Ecole Nationale des Sciences et Technologies Avancées à Borj Cédria

Fluid Mechanics

Fluid Statics

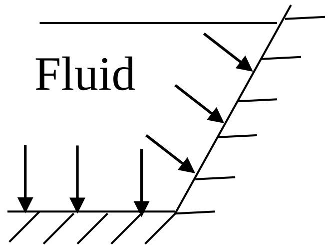
2022 - 23

Introduction

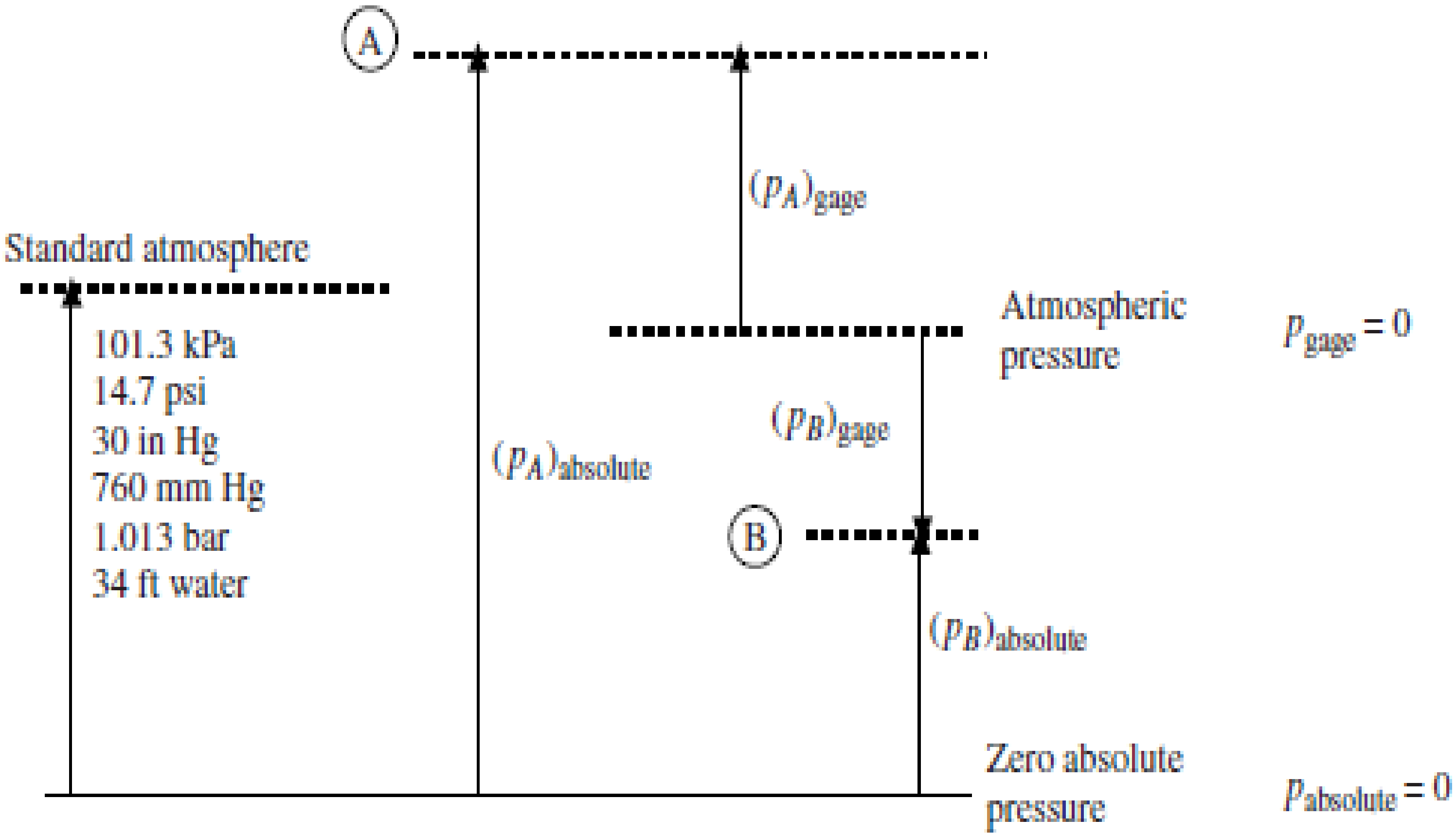
- Fluid static is the study of pressures throughout a fluid at rest and the pressure forces on finite surfaces
- For a fluid at rest

  - No force / shear stress acting on it

  - Any force between the fluid and the boundary must be acting perpendicular to the surface

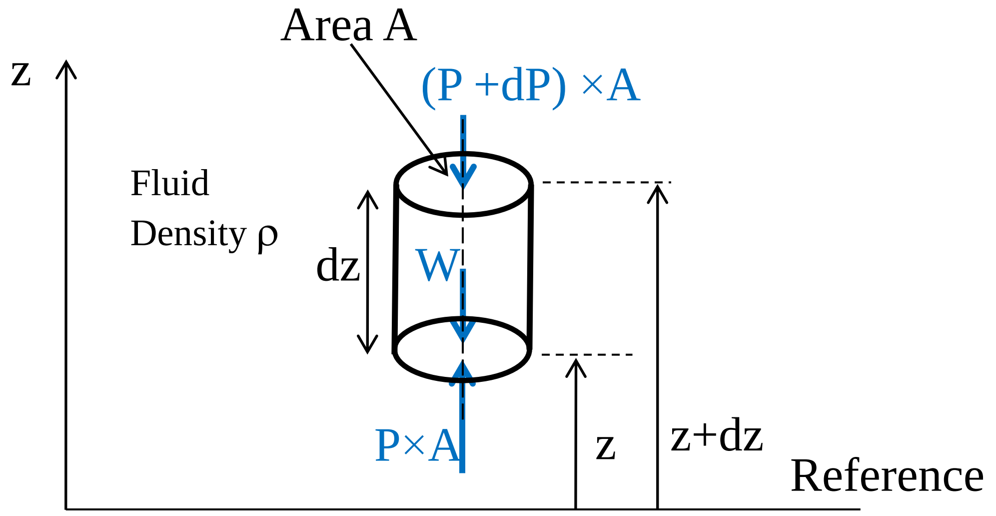


Gage and Absolute Pressure



Pressure variation with elevation

Let's consider a small cylindrical element of fluid with cross-sectional area A, height (dz), surrounded by the same fluid of mass density, ρ.



