Chapter 4 : Aerodynamics of Wind Turbines

1. Power in the Wind

Long in front of the rotor, the wind speed is $v1$. After passing the wind turbine rotor (called the rotor in the following), the wind speed would be reduced to $v3$. The pressure distribution is as follows.
The initial pressure is $p1$. As the air moves towards the rotor, the pressure rises to a pressure $p+$ and by passing the rotor, the pressure suddenly falls by an amount of Δp i.e. the pressure is here $p- = p+ - \Delta p$.
After passing the rotor, and far down stream the pressure again rises to $p3 = p1$. Curves for wind speed and pressure are shown in figure 1.
Bernoulli's equation: If we look at the air moving towards the rotor plane, we can use the Bernoulli's equation to find the relation between the pressure p and the speed v, while we can make the assumption that the flow is frictionless:

$$\frac{1}{2}\rho v^2 + p = p_{tot} \quad \text{(Eq.1)}$$

Where p_{tot} is the total pressure, which is constant. That means, if the speed of flow goes up, the pressure goes and vice versa.

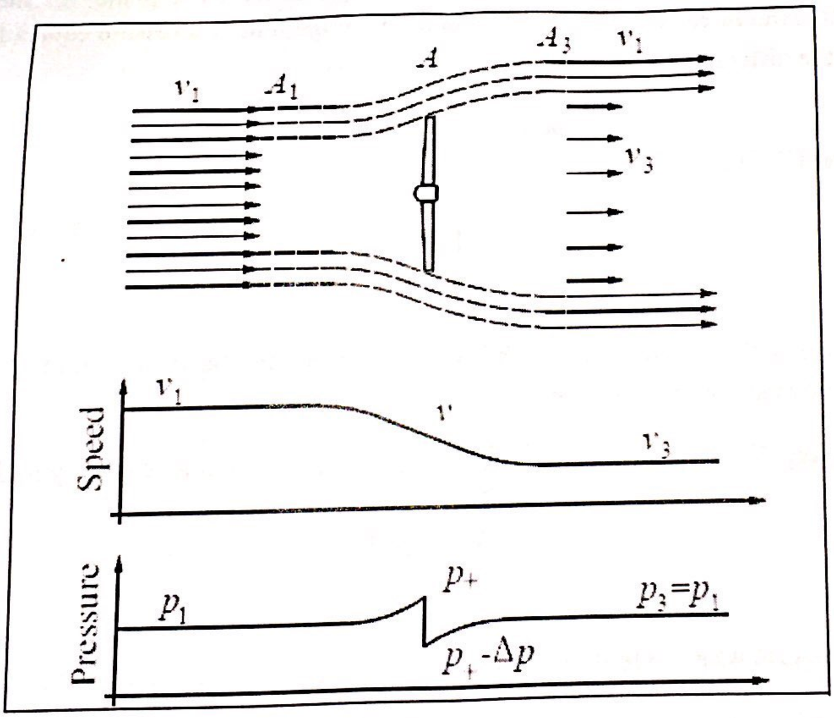

Figure1: Interaction between wind and wind turbine

- v_1 = wind speed upstream rotor
- v_3 = wind speed downstream rotor

Assumption: The pressure changes are relatively small compared to the pressure in the ambient. Therefore we assume the density to be constant.

If we use (Eq1) for the flow up stream of the rotor, we get:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_+ + \frac{1}{2}\rho v^2 \qquad \text{(Eq.2)}$$

If we use (Eq1) down stream of the rotor plane, we get:

$$p_+ - \Delta p + \frac{1}{2}\rho v^2 = p_1 + \frac{1}{2}\rho v_3^2 \qquad \text{(Eq.3)}$$

Subtracting (Eq3) from (Eq2) we get

$$\Delta p = \frac{1}{2}\rho \; v_1^2 - v_3^2 \quad [\text{Pa}] \qquad \text{(Eq.4)}$$

Change of momentum: This differential pressure can also be calculated on the basis of 'change in momentum'. If we look at one square meter of the rotor plane, the mass flow equals ρv. Momentum equals mass times velocity, with the unit N. Pressure equals force per surface, then the differential pressure can be calculated as:

$$\Delta p = \rho v \;\; v_1 - v_3 \quad [\text{Pa}] \qquad \text{(Eq.5)}$$

Now (Eq4) and (Eq5) give

$$v = \frac{1}{2} \;\; v_1 + v_3 \quad [\text{m/s}] \qquad \text{(Eq.6)}$$

This indicates that the speed of air in the rotor plane equals the mean value of the speed upstream and downstream of the rotor.

