

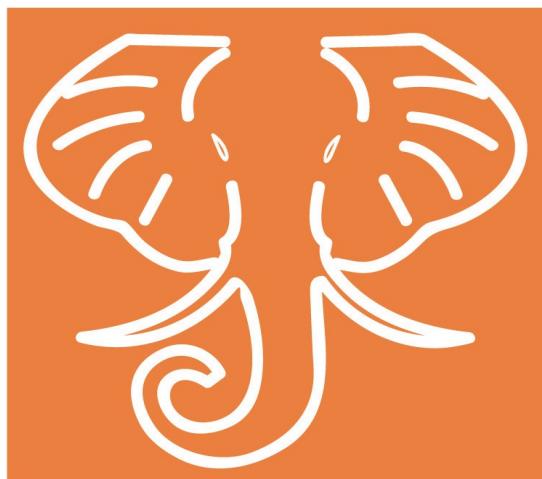
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Fuchel, K.

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# RANDOM MATRIX DIAGONALIZATION – A COMPUTER PROGRAM

K. FUCHEL, RITA J. GREIBACH, AND C.E. PORTER



September 1962

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UPTON, NEW YORK**

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## ABSTRACT

A computer program is described which generates random matrices, diagonalizes them, and sorts appropriately the resulting eigenvalues and eigenvector components. FAP and FORTRAN listings for the IBM 7090 computer are included.



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## I. INTRODUCTION

The purpose of this report is to describe a computing machine program which generates random matrices, diagonalizes them, and sorts appropriately the resulting eigenvalues and eigenvector components. Although stochastic matrices are of interest for their own sake, the development of this program has been motivated primarily by problems associated with physical properties of spectra of highly excited many body systems. The various physical applications of the computations will be discussed elsewhere.

## II. OUTLINE OF PROGRAM

In this program we are concerned entirely with real symmetric matrices with matrix elements distributed normally about zero mean. (The last paragraph of this section mentions briefly another available input matrix element distribution.) Once the dimension of the matrices is specified, there are only two free parameters A and B in the input; they are the root mean square dispersions of the diagonal and off-diagonal matrix elements. Since only dimensionless variables are studied in the output sorts, only the ratio A/B is important. The choice  $A = \sqrt{2}$ ,  $B = 1$  leads to the well-known rotationally invariant Gaussian ensemble of matrices (diagonal matrix element dispersion twice that of the off-diagonal elements).

The input Gaussian distributions for the matrix elements are obtained from a transformation of the random number generator on the library tape. This standard generator provides a sequence of numbers  $\eta$ , distributed such that

$$P(\eta) = \begin{cases} 1 & 0 \leq \eta \leq 1 \\ 0, & \eta < 0 \\ 0, & \eta > 1 \end{cases} . \quad (1)$$

By choosing  $\eta$ 's in pairs called  $\eta_1$  and  $\eta_2$  and making the transformation

$$\begin{aligned} y_1 &= A(-2 \log \eta_1)^{1/2} \cos 2\pi\eta_2, \\ y_2 &= A(-2 \log \eta_1)^{1/2} \sin 2\pi\eta_2, \end{aligned} \tag{2}$$

one obtains a pair of independent normally distributed variables  $y_1$  and  $y_2$ , viz.,

$$P(y_1, y_2) = P(y_1)P(y_2),$$

with

$$\begin{aligned} P(y_1) &= (2\pi A^2)^{-1/2} \exp(-y_1^2/2A^2), \\ P(y_2) &= (2\pi A^2)^{-1/2} \exp(-y_2^2/2A^2). \end{aligned} \tag{3}$$

These variables are then fed into the matrix diagonalization subroutine. ( $A \rightarrow B$  in (2) gives the off diagonal matrix elements.)

The matrix diagonalization subroutine is a standard package which produces eigenvalues  $E_i$  and eigenvector components  $a_{ij}$ . The eigenvectors are arranged to be orthonormal, i.e.,

$$\sum_{j=1}^N a_{ij} a_{jk} = \delta_{ik}, \tag{4}$$

where  $N$  is the dimension of the matrices. Note that the production of the eigenvector components necessarily involves some arbitrary choices of phases so that the signs of the  $a_{ij}$  may not be random. For this reason in the output sorting, only the magnitudes of the components are considered.

Much more output sorting is carried out on the eigenvalues in the present program than on the eigenvector components. The eigenvalues are divided by  $2\sqrt{NB^2}$  (a division motivated by the famous semi-circle law of Wigner) and then sorted according to size to obtain the single eigenvalue distribution (which is multiplied by  $\pi/2$  to correspond to the semi-circle law). They are ordered and can be sorted individually in order

(asymmetric sort) or the counts of highest and lowest, next highest and next lowest, etc.

can be combined in pairs as the plus-minus symmetry of the input Gaussians warrant (symmetric sort). The nearest-neighbor, next-nearest-neighbor, etc., spacings are determined and are divided by the mean nearest-neighbor spacing and the resulting numbers are sorted with respect to size to obtain the various spacing distributions and

their moments (note that  $M_0 = \int_0^\infty P(x)dx$ ,  $M_1 = \int_0^\infty xP(x)dx$ , etc.).

The first type of sort puts all  $N^2$  of the  $a_{ij}$  in one box (general sort). In addition they are arranged eigenvector by eigenvector to correspond to the ordered eigenvalues and sorted either pairwise (symmetric sort – highest and lowest, next highest and next lowest, etc., together) or singly within each eigenvector (asymmetric sort).

Also included is a random matrix loading routine which loads plus and minus one's in a fixed ratio as a first step to examining the effect of breaking systematically the plus-minus symmetry of the input distribution. This loading replaces that described in (2) and (3); the loading ratio is, however, based on (1) by loading +1 if  $\eta$  is greater than an input parameter A and -1 if  $\eta$  is less than A where  $0 \leq A \leq 1$ . A then is the loading ratio directly.

### III. INPUT DATA

#### 1st Card:

I TAPE : tape to be used for eigenvector components

J TAPE : tape to be used for normalized eigenvalues.

(This tape may be a scratch tape.)

K TAPE : tape to be used for eigenvalues.

#### 2nd Card:

N : dimension of matrix

NM : number of matrices desired

NB : last neighbor relation (NB < N)

#### 3rd Card:

WFA : on-diagonal weight factor

WFB : off-diagonal weight factor

#### 4th Card:

DX<sub>i</sub>, (i=1, . . . 4) : grid width for spacings

#### 5th Card:

DE<sub>i</sub>, (i=1, . . . 4) : grid width for eigenvalues

#### 6th Card:

DA<sub>i</sub>, (i=1, . . . 4) : grid width for eigenvector components

#### 7th Card:

Control data:

KONE { = 0 to suppress all sorting of eigenvalues  
          = 1 for general and asymmetric sorting of eigenvalues  
          = 2 for general and symmetric sorting of eigenvalues  
          = 3 for general sorting of eigenvalues only

KONA	$\left\{ \begin{array}{l} = 0 \text{ to suppress all sorting of eigenvector components} \\ = 1 \text{ for general and asymmetric sorting of eigenvectors} \\ = 2 \text{ for general and symmetric sorting of eigenvectors} \\ = 3 \text{ for general sorting of eigenvector components only} \end{array} \right.$
KONN	$\left\{ \begin{array}{l} = 0 \text{ to sort either eigenvalues or their spacings} \\ \neq 0 \text{ to suppress eigenvalues and their spacings} \end{array} \right.$
KONX	$\left\{ \begin{array}{l} = 0 \text{ to compute and sort eigenvalue spacings} \\ \neq 0 \text{ to suppress the spacings} \end{array} \right.$

8th Card:    Restart card,

If the job is not a restart job, this card should be blank.

If it is a restart job, card carries:

W : argument to restart random number generates

M : current matrix index

TOTAL : Sum of nearest neighbor spacings to date.

N. B.

When restarting, change only the last data card.

#### KEY

$E_i$  ( $i=1, \dots, N$ ) = Eigenvalues

$A_i$  ( $i=1, \dots, N^2$ ) = Eigenvector components

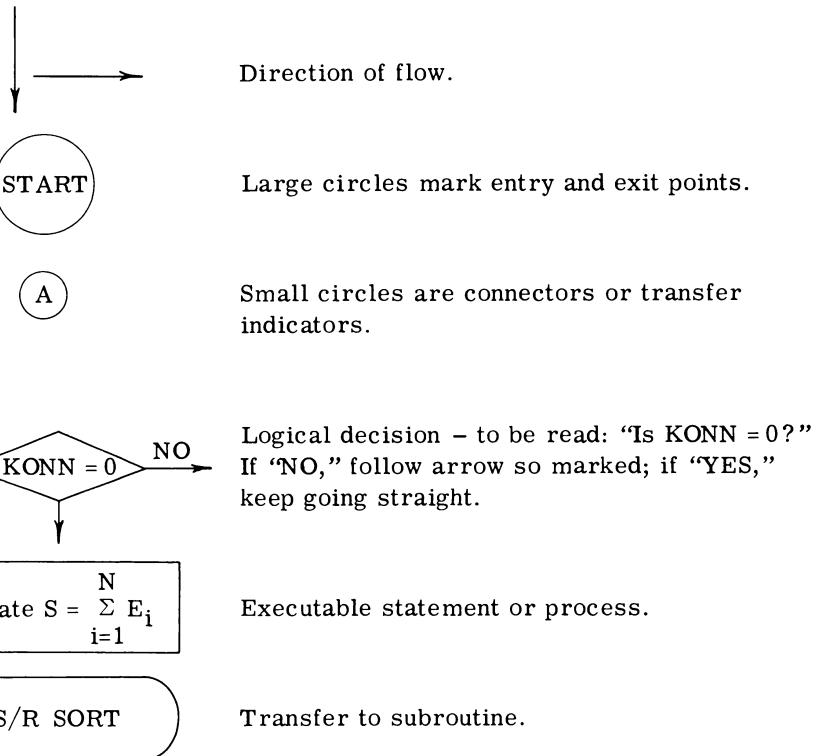
$U_{i,j}$  ( $j=1, \dots, N; i=1, \dots, N$ ) = Matrix array of the  $A_i$

$SPAC_i$  ( $i=1, \dots, N-1$ ) = Nearest neighbor spacing       $= E_{i+1} - E_i$

$X_i$  ( $i=1, \dots, N-INB$ ) = Normalized spacings, INB is index of neighbor

relation.

## KEY TO FLOW CHARTS



### FORMAT FOR DATA CARDS

1 <sup>ST</sup> : 3I2:	I J K	TAPE						
2 <sup>ND</sup> : I2,I10,I2:	N	NM	NB					
3 <sup>RD</sup> : 2F10.4:	WFA		WFB					
4 <sup>TH</sup> : 4F10.4:	DX(1)	DX(2)	DX(3)	DX(4)				
5 <sup>TH</sup> : 4F10.4:	DE(1)	DE(2)	DE(3)	DE(4)				
6 <sup>TH</sup> : 4F10.4:	DA(1)	DA(2)	DA(3)	DA(4)				
7 <sup>TH</sup> : 4I1:	KONA	KONIN						
8 <sup>TH</sup> : 0I2,I10:	W	M						

5    10    12    15    20    25    30    35    40    45    50

N. B.

This program is set up so that after each complete run, it will immediately run again if it encounters another set of eight data cards. Thus the deck may be sent in once to compute short runs with varying combinations of data.

#### OPERATING INSTRUCTIONS

The program is designed to run under the 709-90 FORTRAN II, Version 2 monitor system.

Three tapes are required; if no restart is envisioned, they may be scratch tapes.

If sense switch 1 is depressed, a restart card is punched and the program discontinued. To restart it, simply mount tapes I and K, replace last data card with the restart card, and execute. As a precaution against lost time in the event of machine failure, a restart card is automatically punched every 20 minutes.

## IV. PROGRAM

### A. MAIN PROGRAM

The input data is read. Magnetic tapes used are rewound; if performing a restart run, the tapes are positioned as they were when the run was discontinued.

Subroutine MXGEN sets up the matrix,  $H_{i,j}$  ( $i=1, \dots, N$ ;  $j=1, \dots, N$ ), and calculates the trace,  $T = \sum_{i=1}^N H_{i,i}$ .

Subroutine HDIAG computes the eigenvalues  $E_i$  ( $i=1, \dots, N$ ) and the eigenvector components  $A_i = U_{j,k}$  ( $i=1, \dots, N$ ;  $j=1, \dots, N$ ;  $k=1, \dots, N$ ). The sum of the eigenvalues,  $S = \sum_{i=1}^N E_i$ , is computed and compared with the trace,  $T$ . If  $S \neq T$ , there is an error in the eigenvalues.

The eigenvalues are then ordered ascendingly, after which the eigenvectors are rearranged to match this ordering. The reordered eigenvalues and the absolute values of the components of the reordered eigenvectors are taped for eventual use by the sorting subroutines SPACE, VALUE, VECTOR.

The nearest neighbor spacings of eigenvalues are computed for use in calculating the normalization factor for the spacings.

Tests are made whether to punch a restart card or not. This process is repeated for each of NM matrices.

Finally, options are tested for transfer to the subroutines SPACE, VALUE, VECTOR to sort the spacings, eigenvalues, or eigenvector components, respectively.

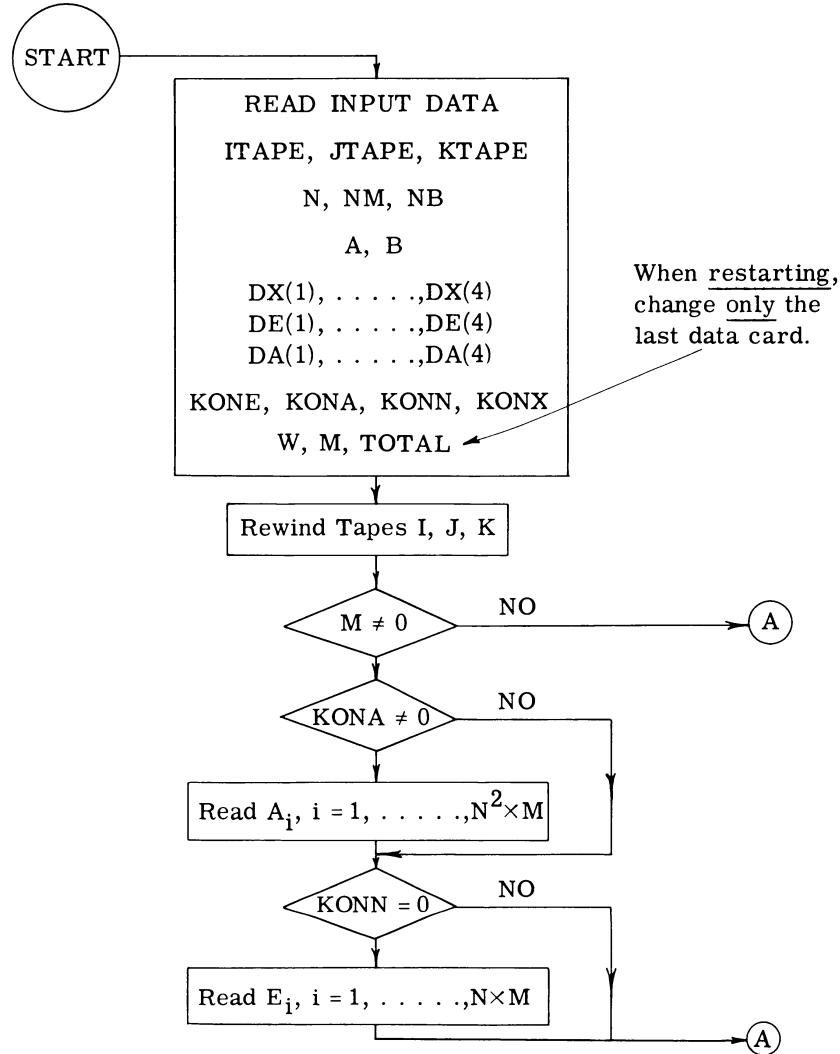
Provision has been made to repeat this entire process for any desired variations in input data if the program encounters another set of eight data cards.

