the conditions under which pairwise majority does not lead to cycles. Dasgupta and Maskin (2008) provide an argument that the method of majority decision is more robust than other voting methods in that it violates the standard axioms on fewer domains.

1.1 Preference Aggregation as a Model of Courts

We interpret preferences as binary comparisons made by judges about the extent to which judicial opinions better represent the law (see Easterbrook, 1982). This idea is consistent with the view, commonly held by lawyers, that multimember courts are a mechanism for resolving differences among judges about the nature and content of the law. However, this is not the only possible interpretation of the model. Political scientists have traditionally taken a more realist perspective in which judges use their power to implement their preferred social policies. In this latter context, preferences are binary comparisons made by judges about the extent to which one opinion will lead to a better social consequence than another.[13] Both of these interpretations are consistent with the formal analysis presented in this paper.

The model assumes an unrestricted preference domain, as in Arrow (1963). That is, judges are unconstrained when comparing alternatives in terms of how well they reflect the law. The assumption may be overly restrictive; conceivably, some certain combinations of preferences may never arise in practice. If so, a rule could condition itself on this fact.

Fortunately, the full strength of the assumption is not required for the proof to hold. In practice, there only need to be three alternatives over which judges' comparisons are unconstrained. This does not seem to be a particularly strong assumption, as the law often seeks to satisfy multiple criteria (see Spitzer, 1979; Katz, 2010). For example, consider the question of whether a state may discriminate against resident aliens when issuing firearms permits.[14] At least two salient issues are relevant: the extent to which the constitution protects the right to a firearm, and the extent to which the government can discriminate against resident aliens. We can imagine three opinions: one which requires the state to issue permits in a nondiscriminatory manner ("P"), one which does not require the state to issue permits, but also does not allow such discrimination ("NP"), and one which allows the state to issue permits only to citizens ("D"). A judge who believes that opinion P best reflects the law may prefer NP to D (because this judges prioritizes equal protection over gun rights) or D to NP (because this judge prioritizes gun rights, and D allows more individuals to have permits). Similarly, a judge who believes that NP best represents the law may prefer D to P, or P to D, for similarly reasons. And a judge who believes that D best represents the law is not constrained on how that judge ranks opinions P and

[13]This generally conforms to a distinction made between preference aggregation and judgment aggregation (see Sen, 1977; Kornhauser and Sager, 1986), in which "preference" refers the judge's policy preferences and "judgment" refers to the judge's belief about the correct interpretation of the law. The terminology can be confusing, as the same language is used to describe the formal models of preference aggregation (see Arrow, 1963) and judgment aggregation (see Kornhauser and Sager, 1986). While this paper relies on the *formal* model of preference aggregation, the underlying binary relation can be interpreted in terms of the the judge's beliefs about the law or in terms of the judge's policy preferences.

[14]See *Fotoudis v. City & County of Honolulu*, 54 F. Supp. 3d 1136 (D. Haw. 2014).

NP. Similarly, in many cases, the set of possible opinions may include one that is ground in originalism, another in textualism, and a third in contextualism; there is no a priori reason that a judge must rank these in any particular way.

The model of external consistency involves judges making binary comparisons between judicial opinions across cases, over time. Of course, each opinion is tailored to the case, and no two cases are identical. The model can account some of the differences; for example, we can deal with time inconsistency by reinterpreting the binary relation as "reflects the law at time z at least as well as." However, as with every model of external consistency, we cannot make comparisons across cases unless some minor differences are treated as irrelevant.

1.2 Why should courts be consistent?

One may ask whether a lack of consistency is a problem. Inconsistency on an individual level, of course, reflects poorly on a judge. But there is nothing fundamentally wrong with a inconsistency at the level of a multimember court, as such inconsistency can arise despite the consistency of the individual judges. As long as we understand that judicial inconsistency is unavoidable, we should not view it as normatively undesirable.

Nonetheless, there are several reasons why inconsistency of a court can be problematic. To begin, not everyone is familiar with Easterbrook (1982). Inconsistent decisions may be viewed negatively by the general public, and such inconsistencies may harm the reputation and perceived legitimacy of the court.

Second, inconsistency of court decisions leads to problem of indeterminacy: a judgment cannot be the opinion of a court, because the court does not have an opinion in any meaningful set. Of course, the indeterminacy problem does not mean that courts cannot adjudicate cases brought before it, but only that their judgments are at times akin to a coin flip.