Pressure variation with elevation

The fluid is at rest and in equilibrium

$$\sum \vec{F} = \vec{0} \quad \text{at every point in the fluid}$$

Projection along the z-axis

$$PA - (P + dP)A - \rho g A dz = 0$$

$$\Rightarrow -dP - \rho g dz = 0$$

The fundamental equation of hydrostatic states that:

$$\frac{dP}{dz} = -\rho g = -\gamma$$

Pressure variation with elevation

If we integrate the previous equation (for incompressible fluid and constant g)

$$\int dP = -\gamma \int dz$$

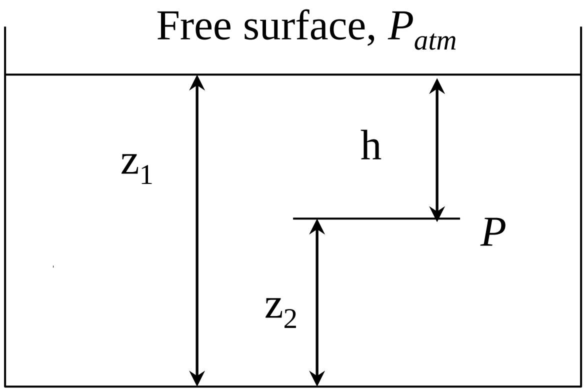
$$P(z) = -\gamma z + P_0$$

where P_0 is the pressure at z = 0

In liquid with free surface it is more practicle to use the depth of the liquid from the free surface, $h = -z = z_1 - z_2$

Then we have

$$P(h) = P_{atm} + \gamma h$$



Pressure and head

- From the above equations, it can be concluded that the change in pressure is directly proportional to the specific weight of the liquid, and pressure varies linearly with the change of elevation or depth.

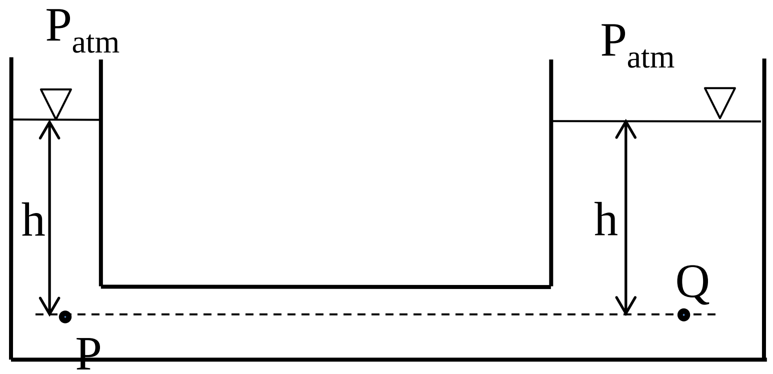
- The linear variation with depth below the free surface is known as **hydrostatic** pressure distribution.

- As g is assumed constant, the gauge pressure can be given by stating the vertical height, h, of any fluid density, ρ, which would be necessary to produce this pressure. This vertical height, ***h***, is known as **pressure head** or just **head** of fluid, and can be written as:

$$h = P/\rho g$$

Pressure at the same level

Consider two tanks of different cross-sections connected by a pipe



The pressure at P and Q can be computed as follows

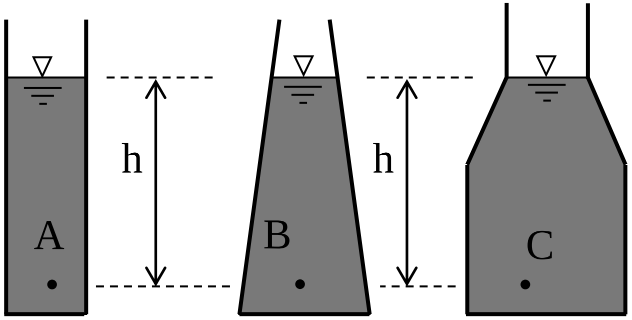
$$\begin{aligned} P_P &= P_{atm} + \rho g h \\ P_Q &= P_{atm} + \rho g h \end{aligned} \implies P_P = P_Q$$

This shows that the pressure is the same at two points

P & Q at equal levels

Pressure at the same level

- The change in pressure depends only on the change of elevation, the free surface pressure and the type of fluid,

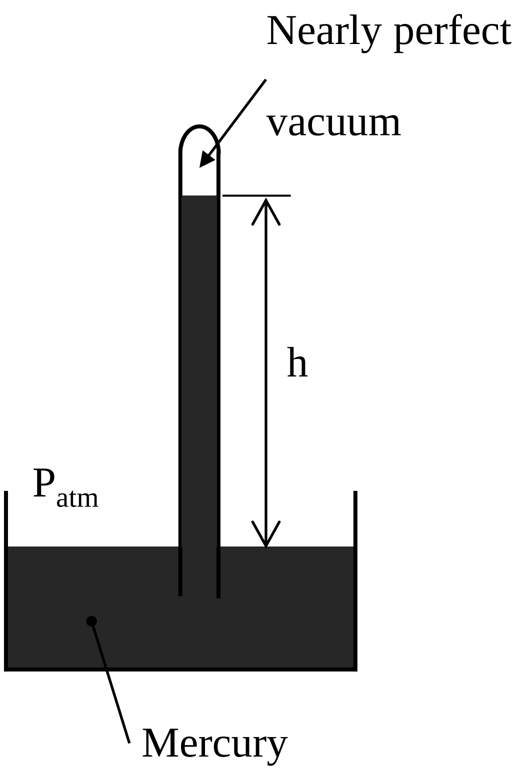


$P_A = P_B = P_C$ no matter the size and the shape of the container, since the depth (h) and the fluid are the same in each container.

Pressure measurement

- Atmospheric pressure is usually measured by a mercury barometer
- A void is produced at the top of the tube which is very nearly a perfect vacuum.
- The height of the mercury column in the tube is approximately 760 mm (30 in.) at sea level.
- The atmospheric pressure can be deduced directly from the reading of the mercury level using the relation:

$$P_{atm} = \rho g h$$



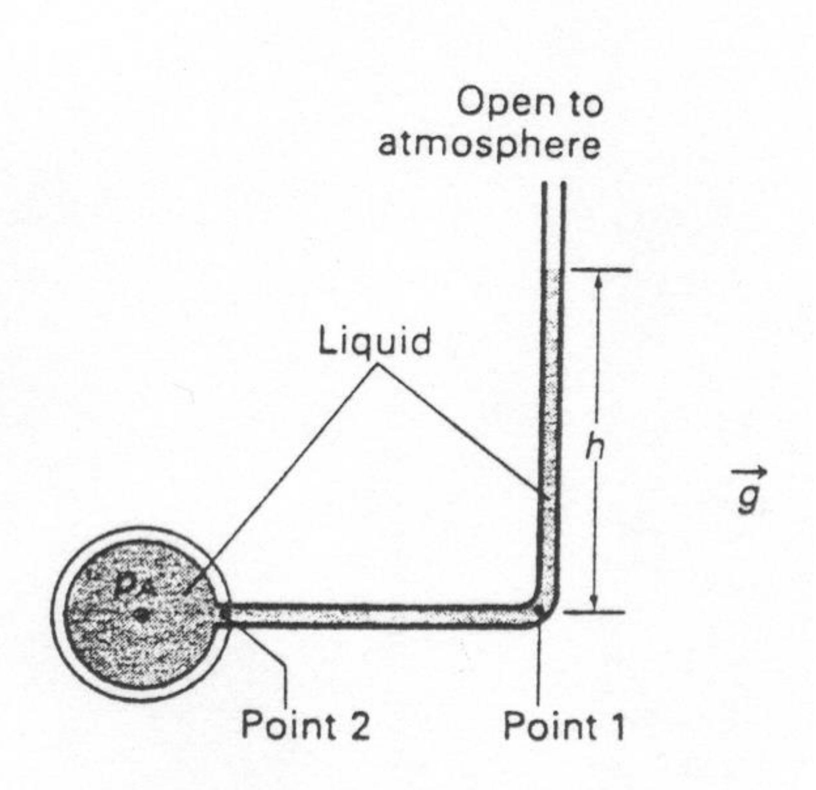
Pressure measurement

- Manometer

$$P_1 = P_{atm} + \rho_{liq} g h$$

$$P_1 = P_2 = P_A$$

$$\Rightarrow P_A = P_{atm} + \rho_{liq} g h$$

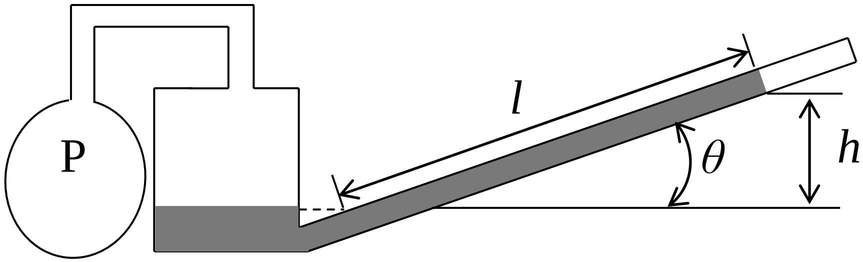


Pressure measurement

❑ Inclined manometer

Common problems when measuring pressure differences in low velocity systems as air ventilation system are the low column heights and satisfying accurately

$$P = \rho g l sin(\theta)$$

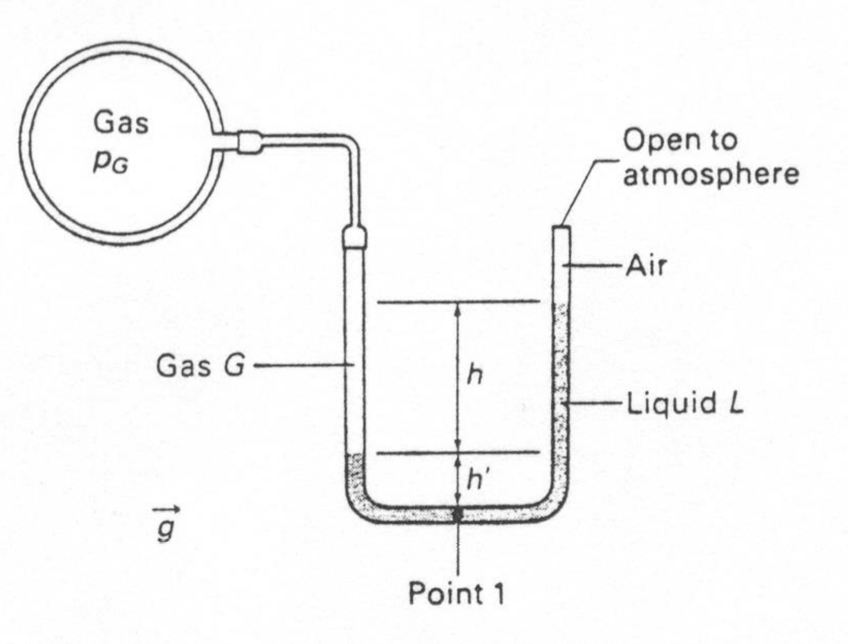


Inclining the tube manometer will increase the accuracy of the measurement.

Pressure measurement

❑ U-tube manometer

The U-Tube contains water or mercury in a U-shaped tube, and is usually used to measure gas pressure.



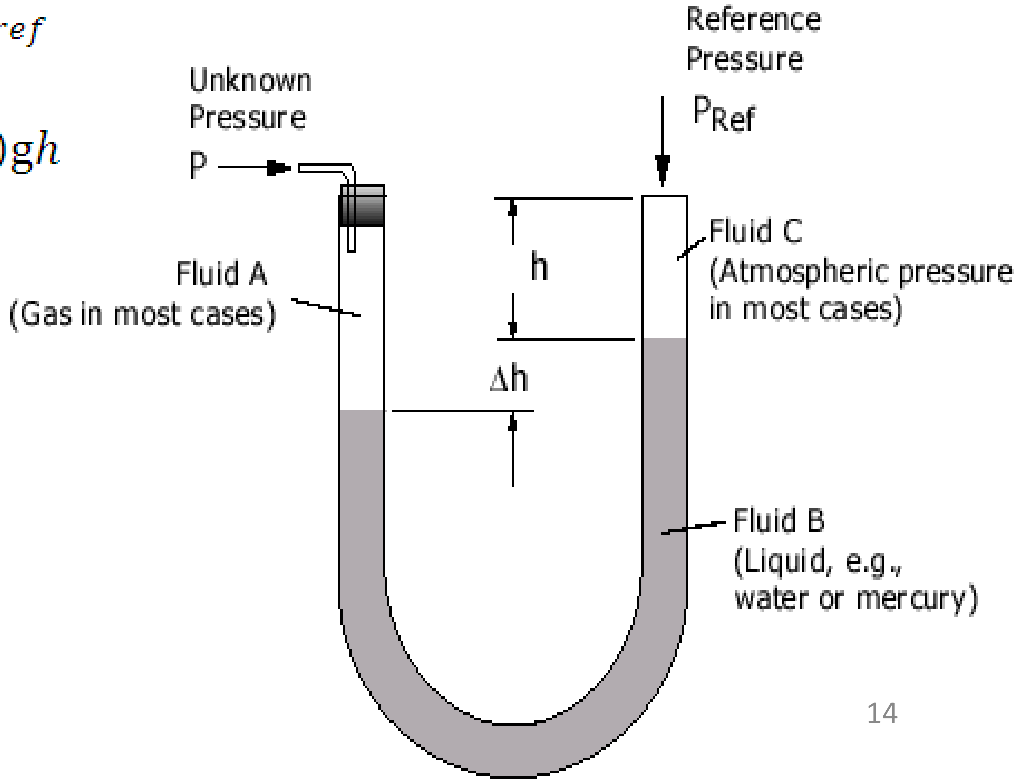
Pressure measurement

❑ U-tube manometer

By comparing the level of the liquid on both sides of the U tube, the unknown pressure can be obtained from fluid statics,

$$P + \rho_A g(h + \Delta h) = \rho_B g\Delta h + \rho_C g h + P_{ref}$$

$$\Rightarrow P = P_{ref} + (\rho_B - \rho_A) g\Delta h + (\rho_C - \rho_A) g h$$