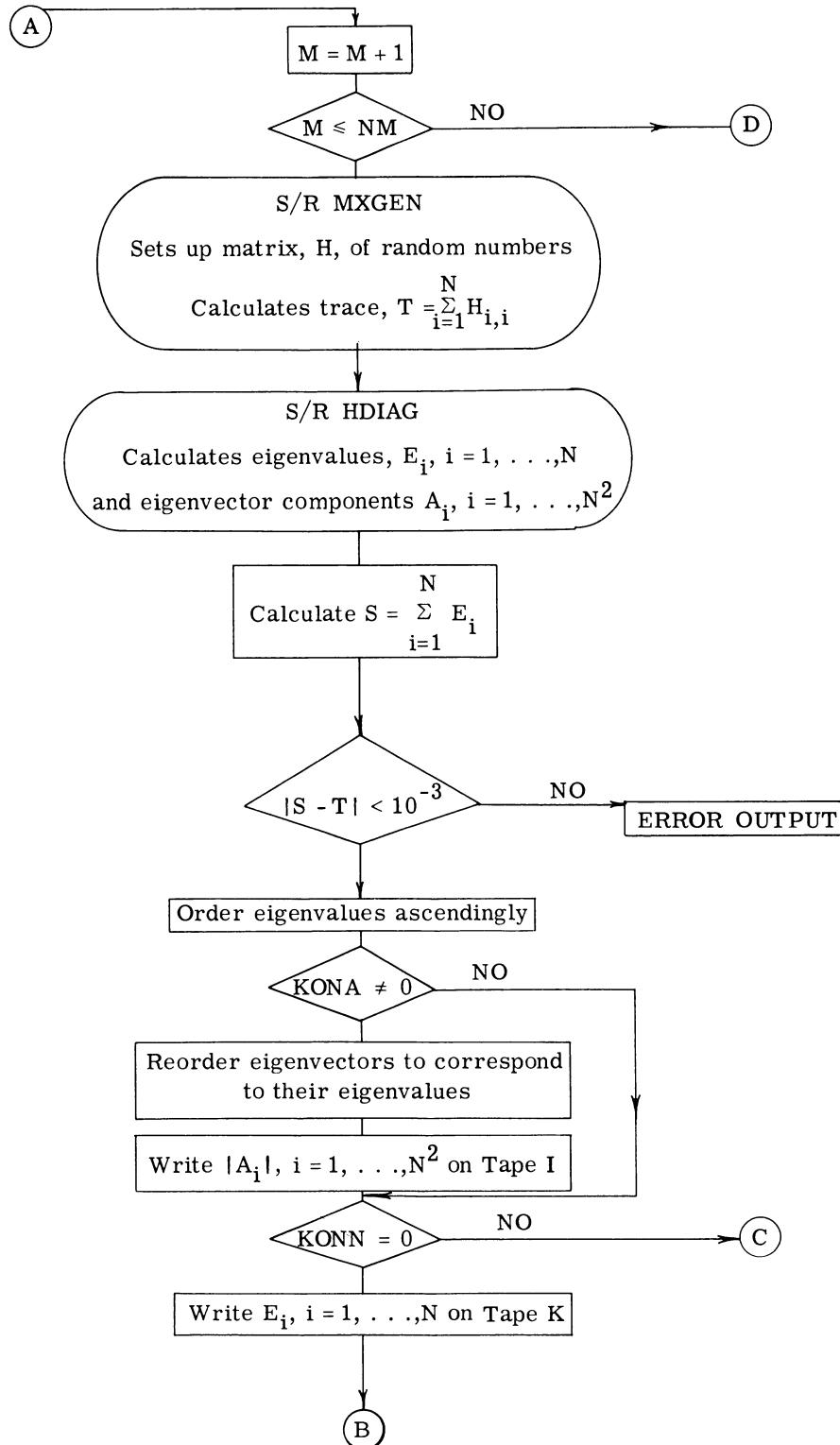
Provision has also been made to stop the program while still diagonalizing the NM matrices and then restart it at the same point without reinitializing the random number generator. This is done by replacing the last data card by a similar card

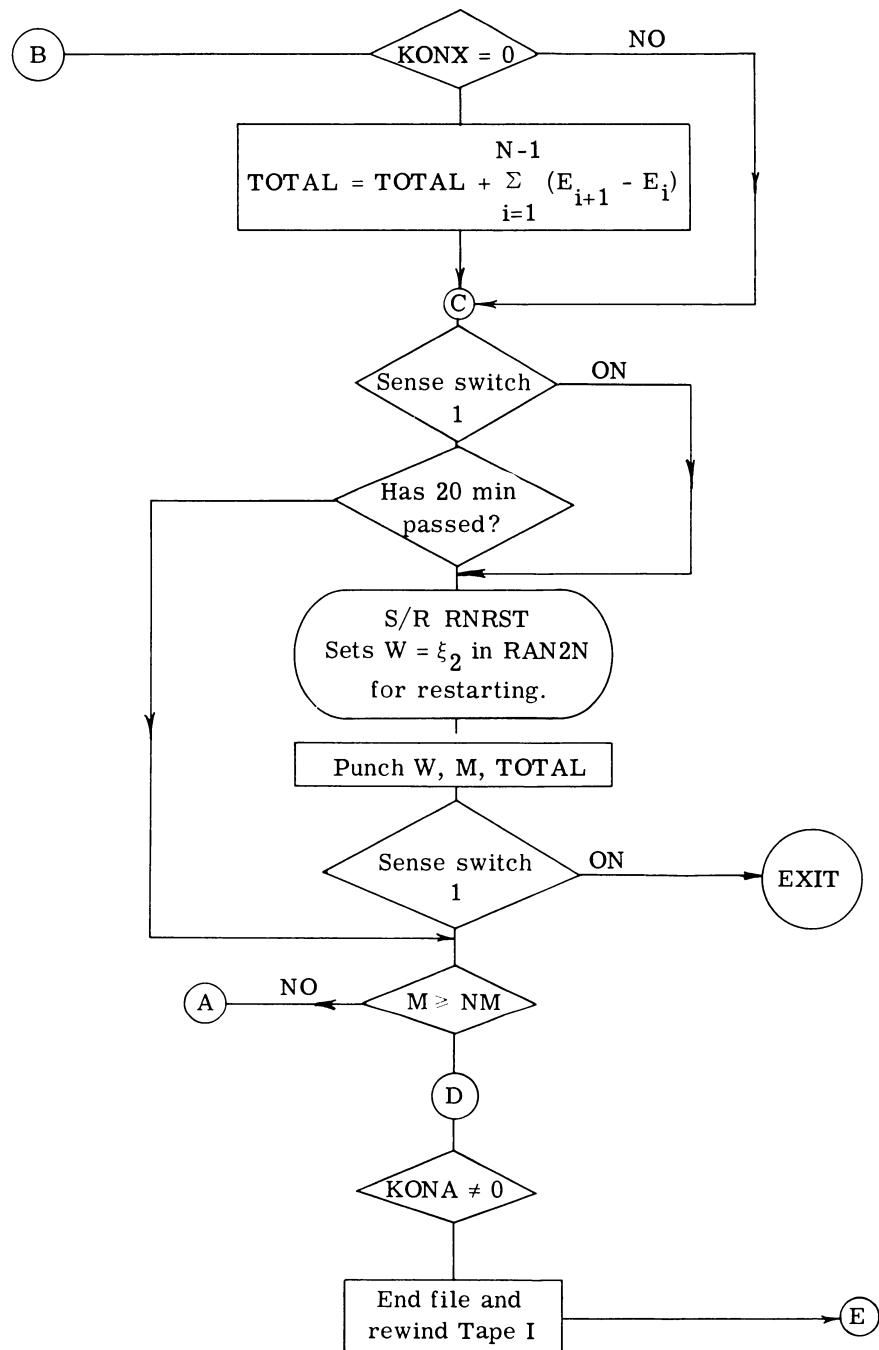
punched by the program containing W, the argument to restart RAN2N or RANUN M,  
the number of matrices already diagonalized, and TOTAL, the sum of the nearest  
neighbor spacings. Such a card is punched after every 20 min of diagonalization.

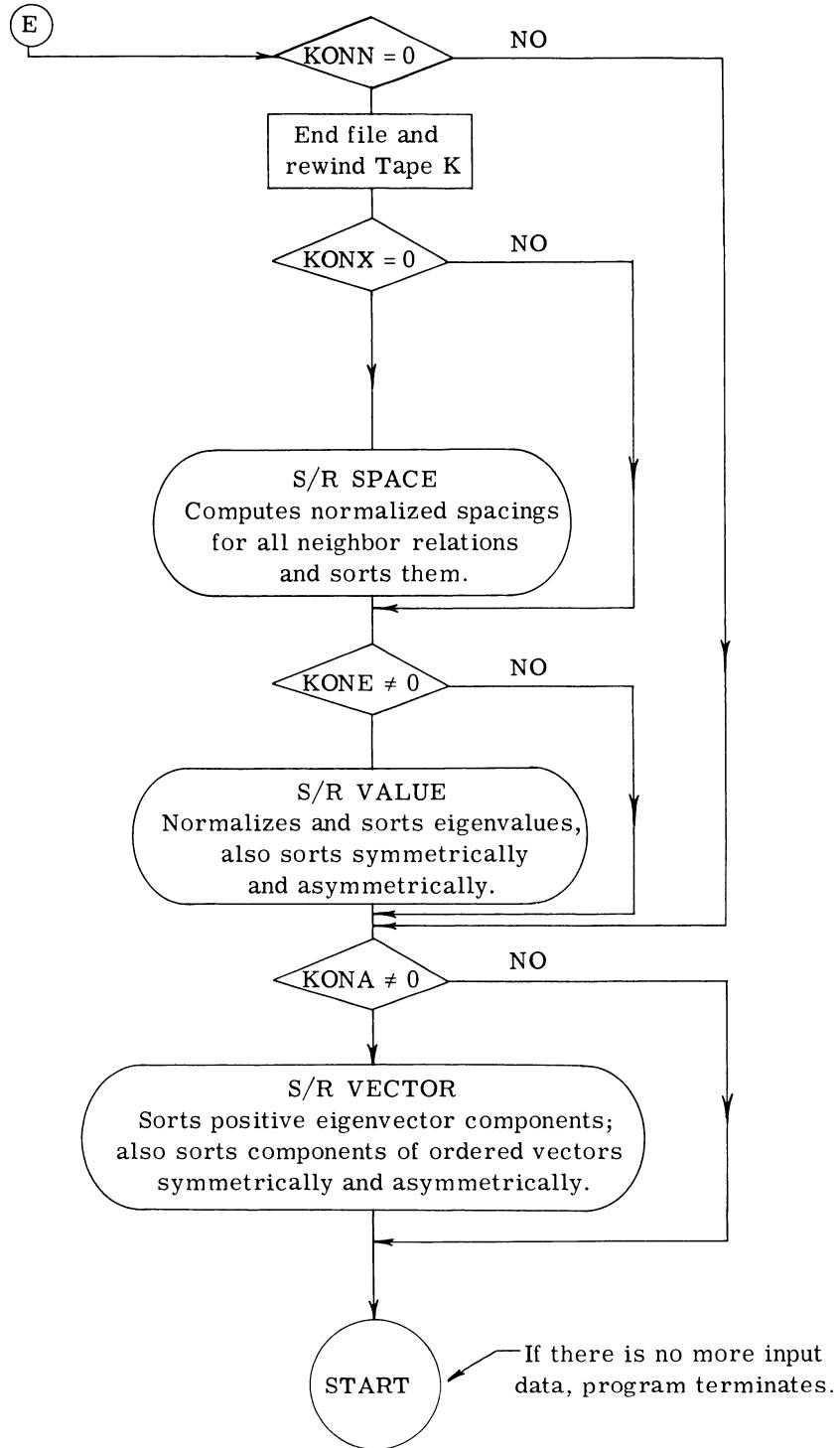
(This feature can only be used on a 7090 equipped with a core clock.) If sense switch 1  
is turned on, a restart card is punched, and the program exits: control returns to the  
monitor.

RANDOM MATRIX DIAGONALIZATION  
MAIN PROGRAM









## RMD RANDOM MATRIX DIAGONALIZATION

```

DIMENSIØN A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275)RMD00001
1           ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003)RMD00002
CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,      RMD00003
1 XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB,   RMD00004
2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM                 RMD00005
C
C
C ITAPE IS USED FØR THE EIGENVECTØR CØMPØNENTS
C JTAPE IS USED FØR THE NØRMALIZED EIGENVALUES
C KTAPE IS USED FØR THE EIGENVALUES
C
10 READINPUTTAPES,20,ITAPE,JTAPE,KTAPE
20 FØRMAT(3I2)
C
C N IS DIMENSIØN ØF MATRIX
C NM IS THE NUMBER ØF MATRICES DESIRED
C NB IS LAST NEIGHBOR RELATØN (N-1)
C
READINPUTTAPES,30,N,NM,NB
30 FØRMAT(I2,I10,I2)
C
C WFA IS ØN-DIAGONAL WEIGHTING FACTØR
C WFB IS ØFF-DIAGONAL WEIGHTING FACTØR
C
READINPUTTAPES,40,WFA,WFB
40 FØRMAT(2F10.4)
C
C DX IS GRID WIDTH FØR SPACINGS
C DE IS GRID WIDTH FØR EIGENVALUES
C DA IS GRID WIDTH FØR EIGENVECTØR CØMPØNENTS
C
READINPUTTAPES,50,(DX(I),I=1,4)
READINPUTTAPES,50,(DE(I),I=1,4)
READINPUTTAPES,50,(DA(I),I=1,4)
50 FØRMAT(4F10.4)
C
C KØNE EQUALS 0 TØ SUPPRESS ALL SØRTING ØF EIGENVALUES
C EQUALS 1 FØR GENERAL AND ASYMMETRIC SØRTING ØF EIGENVALUES
C EQUALS 2 FØR GENERAL AND SYMMETRIC SØRTING ØF EIGENVALUES
C EQUALS 3 FØR JUST GENERAL SØRTING ØF EIGENVALUES
C KØNA EQUALS 0 TØ SUPPRESS ALL SØRTING ØF EIGENVECTØR CØMPØNENTS
C EQUALS 1 FØR GENERAL AND ASYMMETRIC SØRTING ØF EIGENVECTØRS
C EQUALS 2 FØR GENERAL AND SYMMETRIC SØRTING ØF EIGENVECTØRS
C EQUALS 3 FØR JUST GENERAL SØRTING ØF EIGENVECTØR CØMPØNENTS
C KØNN EQUALS 0 TØ SØRT EITHER EIGENVALUES ØR THEIR SPACINGS
C IS UNEQUAL TØ 0 TØ SUPPRESS EIGENVALUES AND THEIR SPACINGS
C KØNX EQUALS 0 TØ CØMPUTE AND SØRT EIGENVALUE SPACINGS
C IS UNEQUAL TØ 0 TØ SUPPRESS THE SPACINGS
C
READINPUTTAPES,60,KØNE,KØNA,KØNN,KØNX
60 FØRMAT(4I1)
C
C W IS RESET ARGUMENT FØR REENTRY INTØ GENERATING SUBROUTINE
C M IS MATRIX INDEX
C TØTAL SUM ØF NEAREST NEIGHBOR SPACINGS FØR NØRMALIZATION FACTØR

