Introduction to quiz: Wind turbine blade

This document contains all the information you need to solve the quiz called 'Wind turbine blade'. Please calculate your answers to the questions at the bottom of this page. You will then be prepared to start the quiz.

We recommend using a calculation tool such as a spread sheet or a computer programming tool to solve the exercise.

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Learning Objectives

When you have completed this exercise, you will be able to explain the main load working on a wind turbine blade and calculate how a material choice will affect the resulting weight of the blade.

Description

One of the challenges designing long wind turbine blades is to keep the weight down. Based on the information given in the "Materials" lectures, this will be studied. The aim of the exercise is to estimate the weight of an $86m$ long wind turbine blade made of aluminum, glass fiber composite or carbon fiber composite.

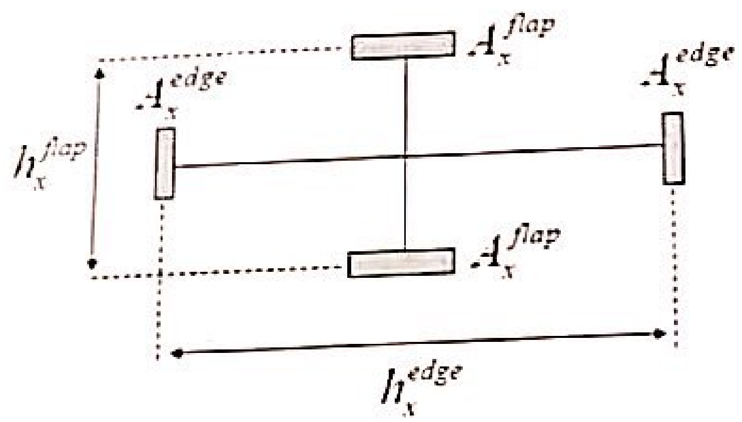

Figure 1. Illustration of the flapwise and edgewise materials cross sections inside a turbine blade.

In the following the index $()_x$ indicates a value at a coordinate x along the blade length while $()_0$ the value at the root section, x=0.

In order to utilize the full material strength potential along the blade length, the material cross section will vary such that the stresses in the blade material are kept more are less constant. Th stresses in the blade cross-section was in the "Material requirements for wind turbine blades"

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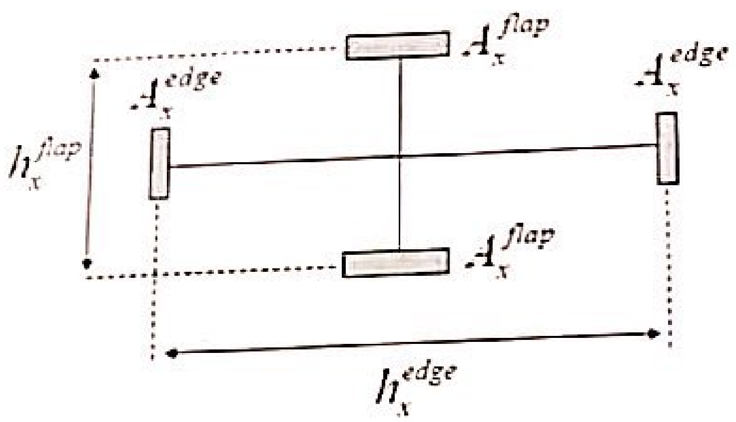

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$$\sigma_x^{flap} = \frac{4}{81}\frac{\rho_{air}V_r^2\pi(2L+x)(L-x)^2}{A_x^{flap}h_x^{flap}} \tag{1}$$

$$\sigma_x^{edge} = \frac{1}{2}\frac{\rho_{mat}A_x^{mat}g(L-x)^2}{A_x^{edge}h_x^{edge}} \tag{2}$$

From the figure of the simplified cross section it can be seen that:

$$A_x^{mat} = 2A_x^{flap} + 2A_x^{edge} \tag{3}$$

Keeping both the flap- and edge-wise material cross section constant:

$$A_x^{flap} \equiv A_0^{flap}; A_x^{edge} \equiv A_0^{edge} \tag{4}$$

it is possible to choose a blade height variation h_x^{flap} and blade width variation h_x^{edge} resulting in a constant stress state:

$$\sigma_x^{flap} \equiv \sigma_0^{flap}; \sigma_x^{edge} \equiv \sigma_0^{edge} \tag{5}$$

In the addition it is now possible to find the mass of the wind turbine blade as:

$$M_{blade} = A_{mat}L\rho_{mat} = 2(A_0^{flap} + A_0^{edge})L\rho_{mat} \tag{6}$$

Data

The properties of the three materials to be investigated are given in Table 1. The composite material data is for unidirectional (UD) composites with a fibre volume fraction of $55-60\%$. The carbon fibre composite data is for a low cost carbon fibre. The provided data for fatigue limits corresponds to approximately $10^8 - 10^9$ load cycles.

Material	Density ρ	Stiffness	Strength	Fatigue limit
Aluminium	$2700kg/m^3$	$70GPa$	$300MPa$	$150MPa$
UD glass fibre composite	$1900kg/m^3$	$44GPa$	$1000MPa$	$180MPa$
UD carbon fibre composite	$1600kg/m^3$	$120GPa$	$2000MPa$	$300MPa$

Table 1. Properties of three different materials.

For all three materials, it can be seen that the fatigue limit is significantly lower than the static strength. Therefore, the fatigue limit will be used as the material limiting stress through out the exercises.

Carbon fibers are approximately 10 times more expensive per weight unit compared with glass fibers. With a fiber volume fraction on approximately 50% this will result in a carbon fiber composite which will be 7.4 times as expensive as the corresponding glass fiber composite per weight unit.