In most applications

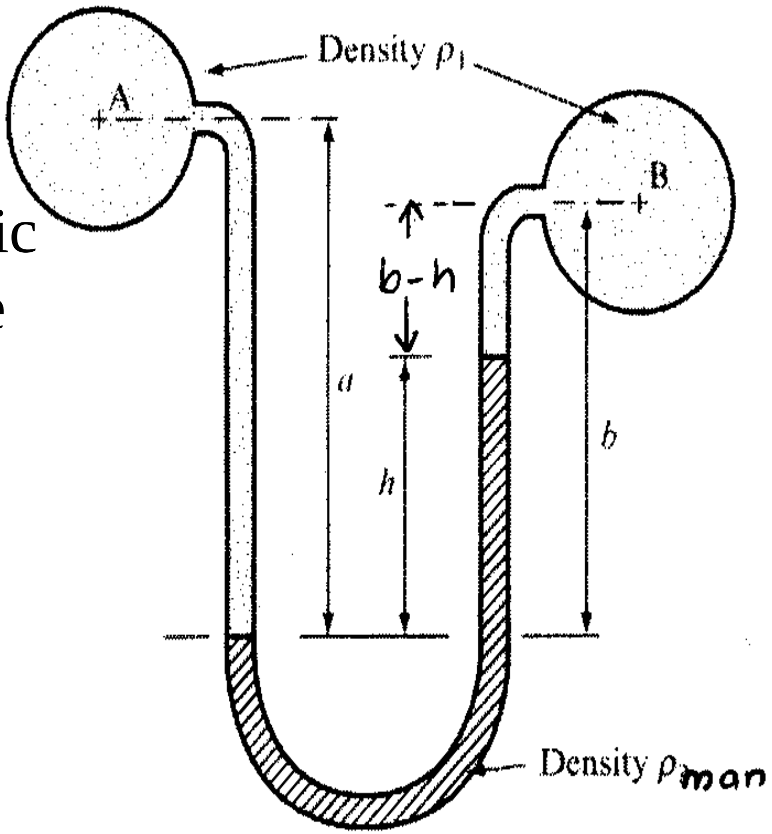
$$\rho_B \gg \rho_A, \rho_C$$

Then $P \approx P_{ref} + \rho_B g\Delta h$

Pressure measurement

❑ Differential Manometer

Standard manometers are used to measure the pressure in a container by comparing it to normal atmospheric pressure. Differential manometers are also used to compare the pressure of two different containers.



$$P_A + \rho_1 g a = P_B + \rho_1 g(b-h) + \rho_{liq} g h$$

$$P_A - P_B = \rho_1 g(b-a) + gh\left(\rho_{liq} - \rho_1\right)$$

Pressure measurement

❑ Pressure Gages (Bourdon tube pressure gauges)

The **Bourdon Tube** is a nonliquid pressure measurement device. It is widely used in applications where inexpensive static pressure measurements are needed.

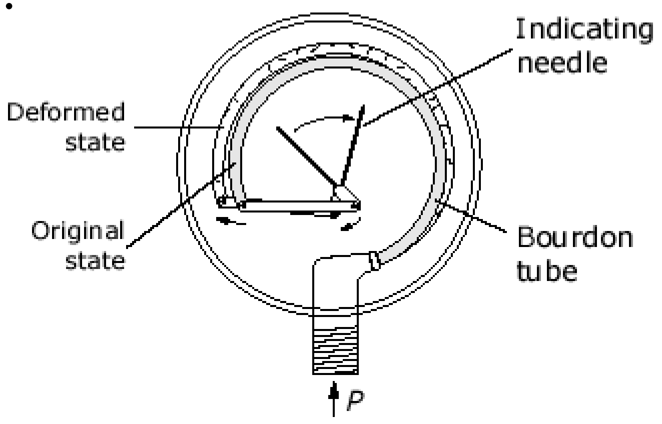
Pressure applied to the tube tends to cause the tube to straighten out, and the deflection of the end of the tube is communicated through a system of levers to a recording needle, as shown schematically below.

Pros:

- Portable
- Convenient
- No leveling required

Cons:

- Limited to static or quasi-static measurement.
- Accuracy may be insufficient for many applications. A mercury barometer can be used to calibrate.



Typical Bourdon Tube Pressure Gages

Hydrostatic Forces on immersed surfaces

Pressure has been defined as force per unit area. If the pressure acts on a small area δS then the force exerted on this area is

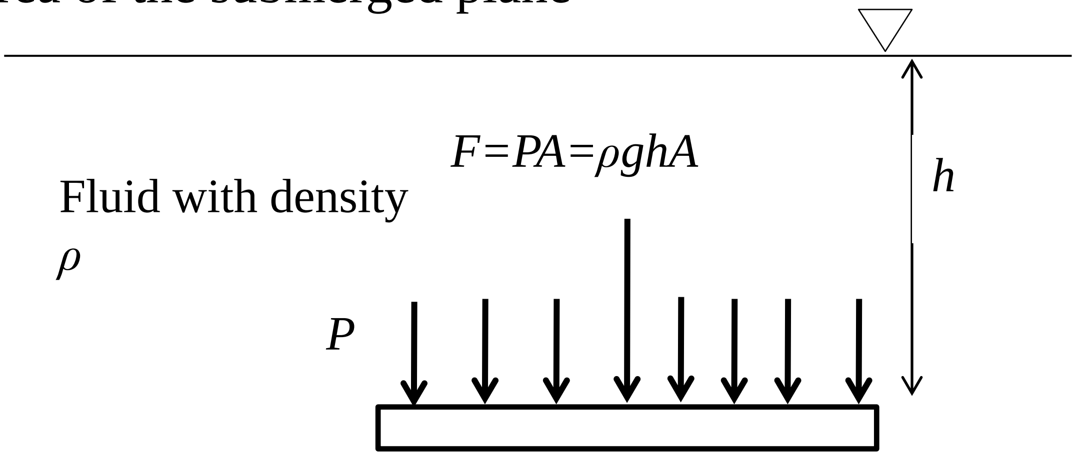
$$F = P \times \delta S$$

The fluid is at rest the force will act at right-angles to the surface.