```

RMD RANDØM MATRIX DIAGØNALIZATØN

C INITIALIZE AS A BLANK CARD RMD00056  
C TO RESTART, USE CARD PUNCHED IN LAST RUNNING ØF PRØGRAM RMD00057  
C  
C READINPUTTAPES,70,W,M,TØTAL RMD00058  
7C FØRMAT(Ø12,I10,Ø15) RMD00059  
C  
C PRINT HEADINGS RMD00060  
C  
80 CALLDATØ(DAY) RMD00061  
WRITEØUTPUTTAPØ6,90,DAY RMD00062  
90 FØRMAT(1H1/1H345X30H RANDØM MATRIX DIAGØNALIZATØN///49X15H PERFOR RMD00066  
1RMED ØN 2A6///) RMD00067  
WRITEØUTPUTTAPØ6,100,N,WFA,WFB RMD00068  
100 FØRMAT(49XI2,21H - DIMENSIØNAL MATRIX///39X6H WFA =1PE14.7,  
1 10H WFB =1PE14.7///) RMD00069  
WRITEØUTPUTTAPØ6,110 RMD00070  
110 FØRMAT(1H1/1H454X11H INPUT DATA///) RMD00071  
WRITEØUTPUTTAPØ6,120,ITAPE,JTAPE,KTAPE RMD00072  
120 FØRMAT(38X8H ITAPE =I3,5X8H JTAPE =I3,5X8H KTAPE =I3//) RMD00073  
WRITEØUTPUTTAPØ6,130,N,NM,NB RMD00074  
130 FØRMAT(39X4H N =I3,5X5H NM =I11,5X5H NB =I3//) RMD00075  
WRITEØUTPUTTAPØ6,140,WFA,WFB RMD00076  
140 FØRMAT(40X7H WFA = F10.4,5X7H WFB = F10.4//) RMD00077  
WRITEØUTPUTTAPØ6,150,(DX(I),I=1,4) RMD00078  
150 FØRMAT(33X7H DX ARE4F10.4//) RMD00079  
WRITEØUTPUTTAPØ6,160,(DE(I),I=1,4) RMD00080  
160 FØRMAT(33X7H DE ARE4F10.4//) RMD00081  
WRITEØUTPUTTAPØ6,170,(DA(I),I=1,4) RMD00082  
170 FØRMAT(33X7H DA ARE4F10.4//) RMD00083  
WRITEØUTPUTTAPØ6,180,KØNE,KØNA,KØNN,KØNX RMD00084  
180 FØRMAT(34X7H KØNE =I2,9H KØNA =I2,9H KØNN =I2,9H KØNX =I2//) RMD00085  
WRITEØUTPUTTAPØ6,190,W,M,TØTAL RMD00086  
190 FØRMAT(41X5H W = Ø12,5X4H M =I11//50X8HTØTAL = Ø12///) RMD00087  
C  
KN=1 RMD00088  
KE=M RMD00089  
NS=N\*N RMD00090  
NT=N-1 RMD00091  
FNM=NM RMD00092  
REWINDITAPE RMD00093  
REWINDJTAPE RMD00094  
REWINDKTAPE RMD00095  
CALLRCLØK RMD00096  
IF(M)200,280,200 RMD00097  
200 IF(KØNA)210,230,210 RMD00098  
210 DØ220I=1,M RMD00099  
220 READTAPEITAPE,(A(L),L=1,NS) RMD00100  
230 IF(KØNN)280,240,280 RMD00101  
240 DØ250I=1,M RMD00102  
250 READTAPEKTAPE,(E(I),I=1,N) RMD00103  
KE=KE+1 RMD00104  
280 M=M+1 RMD00105  
IF(M-NM)290,290,610 RMD00106  
C  
C PRØDUCE RANDØM NUMBER MATRIX RMD00107  
C RMD00108  
C RMD00109  
C RMD00110  
C RMD00111

## RMD RANDOM MATRIX DIAGONALIZATION

```

290 CALLMXGEN(W) RMD00112
C
C      DIAGONALIZE AND PRODUCE EIGENVALUES RMD00113
C
C      300 CALLHDAG(0,NR) RMD00114
C          S=0. RMD00115
C          310 D0320I=1,N RMD00116
C              E(I)=H(I,I) RMD00117
C              320 S=S+E(I) RMD00118
C
C      CHECK AND ERROR PRINT RMD00119
C
C      330 IF(ABSF(S-T)-0.001)440,440,340 RMD00120
C      340 WRITE0UTPUTTAPE6,350,KN,M RMD00121
C      350 F0RFORMAT(19HO S-T ERR0R,I3,10H ON MATRIX,I3//) RMD00122
C          WRITE0UTPUTTAPE6,360,S,T RMD00123
C      360 F0RFORMAT(10X,4H S =,E11.4,8H T =,E11.4//) RMD00124
C          WRITE0UTPUTTAPE6,370,(E(I),I=1,N) RMD00125
C      370 F0RFORMAT(10X,12H EIGENVALUES/(10X,7E15.4//)) RMD00126
C          WRITE0UTPUTTAPE6,380,(HD(I),I=1,N) RMD00127
C      380 F0RFORMAT(10X,27H ORIGINAL DIAGONAL ELEMENTS/(10X,7E15.4//)) RMD00128
C          WRITE0UTPUTTAPE6,390 RMD00129
C      390 F0RFORMAT(10X,7H MATRIX) RMD00130
C          D0400I=1,N RMD00131
C      400 WRITE0UTPUTTAPE6,410,(H(I,J),J=1,N) RMD00132
C      410 F0RFORMAT(1H08E15.4/(8E15.4)) RMD00133
C          IF(KN-5)430,420,420 RMD00134
C      420 CALLDUMP RMD00135
C      430 KN=KN+1 RMD00136
C
C      ORDER EIGENVALUES RMD00137
C
C      440 D0450K=1,N RMD00138
C      450 IGRID(K)=K RMD00139
C          D0470L=1,NT RMD00140
C              LJ=L+1 RMD00141
C              D0470J=LJ,N RMD00142
C              IF(E(L)-E(J))470,470,460 RMD00143
C      460 TEMP=E(L) RMD00144
C          E(L)=E(J) RMD00145
C          E(J)=TEMP RMD00146
C          INTER=IGRID(L) RMD00147
C          IGRID(L)=IGRID(J) RMD00148
C          IGRID(J)=INTER RMD00149
C      470 CONTINUE RMD00150
C
C      PRODUCE EIGENVECTOR COMPONENTS BY VECTORS CORRESPONDING TO RMD00151
C          THE ORDERED EIGENVALUES RMD00152
C
C      480 KA=0 RMD00153
C          D0490L=1,N RMD00154
C              LT=IGRID(L) RMD00155
C              D0490K=1,N RMD00156
C              KA=KA+1 RMD00157
C      490 A(KA)=ABSF(U(K,LT)) RMD00158

```

```

RMD          RANDØM MATRIX DIAGØNALIZATION

      WRITETAPEITAPE,(A(I),I=1,NS)                      RMD00168
C
  500 IF(KØNN)540,510,540                                RMD00169
  510 WRITETAPEKTAPE,(E(L),L=1,N)                        RMD00170
C
C     COMPUTE FIRST SPACINGS FØR NØRMALIZATION FACTØR   RMD00171
C
  520 IF(KØNX)540,520,540                                RMD00172
  520 DØ530I=1,NT                                         RMD00173
  530 TØTAL=TØTAL+E(I+1)-E(I)                           RMD00174
  540 IF(SENSESWITCH1)570,550                            RMD00175
  550 CALLCLØK(ITIME)                                    RMD00176
  560 IF(72000-ITIME)570,570,600                         RMD00177
  570 CALLNRNST(W)                                       RMD00178
    PUNCH580,W,M,TØTAL,DAY(1)                           RMD00179
  580 FØRFORMAT(Ø12,I10,Ø15,35XA5,3HRMD)                RMD00180
    WRITEØUTPUTTAPE6,70,W,M,TØTAL                         RMD00181
    CALLRCLOK
    IF(SENSESWITCH1)590,600                            RMD00182
  590 REWINDITAPE                                     RMD00183
    REWINDKTAPE                                     RMD00184
    CALLEXIT                                         RMD00185
  600 IF(M-NM)280,610,610                            RMD00186
  610 N2=N/2                                           RMD00187
    N21=N2+1                                         RMD00188
    IF(KØNA)620,630,620                            RMD00189
  620 ENDFILEITAPE                                    RMD00190
    REWINDITAPE                                     RMD00191
  630 IF(KØNN)680,640,680                            RMD00192
  640 ENDFILEKTape                                    RMD00193
    REWINDKTape                                     RMD00194
C
C     TEST ØPTIØNS FØR SØRTINGS                       RMD00195
C
  650 IF(KØNX)660,650,660                            RMD00196
  650 CALLSPACE                                      RMD00197
  660 IF(KØNE)670,680,670                            RMD00198
  670 CALLVALUE                                       RMD00199
  680 IF(KØNA)690,700,690                            RMD00200
  690 CALLVECTØR                                     RMD00201
  700 WRITEØUTPUTTAPE6,710                           RMD00202
  710 FØRFORMAT(1H1)
    IFACCUMULATØRØVERFLØW720,740                  RMD00203
  720 WRITEØUTPUTTAPE6,730                           RMD00204
  730 FØRFORMAT(21H3ACCUMULATØR ØVERFLØW)           RMD00205
  740 IFDIVIDECHECK750,10                           RMD00206
  750 WRITEØUTPUTTAPE6,760                           RMD00207
  760 FØRFORMAT(13H3DIVIDE CHECK)
    GØ TØ 10                                         RMD00208
  END

```

## A1. SUBROUTINE MXGEN

Calling sequence: CALL MXGEN (W)

W =  $\xi_2$  in RAN2N and RANUN – argument to restart the generating sequence of random numbers.

This routine was separated from the main program to facilitate changing the type of matrix used. Thus there are two routines called by the same calling sequence, but with differently labeled binary decks and listings:

MXNORM      - Uses RAN2N to generate  $\frac{1}{2}(N^2 + N)$  normally distributed random numbers. These numbers are then used to set up an  $N \times N$  real symmetric matrix, H. The random numbers used for the diagonal elements are multiplied by WFA; those for the off-diagonal elements, by WFB.

MXONES      - Uses RANUN to generate  $\frac{1}{2}(N^2 + N)$  random numbers,  $\eta$ , uniformly distributed between 0 and 1. The matrix H is then loaded with plus or minus ones depending on whether  $\eta$  is, respectively, greater or less than an arbitrary parameter, WFA, with  $H_{ij} = H_{ji}$ .

Both routines compute the trace,  $T = \sum_{i=1}^N H_{i,i}$ . Both use RNSET to restart the generating subroutine.

```

MXNØRM
C      RØLTINE TØ GENERATE RANDØM NUMBER MATRICES
C
C      CALLING SEQUENCE IS
C          CALL MXGEN(W)
C
C      W IS ARGUMENT FØR REENTRY INTØ RANDØM NUMBER GENERATØR
C
C      SUBROUTINE MXGEN(W)
C
C      DIMENSIØN A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275)
C      ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003)
C      CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,
C      XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB,
C      KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM
C
C      GENERATE (N**2+N)/2 NØRMALLY DISTRIBUTED RANDØM NUMBERS
C
C      LD=(N**2+N)/2
C      IF(M-KE)20,10,20
10  CALLRNSET(W)
20  DØ30 I=1,LD,2
     CALLRAN2N(Y1,Y2)
     BL(I)=Y1
30  BL(I+1)=Y2
C
C      SET UP MATRIX AND CØMPUTE TRACE
C
C      T=0.
C      L=1
DØ60 I=1,N
DØ60 J=L,N
IF(I-J)50,40,50
40  H(I,I)=BL(L)*WFA
    HD(I)=H(I,I)
    T=T+HD(I)
    GØTØ60
50  H(I,J)=BL(L)*WFB
    H(J,I)=H(I,J)
60  L=L+1
    RETURN
    END

```

MXNØRM01  
MXNØRM02  
MXNØRM03  
MXNØRM04  
MXNØRM05  
MXNØRM06  
MXNØRM07  
MXNØRM08  
MXNØRM09  
MXNØRM10  
MXNØRM11  
MXNØRM12  
MXNØRM13  
MXNØRM14  
MXNØRM15  
MXNØRM16  
MXNØRM17  
MXNØRM18  
MXNØRM19  
MXNØRM20  
MXNØRM21  
MXNØRM22  
MXNØRM23  
MXNØRM24  
MXNØRM25  
MXNØRM26  
MXNØRM27  
MXNØRM28  
MXNØRM29  
MXNØRM30  
MXNØRM31  
MXNØRM32  
MXNØRM33  
MXNØRM34  
MXNØRM35  
MXNØRM36  
MXNØRM37  
MXNØRM38  
MXNØRM39  
MXNØRM40

## MXØNES

```

C   RØLTINE TØ GENERATE RANDØM NUMBER MATRICES          MXØNES01
C
C   CALLING SEQUENCE IS                                     MXØNES02
C           CALL MXGEN(W)                                    MXØNES03
C
C   W IS ARGUMENT FØR REENTRY INTØ RANDØM NUMBER GENERATØR MXØNES04
C
C   SUBRØUTINETMXGEN(W)                                    MXØNES05
C
C   DIMENSIØN A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275)MXØNES10
C   ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003)MXØNES11
C   CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID, MXØNES12
C   1 XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB, MXØNES13
C   2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM      MXØNES14
C
C   GENERATE (N**2+N)/2 RANDØM NUMBERS UNIFØRMLY DISTRIBUTED MXØNES15
C   BETWEEN 0 AND 1                                         MXØNES16
C
C   T=C.                                                 MXØNES17
C   IF(M-KE)20,10,20                                     MXØNES18
10  CALLRNSET(W)                                       MXØNES19
20  DØ80I=1,N                                         MXØNES20
    SENSELIGHT3                                         MXØNES21
    DØ80J=I,N                                         MXØNES22
    CALLRANUN(ETA)                                     MXØNES23
C
C   SET UP WEIGHTED MATRIX ØF PLUS AND MINUS ØNES AND CØMPUTE TRACE MXØNES24
C
C   IF(ETA-WFA)30,30,40                                 MXØNES25
30  ETA=-1.                                            MXØNES26
    GØ TØ 50                                           MXØNES27
40  ETA=+1.                                            MXØNES28
50  H(I,J)=ETA                                         MXØNES29
    IF(SENSELIGHT3)60,70                               MXØNES30
60  T=T+H(I,I)                                         MXØNES31
7C  H(J,I)=H(I,J)                                     MXØNES32
8C  CØNTINUE                                         MXØNES33
    RETURN                                              MXØNES34
    END                                                 MXØNES35

```

## A2. SUBROUTINE RAN2N

### Calling sequence:

CALL RAN2N (Y1, Y2)      Produces two normally distributed random numbers

of mean zero and standard deviation one:

$$Y_1 = \sqrt{-2 \ln \eta_1} \cos 2\pi\eta_2$$

$$Y_2 = \sqrt{-2 \ln \eta_1} \sin 2\pi\eta_2$$

$\eta_1, \eta_2$  are random numbers uniformly distributed on  
(0,1), produced as follows:

two constants,  $\xi_1, \xi_2$  are multiplied together. The low order bits of the product replace  $\xi_2$ , and are converted to a normalized, floating point number between 0 and 1.  $\eta_2$  is generated in the same way, using the new  $\xi_2$ .

Provision has been made to record the variable factor  $\xi_2$  and to restart the sequence of random numbers at any point:

CALL RANUN (ETA)      Produces a uniformly distributed random number,  $\eta$ , such that  $0 < \eta < 1$ .

CALL RNRST (W)      sets  $W = \xi_2$

CALL RNSET (X)      sets  $\xi_2 = X$ , if  $X \neq 0$ ;  
if  $X = 0$ ,  $\xi_2$  is restored to its value at load time.