Questions

Please use the information given above to complete the following tasks. Write down your answers as you go. You will need them when you answer the quiz.

Stresses in the blade cross section

In this first part of the quiz you will work conceptually with the stresses in a blade cross section.

For simplification the edge-wise and the flap-wise cross material section in equation (4) are kept constant. $A_x^{flap} \equiv A_0^{flap}$, $A_x^{edge} \equiv A_0^{edge}$

1. How is the relationship h_x^{flap}/h_0^{flap} expressed if the stress state is constant regarding the flap-wise loading of the cross section (i.e. $\sigma_x^{flap} \equiv \sigma_0^{flap}$)? *Hint: Use Eq. 1 and solve it for both h_x^{flap} and h_0^{flap} (setting X=0); then divide the two expressions.* One of the formulas below is correct:

$$\frac{h_x^{flap}}{h_0^{flap}} = \frac{(2L+x)(L-x)}{2L^2} \quad (1)$$

$$\frac{h_x^{flap}}{h_0^{flap}} = \frac{(2L+x)(L-x)^2}{2L^3} \quad (2)$$

$$\frac{h_x^{flap}}{h_0^{flap}} = \frac{(2L+x)(L-x)^3}{2L^4} \quad (3)$$

2. How is the relationship h_x^{edge}/h_0^{edge} expressed if the stress state is constant regarding the edge-wise loading of the cross section(i.e. $\sigma_x^{edge} \equiv \sigma_0^{edge}$)? *Hint: Use Eq. 2 and solve it for both h_x^{edge} and h_0^{edge} (setting X=0); then divide the two expressions.* One of the formulas below is correct:

$$\frac{h_x^{edge}}{h_0^{edge}} = \frac{(L-x)}{L} \quad (1)$$

$$\frac{h_x^{edge}}{h_0^{edge}} = \frac{(L-x)^2}{L^2} \quad (2)$$

$$\frac{h_x^{edge}}{h_0^{edge}} = \frac{(L-x)^3}{L^3} \quad (3)$$

3. Which is the correct expression for the constant material cross section in the flap direction? *Hint: Use the expression for h_0^{flap}, which you derived previously, and isolate A_0^{flap}.*

$$A_{flap} = \frac{2\pi}{81}\frac{\rho_{air} V_r^2 L^3}{\sigma_0^{flap} h_0^{flap}} \quad (1$$

$$A_{flap} = \frac{4\pi}{81}\frac{\rho_{air} V_r^2 L^3}{\sigma_0^{flap} h_0^{flap}} \quad ($$

$$A_{flap} = \frac{8\pi}{81}\frac{\rho_{air} V_r^2 L^3}{\sigma_0^{flap} h_0^{flap}} \quad ($$

4. Which is the correct expression for the constant material cross section in the edge direction? **Hint:** Use eq. 2 to derive an expression for σ_0^{edge} (setting x=0). Replace A_0^{mat} with the expression in eq. 3 and solve for A_0^{edge}

$$A_s^{edge} = A_0^{edge} - \frac{\rho_{mat} g L^2}{\sigma_0^{edge} h_0^{edge} - \rho_{mat} g L^2} A_0^{flap} \tag{1}$$

$$A_s^{edge} = A_0^{edge} - \frac{2\rho_{mat} g L^2}{\sigma_0^{edge} h_0^{edge} - 2\rho_{mat} g L^2} A_0^{flap} \tag{2}$$

$$A_s^{edge} = A_0^{edge} - \frac{4\rho_{mat} g L^2}{\sigma_0^{edge} h_0^{edge} - 4\rho_{mat} g L^2} A_0^{flap} \tag{3}$$

The weight of a wind turbine blade

You will now use the equations derived above to calculate the weight of a turbine blade. In the further questions, use the following information together with the information shown in Table 1:

Length of the blade	$L = 86m$
Rated wind speed	$V_r = 11m/s$
Density of air	$\rho_{air} = 1.2kg/m^3$
Acceleration of gravity	$g = 10m/s^2$
Blade height at root section	$h_0^{flap} = 2m$
Blade width at root section	$h_0^{edge} = 4m$

1. Calculate the mass (in metric tons) of a blade made of Aluminum, M_{blade}^{Al}, setting the stress limit equal to fatigue limit of the material, $\sigma_0^{Al} = 150MPa$. ***Hint:** Use the expressions you have derived in question 3. and 6. to calculate A_0^{flap} and A_0^{edge}. Insert these material cross sections in eq. 6, with $\rho_{mat} = \rho_{Al}$*
2. Calculate the mass (in metric tons) of a blade made of glass fiber reinforced polymers, M_{blade}^{GFRP}, setting the stress limit equal to fatigue limit of the material $\sigma_0^{GFRP} = 180MPa$. ***Hint:** Use the expressions you have derived in question 3. and 6. to calculate A_0^{flap} and A_0^{edge}. Insert these material cross sections in eq. 6, with $\rho_{mat} = \rho_{GFRP}$*
3. Calculate the mass (in metric tons) of a blade made of carbon fiber reinforced polymers, M_{blade}^{CFRP}, setting the stress limit equal to fatigue limit of the material $\sigma_0^{CFRP} = 300MPa$. ***Hint:** Use the expressions you have derived in question 3. and 6. to calculate A_0^{flap} and A_0^{edge}. Insert these material cross sections in eq. 6, with $\rho_{mat} = \rho_{CFRP}$*
4. Calculate an approximate factor, which the carbon fiber composite blade will cost more in material price compared with the glass fiber composite blade using the weight of the two blades found in the two previous questions. Material-wise, CFRP is 7.4 times as expensive as GFRP. ***Hint:** Calculate the ratio of MCFRP to MGFR and multiply with the cost factor of 7.4.*

Summary