For horizontal plane submerged in a liquid, the pressure, P, will be equal at all points of the surface.

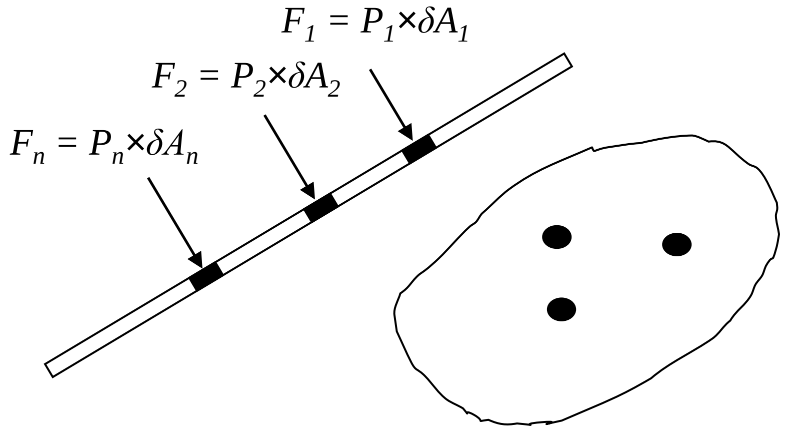
$$F = P \times A$$

Where A is the area of the submerged plane



Hydrostatic Forces on immersed surfaces

Consider the plane surface shown in the figure below. The total area is made up of many elemental areas.



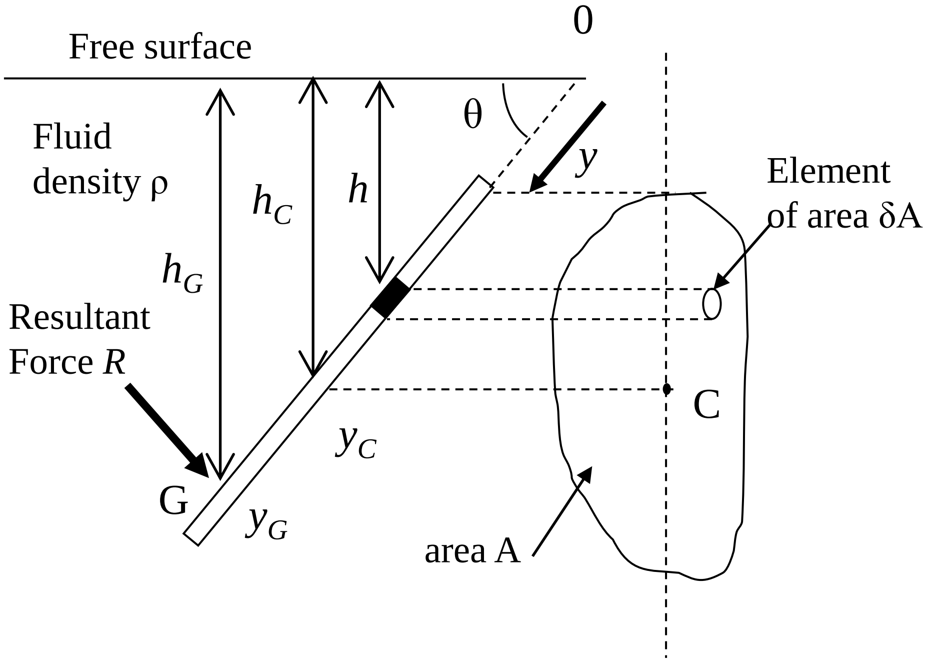
The force on each elemental area is always normal to the surface but, in general, each force is of different magnitude as the pressure usually varies.

The total or ***resultant*** force is then:

$$R = P_1 \times \delta A_1 + P_2 \times \delta A_2 + \cdots + P_n \times \delta A_n$$

This resultant force will act through the centre of pressure

Hydrostatic Forces on immersed surfaces



Consider a plane totally submerged in a liquid of density ρ, at an angle of θ to the horizontal.

Taking $P = 0$ at the free surface, the pressure on an element δA, submerged a distance h

$$P = \rho g h$$

and therefore the force on the element is

$$F = P\,\delta A = \rho g h\,\delta A$$

The resultant force: $R = \rho g \sum h\,\delta A$

The term $\sum h\,\delta A$ is known as the *1st Moment of Area* of the plane about the free surface.

$$\sum h\,\delta A = h_C A$$

where A is the area of the plane and h_C is the depth to the centroid, C

Thus, $\boldsymbol{R = \rho g\, h_C A = \rho g\, y_C \sin(\theta) A = P_C A}$ $\quad$ ($\sum y\,\delta A = y_C A$ and $P_C = \rho g\, h_C$)

Hydrostatic Forces on immersed surfaces

This resultant force acts at right angles to the plane through the centre of pressure, G, at a depth h_G.

The vertical depth of the center of pressure, G, below the free surface can be found using the following equation for vertical plate ($\theta = 90°$):

$$h_G = h_C + (Ig) /(A*h_C)$$

where: I_g is the second moment of plane area about its center of gravity . The above equation implies that the center of pressure is always below the centroid.

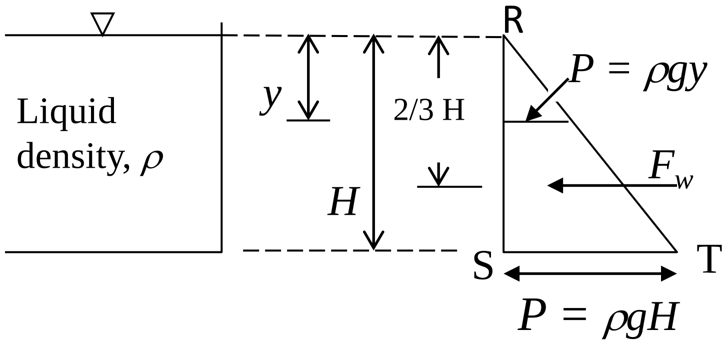
In general for an inclined plate with angle θ, we have:

$$h_C = y_C * \sin(\theta) \; ; \; h_G = y_G * \sin(\theta)$$

$$y_G = y_C + (Ig) /(A*y_C)$$

Pressure Diagram

The triangle (RST) is a graphical representation of the (gauge) pressure change with depth on one side of the vertical wall of the tank containing a liquid with density ρ. At the free surface the gauge pressure is zero. It increases linearly from zero at the surface by $P = \rho g y$, to a maximum of at the base of $P = \rho g H$.