RANZN

GENERATES 2 NORMALLY DISTRIBUTED RANDOM NUMBERS

00006	ENTRY	RANZN	RAN2N003		
00075	ENTRY	RANUN	RAN2N004		
00064	ENTRY	RNSET	RAN2N005		
00072	ENTRY	RNRST	RAN2N006		
00000	434627606060	LØG			
00001	625051636060	SQRT			
00002	234662606060	CØS			
00003	623145606060	SIN			
00006	0634 00 4 00051	RANZN SXA	BK,4	RAN2N007	
00007	0560 00 0 00057	LDQ	RA	RAN2N008	
00010	0200 00 0 00060	MPY	RB	RAN2N009	
00011	0763 00 0 00005	LLS	5	RAN2N010	
00012	-0754 00 0 00000	ZAC		RAN2N011	
00013	0765 00 0 00005	LRS	5	RAN2N012	
00014	-0600 00 0 00057	STQ	RA	RAN2N013	
00015	0763 00 0 00040	LLS	32	RAN2N014	
00016	-0501 00 0 00056	ØRA	CH	RAN2N015	
00017	0300 00 0 00056	FAD	CH	RAN2N016	
00020	0074 00 4 00000	TSX	\$LØG,4	RAN2N017	
00021	0131 00 0 00000	XCA		RAN2N018	
00022	0260 00 0 00112	FMP	=-2.	RAN2N019	
00023	0074 00 4 00001	TSX	\$SQRT,4	RAN2N020	
00024	0601 00 0 00061	STØ	WS1	RAN2N021	
00025	0560 00 0 00057	LDQ	RA	RAN2N022	
00026	0200 00 0 00060	MPY	RB	RAN2N023	
00027	0763 00 0 00005	LLS	5	RAN2N024	
00030	-0754 00 0 00000	ZAC		RAN2N025	
00031	0765 00 0 00005	LRS	5	RAN2N026	
00032	-0600 00 0 00057	STQ	RA	RAN2N027	
00033	0763 00 0 00040	LLS	32	RAN2N028	
00034	-0501 00 0 00056	ØRA	CH	RAN2N029	
00035	0300 00 0 00056	FAD	CH	RAN2N030	
00036	0131 00 0 00000	XCA		RAN2N031	
00037	0260 00 0 00111	FMP	=6.2831853	RAN2N032	
00040	0601 00 0 00062	STØ	WS2	RAN2N033	
00041	0074 00 4 00002	TSX	\$CØS,4	RAN2N034	
00042	0131 00 0 00000	XCA		RAN2N035	
00043	0260 00 0 00061	FMP	WS1	RAN2N036	
00044	0601 00 0 00063	STØ	WS3	RAN2N037	
00045	0500 00 0 00062	CLA	WS2	RAN2N038	
00046	0074 00 4 00003	TSX	\$SIN,4	RAN2N039	
00047	0131 00 0 00000	XCA		RAN2N040	
00050	0260 00 0 00061	FMP	WS1	RAN2N041	
00051	0774 00 4 00000	BK AXT	**,4	RAN2N042	
00052	0601 60 4 00002	STØ*	2,4	RAN2N043	
00053	0500 00 0 00063	CLA	WS3	RAN2N044	
00054	0601 60 4 00001	STØ*	1,4	RAN2N045	
00055	0020 00 4 00003	TRA	3,4	RAN2N046	
00056	2 00000 0 00000	CH	TWØ	RAN2N047	
00057	+005343277245	RA	ØCT	005343277245	RAN2N048
00060	+005343277245	RB	ØCT	005343277245	RAN2N049
00061	0 00000 0 00000	WS1	PZE	0	RAN2N050
00062	0 00000 0 00000	WS2	PZE	0	RAN2N051

## RAN2N                    GENERATES 2 NORMALLY DISTRIBUTED RANDOM NUMBERS

00063	0 00000	0 00000	WS3	PZE	0	RAN2N052
00064	0500 60 4	00001	RNSET	CLA*	1,4	RAN2N053
00065	-0320 00 0	00110		ANA	=077777777777	RAN2N054
00066	-0100 00 0	00070		TNZ	*+2	RAN2N055
00067	0500 00 0	00060		CLA	RB	RAN2N056
00070	0601 00 0	00057		STØ	RA	RAN2N057
00071	0020 00 4	00002		TRA	2,4	RAN2N058
00072	0500 00 0	00057	RNRST	CLA	RA	RAN2N059
00073	0601 60 4	00001		STØ*	1,4	RAN2N060
00074	0020 00 4	00002		TRA	2,4	RAN2N061
00075	0560 00 0	00057	RANUN	LDQ	RA	RAN2N062
00076	0200 00 0	00060		MPY	RB	RAN2N063
00077	0763 00 0	00005		LLS	5	RAN2N064
00100	-0754 00 0	00000		ZAC		RAN2N065
00101	0765 00 0	00005		LRS	5	RAN2N066
00102	-0600 00 0	00057		STQ	RA	RAN2N067
00103	0763 00 0	00040		LLS	32	RAN2N068
00104	-0501 00 0	00056		ØRA	CH	RAN2N069
00105	0300 00 0	00056		FAD	CH	RAN2N070
00106	0601 60 4	00001		STØ*	1,4	RAN2N071
00107	0020 00 4	00002		TRA	2,4	RAN2N072
				END		RAN2N073

### A3. SUBROUTINE HDIAG

Calling sequence: CALL HDIAG (IEGEN, NR)

IEGEN is set unequal to zero if only eigenvalues are to be computed;

is set equal to zero if eigenvalues and eigenvectors are to be computed.

NR is the number of rotations.

In addition, HDIAG expects to find the following arguments in COMMON:

H : the matrix to be diagonalized. Only the elements to the right of the main diagonal are operated on; at exit the eigenvalues are along the diagonal of H;

U : will contain the eigenvectors stored columnwise;

N : the order of the matrix H.

This subroutine is an adaptation of MI HDI3 by M. Merwin, distributed through SHARE.

The Jacobi method is used.

## HDIAG      DIAGONALIZATION ØF REAL SYMMETRIC MATRIX

```

SUBROUTINE HDIAG(IEGEN,NR)                               HDIAG001
DIMENSION A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275) HDIAG002
1   ,IQ(50),E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003) HDIAG003
CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,      HDIAG004
1 XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB,    HDIAG005
2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM             HDIAG006
C
15   IF (IEGEN) 15,10,15                                HDIAG007
10   DØ 14 I=1,N                                      HDIAG008
10   DØ 14 J=1,N                                      HDIAG009
11   IF(I-J)12,11,12                                 HDIAG010
11   U(I,J)=1.0                                     HDIAG011
12   GØ TØ 14                                         HDIAG012
12   U(I,J)=0.                                       HDIAG013
14   CØNTINUE                                         HDIAG014
C
15   NR = 0                                           HDIAG015
15   IF (N-1) 1000,1000,17                           HDIAG016
C
C SCAN FØR LARGEST ØFF DIAGONAL ELEMENT IN EACH RØW      HDIAG017
C X(I) CØNTAINS LARGEST ELEMENT IN ITH RØW            HDIAG018
C IQ(I) HØLDS SECØND SUBSCRIPT DEFINING PØSITIONØN ØF ELEMENT HDIAG019
C
17   NMII=N-1                                         HDIAG020
17   DØ 30 I=1,NMII                                  HDIAG021
17   X(I) = 0.                                       HDIAG022
17   IPLI=I+1                                         HDIAG023
17   DØ 30 J=IPLI,N                                 HDIAG024
17   IF ( X(I) - ABSF( H(I,J)) ) 20,20,30          HDIAG025
20   X(I)=ABSF(H(I,J))                            HDIAG026
20   IQ(I)=J                                         HDIAG027
30   CØNTINUE                                         HDIAG028
C
C SET INDICATØR FØR SHUT-ØFF. RAP=2***-27, NR=NØ. ØF RØTATIØNS HDIAG029
C RAP=7.450580596E-9                                HDIAG030
C HDTEST=1.0E38                                         HDIAG031
C
C FIND MAXIMUM ØF X(I) S FØR PIVØT ELEMENT AND       HDIAG032
C TEST FØR END ØF PRØBLEM                          HDIAG033
C
40   DØ 70  I=1,NMII                                  HDIAG034
40   IF (I-1) 60,60,45                                HDIAG035
45   IF ( XMAX- X(I) ) 60,70,70                      HDIAG036
60   XMAX=X(I)                                       HDIAG037
60   IPIV=I                                         HDIAG038
60   JPIV=IQ(I)                                     HDIAG039
70   CØNTINUE                                         HDIAG040
C
C IS MAX. X(I) EQUAL TØ ZERØ, IF LESS THAN HDTEST, REVISE HDTEST HDIAG041
C IF ( XMAX) 1000,1000,80                            HDIAG042
80   IF (HDTEST) 90,90,85                            HDIAG043
85   IF (XMAX - HDTEST) 90,90,148                  HDIAG044
90   HDIMIN = ABSF( H(1,1) )                         HDIAG045
90   DØ 110  I= 2,N                                 HDIAG046
90   IF (HDIMIN- ABSF( H(I,I))) 110,110,100        HDIAG047

```

## HDIAG      DIAGONALIZATION ØF REAL SYMMETRIC MATRIX

```

100  HDIMIN=ABSF(H(I,I))          HDIAG056
110  CØNTINUE                      HDIAG057
C
120  HDTEST=HDIMIN*RAP             HDIAG058
C
130  RETURN IF MAX.H(I,J)LESS THAN(2**-27)AHSF(H(K,K)-MIN)   HDIAG059
    IF (HDTEST- XMAX) 148,1000,1000                           HDIAG060
148  NR = NR+1                      HDIAG061
C
150  COMPUTE TANGENT, SINE AND CØSINE,H(I,I),H(J,J)           HDIAG062
    TANG=SIGNF(2.0,(H(IPIV,IPIV)-H(JPIV,JPIV)))*H(IPIV,JPIV)/(ABSF(H(IHDIAG066
    IPIV,IPIV)-H(JPIV,JPIV))+SQRTF((H(IPIV,IPIV)-H(JPIV,JPIV))**2+4.0*HDIAG067
    2(IPIV,JPIV)**2))                                         HDIAG068
    CØSINE=1.0/SQRTF(1.0+TANG**2)                            HDIAG069
    SINE=TANG*CØSINE                                         HDIAG070
    HII=H(IPIV,IPIV)                                         HDIAG071
    H(IPIV,IPIV)=CØSINE**2*(HII+TANG*(2.*H(IPIV,JPIV)+TANG*H(JPIV,JPIV)HDIAG072
    1)))                                                 HDIAG073
    H(JPIV,JPIV)=CØSINE**2*(H(JPIV,JPIV)-TANG*(2.*H(IPIV,JPIV)-TANG*H
    1II))                                                 HDIAG074
    H(IPIV,JPIV)=0.                                         HDIAG075
C
160  PSEUDØ RANK THE EIGENVALUES                         HDIAG076
C
170  ADJUST SINE AND CØS FOR COMPUTATION ØF H(IK) AND U(IK)  HDIAG077
    IF ( H(IPIV,IPIV) - H(JPIV,JPIV) ) 152,153,153          HDIAG078
152  HTEMP = H(IPIV,IPIV)                                 HDIAG079
    H(IPIV,IPIV) = H(JPIV,JPIV)                           HDIAG080
    H(JPIV,JPIV) = HTEMP                                HDIAG081
C
180  RECOMPUTE SINE AND CØS                               HDIAG082
    HTEMP = SIGNF (1.0, -SINE) * CØSINE                  HDIAG083
    CØSINE = ABSF (SINE)                                 HDIAG084
    SINE = HTEMP                                         HDIAG085
153  CØNTINUE                                         HDIAG086
C
190  INSPECT THE IQS BETWEEN I+1 AND N-1 TO DETERMINE      HDIAG087
    WHETHER A NEW MAXIMUM VALUE SHØULD BE CØMPUTED SINCE   HDIAG088
    THE PRESENT MAXIMUM IS IN THE I ØR J RØW.            HDIAG089
C
200  DØ 350 I=1,NMI1                                     HDIAG090
    IF(I-IPIV)210,350,200                                 HDIAG091
210  IF(I-JPIV)210,350,210                                 HDIAG092
230  IF(IQ(I)-IPIV)230,240,230                           HDIAG093
240  K=IQ(I)                                              HDIAG094
250  HTEMP=H(I,K)                                         HDIAG095
    H(I,K)=0.                                              HDIAG096
    IPL1=I+1                                              HDIAG097
    X(I) =0.                                              HDIAG098
C
260  SEARCH IN DEPLETED RØW FOR NEW MAXIMUM               HDIAG099
C
270  DØ 320 J=IPL1,N                                     HDIAG100
    IF ( X(I)- ABSF( H(I,J)) ) 300,300,320             HDIAG101
300  X(I) = ABSF(H(I,J))                                HDIAG102
    IQ(I)=J                                              HDIAG103
320  CØNTINUE                                         HDIAG104

```

HDIAG      DIAGONALIZATION ØF REAL SYMMETRIC MATRIX

```
      H(I,K)=HTEMP
350  CØNTINUE
C
X(IPIV) =0.
X(JPIV) =0.
C
C    CHANGE THE ØTHER ELEMENTS ØF H
C
DØ 530 I=1,N
C
IF(I-IPIV)370,530,420
370  HTEMP = H(I,IPIV)
H(I,IPIV) = CØSINE*HTEMP + SINE*H(I,JPIV)
IF ( X(I) - ABSF( H(I,IPIV)) )380,390,390
380  X(I) = ABSF(H(I,IPIV))
IQ(I) = IPIV
390  H(I,JPIV) = -SINE*HTEMP + CØSINE*H(I,JPIV)
IF ( X(I) - ABSF( H(I,JPIV)) ) 400,530,530
400  X(I) = ABSF(H(I,JPIV))
IQ(I) = JPIV
GØ TØ 530
C
420  IF(I-JPIV)430,530,480
430  HTEMP = H(IPIV,I)
H(IPIV,I) = CØSINE*HTEMP + SINE*H(I,JPIV)
IF ( X(IPIV) - ABSF( H(IPIV,I)) ) 440,450,450
440  X(IPIV) = ABSF(H(IPIV,I))
IQ(IPIV) = I
450  H(I,JPIV) = -SINE*HTEMP + CØSINE*H(I,JPIV)
IF ( X(I) - ABSF( H(I,JPIV)) ) 400,530,530
C
480  HTEMP = H(IPIV,I)
H(IPIV,I) = CØSINE*HTEMP + SINE*H(JPIV,I)
IF ( X(IPIV) - ABSF( H(IPIV,I)) ) 490,500,500
490  X(IPIV) = ABSF(H(IPIV,I))
IQ(IPIV) = I
500  H(JPIV,I) = -SINE*HTEMP + CØSINE*H(JPIV,I)
IF ( X(JPIV) - ABSF( H(JPIV,I)) ) 510,530,530
510  X(JPIV) = ABSF(H(JPIV,I))
IQ(JPIV) = I
530  CØNTINUE
C
C    TEST FØR CØMPUTATION ØF EIGENVECTØRS
C
IF(IEGEN)40,540,40
540  DØ 550 I=1,N
HTEMP=U(I,IPIV)
U(I,IPIV)=CØSINE*HTEMP+SINE*U(I,JPIV)
550  U(I,JPIV)=-SINE*HTEMP+CØSINE*U(I,JPIV)
GØ TØ 40
1000 RETURN
END
```

HDIAG112  
HDIAG113  
HDIAG114  
HDIAG115  
HDIAG116  
HDIAG117  
HDIAG118  
HDIAG119  
HDIAG120  
HDIAG121  
HDIAG122  
HDIAG123  
HDIAG124  
HDIAG125  
HDIAG126  
HDIAG127  
HDIAG128  
HDIAG129  
HDIAG130  
HDIAG131  
HDIAG132  
HDIAG133  
HDIAG134  
HDIAG135  
HDIAG136  
HDIAG137  
HDIAG138  
HDIAG139  
HDIAG140  
HDIAG141  
HDIAG142  
HDIAG143  
HDIAG144  
HDIAG145  
HDIAG146  
HDIAG147  
HDIAG148  
HDIAG149  
HDIAG150  
HDIAG151  
HDIAG152  
HDIAG153  
HDIAG154  
HDIAG155  
HDIAG156  
HDIAG157  
HDIAG158  
HDIAG159  
HDIAG160  
HDIAG161  
HDIAG162