<u>Power production</u>: The power of the turbine equals the change in kinetic energy in the air.

$$P = \frac{1}{2}\rho v \;\; v_1^2 - v_3^2 \; .A \;\; [\text{W}] \qquad \text{(Eq.7)}$$

Here A is the surface area swept by the rotor.

The axial force (Thrust) on the rotor can be calculated as:

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$$T = \Delta p . A \quad [\mathrm{N}] \qquad \text{(Eq.8)}$$

We now define 'the axial interference factor' a such that:

$$v = (1-a)v_1 \quad [\mathrm{m/s}] \qquad \text{(Eq.9)}$$

Using (Eq6) and (Eq9) we get

$$v_3 = (1-2a)v_1 \qquad \text{(Eq. 10)}$$

And (Eq7) and (Eq8) can be written as

$$P = 2\rho a \;\; 1-a \;^{2}\, v_1^3 A \quad [\mathrm{W}] \qquad \text{(Eq. 11)}$$

$$T = 2\rho a \;\; 1-a \;\; v_1^2 A \quad [\mathrm{N}] \qquad \text{(Eq.12)}$$

We now define two coefficients , one of the power production and one of the axial forces as:

$$C_P = 4a \;\; 1-a \;^{2} \qquad \text{(Eq. 13)}$$

$$C_T = 4a \;\; 1-a \qquad \text{(Eq.14)}$$

Then (Eq10) and (Eq11) can be written as

$$P = \frac{1}{2}\rho v_1^3 A C_P \qquad \text{(Eq.15)}$$

$$T = \frac{1}{2}\rho v_1^2 A C_T \qquad \text{(Eq.16)}$$

In Figure 2, curves for C_P and C_T are shown

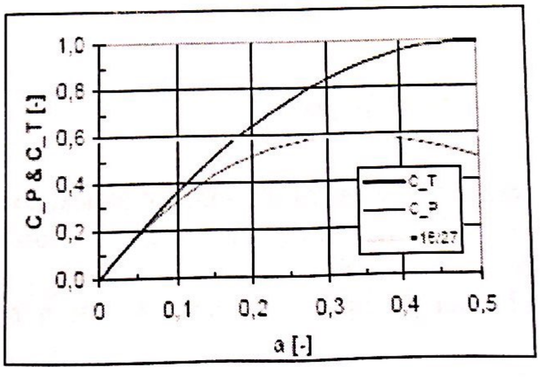


Figure 2. Coefficient of power C_P and coefficient of axial force C_T for an idealized wind Turbine

As shown, C_P has an optimum at about 0.59 (exactly 16/27) at an axial interference factor of 0.33 (exactly 1/3). According to Betz have

$$P_{Betz} = C_{P,Betz}(1/2)\rho v_1^3 A \text{ avec } C_{P,Betz} = \frac{16}{27} \text{ [W]} \qquad \text{(Eq.17)}$$

2. Rotor Design

2.1. Air Foil Theory

Figure 3 shows a typical wing section of the blade.
The air hits the blade in an angle α_A which is called the "angle of attack". The reference line" for the angle on the blade is most often "the chord line". The force on the blade F can be divided into two components – the lift force F_L and the drag force F_D and the lift force is – per definition – perpendicular to the wind direction.