- The area of this triangle (RST) represents the resultant force per unit width on the vertical wall. So;
- Area of pressure diagram = Resultant force per unit width,

$$F_w = \frac{1}{2} \rho g H^2 \text{ (N/m)}$$

- This force acts through the centroid of the pressure diagram. For a triangle, the centroid is located at 2/3 its height, thus the resultant force acts at a depth of 2/3 H from R.
- The total resultant force can be obtained by multiplying the above equation with the width of the surface, B.

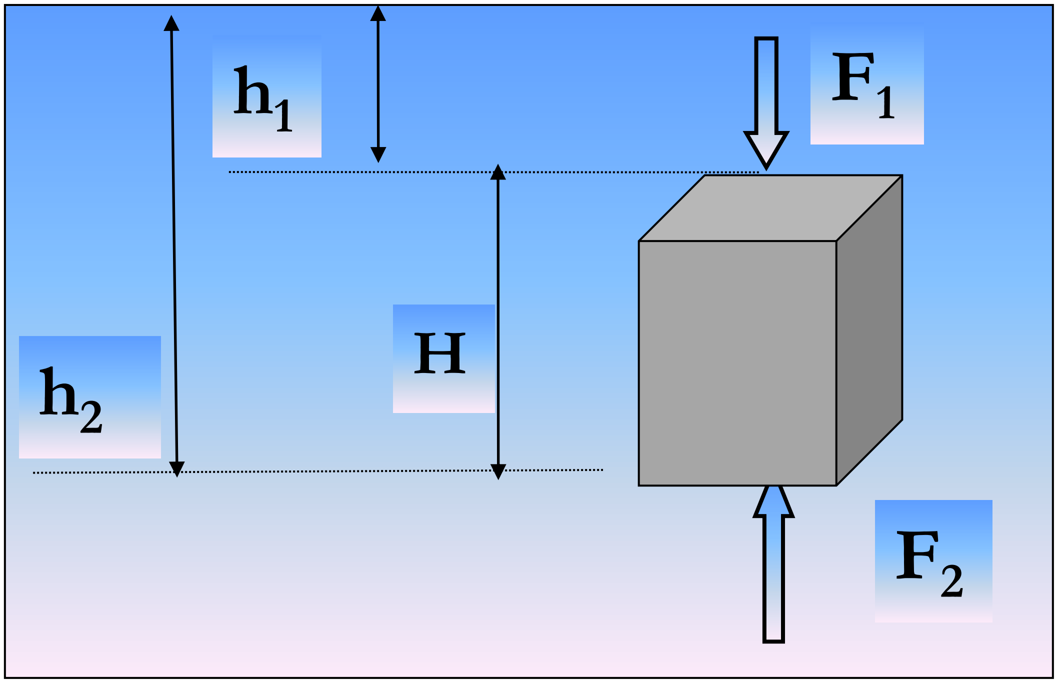
$$F = \frac{1}{2} \rho g H^2 B$$

- The same pressure diagram technique can be used when combinations of liquid are held in tanks (e.g. oil floating on water).

Buoyancy: Archimedes' Principle

- **A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces**
- **A floating body displaces its own weight in the fluid in which it floats**

Free liquid surface



- **The upper surface of the body is subjected to a smaller force than the lower surface**
- **A net force is acting upwards**

Buoyancy: Archimedes' Principle

The net force due to pressure in the vertical direction is:

$$\boxed{F_B = F_2 - F_1 = (\mathrm{P}_{\mathrm{bottom}} - \mathrm{P}_{\mathrm{top}})(\Delta \mathrm{x} \Delta y)}$$

The pressure difference is:

$$\boxed{\mathrm{P}_{\mathrm{bottom}} - \mathrm{P}_{\mathrm{top}} = \rho\, g\, (h_2 - h_1) = \rho\, g\, H}$$

Combining:

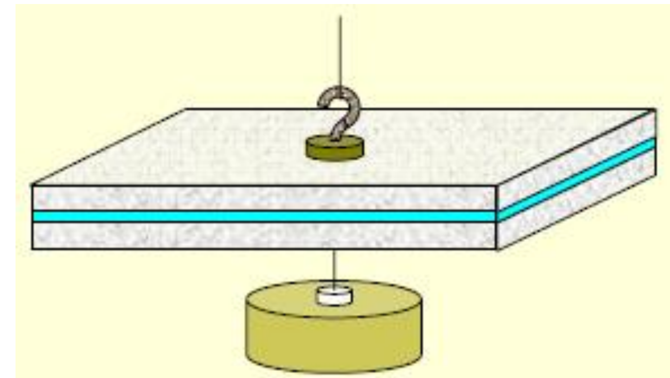
$$\boxed{\mathrm{F}_{\mathrm{B}} = \rho\, g\, H (\Delta x \Delta y)}$$

Thus the buoyant force is:

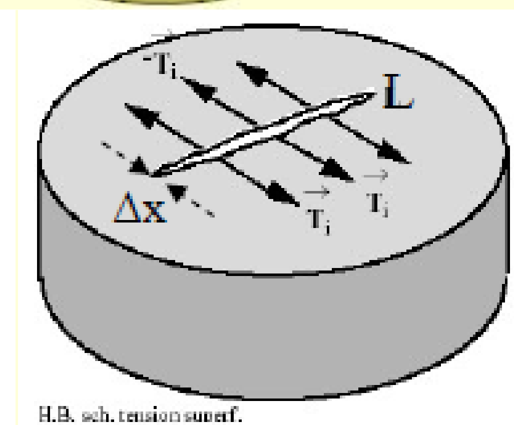
$$\boxed{\mathrm{F}_{\mathrm{B}} = \rho\, g\, \mathrm{V}}$$

Archimedes' Principle : The buoyant force is equal to the weight of the volume of fluid displaced by the object

Surface tension and capillarity effect



Two glass plates between which a thin film of water was deposited seem bonded to each other. The bottom plate can support a weight of several hundred grams before falling.



Imagine that we want to create the free surface of a liquid an opening slit of length L and width Δx very small: this requires exercise in several points of the opening of Ti forces shall be tensile forces: Indeed, the liquid tends to oppose this by developing a standard force F that opposes the Ti forces.