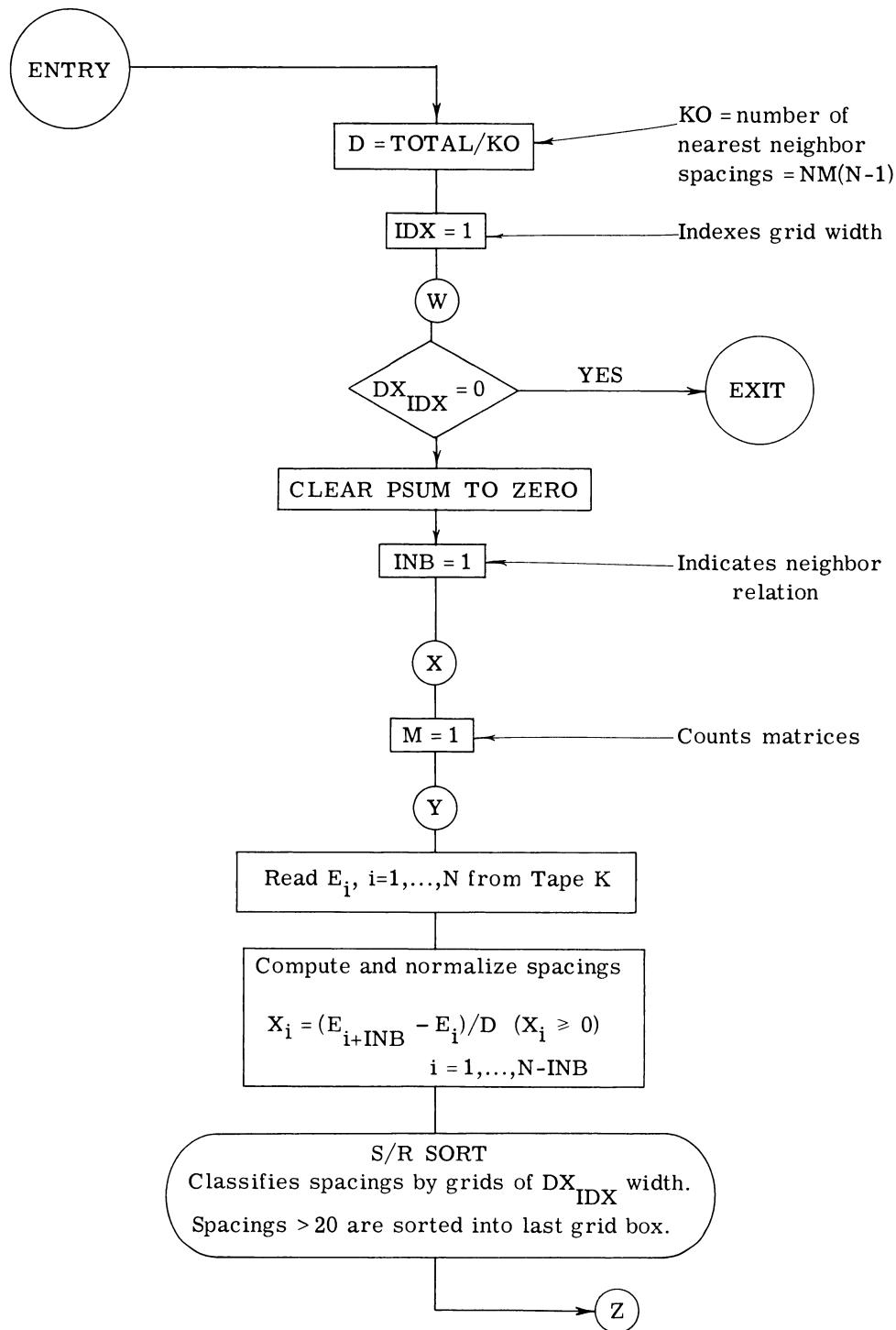
## B. SUBROUTINE SPACE

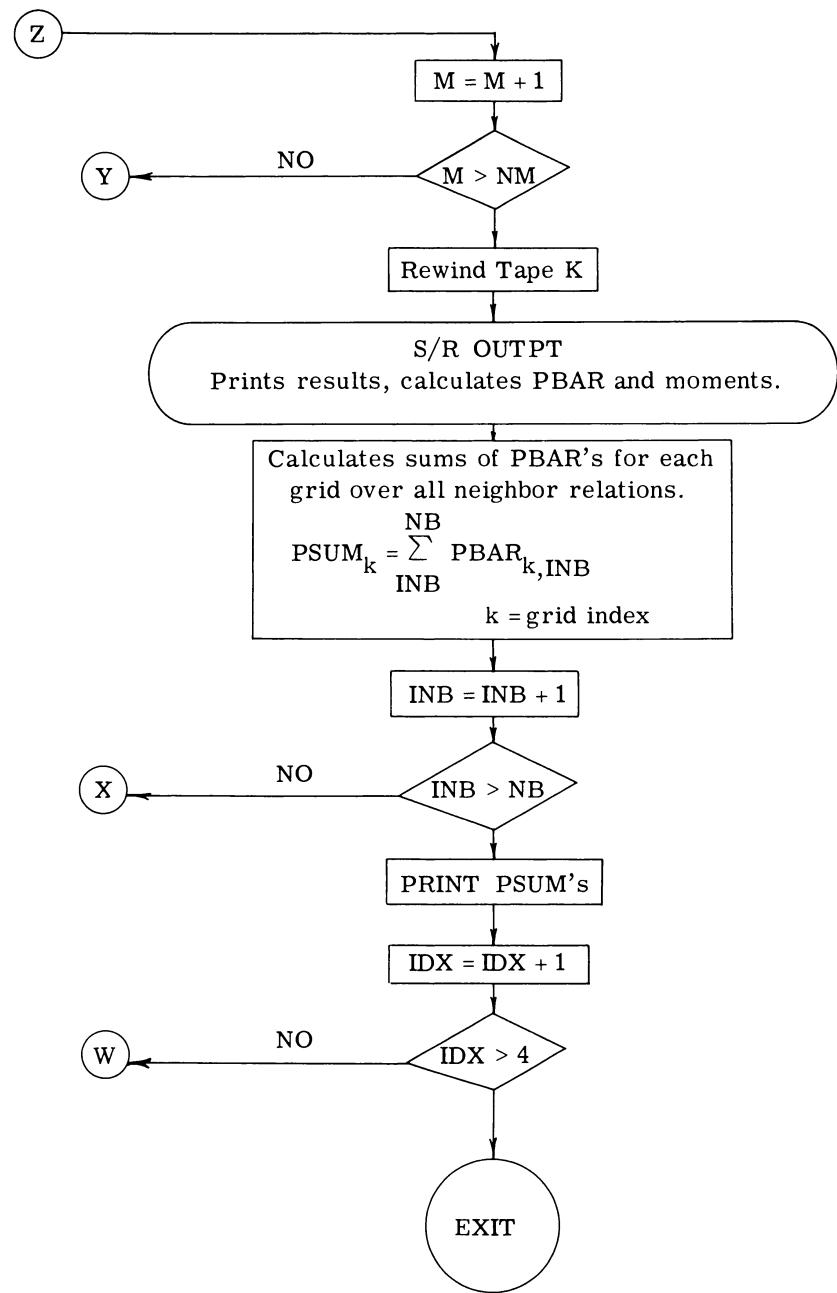
Calling sequence: CALL SPACE

1. Calculates the normalization factor,  $D = (\sum_{m=1}^{NM} \sum_{i=1}^{N-1} SPAC_{i,m})/[NM*(N-1)]$ , where  $SPAC_{i,m}$  are the nearest neighbor spacings of eigenvalues,  $E_i$  ( $i=1, \dots, N$ ), for the  $m^{\text{th}}$  matrix, NM the number of matrices, N the dimension of the matrices.
2. For each neighbor relation, INB = 1, ..., NB, the eigenvalue spacings are computed and normalized:  $X_i = (E_{i+INB} - E_i)/D$ , ( $i=1, \dots, N-INB$ ). Since the eigenvalues were ordered from smallest up, the spacings are all non-negative.
3. For each neighbor relation separately, subroutine SORT classifies the spacings by grids of width DX, putting spacings  $> 20$  into the last grid box. The results are printed and PBAR and the moments computed by subroutine OUTPT for each neighbor relation.
4. Calculates the sums of PBAR's for each grid over all neighbor relations:  
$$PSUM_K = \sum_{INB=1}^{NB} PBAR_{K,INB}$$
 K is index of grids. These results are printed.

Provision has been made to repeat this process for up to four different grid widths.

SUBROUTINE SPACE





ROUTINE TO HANDLE SPACINGS

CALLING SEQUENCE IS  
CALL SPACE

SUBROUTINESPACE

DIMENSIØN A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275)SPACE007  
1 ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003)SPACE008  
C ØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,  
C 1 XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB,  
C 2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM  
C  
C ØMPUTE NØRMALIZATION FACTØR  
C  
AKØ=NM\*NT  
D=TØTAL/AKØ  
GRID=0.  
CØNST=1.

C ØMPUTE, NØRMALIZE AND SØRT SPACINGS

10 DØ170IDX=1,4  
IF(DX(IDX))20,180,20  
20 XMID=DX(IDX)/2.  
NGS=20./DX(IDX)+1.  
DØ30I=1,NGS  
30 PSUM(I)=0.  
MAXG=1  
DØ70INB=1,NB  
NUM=N-INB  
TØTAL=FLØATF(NUM)\*FNM  
REWIND KTAPE  
DØ60M=1,NM  
M=M  
READTAPEKTAPE,(E(I),I=1,N)  
DØ50I=1,NUM  
IJ=I+INB  
50 X(I)=(E(IJ)-E(I))/D  
60 CALLSØRT(DX(IDX),X)  
REWINDKTAPE

C ØUTPUT AND CØMPUTING MØMENTS

KNB=INB-1  
WRITEØUTPUTTAP6,100,DX(IDX),KNB  
100 FØRFORMAT(1H116X9H DELTAX =F6.3,9X8HNEIGHBØRI3///)  
160 FØRFORMAT(18XF6.3,7X1PE12.4)  
WRITEØUTPUTTAP6,110  
110 FØRFORMAT(17X32H X MIDPT CØUNT P-BAR//)  
CALLØUTPT(DX(IDX))

C ØMPUTE SUMS ØF PBARS

DØ70I=1,MAXG  
70 PSUM(I)=PSUM(I)+PBAR(I)



### C. SUBROUTINE VALUE

Calling sequence: CALL VALUE

Calculates the normalization factor,  $SDEN = 2*WFB*\sqrt{N}$ , where WFB is the off-diagonal scale factor, N the matrix dimension, and normalizes the eigenvalues,  $E_i = \frac{E_i}{SDEN}$  ( $i=1, \dots, N$ ). Calls subroutine SORT to classify these eigenvalues by grids of width DE.

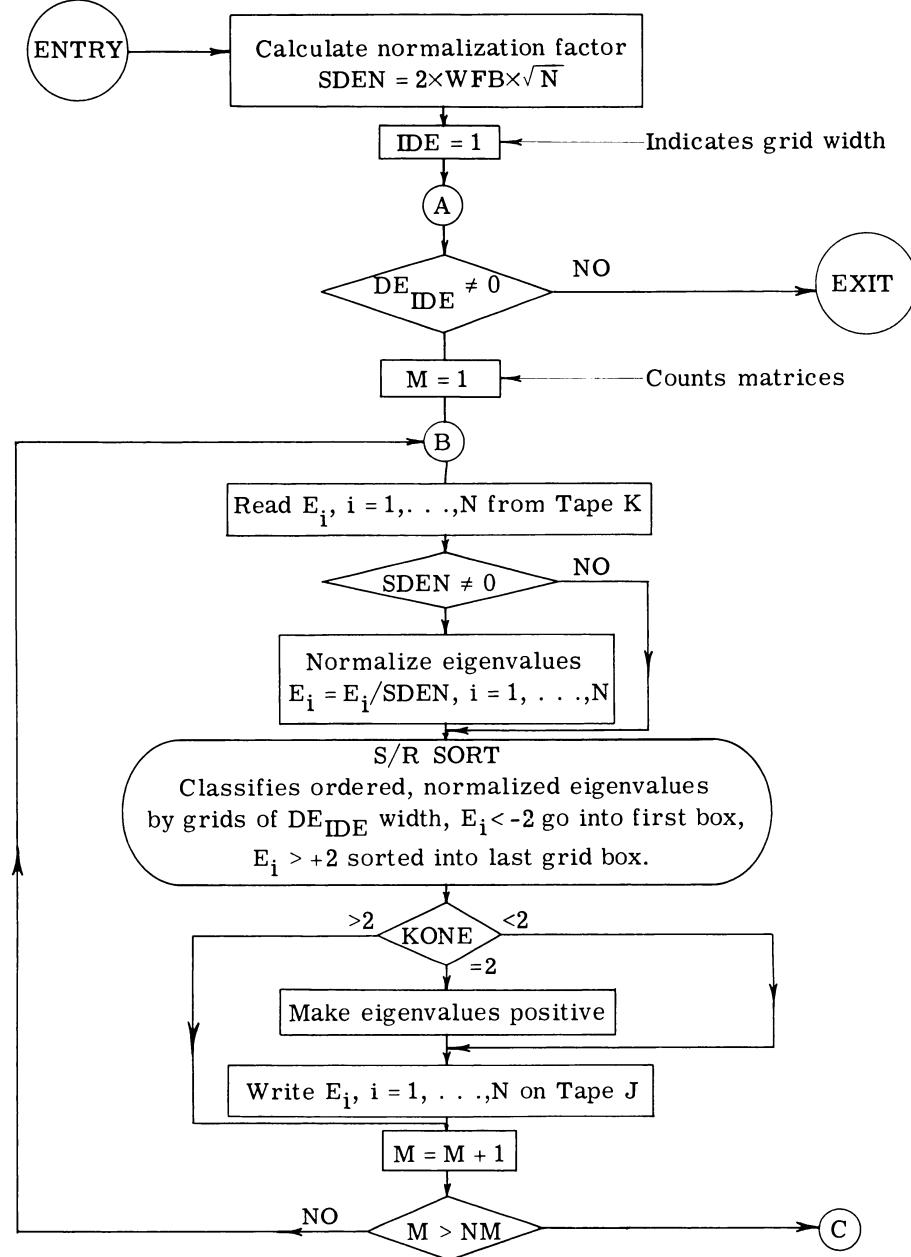
If a symmetric sorting is required, the eigenvalues are now made positive. Next they are taped for use, if desired, by subroutine SYMM or ASYM.

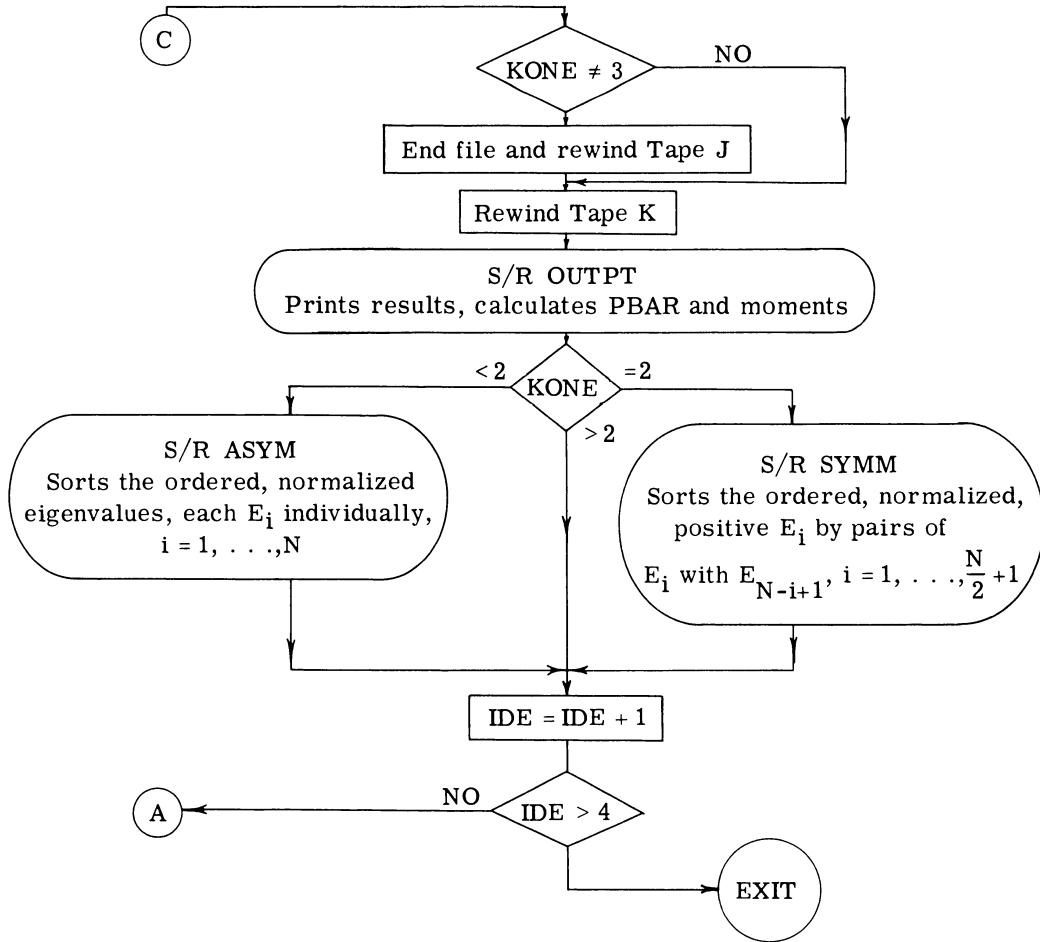
The results of the general sorting are printed and PBAR and the moments computed by subroutine OUTPT.

Finally options are tested for transfer to subroutine SYMM or ASYM for symmetric or asymmetric sorting, respectively.

Provision has been made to repeat this process for up to four different grid widths.