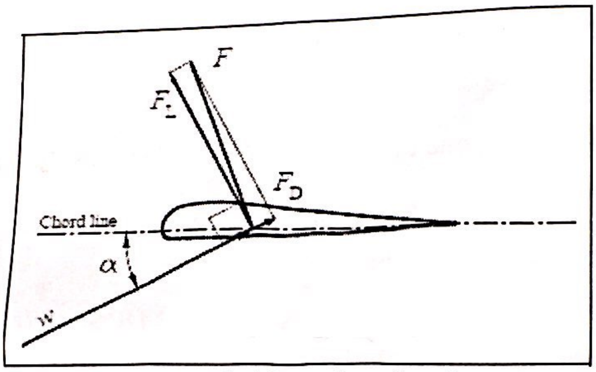


Figure 3: Definition of angle of attack

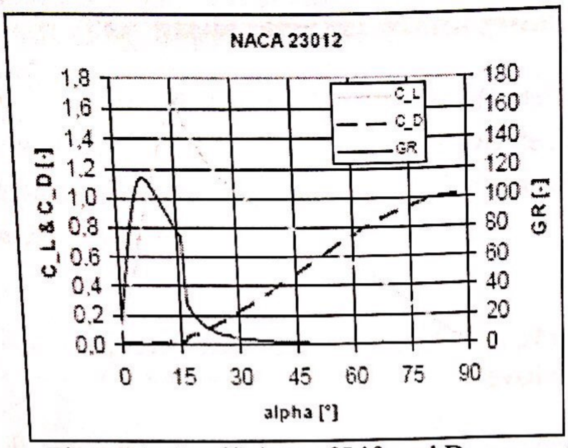
The Lift Force can be calculated as

$$F_L = C_L(\frac{1}{2})\rho w^2 (bc) \qquad \text{(Eq.18)}$$

where C_L is the "coefficient of lift", ρ is the density of air, w the relative wind speed, b the width of the blade section and c the length of the chord line.
Similar for the drag force

$$F_D = C_D(\frac{1}{2})\rho w^2 (bc) \qquad \text{(Eq.19)}$$

The coefficient of lift and drag both depend of the angle of attack, see figure 4.
The ratio CL/CD is called the "glide ratio", i.e. $GR = CL/CD$. Normally we are interested in at high glide ratio for wind turbines as well as for air planes. Values up to 100 or higher is not uncommon and the angles of attack giving maximum are typical in the range 5 – 10°.



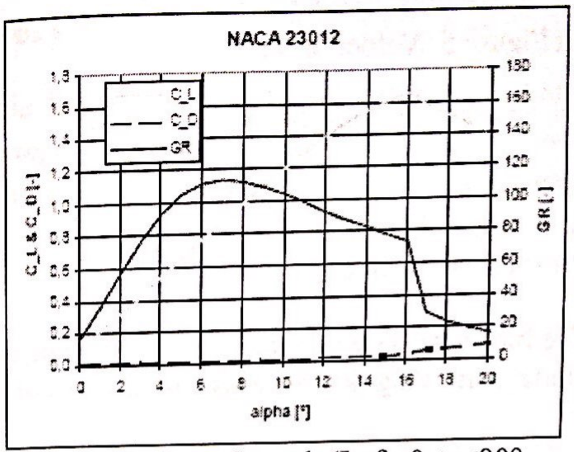


Figure 4: Coefficient of Lift and Drag as a function of the angle of attack (Left: 0<α<90°; right: 0< α <20°)

2.2. Velocity Triangle
After Betz

Figure 5 shows the velocities and the angles in a given distance, r, from the rotor axis. The rotor shown on the figure is with two blades, i.e. $B = 2$. To design the rotor we have to define the pitch angle β and the chord length c. Both of them depend on the given radius, that we are looking at therefore we sometimes write $\beta(r)$ and $c(r)$.