The first problem can be avoided, to some extent, through the principle of stare decisis. Courts that follow precedent will make decisions that are observably consistent. The second problem, however, is made worse. An inconsistent court that adheres to stare decisis will be subject to two problems not exhibited by a coin flip. The first of these is path dependence: the order in which questions are presented to the court may affect the determination of subsequent cases. A skilled advocate with a goal of legal reform—a cause lawyer—may strategically choose cases so as to present questions to the court in a particular order.[15] The second problem is strategic voting: judges may issue 'false' opinions to limit the implications of stare decisis down the road. A judge who understands the strategy of the cause lawyer may seek to thwart it in advance by writing opinions with which the judge does not agree. False opinions

[15] Stearns (1995a,b) argue that doctrines of standing and justiciability were developed to avoid the problem posed by path-dependence; in essence, making decisions more similar to coin flips. For more on cause lawyering, see Baker and Biglaiser (2014).

misrepresent the law and are consequently of limited value as precedent. If we cannot tell which opinions are false, the rule of law suffers as a whole.

1.3 Related literature.

This paper is closely related to the formal literature in social choice theory. Previous studies have sought to weaken the assumption of rationality in Arrow (1963) by permitting a wider range of collective preferences. The case of quasi-transitive collective preferences was studied by many including Gibbard (1969), Sen (1969, 1970), Schick (1969), and Mas-Colell and Sonnenschein (1972), and that of acyclic preferences by Mas-Colell and Sonnenschein (1972), Blau and Deb (1977), Deb (1981), and Blair and Pollak (1982). For more see Sen (1977).[16]

Other scholars have tried to avoid the negative conclusions of Arrow (1963) by moving in the opposite direction. Instead of expanding the range of admissible collective preferences, these studies restrict the domain of allowable preferences. The most prominent example is that of the single-peaked preference restriction of Black (1948a,b) and Arrow (1963).

The most closely related formal literature is the study of tournaments, which are described by binary relations which are antisymmetric and complete. Unlike the preferences that we study, tournaments do now allow for the possibility of ties. In this context, Monjardet (1978) shows that a collective choice rule that (a) maps every profile of transitive preferences into a transitive preference, (b) satisfies the independence of irrelevant alternatives axiom and (c) satisfies a non-imposition axiom is either dictatorial or "persecutive." Roughly speaking, persecutive means that the decisive coalitions are all the coalitions that do not contain a certain individual i. A related result can be found in Barthelemy (1982). As far as we can tell, the monotonicity in rationality axiom that we present is new to this paper.

2 Model and result

Let X be a set of alternatives, $|X| \geq 3$. A preference relation R on X is (a) **complete** if for all $x, y \in X$, either xRy or yRx,[17] and (b) **transitive** if for all $x, y, z \in X$, xRy and yRz implies that xRz. Let $\mathscr{R}$ be the set of all complete preference relations on X. A **preference ordering** is a preference relation which is complete and transitive. Let $\mathscr{R}^* \subseteq \mathscr{R}$ be the set of preference orderings on X.

For a preference relation $R \in \mathscr{R}$ we denote by P its asymmetric component; that is, xPy if xRy but not yRx. A preference relation is **acyclic** if, for every $k \geq 3$ and every $x^1, ..., x^k \in X$, $x^i P x^{i+1}$ for all $i < k$ implies that $x^1 R x^k$.[18] Let $\mathscr{R}^a \subseteq \mathscr{R}$ be

[16] A related literature looks at the case in which preferences are transitive but not necessarily complete (see Baucells and Shapley, 2008; Pini et al., 2009).

[17] This definition of completeness subsumes reflexiveness for the sake of simplicity.

[18] For ease of exposition, we have chosen a definition that assumes completeness.

the set of preference relations which are complete and acyclic. It is well known that $\mathscr{R}^* \subsetneq \mathscr{R}^a \subsetneq \mathscr{R}$ (see Suzumura, 1983).

For $Y \subseteq X$, denote by $\mathscr{R}|_Y$ the set all complete preference relations on Y, and denote by $\mathscr{R}^*|_Y$ the set all preference orderings in $\mathscr{R}|_Y$. For $R \in \mathscr{R}$ and $Y \subseteq X$, denote by $R|_Y \in \mathscr{R}|_Y$ the restriction of R to Y.