SUBROUTINE VALUE





```

VALUE
C      RØUTINE TØ HANDLE THE EIGENVALUES          VALUE001
C
C      CALLING SEQUENCE IS                      VALUE002
C          CALL VALUE                         VALUE003
C
C      SUBRØUTINE VALUE                      VALUE004
C
C      DIMENSØN A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275) VALUE005
1      ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003) VALUE006
CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,          VALUE010
1 XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB,          VALUE011
2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM             VALUE012
C
C      AN=N                                     VALUE013
SDEN=(WFB*SQRTF(AN))+(WFB*SQRTF(AN))           VALUE014
DØ140 IDE=1,4                                    VALUE015
IF(DE(IDE))10,150,10                           VALUE016
10 NGS=4./DE(IDE)+3.                          VALUE017
CØNST=1.5707963268                            VALUE018
GRID=-2.                                         VALUE019
NUM=N                                           VALUE020
TØTAL=FLØATF(NUM)*FNM                         VALUE021
MAXG=1                                         VALUE022
DØ60M=1,NM                                      VALUE023
M=M                                           VALUE024
READTAPEKTAPE,(E(I),I=1,N)                     VALUE025
C
C      NØRMALIZE THE EIGENVALUES               VALUE026
C
C      DØ20I=1,N                                     VALUE027
20 E(I)=E(I)/SDEN                            VALUE028
C
C      SØRTING THE EIGENVALUES                  VALUE029
C
CALLSØRT(DE(IDE),E)                           VALUE030
IF(KØNE-2)50,30,60                           VALUE031
C
C      MAKE EIGENVALUES PØSITIVE                VALUE032
C
C      DØ40I=1,N21                                VALUE033
30 E(I)=ABSF(E(I))                           VALUE034
40 WRITETAPEJTAPE,(E(L),L=1,N)              VALUE035
50 CØNTINUE                                     VALUE036
60 IF(KØNE-3)70,80,70                           VALUE037
70 ENDFILEJTAPE                               VALUE038
REWINDJTAPE                                  VALUE039
80 REWINDKTAPE                                VALUE040
C
C      ØUTPUT AND CØMPUTING MØMENTS            VALUE041
C
WRITEØUTPUTTAPE6,90,DE(IDE)                   VALUE042
90 FØRFORMAT(1H115X9H DELTAE =F6.3,10X12H EIGENVALUES)          VALUE043
WRITEØUTPUTTAPE6,100,SDEN                      VALUE044
100 FØRFORMAT(1H011X46H EIGENVALUES NØRMALIZED BY 2*SQRTF(N*B**2) =1PE1)    VALUE045
11,4//)                                         VALUE046

```

```

VALUE

      WRITEØUTPUTTAPE6,110          VALUE056
110  FØRFORMAT(17X36H E MIDPT      COUNT      P-BAR*(PI/2)//)  VALUE057
      XMID=-2.+DE(IDE)/2.          VALUE058
      CALLØUTPT(DE(IDE))          VALUE059
C                                         VALUE060
C   ØPTION FØR SYMMETRIC ØR ASYMMETRIC SØRTING  VALUE061
C                                         VALUE062
C   IF(KØNE-2)120,130,140          VALUE063
120  CALLASYM(DE(IDE))          VALUE064
GØTØ140                           VALUE065
130  CALLSYMM(DE(IDE))          VALUE066
140  CØNTINUE                     VALUE067
150  RETURN                         VALUE068
END

```

## C1. SUBROUTINE SYMM

Calling sequence: CALL SYMM (DEL)

DEL = grid width

Uses subroutine SORT to classify the ordered, positive eigenvalues,  $E_i$  ( $i=1, \dots, N$ ) in pairs of the highest and lowest,  $E_1$  and  $E_N$ , the second highest and second lowest,  $E_2$  and  $E_{N-1}, \dots, E_i$  and  $E_{N-i+1}$ , as the plus-minus symmetry of the input Gaussians warrants.

After sorting each pair from all NM matrices, subroutine OUTPT is used to compute PBAR and the moments and print the results.

If the dimension N, of the matrix is odd, the middle eigenvalues are sorted alone.

```

SYMM

C      RØLTINE FØR SYMMETRIC SØRTING ØF EIGENVALUES BY PAIRS          SYMM0001
C
C      CALLING SEQUENCE IS                                         SYMM0002
C          CALL SYMM(DEL)                                         SYMM0003
C
C      DEL IS GRID WIDTH                                         SYMM0004
C
C      SUBROUTINESYMM(DEL)                                         SYMM0005
C
C      DIMENSIØN A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275)SYMM0010
C          ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003)SYMM0011
C      CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,   SYMM0012
C          XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB,SYMM0013
C          1 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM           SYMM0014
C
C      CØNST=1.                                                 SYMM0015
C      GRID=0.                                                 SYMM0016
C      NGS=2./DEL+2.                                         SYMM0017
C      NUM=2                                                 SYMM0018
C      TØTAL=FNM+FNM                                         SYMM0019
C      DØ70J=1,N21                                         SYMM0020
C      LJ=N-J+1                                         SYMM0021
C      MAXG=1                                         SYMM0022
C
C      SØRT EIGENVALUES BY PAIRS                               SYMM0023
C
C      CØ10M=1,NM                                         SYMM0024
C      M=M                                         SYMM0025
C      READTAPE JTAPE,(E(L),L=1,N)                         SYMM0026
C      E(1)=E(J)                                         SYMM0027
C      E(2)=E(LJ)                                         SYMM0028
C      10 CALLSØRT(DEL,E)                                     SYMM0029
C      REWIND JTAPE                                         SYMM0030
C
C      ØUTPUT AND CØMPUTING MØMENTS                         SYMM0031
C
C      WRITEØUTPUTTAPE6,20,DEL,J,LJ                         SYMM0032
C      20 FØRMAT(1H111X9H DELTAE =F6.3,10X13H EIGENVALUES(I2,4H), (I2,1H)) SYMM0033
C      WRITEØUTPUTTAPE6,30,SDEN                           SYMM0034
C      30 FØRMAT(1H011X46H EIGENVALUES NØRMALIZED BY 2*WFB*SQRTF(N) = 1PE1SYMM0040
C          10.4//)
C      WRITEØUTPUTTAPE6,40                                         SYMM0041
C      40 FØRMAT(17X32H E MIDPT      COUNT      P-BAR//)          SYMM0042
C          XMID=DEL/2.                                         SYMM0043
C          CALLØUTPT(DEL)                                     SYMM0044
C          IF(J-N2)70,50,70                                 SYMM0045
C          50 IF(N2+N2-N)60,80,80                           SYMM0046
C          60 NUM=1                                         SYMM0047
C          TØTAL=FNM                                         SYMM0048
C          70 CØNTINUE                                         SYMM0049
C          80 RETURN                                         SYMM0050
C          END                                           SYMM0051

```

## C2. SUBROUTINE ASYM

Calling sequence: CALL ASYM (DEL)

DEL = grid width

Uses subroutine SORT to classify the ordered eigenvalues,  $E_i$  ( $i=1, \dots, N$ ), individually;  $E_1$  from all NM matrices, then  $E_2$ , etc. The result of each sorting is printed, and PBAR and the moments computed by subroutine OUTPT before the next set of  $E_i$ 's is sorted.

## ASYM

```

C      RØUTINE FØR ASYMMETRIC SØRTING ØF EIGENVALUES BY SINGLES          ASYM0001
C
C      CALLING SEQUENCE IS                                              ASYM0002
C          CALL ASYM(DEL)
C
C      DEL   IS GRID WIDTH                                              ASYM0003
C
C      SUBRØUTINEASYM(DEL)                                              ASYM0004
C
C      DIMENSION A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275) ASYM0010
C          ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003) ASYM0011
C      CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID, ASYM0012
C          1 XMid,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB, ASYM0013
C          2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM             ASYM0014
C
C      SØRT EIGENVALUES BY SINGLES                                         ASYM0015
C
C      CØNST=1.                                                       ASYM0016
C      GRID=-2.                                                       ASYM0017
C      NUM=1                                                        ASYM0018
C      TØTAL=FNM                                              ASYM0019
C      DØ50J=1,N                                              ASYM0020
C      MAXG=1                                              ASYM0021
C      DØ10M=1,NM                                              ASYM0022
C      M=M                                                        ASYM0023
C      READTAPEJTAPE,(E(L),L=1,N)                                     ASYM0024
C
10     CALLSØRT(DEL,E(J))
C      REWINDJTAPE                                         ASYM0025
C
C      ØUTPUT AND CØMPUTING MØMENTS                                ASYM0026
C
C      WRITEØUTPUTTAPE6,20,DEL,J                                     ASYM0027
C
20     FØRMAT(1H114X9H DELTAE =F6.3,10X12H EIGENVALUE(I2,1H))       ASYM0028
C      WRITEØUTPUTTAPE6,30,SDEN                                         ASYM0029
C
30     FØRMAT(1H011X46H EIGENVALUES NØRMALIZED BY 2*WFB*SQRTF(N) = 1PE1 ASYM0030
C          10.4//)                                              ASYM0031
C      WRITEØUTPUTTAPE6,40                                         ASYM0032
C
40     FØRMAT(17X32H E MIDPT      COUNT      P-BAR//)           ASYM0033
C      XMid=-2.+DEL/2.                                              ASYM0034
C
50     CALLØUTPT(DEL)
C      RETURN
C      END

```

#### D. SUBROUTINE VECTOR

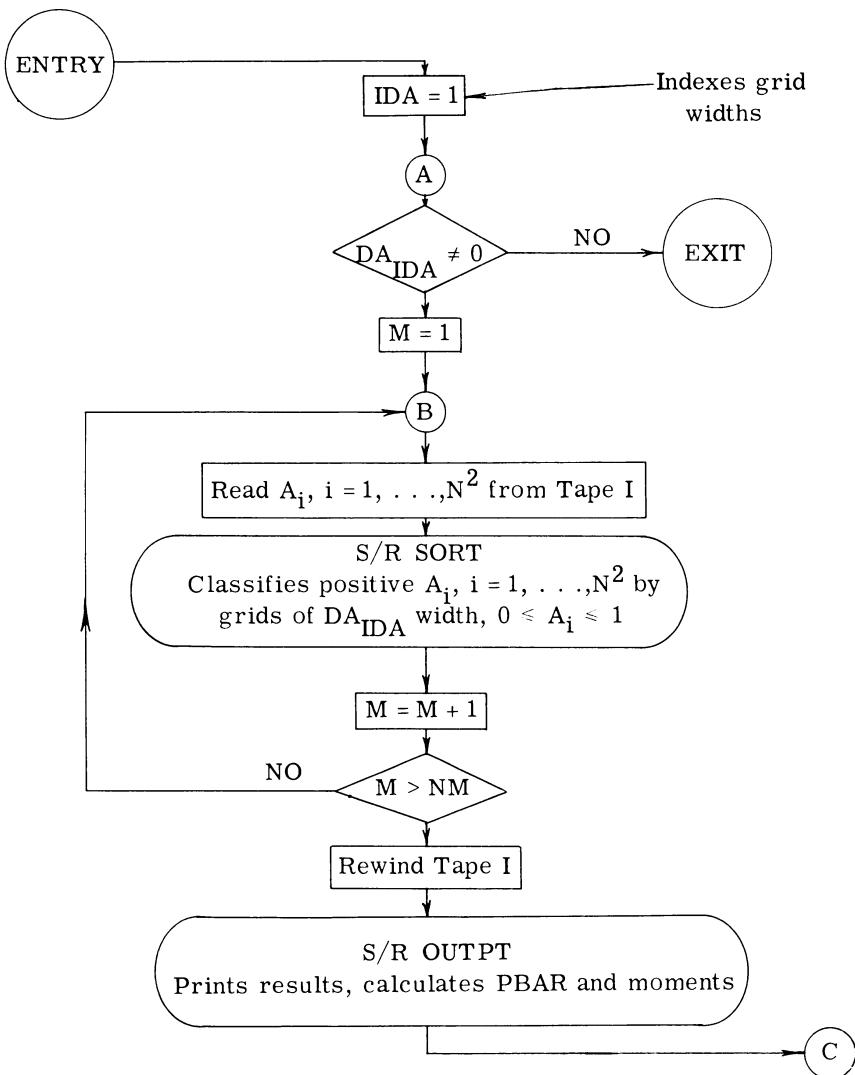
Calling sequence: CALL VECTOR

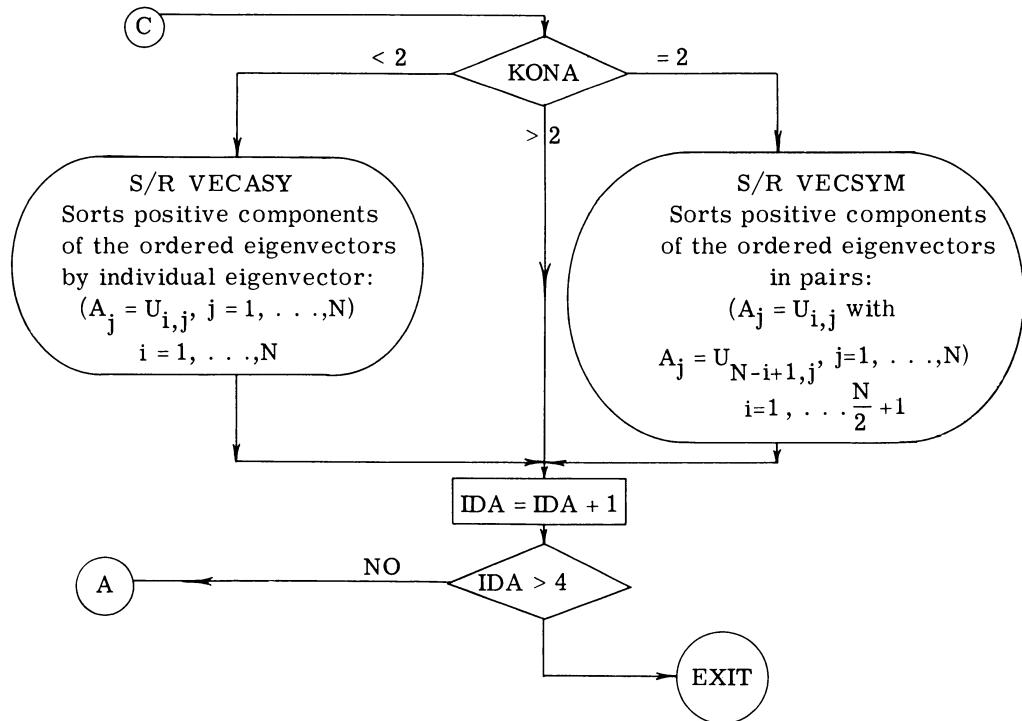
Classifies the positive eigenvector components,  $A_i$  ( $i=1, \dots, N^2$ ) by grids of width DA using subroutine SORT. The results are printed and PBAR and the moments computed by subroutine OUTPT.

Then options are tested for transfer to subroutine VECASY or VECASY for symmetric or asymmetric sorting of the eigenvectors, respectively.

Provision has been made to repeat this process for up to four different grid widths.

SUBROUTINE VECTOR





## VECTØR

```

C      ROUTINE TØ HANDLE THE EIGENVECTØR CØMPØNENTS          VECTØR01
C
C      CALLING SEQUENCE IS                                     VECTØR02
C          CALL VECTØR
C
C      SUBROUTINE VECTØR                                     VECTØR03
C
C      DIMENSIØN A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275)VECTØR08
C      1           ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003)VECTØR09
C      CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,          VECTØR10
C      1 XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,KTAPE,M,N,NM,NB,          VECTØR11
C      2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM          VECTØR12
C
C      CØNST=1.                                              VECTØR13
C      GRID=0.                                               VECTØR14
C      DØ70IDA=1,4                                         VECTØR15
C      IF(DA(IDA))10,80,10                                 VECTØR16
C      10 NGS=1./DA(IDA)+1.                                VECTØR17
C      NUM=NS                                              VECTØR18
C      TØTAL=FLØATF(NUM)*FNM                            VECTØR19
C      MAXG=1                                              VECTØR20
C
C      SØRTING THE EIGENVECTØR CØMPØNENTS                VECTØR21
C
C      DØ20M=1,NM                                         VECTØR22
C      M=M                                              VECTØR23
C      READTAPEITAPE,(A(L),L=1,NS)                      VECTØR24
C      20 CALLSØRT(DA(IDA),A)                           VECTØR25
C      REWINDITAPE
C
C      ØUTPUT AND CØMPUTING MØMENTS                     VECTØR26
C
C      WRITEØUTPUTTAPE6,30,DA(IDA)                      VECTØR27
C      30 FØRFORMAT(1H110X9H DELTAA =F6.3,10X23H EIGENVECTØR CØMPØNENTS//)    VECTØR28
C      WRITEØUTPUTTAPE6,40                               VECTØR29
C      40 FØRFORMAT(17X32H A MIDPT          COUNT          P-BAR//)        VECTØR30
C      XMID=DA(IDA)/2.                                 VECTØR31
C      CALLØUTPT(DA(IDA))                           VECTØR32
C      IF(KØNA-2)50,60,70                           VECTØR33
C      50 CALLVECASY(DA(IDA))                      VECTØR34
C      GØ TØ 70                                     VECTØR35
C      60 CALLVECSYM(DA(IDA))                      VECTØR36
C      70 CØNTINUE                                    VECTØR37
C      80 RETURN                                     VECTØR38
C      END                                         VECTØR39

```

## D1. SUBROUTINE VECSYM

Calling sequence: CALL VECSYM (DAL)

DAL = grid width

Sorts the positive components,  $A_i$  ( $i=1, \dots, N$ ), of symmetric pairs of the eigenvectors which have been reordered to correspond with the ordering of the eigenvalues. The components of the first and last vector are sorted together by subroutine SORT, the results are printed, and PBAR and the moments computed by subroutine OUTPT. Then the components of the second and next to last vector are sorted and the results printed, etc.