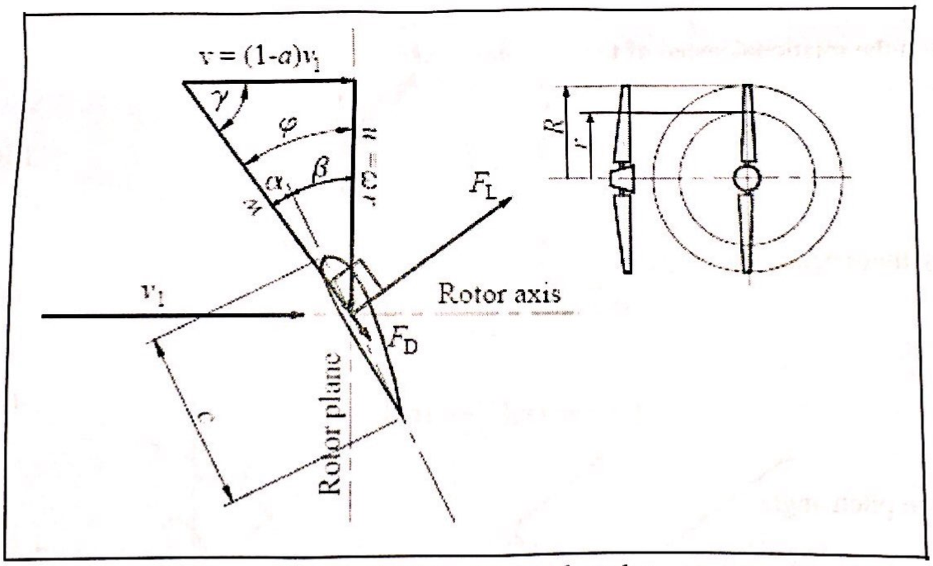


Figure 5: Velocities and angles

Figure 5 shows the profile and the wind speeds in one ring. The angle of attack is given **α=φ-β**

Angles, that all depend on the given radius
- $\gamma(r)$ = angle of relative wind to rotor axis
- $\varphi(r)$ = angle of relative wind to rotor plane
- $\beta(r)$ = pitch angle of the blade

On Figure 5, Velocities are

- v_1 = wind speed upstream rotor (m/s)

- v = axial wind speed in the rotor plane
- u = tangential wind speed
- w = relative wind speed

The blade, as shown on the figure is moving up wards, thus the wind speed, seen from the blade, is moving down wards with a speed of u. We have

$$w^2 = v^2 + u^2 \qquad \text{(Eq.20)}$$

NB Betz does not include rotation of the wind, i.e. $a' = 0$ (see definition of a' later)
Therefore

$$u = \omega r \quad \text{[m/s]} \qquad \text{(Eq.21)}$$

Here , ω is the angular speed of the rotor given by

$$\omega = 2\pi n \quad \text{[rad/s]} \qquad \text{(Eq.22)}$$

where , n is the rotational speed of the rotor in round per second.
Now we define the "tip speed ratio", i.e.

$$\lambda = \frac{v_{tip}}{v_1} = \frac{\omega R}{v_1} \quad [-] \qquad \textit{(Eq.23)}$$

Combining these equations we get

$$\gamma(r) = \arctan(\frac{3r\lambda}{2R}) \quad \text{[rad]} \qquad \text{(Eq.24)}$$

Or

$$\varphi(r) = \arctan(\frac{2R}{3r\lambda}) \quad \text{[rad]} \qquad \text{(Eq.25)}$$

And then the pitch angle

$$\beta(r) = \arctan(\frac{2R}{3r\lambda}) - \alpha_D \quad \text{[rad]} \qquad \text{(Eq.26)}$$

where α_D is the angle of attack, used for the design of the blade. Most often the angle is chosen to be close to the angle, that gives maximum glide ration, see figure 4 that means in the range from 5 to 10°, but near the tip of the blade the angle is sometimes reduced.

3. The Blade element Momentum Theory

In the blade element momentum (BEM) method the flow area swept by the rotor is divided into a number of concentric ring elements. The rings are considered separately under the assumption that there is no radial interference between the flows in one ring to the two neighbouring rings.

Chord length $c(r)$:

If we look at one blade element in the distance r from the rotor axis with the thickness dr the lift force is, see formula (Eq18) and (Eq19).