Let $N \equiv \{1, ..., n\}$ be a finite set of agents, $n \geq 2$. A *profile* $\mathbf{R} = (R_1, \cdots, R_n) \in \mathscr{R}^N$ is a vector of preference relations, one for each agent. A **collective choice rule** is a mapping $f: \mathscr{R}^N \to \mathscr{R}$.[19] We define $R_0 \equiv f(\mathbf{R})$ to be the social relation, and we denote by P_0 its asymmetric component. A set of elements $Y \subseteq X$ is **top-ranked** in profile $\mathbf{R}$ if $a \in Y$, $b \in X \setminus Y$, and $i \in N$ implies that aP_ib.

A **rationality measure**[20] is a binary relation $\succcurlyeq$ on $\mathscr{R}$ which satisfies the following properties:

1. For all $R \in \mathscr{R}$, $R \succcurlyeq R$.
2. For all $R' \in \mathscr{R}^*$ and $R \in \mathscr{R}$, $R \succcurlyeq R'$ implies that $R \in \mathscr{R}^*$.
3. For all $R^* \in \mathscr{R}^*$ and $R \in \mathscr{R} \backslash \mathscr{R}^a$: if there is a three-element set $Y \subseteq X$ which is top-ranked in both relations, such that $R^* \big|_{X \setminus Y} = R \big|_{X \setminus Y}$, then $R^* \succcurlyeq R$.

For two profiles $\mathbf{R}, \mathbf{R}' \in \mathscr{R}^N$ we write $\mathbf{R} \succcurlyeq \mathbf{R}'$ if $R_i \succcurlyeq R_i'$ for all $i \in N$.

Property 1, known as reflexivity, requires each preference relation to be "at least as rational" as itself. Property 2 requires that only a transitive preference ordering can be at least as rational as another transitive preference ordering. Property 3 requires that every transitive preference ordering must be at least as rational as every cyclic preference relation, provided that the two relations are identical except for the three top-ranked elements.

A wide range of rationality measures satisfies these conditions. We provide two examples. The simplest rationality measure $\succcurlyeq'$ is one for which $R^* \succcurlyeq' R$ if and only if (a) $R^* \in \mathscr{R}^*$ and $R \in \mathscr{R} \setminus \mathscr{R}^*$ or (b) $R^* = R$. A more complicated rationality measure can incorporate the structure of coherence properties studied in the social choice literature. For example, we can define a rationality measure $\succcurlyeq''$ such that $R^* \succcurlyeq'' R$ if and only if (a) there exists an $\mathscr{C} \in \{\mathscr{R}^*, \mathscr{R}^a\}$ such that $R^* \in \mathscr{C}$ but $R \notin \mathscr{C}$ or (b) $R^* = R$.[21]

[19] This is a slight change from the standard definition, in which the domain of a collective choice rule is a set of preference orderings (see Sen, 1970).

[20] This definition is broad and only contains minimal conditions for a rationality measure. Arguably, a measure should also be transitive. The breadth of the definition is desirable because it implies a weaker monotonicity in rationality axiom, below. The definition is not used for any other purpose in this paper.

[21] The three classes were chosen for the ease of the exposition. Clearly a rationality measure can incorporate any number of classes, and these not be totally ordered through set inclusion. In particular, the rationality measure can incorporate the coherence properties of quasi-transitivity, semi-transitivity and the interval order. See Cato (2012).

Our first axiom, monotonicity in rationality, requires that if preference relations change, and each individual's new preference relation stays at least as rational as it was before the change, then the social preference must stay at least as rational.

Monotonicity in rationality: For all $\mathbf{R}, \mathbf{R}' \in \mathscr{R}^N$, if $\mathbf{R} \succcurlyeq \mathbf{R}'$ then $R_0 \succcurlyeq R_0'$.

The following three axioms were introduced by Arrow (1963); for brevity, we will not discuss them.

Weak Pareto: For every $\mathbf{R} \in \mathscr{R}^N$ and $x, y \in X$, if xP_iy for all $i \in N$, then xP_0y.