La norme F de la force est proportionnelle à la longueur L de la fente. On peut donc écrire

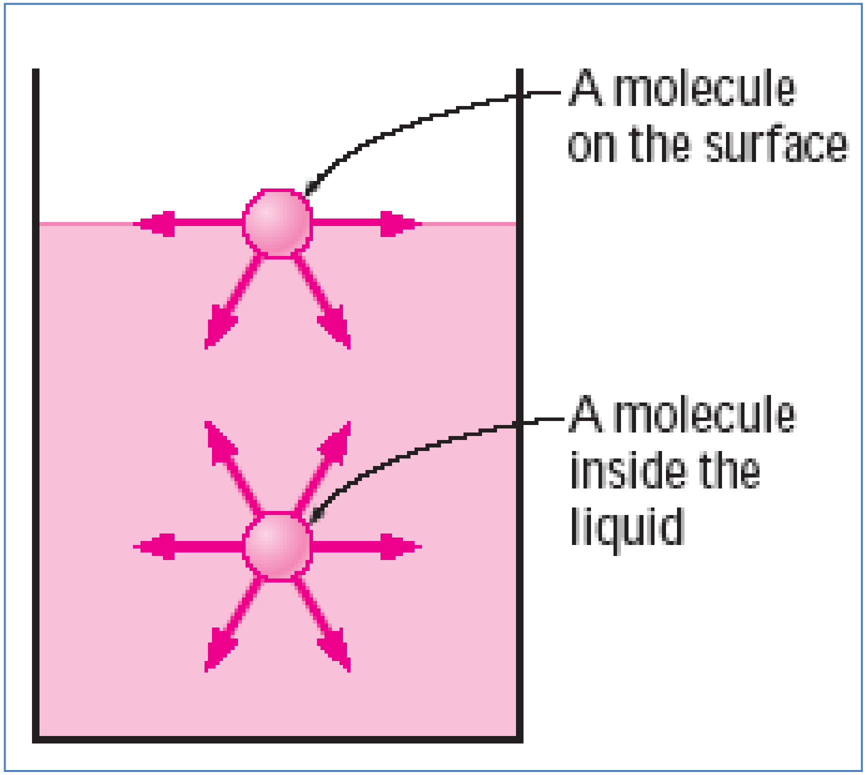
$$\boxed{F = \sigma_s L}$$

The coefficient σ_s is called **surface tension** mesured inN/m.

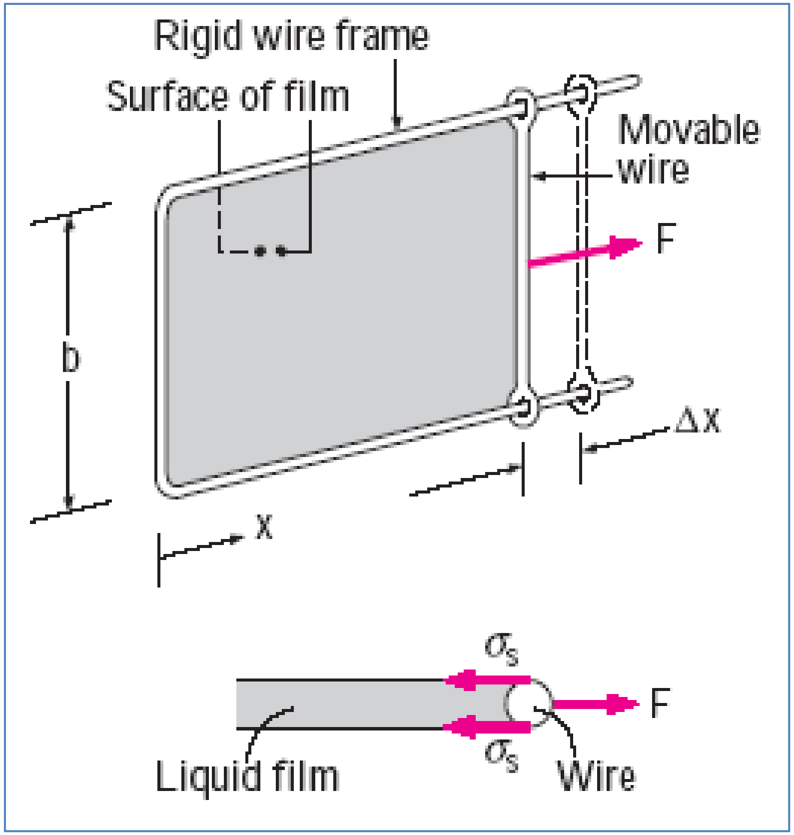
Surface tension and capillarity effect

Some consequences of surface tension

Surface tension and capillarity effect



Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.



Stretching a liquid film with a U-shaped wire,and the forces actingon the movable wire of length b.

Surface tension and capillarity effect

Consider a liquid film (such as the film of a soap bubble) suspended on a U-shaped wire frame with a movable side.

Normally, the liquid film tends to pull the movable wire inward in order to minimize its surface area. A force F needs to be applied on the movable wire in the opposite direction to balance this pulling effect. The thin film in the device has two surfaces (the top and bottom

$$\sigma_s = \frac{F}{2b}$$

$$W = Force \times Dis\tan ce = F\Delta x = 2b\sigma_s \Delta x = \sigma_s \Delta A$$

Surface tension: The work done per unit increase in the surface area of the liquid

Surface tension and capillarity effect

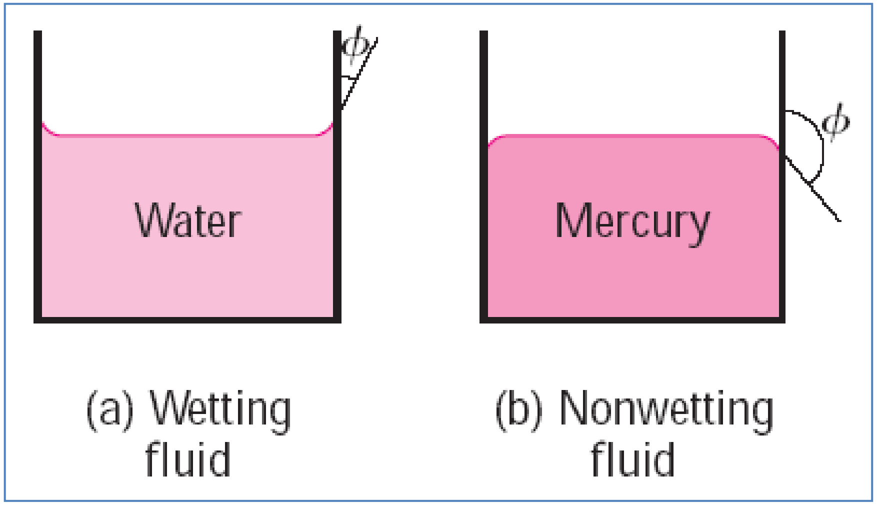
Capillary effect: The rise or fall of a liquid in a small-diameter tube inserted into the liquid.