If the dimension,  $N$ , of the matrix is odd, the components of the middle vector are sorted by themselves.

## VECSYM

```

C      RØUTINE FØR SYMMETRIC SØRTING ØF EIGENVECTØRS BY PAIRS          VECSYM01
C
C      CALLING SEQUENCE IS                                              VECSYM02
C          CALL VECASYM(DAL)                                              VECSYM03
C
C      DAL IS GRID WIDTH                                                 VECSYM04
C
C      SUBROUTINE VECASYM(DAL)                                             VECSYM05
C
C      DIMENSIØN A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275)VECSYM10
C          ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003)VECSYM11
C      CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,    VECSYM12
C      1 XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB,  VECSYM13
C      2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM                 VECSYM14
C
C      NUM=N+N                                              VECSYM15
C      TØTAL=FLØATF(NUM)*FNM                                         VECSYM16
C      CØNST=1.                                              VECSYM17
C      DØ70K=1,N21                                              VECSYM18
C      MAXG=1                                              VECSYM19
C      NK=N*(K-1)                                              VECSYM20
C      NKN=NK+N                                              VECSYM21
C      DØ20M=1,NM                                              VECSYM22
C      M=M                                              VECSYM23
C      READTAPEITAPE,(A(L),L=1,NS)                                     VECSYM24
C      DØ10I=1,N                                              VECSYM25
C      NK I=NK+I                                              VECSYM26
C      A(I)=A(NK I)                                              VECSYM27
C      IN=I+N                                              VECSYM28
C      NS I=NS-NKN+I                                              VECSYM29
C      10 A(IN)=A(NS I)                                              VECSYM30
C      20 CALLSØRT(DAL,A)                                              VECSYM31
C      REWINDITAPE                                              VECSYM32
C      LN=N-K+1                                              VECSYM33
C      WRITEØUTPUTTAPE6,30,DAL,K,LN                                VECSYM34
C      30 FØRFORMAT(1H113X9H DELTAA =F6.3,10X14H EIGENVECTØRS(I2,4H), (I2,1H))  VECSYM35
C      WRITEØUTPUTTAPE6,40                                              VECSYM36
C      40 FØRFORMAT(1H016X32H A MIDPT      CØUNT      P-BAR//)        VECSYM37
C      XMID=DA/2.                                              VECSYM38
C      CALLØUTPT(DAL)                                              VECSYM39
C      1F(K-N2)70,50,70                                              VECSYM40
C      50 IF(N2+N2-N)60,80,80                                         VECSYM41
C      60 NUM=N                                              VECSYM42
C      TØTAL=FLØATF(NUM)*FNM                                         VECSYM43
C      70 CØNTINUE                                              VECSYM44
C      80 RETURN                                              VECSYM45
C      END                                              VECSYM46

```

## D2. SUBROUTINE VECASY

Calling sequence: CALL VECASY (DAL)

DAL = grid width

Sorts individually, using subroutine SORT, the positive components,  $A_i$  ( $i=1, \dots, N$ ), of each of the eigenvectors which have been reordered to correspond with the ordering of the eigenvalues. Uses subroutine OUTPT to print results and compute PBAR and the moments for each vector before sorting the next one.

## VECASY

```

C      ROUTINE FØR ASYMMETRIC SØRTING ØF EIGENVECTØRS BY SINGLES      VECASY01
C
C      CALLING SEQUENCE IS                                              VECASY02
C          CALL VECASY(DAL)                                               VECASY03
C
C      DAL    IS GRID WIDTH                                              VECASY04
C
C      SUBROUTINE VECASY(DAL)                                              VECASY05
C
C      DIMENSION A(2500),PBAR(2003),PSUM(2003),H(50,50),U(50,50),BL(1275) VECASY10
C          ,E(50),HD(50),X(50),DX(4),DE(4),DA(4),DAY(2),IGRID(2003) VECASY11
C      CØMMØN A,PBAR,PSUM,H,U,BL,E,HD,X,DX,DE,DA,DAY,W,WFA,WFB,GRID,      VECASY12
C      1 XMID,SDEN,T,CØNST,FNM,TØTAL,IGRID,ITAPE,JTAPE,KTAPE,M,N,NM,NB,   VECASY13
C      2 KØNE,KØNA,KØNN,KØNX,MAXG,NGS,NS,NT,KE,N2,N21,NUM                 VECASY14
C
C      NUM=N                                              VECASY15
C      TØTAL=FLØATF(NUM)*FNM                                         VECASY16
C      CØNST=1.                                              VECASY17
C      DØ50K=1,N                                              VECASY18
C      MAXG=1                                              VECASY19
C      NK=N*(K-1)                                              VECASY20
C      DØ20M=1,NM                                              VECASY21
C      M=M                                              VECASY22
C      READTAPEITAPE,(A(L),L=1,NS)                                     VECASY23
C      DØ10I=1,N                                              VECASY24
C      NKI=NK+I                                              VECASY25
C      10 A(I)=A(NKI)                                              VECASY26
C      20 CALLSØRT(DAL,A)                                              VECASY27
C      REWINDITAPE                                              VECASY28
C      WRITEØUTPUTTAPE6,30,DAL,K                                     VECASY29
C      30 FØRFORMAT(1H113X9H DELTAA =F6.3,10X13H EIGENVECTØR(I2,1H)) VECASY30
C      WRITEØUTPUTTAPE6,40                                         VECASY31
C      40 FØRFORMAT(1H016X32H A MIDPT      CØUNT      P-BAR//) VECASY32
C      XMID=DA/2.                                              VECASY33
C      50 CALLØUTPT(DAL)                                              VECASY34
C      RETURN                                              VECASY35
C      END                                              VECASY36

```

## E. SUBROUTINE SORT

Calling sequence: CALL SORT (DD, V)

DD = grid width

V = array to be sorted

In addition, SORT expects to find the following arguments in COMMON:

NUM = number of values in V to be sorted

M = matrix count

NGS = number of grids

GRID = origin of grids

IGRID = array for counts

MAXG = will be set to highest index used in IGRID

Counts how many elements of V fall into each of NGS intervals of origin GRID and width DD, and stores these counts into an array  $IGRID_i$ ,  $i = 1, \dots, MAXG$ . If any element of V is less than GRID, or greater than GRID + NGS \* DD, the count will go into the first or last box of IGRID respectively.

## SØRT

## SØRTING SUBROUTINE

		*	CALLING SEQUENCE IS	SØRT0001	
		*	CALLSØRT(DD,V)	SØRT0002	
		*	DD=GRID WIDTH	SØRT0003	
		*	V=ARRAY TO BE SØRTED	SØRT0004	
	00002	ENTRY	SØRT	SØRT0005	
00002	0634 00 1 0C052	SØRT	SXA	SØRT0008	
00003	0634 00 2 00053		SXA	SØRT0009	
00004	0500 00 0 42300		CLA	SØRT0010	
00005	0402 00 0 00055		SUB	SØRT0011	
00006	-0100 00 0 00022		TNZ	SØRT0012	
00007	0500 00 4 00002		CLA	SØRT0013	
00010	0400 00 0 00056		ADD	SØRT0014	
00011	0621 00 0 00023		STA	SØRT0015	
00012	0500 00 0 42267		CLA	SØRT0016	
00013	0622 00 0 00021		STD	SØRT0017	
00014	0500 00 0 42261		CLA	SØRT0018	
00015	0622 00 0 00051		STD	SØRT0019	
00016	0774 00 1 00001		AXT	SØRT0020	
00017	0600 00 1 46227		STZ	SØRT0021	
00020	1 00001 1 00021		TXI	SØRT0022	
00021	-3 00000 1 00017	NG	TXL	SØRT0023	
00022	0774 00 1 00001	NØT	AXT	SØRT0024	
00023	0500 00 1 00000	V	CLA	SØRT0025	
00024	0302 00 0 46235		FSB	SØRT0026	
00025	0120 00 0 00030		TPL	SØRT0027	
00026	0500 00 0 00055		CLA	SØRT0028	
00027	0020 00 0 00041		TRA	SØRT0029	
00030	0241 60 4 00001		FDP*	SØRT0030	
00031	0131 00 0 00000		XCA	SØRT0031	
00032	0300 00 0 00060		FAD	=02334000000000	SØRT0032
00033	-0320 00 0 00057		ANA	=077777	SØRT0033
00034	0767 00 0 00022		ALS	18	SØRT0034
00035	0400 00 0 00055		ADD	ØNED	SØRT0035
00036	0340 00 0 42267		CAS	NGS	SØRT0036
00037	0500 00 0 42267		CLA	NGS	SØRT0037
00040	0761 00 0 00000		NØP		SØRT0038
00041	-0734 00 2 0C000	SX	PDX	,2	SØRT0039
00042	0340 00 0 42270		CAS	MAXG	SØRT0040
00043	0601 00 0 42270		STØ	MAXG	SØRT0041
00044	0761 00 0 00000		NØP		SØRT0042
00045	0500 00 2 46227		CLA	IGRID+1,2	SØRT0043
00046	0400 00 0 00056		ADD	=1	SØRT0044
00047	0601 00 2 46227		STØ	IGRID+1,2	SØRT0045
00050	1 00001 1 00051		TXI	*+1,1,1	SØRT0046
00051	-3 00000 1 00023	NU	TXL	V,1,**	SØRT0047
00052	0774 00 1 00000	ØUT	AXT	**,1	SØRT0048
00053	0774 00 2 00000		AXT	**,2	SØRT0049
00054	0020 00 4 00003		TRA	3,4	SØRT0050
00055	0 00001 0 00000	ØNED	PZE	,,1	SØRT0051
	042261	NUM	BØØL	42261 NØ.TØ BE SØRTED	SØRT0052
	042300	M	BØØL	42300 MATRIX CØUNT	SØRT0053
	042267	NGS	BØØL	42267 NØ.ØF GRIDS	SØRT0054
	042270	MAXG	BØØL	42270	SØRT0055
	046235	GRID	BØØL	46235 ØRIGIN	SØRT0056
	046226	IGRID	BØØL	46226 ARRAY ØF CØUNTS	SØRT0057
			END		SØRT0058

## F. SUBROUTINE OUTPT

Calling sequence: CALL OUTPT (DD)

DD = grid width

OUTPT also expects to find the following arguments in COMMON:

MAXG: Highest grid index

TOTAL: Total number of counts

IGRID: Array for counts

PBAR: Corresponding array for normalized counts

CONST: A constant used to calculate PBAR

XMID: The mid-point of the first interval.

Calculates the PBAR corresponding to each count in IGRID, where

$$\text{PBAR} = \frac{\text{CONST} * \text{COUNT}}{\text{TOTAL} * \text{DD}}$$

and the moments  $M_0, M_1, M_2, M_3, M_4$

$$M_k = \frac{1}{\text{DD}} \sum_{i=1}^{\text{TOTAL}} \text{PBAR}_i * \text{XMID}_i^{k-1}$$

These results are written on the output tape for printing as follows

- A table with columns for:

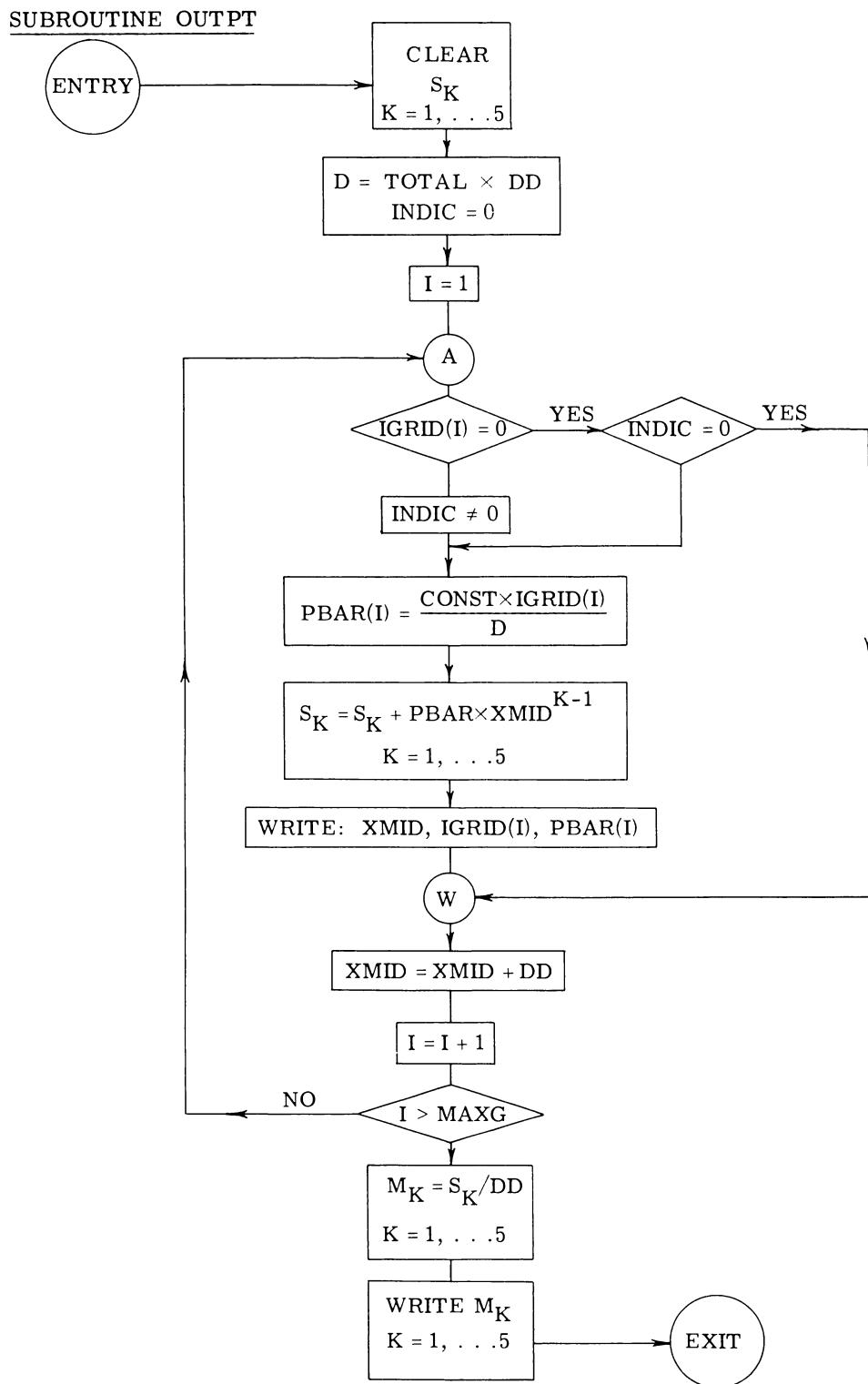
XMID: mid-point of interval

IGRID: the count corresponding to XMID

PBAR: corresponding to IGRID

This table starts at the first non-zero IGRID and ends at IGRID<sub>MAXG</sub>

- The TOTAL count, and the moments .