$$dF_L = (\frac{1}{2})\rho w^2 c dr\, C_L \quad [N] \tag{Eq.27}$$

And the drag Force

$$dF_D = (\frac{1}{2})\rho w^2 c dr\, C_D \quad [N] \tag{Eq.28}$$

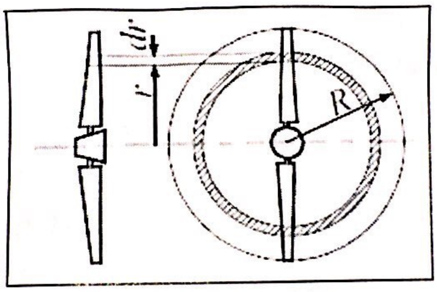


Figure 6: Blade section

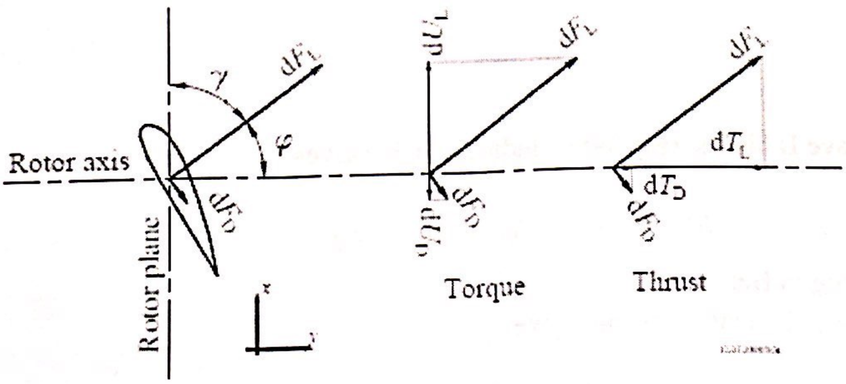


Figure 7 : Forces on the blade element decomposed on the rotor plane , dU (Torque) And in the rotor axis, dT (Thrust)

- For the rotor plane (torque), we have

(Eq.29)

$$dU = (\frac{1}{2})\rho w^2 cdr\, C_x \quad [N]$$

With

(Eq.30)

$$C_x = C_L \sin(\varphi) - C_D \cos(\varphi) \quad [-]$$

If we have B blades, (Eq. 27) becomes

(Eq.31)

$$dU = B(\frac{1}{2})\rho w^2 cdr\, C_x \quad [Nm]$$

- For the rotor axis (thrust), we have

(Eq.32)

$$dT = (\frac{1}{2})\rho w^2 cdr\, C_y \quad [N]$$

With

(Eq.32)

$$C_y = C_L \cos(\varphi) + C_D \sin(\varphi) \quad [-]$$

If we have B blades, (Eq. 30) becomes

(Eq.33)

$$dT = B(\frac{1}{2})\rho w^2 cdr\, C_y \quad [N]$$

- The Power produced
  Now in the design situation, we have $C_L >> C_D$, then (Eq29) and (Eq30) become

(Eq.34)

$$dU = (\frac{1}{2})\rho w^2 cdr\, C_L \cos(\gamma) \quad [Nm]$$

And then the power produced

(Eq.35)

$$dP = dUr\omega \quad [N]$$

If we have B blades, (Eq. 34) including (Eq.35) gives

(Eq.36)

$$dP = B\frac{1}{2}\rho w^2 cdr C_L \cos(\gamma) r\omega \quad [W]$$

According to Betz
The Blade element would also give

(Eq.37)

$$dP = \frac{16}{27}\frac{1}{2}\rho v_1^3 2\pi r dr \quad [W]$$

Using $v_1 = \frac{3}{2} w \cos(\gamma)$ and $u = w \sin(\gamma)$, then (Eq.36) and (Eq.37) gives

$$c(r)_{Betz} = \frac{16\pi R}{9BC_{L,D}} \frac{1}{\lambda\sqrt{\lambda^2\left(\frac{r}{R}\right)+\frac{4}{9}}} \quad [\text{m}] \qquad \text{(Eq.38)}$$

where $C_{L,D}$ is the coefficient of lift at the chosen design angle of attack, $\alpha_{A,D}$.

Remark:

After Schmitz , we define the tangential interference factor a', such that

$$u = r\omega(1+a') \quad [\text{m/s}] \qquad \text{(Eq.39)}$$

The flow angle φ is such that:

$$\tan(\varphi) = \frac{1-a}{1+a'} \frac{v_1}{r\omega} \quad [\text{-}] \qquad \text{(Eq.40)}$$

References

[1] Soren Gundtoft, « Wind Turbines », University College of Aarhus, June 2009.

[2] Martin Otto Laver Hansen, André Ferrand « Répartition de la puissance sur un rotor *d'éolienne* à axe horizontal », DTU Vindemeri Lyngby Denmark, INSA Toulouse.