Independence of Irrelevant Alternatives: For all $Y \subseteq X$ and $\mathbf{R}, \mathbf{R}' \in \mathscr{R}^N$, if $\mathbf{R}|_Y = \mathbf{R}'|_Y$, then $R_0|_Y = R_0'|_Y$

An individual $d \in N$ is a *dictator* if, for all $\mathbf{R} \in \mathscr{R}^N$, xP_dy implies that xP_0y.

Non-Dictatorship: There does not exist a dictator.

We can now turn to the main result. The proof is given in the appendix.

Theorem 1. *There does not exist a collective choice rule that satisfies monotonicity in rationality, weak Pareto, independence of irrelevant alternatives, and non-dictatorship. Furthermore, the axioms are independent.*

3 Conclusion

We argue an increase in the consistency of individual judges can lead to a decrease in the consistency of courts. We do so by identifying consistency with transitivity of a preference relation, and departing from the standard approach to preference aggregation in three ways. First, we modify the standard model to study the case in which neither individual nor collective preferences are required to satisfy transitivity or other coherence conditions. Second, we introduce the concept of an ordinal rationality measure which can be used to compare preference relations in terms of their level of coherence. Third, using this measure, we introduce a monotonicity in rationality axiom that requires the collective preference to become more rational when the individual preferences become more rational. We show that for any ordinal rationality measure, it is impossible to find a collective choice rule which satisfies the monotonicity in rationality axiom and the other standard assumptions introduced by Arrow (1963): unrestricted domain, weak Pareto, independence of irrelevant alternatives, and non-dictatorship.

One might argue that inconsistencies in court decisions are not a problem if these inconsistencies arise from the multi-member nature of appellate courts. As long as the public can see that the individual judges are behaving in a consistent manner, inconsistent judgments may be thought of as a form of bad luck, rather than as unfair. However, there are still several problems that remain. First, judges will still be susceptible to problems akin to agenda manipulation (See Levine and Plott, 1977; Plott and Levine, 1978), where one might try to make a judge more consistent in order to get a less consistent result. Second, the source of the inconsistency is not always apparent; judges do not always write concurring and dissenting opinions, stating their reasons in full, and on some courts, judges are not allowed to write concurring or dissenting opinions at all. Third, a similar aggregation problem exists with jurors, whose decision process is not at all visible to the outside world.

Our primary interpretation of this model is in terms of judicial consistency. However, there may also be implications outside of the legal context. For example, a group of people may become more susceptible to "Dutch books" when the individuals' susceptibility lessens.

A lesson of the Arrow theorem is that groups can be inconsistent even though each member is individually consistent. As our proof illustrates, groups can be consistent even though some members are individually inconsistent. This corresponds with some results in the experimental literature that suggests that groups are more rational than individuals (see Charness and Sutter, 2012).[22] In much of this literature, individuals are presumed to have preferences over money where they prefer more to less; behavior inconsistent with such preferences are generally viewed as cognitive mistakes. This is a different notion of rationality than in found our setup, where judges have beliefs over the degree to which different judicial opinions represent the law. When dealing with cognitive mistakes, it seems natural that increases in individual rationality would lead to increase in group rationality; nonetheless, our result suggests that one may want to test this proposition experimentally.

On a more technical level, a natural question involves the extent to which the monotonicity in rationality axiom introduced in this paper substitutes for the standard assumption of transitivity. For example, consider an axiom, "transitive-to-transitive" which requires every profile of transitive preference relations to map to a transitive social relation.[23] There is no logical relation between this axiom and the monotonicity in rationality axiom we propose. For example, a constant rule that maps all profiles to the same non-transitive social preference satisfies monotonicity in rationality but not this axiom. To see that a rule may satisfy the transitive-to-transitive axiom but not monotonicity in rationality, consider a rule in which the social preference coincides with that of the first agent when that agent, and only that agent, has a non-transitive preference relation, and which otherwise maps to a fixed transitive social preference. When all agents' preferences are transitive, this rule will lead to a transitive social preference, and thus it satisfies the transitive-to-transitive axiom. If the first agent's preferences change and become non-transitive, the social preference will clearly become non-transitive. However, if a second agent's prefer-

[22] We thank an anonymous referee for this insight.
[23] We thank Eric Maskin for suggesting this idea.

ences change and become non-transitive, the social preference will change back to the original transitive preference, thus violating monotonicity in rationality.