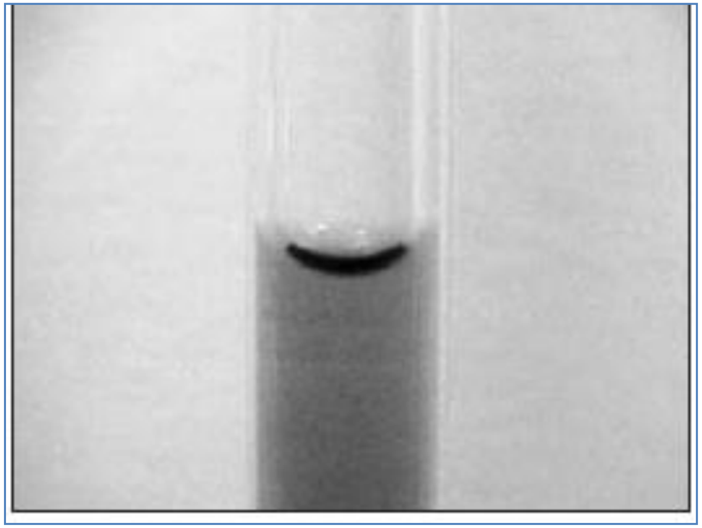
Capillaries: Such narrow tubes or confined flow channels. The capillary effect is partially responsible for the rise of water to the top of tall trees.

Meniscus: The curved free surface of a liquid in a capillary tube.

The strength of the capillary effect is quantified by the **contact (or wetting) angle**, defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.

Surface tension and capillarity effect

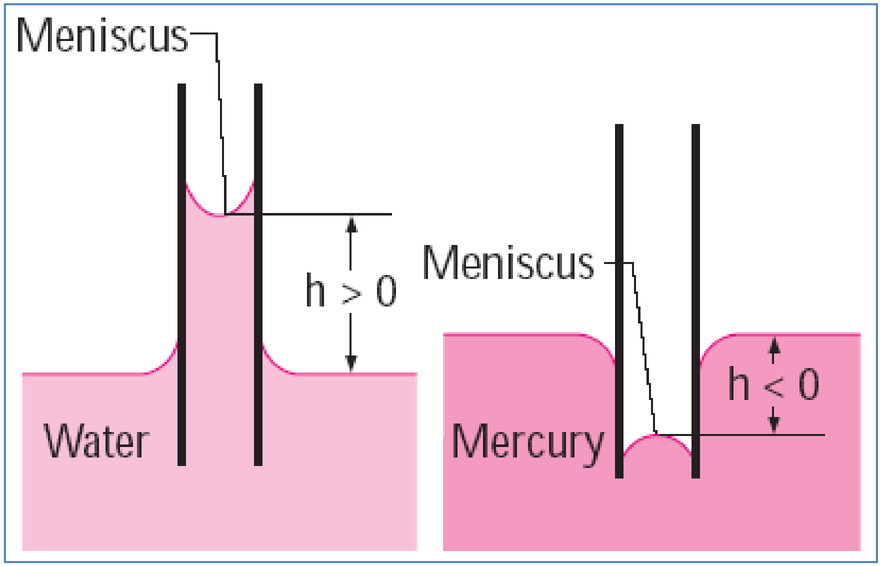


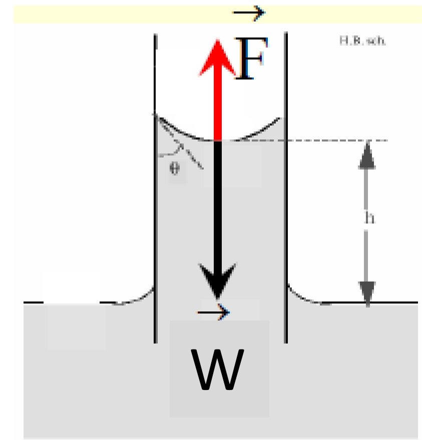


The contact angle for wetting and nonwetting fluids

The meniscus of colored water in a 4-mm-inner-diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle

Surface tension and capillarity effect





The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.

Weight W $= mg = \pi R^2 h \rho g$
Is balanced by the force
F = F = 2πRσ$_s$ cosθ

Jurin law : $$h = \frac{2\sigma_s}{\rho g R} Cos\phi$$

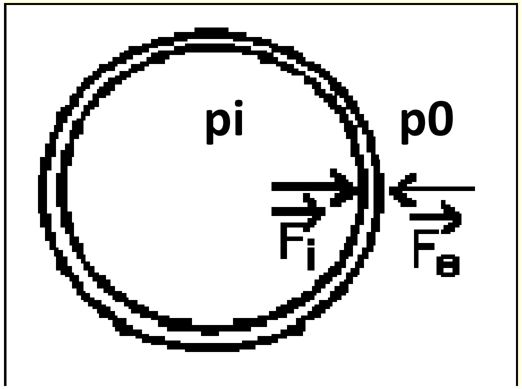
$(R \text{ Constant})$

Capillary rise is inversely proportional to the radius of the tube and density of the liquid

Surface tension and capillarity effect

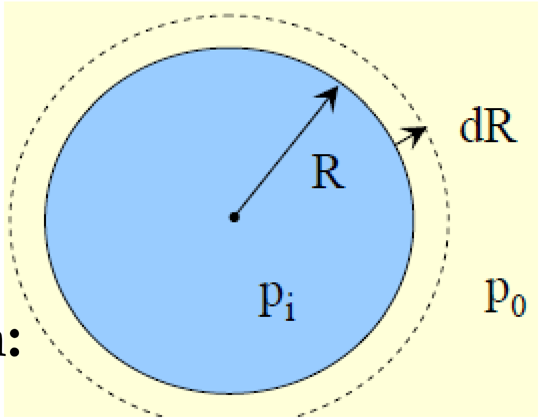
Let a spherical drop of radius R: surface tension forces, which are directed towards the interior of the drop, perform compression inside of it. The pressure pi in the drop is therefore higher than the external environment, p0. This compression is, of course, even greater than the surface forces are large, so that the surface tension σ_s is high.

Laplace's law to calculate the difference pi - p0 = Δp according to R and σ_s.



Surface tension and capillarity effect

By increasing the radius R of the drop of dR, its volume increases $S.dr = 4\pi R^2 dR$ **where S is the surface of the drop.**



Work pressure forces during this operation:

$$\begin{cases} dW_0 = -\,p_0\, 4\pi R^2 dR \\ dW_i = \;\; p_i\, 4\pi R^2 dR \end{cases}$$

The net work is then : $dW = (p_i - p_0) 4\pi R^2 dR$

This work is equal to the work of tension surface : $dW = \sigma_s\, dS$

The surface of the sphere $S = 4\pi R^2$ **Then** $dS = 8\pi R dR$.

Laplace law : $\boxed{\Delta p = pi - p0 = 2\,\sigma_s / R}$