However, in the presence of weak Pareto, independence of irrelevant alternatives, and non-dictatorship, the transitive-to-transitive axiom would also lead to an impossibility result. To see this, note that in the context of Arrow (1963), the transitive-to-transitive axiom implies that Arrow's condition of unrestricted domain is satisfied on the set of transitive profiles. Consequently, when combined with weak Pareto and independence of irrelevant alternatives, this axiom implies the existence of an individual d who is decisive over every pair of alternatives for every transitive profile. That is xP_dy implies xP_0y for every transitive profile. By the independence of irrelevant alternatives axiom, however, it becomes irrelevant whether the profile is transitive; and hence individual d is a dictator.

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Appendix

To prove Theorem 1 we make use of the following lemma. For a coalition $K \subseteq N$, we define $x\bar{D}_K y$ as the statement that the coalition K is decisive for x over y; that is, if xP_iy for all $i \in K$, then xP_0y. Similarly, we define xD_Ky as the statement that the coalition K is decisive for x over y when all others are opposed; that is, if xP_iy for all $i \in K$ and yP_ix for all $i \notin K$, then xP_0y.

Lemma 1. *If a collective choice rule f satisfies monotonicity in rationality, weak Pareto, and independence of irrelevant alternatives, then whenever xD_Ky for a coalition $K \subseteq N$ and some pair of alternatives $x, y \in X$, it follows that $w\bar{D}_Kz$ for every pair $w, z \in X$.*

Proof of Lemma 1. Let the collective choice rule f satisfy the monotonicity in rationality, weak Pareto, and independence of irrelevant alternatives axioms. Let $K \subseteq N$ and $x, y \in X$ such that xD_Ky.

Step one. We claim that, for all $z \in X \setminus \{x, y\}$, if $\mathbf{R} \in \mathscr{R}^N$ such that (a) $R_i\big|_{\{x,y,z\}} \in \mathscr{R}^*\big|_{\{x,y,z\}}$ for all $i \in N$, (b) xP_iy for all $i \in K$, and (c) $R_i\big|_{\{x,y,z\}} = R_j\big|_{\{x,y,z\}}$ for all $i, j \in K$, then $R_0\big|_{\{x,y,z\}} \in \mathscr{R}^*\big|_{\{x,y,z\}}$.

To prove this claim, let $z \in X \setminus \{x, y\}$ and let $\mathbf{R} \in \mathscr{R}^N$ satisfying (a), (b), and (c). From the independence of irrelevant alternatives axiom we can assume, without loss of generality, that the set $\{x, y, z\}$ is top-ranked in each R_i and that $R \in \mathscr{R}^{*N}$. Let $\mathbf{R}^\circ \in \mathscr{R}^N$ such that (i) $R_i^\circ = R_i$ for all $i \in K$, (ii) $yP_i^\circ x$, $xP_i^\circ z$, and $zP_i^\circ y$ for all $i \notin K$, and (iii) $\mathbf{R} \succcurlyeq \mathbf{R}^\circ$. Because xD_Ky it follows that $xP_0^\circ y$.

From condition (c) it follows that there are two cases: either $xP_i^\circ z$ for all $i \in K$, or $zR_i^\circ x$ for all $i \in K$. In the former case, $xP_i^\circ z$ for all $i \in N$, which implies (by weak Pareto), that $xP_0^\circ z$. Because $xP_0^\circ y$ and $xP_0^\circ z$, it follows that $R_0^\circ\big|_{\{x,y,z\}} \in \mathscr{R}^*\big|_{\{x,y,z\}}$. In the latter case, $zP_i^\circ y$ for all $i \in N$, which implies (by weak Pareto), that $zP_0^\circ y$. Because $xP_0^\circ y$ and $zP_0^\circ y$, it follows that $R_0^\circ\big|_{\{x,y,z\}} \in \mathscr{R}^*\big|_{\{x,y,z\}}$. Because $R_0^\circ\big|_{\{x,y,z\}} \in \mathscr{R}^*\big|_{\{x,y,z\}}$ it follows from monotonicity in rationality and independence of irrelevant alternatives that $R_0\big|_{\{x,y,z\}} \in \mathscr{R}^*\big|_{\{x,y,z\}}$, proving the claim.

Step two. Let $\mathbf{R}' \in \mathscr{R}^{*N}$ such that, for all $i \in K$, $xP_i'y$ and $yP_i'z$ and, for all $i \notin K$, $yP_i'x$ and $yP_i'z$. Because xD_ky it follows that $xP_0'y$, and because $yP_i'z$ for all $i \in N$ it follows from weak Pareto that $yP_0'z$. Because $\mathbf{R}'$ satisfies requirements (a), (b), and (c) of step one, it follows that $R_0'\big|_{\{x,y,z\}} \in \mathscr{R}^*\big|_{\{x,y,z\}}$ and therefore $xP_0'z$. By the independence of irrelevant alternatives axiom, this implies that $x\bar{D}_Kz$. In other words:

$$xD_Ky \text{ implies that } x\bar{D}_Kz. \tag{1}$$

Now, let $\mathbf{R}'' \in \mathscr{R}^{*N}$ such that, for all $i \in K$, $zP_i''x$ and $xP_i''y$ and, for all $i \notin K$, $zP_i''x$ and $yP_i''x$. Because xD_ky it follows that $xP_0''y$, and because $zP_i''x$ for all $i \in N$ it follows from weak Pareto that $zP_0''x$. Because $\mathbf{R}''$ satisfies requirements (a), (b), and (c) of step one, it follows that $R_0''\big|_{\{x,y,z\}} \in \mathscr{R}^*\big|_{\{x,y,z\}}$ and therefore $zP_0''y$. By the independence of irrelevant alternatives axiom, this implies that $z\bar{D}_K y$. In other words:

$$xD_K y \text{ implies that } z\bar{D}_K y. \tag{2}$$

By interchanging y and z in statement (2) it follows that $xD_K z$ implies that $y\bar{D}_K z$, and by replacing x by y, y by z, and z by x in statement (1) it follows $yD_K z$ implies that $y\bar{D}_K x$. As a consequence, it follows that

$$xD_K y \text{ implies that } y\bar{D}_K x. \tag{3}$$

By interchanging x and y in statements (1), (2), and (3), it follows that $yD_K x$ implies that $y\bar{D}_K z$, $z\bar{D}_K x$, and $x\bar{D}_K y$. As a consequence, we are led to the implication that for every $\{x, y, z\} \subseteq X$, if $xD_K y$ then $a\bar{D}_K b$ for every $a, b \in \{x, y, z\}$. If $|X| = 3$ this completes the proof.

If $|X| \geq 4$, then let $w \in X \setminus \{x, y, z\}$. By replacing y with z and z with w in statement (2), it follows that $xD_K z$ implies that $w\bar{D}_K z$, concluding the proof.[24] $\square$

Proof of Theorem 1. Let f be a collective choice rule that satisfies the monotonicity in rationality, weak Pareto, independence of irrelevant alternatives, and non-dictatorship axioms. We will derive a contradiction.

Let $S \subseteq N$ be a decisive coalition of minimal size, so that $|T| < |S|$ implies that $xD_T y$ is false for all $x, y \in X$. By the weak Pareto axiom, such a coalition S exists. By the non-dictatorship axiom and Lemma 1, $|S| \geq 2$. Without loss of generality, let $xD_S y$. Let $S_1 \subseteq S$ such that $|S_1| = 1$, let $S_2 \equiv S \setminus S_1$, and let $S_3 \equiv N \setminus S$.

Let $\mathbf{R} \in \mathscr{R}^{*N}$ be a transitive profile such that (a) $xP_i y$, $yP_i z$, and $xP_i z$ for all $i \in S_1$, (b) $zP_i x$, $xP_i y$, and $zP_i y$ for all $i \in S_2$, and (c) $yP_i z$, $zP_i x$, and $yP_i x$ for all $i \in S_3$.

Let $R_\times \in \mathscr{R}$ such that $xP_\times y$, $yP_\times z$, and $zP_\times x$. Let $R_+ \in \mathscr{R}$ such that $xP_+ z$, $zP_+ y$, and $yP_+ x$.

Let $\mathbf{R}^A, \mathbf{R}^B, \mathbf{R}^C \in \mathscr{R}^N$ be profiles such that (a) $R_i^A = R_i^B = R_i^C = R_\times$ for all $i \in S_1$, (b) $R_i^A = R_i^B = R_i^C = R_+$ for all $i \in S_2$, and (c) $R_i^A = R_\times$, $R_i^B = R_+$, and $R_i^C = R_i$ for all $i \in S_3$.

Because of the independence of irrelevant alternatives axiom, we can assume, without loss of generality, that the elements $x, y, z \in X$ are top-ranked in profiles $\mathbf{R}$, $\mathbf{R}^A$, $\mathbf{R}^B$, and $\mathbf{R}^C$ and that $\mathbf{R}\big|_{X\setminus\{x,y,z\}} = \mathbf{R}^A\big|_{X\setminus\{x,y,z\}} = \mathbf{R}^B\big|_{X\setminus\{x,y,z\}} = \mathbf{R}^C\big|_{X\setminus\{x,y,z\}}$. It follows that $\mathbf{R} \not\succcurlyeq \mathbf{R}^A$, $\mathbf{R} \not\succcurlyeq \mathbf{R}^B$, and $\mathbf{R} \not\succcurlyeq \mathbf{R}^C$. Suppose, contrariwise, that R_0 is not transitive. It follows from monotonicity in rationality that neither R_0^A, R_0^B, nor R_0^C may be transitive. Because S_2 is not a decisive coalition, it follows that $xR_0^A y$, $yR_0^A z$, and $zR_0^A x$. Because R_0^A is not transitive it follows that $S_1 \cup S_3$ must be decisive for at least one of the three pairs x over y, y over z, or z over x. By Lemma 1, it follows that $xD_{S_1 \cup S_3} y$ for all $x, y \in X$.

[24] We thank an anonymous referee for simplifying the proof.

Because S_1 is not a decisive coalition, it follows that xR_0^Bz, zR_0^By, and yR_0^Bx. Because R_0^B is not transitive it follows that $S_2 \cup S_3$ must be decisive for at least one of the three pairs x over z, z over y, or y over x. By Lemma 1, it follows that $xD_{S_2 \cup S_3}y$ for all $x, y \in X$.

Because $xD_{S_1 \cup S_3}y$ for all $x, y \in X$ it follows that yP_0^Cz and zP_0^Cx. Because $xD_{S_2 \cup S_3}y$ for all $x, y \in X$ it follows that yP_0^Cx. Therefore, R_0^C is transitive, which is a contradiction that proves that R_0 must be transitive.

By assumption, the coalition $S = S_1 \cup S_2$ is decisive for x over y. This implies that xP_0y. Because zP_iy only for $i \in S_2$ and S_2 is not decisive, it follows that yR_0z. Because R_0 is transitive, it follows that xP_0z. But this means that $xD_{S_1}z$, which implies, by Lemma 1, that S_1 is a dictator. This violates the non-dictatorship axiom, and concludes the impossibility proof.

Independence of the Axioms. We describe four collective choice rules. Each of the rules satisfies three of the axioms while violating the fourth. This is sufficient to prove the independence of the axioms.

Rule 1. For all $x, y \in X$, let xR_0y if and only if $|\{i \in N : xR_iy\}| \geq |\{i \in N : yR_ix\}|$. This rule clearly satisfies weak Pareto, independence of irrelevant alternatives, and non dictatorship, but violates monotonicity in rationality.

Rule 2. Let $d \in N$. For all $x, y \in X$, let xR_0y if and only if xR_dy. This rule clearly satisfies monotonicity in rationality, weak Pareto, and independence of irrelevant alternatives, but violates non-dictatorship.

Rule 3. Let $\mathscr{R}^T$ be the set of preference relations such that $R \in \mathscr{R}^T$ and $R' \succcurlyeq R$ implies that $R' \in \mathscr{R}^T$. If $R_1, R_2 \in \mathscr{R}^T$, let $f(R_1, ..., R_n) = R_1$, otherwise, let $f(R_1, ..., R_n) = R_2$. This rule satisfies monotonicity in rationality, weak Pareto, and non dictatorship, but violates independence of irrelevant alternatives.

Rule 4. For all $x, y \in X$, let xR_0y. This rule clearly satisfies monotonicity in rationality, independence of irrelevant alternatives, and non-dictatorship, but violates weak Pareto. $\square$