## OUTPT

## COMPUTES MOMENTS AND WRITES OUTPUT

		*	CALLING SEQUENCE IS		OUTPT001
		*	CALLOUTPT(DD)		OUTPT002
		*	DD=GRID WIDTH		OUTPT003
	00004	ENTRY	OUTPT		OUTPT004
00000	746263303460		(STH)		
00001	742631433460		(FIL)		
00004	0634 00 4 00124	OUTPT	SXA	EXIT+1,4	OUTPT007
00005	0634 00 2 00123		SXA	EXIT,2	OUTPT008
00006	0500 60 4 00001		CLA*	1,4	OUTPT009
00007	0601 00 0 00152		STØ	DD	OUTPT010
00010	0500 00 0 46234		CLA	XMID	OUTPT011
00011	0601 00 0 00163		STØ	XCMID	OUTPT012
00012	0774 00 2 00005		AXT	5,2	OUTPT013
00013	0600 00 2 00163		STZ	SIG+1,2	OUTPT014
00014	2 00001 2 00013		TIx	*-1,2,1	OUTPT015
00015	0560 00 0 46227		LDQ	TOTAL	OUTPT015
00016	0260 00 0 00152		FMP	DD	OUTPT017
00017	0601 00 0 00154		STØ	WS+1	OUTPT018
00020	0600 00 0 00155		STZ	INDIC	OUTPT019
00021	0500 00 0 42270		CLA	MAXG	OUTPT020
00022	0622 00 0 00072		STD	ØX	OUTPT021
00023	0774 00 2 00001		AXT	1,2	OUTPT022
00024	0500 00 2 46227	ØA	CLA	IGRID+1,2	OUTPT023
00025	-0100 00 0 00032		TNZ	ØB	OUTPT024
00026	0600 00 2 72556		STZ	PBAR+1,2	OUTPT025
00027	0520 00 0 00155		ZET	INDIC	OUTPT026
00030	0020 00 0 00054		TRA	ØE	OUTPT027
00031	0020 00 0 00066		TRA	ØW	OUTPT028
00032	0601 00 0 00155	ØB	STØ	INDIC	OUTPT029
00033	-0501 00 0 00165		ØRA	=Ø2330000000000	OUTPT030
00034	0300 00 0 00165		FAD	=Ø2330000000000	OUTPT031
00035	0601 00 2 46227		STØ	IGRID+1,2	OUTPT032
00036	0241 00 0 00154		FDP	WS+1	OUTPT033
00037	0260 00 0 46231		FMP	CØNST	OUTPT034
00040	0601 00 2 72556		STØ	PBAR+1,2	OUTPT035
00041	0601 00 0 00153		STØ	WS	OUTPT036
00042	0300 00 0 00162		FAD	SIG	OUTPT037
00043	0601 00 0 00162		STØ	SIG	OUTPT038
00044	0774 00 4 00002		AXT	2,4	OUTPT039
00045	0560 00 0 00153	ØC	LDQ	WS	OUTPT040
00046	0260 00 0 00163		FMP	XCMID	OUTPT041
00047	0601 00 0 00153		STØ	WS	OUTPT042
00050	0300 00 4 00163		FAD	SIG+1,4	OUTPT043
00051	0601 00 4 00163		STØ	SIG+1,4	OUTPT044
00052	1 00001 4 00053		TXI	*+1,4,1	OUTPT045
00053	-3 00005 4 00045		TXL	ØC,4,5	OUTPT046
00054	-0500 00 0 00164	ØE	CAL	=Ø6000000	OUTPT047
00055	0074 00 4 00000		TSX	\$(STH),4	OUTPT048
00056	0 00000 0 00151		PZE	FMG	OUTPT049
00057	0560 00 0 00163		LDQ	XCMID	OUTPT050
00060	-1 00000 0 00000		STR		OUTPT051
00061	0560 00 2 46227		LDQ	IGRID+1,2	OUTPT052
00062	-1 00000 0 00000		STR		OUTPT053

OUTPT COMPUTES MOMENTS AND WRITES OUTPUT						
00063	0560	00	2	72556	LDQ PBAR+1,2	OUTPT054
00064	-1	00000	0	00000	STR	OUTPT055
00065	0074	00	4	00001	TSX \$(FIL),4	OUTPT056
00066	0500	00	0	00163	ØW CLA XCMID	OUTPT057
00067	0300	00	0	00152	FAD DD	OUTPT058
00070	0601	00	0	00163	STØ XCMID	OUTPT059
00071	1	00001	2	00072	TXI *+1,2,1	OUTPT060
00072	-3	00000	2	00024	ØX TXL ØA,2,**	OUTPT061
00073	-0500	00	0	00164	CAL =Ø6000000	OUTPT062
00074	0074	00	4	00000	TSX \$(STH),4	OUTPT063
00075	0	00000	0	00144	PZE FMI	OUTPT064
00076	0560	00	0	46227	LDQ TØTAL	OUTPT065
00077	-1	00000	0	00000	STR	OUTPT066
00100	0074	00	4	00001	TSX \$(FIL),4	OUTPT067
00101	-0500	00	0	00164	CAL =Ø6000000	OUTPT068
00102	0074	00	4	00000	TSX \$(STH),4	OUTPT069
00103	0	00000	0	00140	PZE FMP	OUTPT070
00104	0074	00	4	00001	TSX \$(FIL),4	OUTPT071
00105	0774	00	2	00005	AXT 5,2	OUTPT072
00106	0560	00	2	00163	ØD LDQ SIG+1,2	OUTPT073
00107	0260	00	0	00152	FMP DD	OUTPT074
00110	0601	00	2	00163	STØ SIG+1,2	OUTPT075
00111	2	00001	2	00106	TIX ØD,2,1	OUTPT076
00112	0774	00	2	00001	AXT 1,2	OUTPT077
00113	-0500	00	0	00164	CAL =Ø6000000	OUTPT078
00114	0074	00	4	00000	TSX \$(STH),4	OUTPT079
00115	0	00000	0	00130	PZE FMJ	OUTPT080
00116	0560	00	2	00163	LDQ SIG+1,2	OUTPT081
00117	-1	00000	0	00000	STR	OUTPT082
00120	1	00001	2	00121	TXI *+1,2,1	OUTPT083
00121	-3	00005	2	00116	TXL *-3,2,5	OUTPT084
00122	0074	00	4	00001	TSX \$(FIL),4	OUTPT085
00123	0774	00	2	00000	EXIT AXT **,2	OUTPT086
00124	0774	00	4	00000	AXT **,4	OUTPT087
00125	0020	00	4	00002	TRA 2,4	OUTPT088
00126	346060606060				BCI 1,)	OUTPT089
00127	052501023304				BCI 1,5E12.4	OUTPT090
00130	740100670147				FMJ BCI 1,(10X1P	OUTPT091
00131	613460606060				BCI 1,/)	OUTPT092
00132	033060440461				BCI 1,3H M4/	OUTPT093
00133	306044031167				BCI 1,H M39X	OUTPT094
00134	604402116703				BCI 1, M29X3	OUTPT095
00135	440111670330				BCI 1,M19X3H	OUTPT096
00136	0C1167033060				BCI 1,09X3H	OUTPT097
00137	046703306044				BCI 1,4X3H M	OUTPT098
00140	607401300001				FMP BCI 1, (1H01	OUTPT099
00141	023300346060				BCI 1,2.0)	OUTPT100
00142	466321432601				BCI 1,ØTALF1	OUTPT101
00143	066706306063				BCI 1,6X6H T	OUTPT102
00144	607401300001				FMI BCI 1, (1H01	OUTPT103
00145	346060606060				BCI 1,)	OUTPT104
00146	472501023304				BCI 1,PE12.4	OUTPT105
00147	330073056701				BCI 1,.0,5X1	OUTPT106
00150	330373260101				BCI 1,.3,F11	OUTPT107
00151	740110672606				FMG BCI 1,(18XF6	OUTPT108
00152	+000000000000				DD ØCT 0	OUTPT109

OUTPT COMPUTES MOMENTS AND WRITES OUTPUT  
 00153 +00000000000000 WS OCT 0,0 OUTPT110  
 00155 +0C000000000000 INDIC OCT 0 OUTPT111  
 00156 +0C000000000000 OCT 0,0,0,0 OUTPT112  
 00162 +00000000000000 SIG OCT 0 OUTPT113  
 00163 +00000000000000 XCMID OCT 0 OUTPT114  
     042300 M B00L 42300 MATRIX INDEX OUTPT115  
     046227 TOTAL B00L 46227 TOTAL N0. OF COUNTS OUTPT116  
     046226 IGRID B00L 46226 ARRAY OF COUNTS OUTPT117  
     072555 PBAR B00L 72555 OUTPT118  
     046231 CONST B00L 46231 OUTPT119  
     046234 XMID B00L 46234 OUTPT120  
     042270 MAXG B00L 42270 OUTPT121  
     END OUTPT122

## G. OTHER SUBROUTINES

### 1. DATE: Calling sequence: CALL DATE (DAY)

Can only be used with a monitor which has the date stored as BCD in location  $142_8$ . Fetches date, inserts slashes, and stores, for example:

08/27/ in DAY,

62bbbb in DAY - 1 (b=blank)

DAY can then be printed on the output by means of FORMAT (2A6).

### 2. CLOK: Can only be used on a 7090 equipped with a core clock. This clock adds one to the contents of location 5 every 1/60 sec.

Calling sequence:

CALL RCLOK: Stores the current contents of location 5.

CALL ICLOK (IT): Subtracts from the current contents of location 5 its contents when a previous CALL RCLOK was performed; shifts the difference to the decrement and stores it in T.

CALL FCLOK (T): Same as above except that the difference is converted to a floating point number.

### 3. Subroutines on the Standard FORTRAN Library Tape.

## V. REMARKS

### A. STORAGE REQUIREMENTS AND AVAILABLE CARD DECKS

PROGRAM	STORAGE <sup>1</sup>		DECK LABELS	
	DEC	OCT	SOURCE	BINARY
MAIN	956	1674	RMDOOOO-217	RMDOOOO-47
MXGEN	157	235	<sup>2</sup> MXNORMOO-41	MXNORMOO-08
RAN2N	75	113	<sup>3</sup> RAN2NOOO-73	RAN2NOOO-04
HDIAG	791	1427	HDIAGO00-163	HDIAGO00-36
SPACE	259	403	SPACE000-62	SPACE000-13
VALUE	241	361	VALUE000-69	VALUE000-12
SYMM	181	265	SYMMOO00-52	SYMMOO00-09
ASYM	139	213	ASYMO000-42	ASYMO000-07
VECTOR	158	236	VECTOROO-45	VECTOROO-08
VECSYM	222	336	VECSYMO0-47	VECSYMO0-11
VECASY	168	248	VECASYOO-37	VECASYOO-08
SORT	49	61	<sup>3</sup> SORTOO00-58	SORTOO00-03
OUTPT	118	166	<sup>3</sup> OUTPTOO0-122	OUTPTOO0-06
DATE	19	23	—	DATE0000-01
CLOK	18	22	—	CLOK0000-01
Other	<u>2838</u> 6389	<u>5426</u> 14365	On Standard Library Tape	
COMMON	<u>14977</u> <u>21366</u>	<u>35201</u> <u>51566</u>		

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<sup>1</sup>These storage requirements are based on a maximum matrix dimension of 50×50.

<sup>2</sup>For the alternate matrix loading routine, substitute:

MXGEN	118	166	MXONESOO-39	MXONESOO-06
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<sup>3</sup>These routines are coded in FAP.

## B. TIMING ESTIMATES

The following table contains very rough estimates for generating 1000 matrices with normally distributed elements, diagonalizing each matrix, ordering the eigenvalues and eigenvectors, calculating the nearest neighbor spacings, and taping the eigenvalues, eigenvectors, and spacing.

$\frac{N}{2}$	$\frac{\text{Min}}{2}$	$\frac{N}{20}$	$\frac{\text{Min}}{200}$
3	3	30	630
5	5	40	1500
10	26	50	2900

The time needed to perform the various sortings and produce the output tables is not included; due to tape movement it can amount to 25 to 30% of the total time.

For input matrices containing a large number of zeros, or the  $\pm 1$  input, the time is reduced slightly.

For reference, the time taken for the sample output (2000  $5 \times 5$  matrices) was about 15 minutes; for 15000  $4 \times 4$  matrices about one hour, and for 200  $20 \times 20$  matrices with the  $\pm 1$  input 1 hour.

## C. RESTRICTIONS

1. The punching of a restart card every 20 minutes requires a core clock (RPQ F89349).
2.  $N^2 \times NM$  must be less than  $2^{27}$ .

VI. SAMPLE OUTPUT

RANDOM MATRIX DIAGONALIZATION

PERFORMED ON 09/26/62

5 - DIMENSIONAL MATRIX

WFA = 1.4142136E 00        WFB = 1.0000000E 00

INPUT DATA

ITAPE = 11      JTAPE = 4      KTape = 14  
N = 5      NM = 2000      NB = 4  
WFA = 1.4142      WFB = 1.0000  
DX ARE 0.1000 -0. -0. -0.  
DE ARE 0.1000 0.0500 -0. -0.  
DA ARE 0.0500 -0. -0. -0.  
KØNE = 2      KØNA = 2      KØNN =-0      KØNX =-0  
W = 000000000000      M = -0  
TOTAL = 000000000000

DELTAX = 0.100 NEIGHBOR 0

X MIDPT	COUNT	P-BAR
0.050	61.	7.6250E-02
0.150	195.	2.4375E-01
0.250	312.	3.9000E-01
0.350	427.	5.3375E-01
0.450	470.	5.8750E-01
0.550	562.	7.0250E-01
0.650	560.	7.0000E-01
0.750	625.	7.8125E-01
0.850	613.	7.6625E-01
0.950	595.	7.4375E-01
1.050	544.	6.8000E-01
1.150	471.	5.8875E-01
1.250	446.	5.5750E-01
1.350	392.	4.9000E-01
1.450	334.	4.1750E-01
1.550	271.	3.3875E-01
1.650	255.	3.1875E-01
1.750	191.	2.3875E-01
1.850	171.	2.1375E-01
1.950	111.	1.3875E-01
2.050	101.	1.2625E-01
2.150	81.	1.0125E-01
2.250	51.	6.3750E-02
2.350	42.	5.2500E-02
2.450	35.	4.3750E-02
2.550	26.	3.2500E-02
2.650	19.	2.3750E-02
2.750	10.	1.2500E-02
2.850	11.	1.3750E-02
2.950	3.	3.7500E-03
3.050	5.	6.2500E-03
3.150	4.	5.0000E-03
3.250	1.	1.2500E-03
3.350	3.	3.7500E-03
3.450	0.	0.
3.550	1.	1.2500E-03
3.650	1.	1.2500E-03

TOTAL 8000.

M0 M1 M2 M3 M4

10.0000E-01 1.0005E 00 1.2915E 00 1.9870E 00 3.4955E 00

DELTAX = 0.100

NEIGHBOR 1

X MIDPT	COUNT	P-BAR
0.250	1.	1.6667E-03
0.350	4.	6.6667E-03
0.450	9.	1.5000E-02
0.550	21.	3.5000E-02
0.650	30.	5.0000E-02
0.750	58.	9.6667E-02
0.850	92.	1.5333E-01
0.950	115.	1.9167E-01
1.050	138.	2.3000E-01
1.150	203.	3.3833E-01
1.250	237.	3.9500E-01
1.350	275.	4.5833E-01
1.450	336.	5.6000E-01
1.550	336.	5.6000E-01
1.650	368.	6.1333E-01
1.750	373.	6.2167E-01
1.850	403.	6.7167E-01
1.950	361.	6.0167E-01
2.050	324.	5.4000E-01
2.150	335.	5.5833E-01
2.250	311.	5.1833E-01
2.350	277.	4.6167E-01
2.450	238.	3.9667E-01
2.550	217.	3.6167E-01
2.650	193.	3.2167E-01
2.750	173.	2.8833E-01
2.850	134.	2.2333E-01
2.950	110.	1.8333E-01
3.050	73.	1.2167E-01
3.150	70.	1.1667E-01
3.250	45.	7.5000E-02
3.350	41.	6.8333E-02
3.450	25.	4.1667E-02
3.550	23.	3.8333E-02
3.650	20.	3.3333E-02
3.750	8.	1.3333E-02
3.850	6.	1.0000E-02
3.950	6.	1.0000E-02
4.050	5.	8.3333E-03
4.150	2.	3.3333E-03
4.250	1.	1.6667E-03
4.350	0.	0.
4.450	0.	0.
4.550	1.	1.6667E-03
4.650	1.	1.6667E-03
4.750	1.	1.6667E-03

TOTAL 6000.

M0 M1 M2 M3 M4

10.0000E-01 1.9479E 00 4.1912E 00 9.7982E 00 2.4596E 01

DELTAX = 0.100 NEIGHBOR 2

X MIDPT	COUNT	P-BAR
0.950	2.	5.0000E-03
1.050	0.	0.
1.150	3.	7.5000E-03
1.250	8.	2.0000E-02
1.350	12.	3.0000E-02
1.450	25.	6.2500E-02
1.550	25.	6.2500E-02
1.650	38.	9.5000E-02
1.750	60.	1.5000E-01
1.850	70.	1.7500E-01
1.950	111.	2.7750E-01
2.050	101.	2.5250E-01
2.150	157.	3.9250E-01
2.250	166.	4.1500E-01
2.350	174.	4.3500E-01
2.450	198.	4.9500E-01
2.550	191.	4.7750E-01
2.650	237.	5.9250E-01
2.750	239.	5.9750E-01
2.850	212.	5.3000E-01
2.950	215.	5.3750E-01
3.050	212.	5.3000E-01
3.150	228.	5.7000E-01
3.250	192.	4.8000E-01
3.350	162.	4.0500E-01
3.450	172.	4.3000E-01
3.550	121.	3.0250E-01
3.650	130.	3.2500E-01
3.750	103.	2.5750E-01
3.850	87.	2.1750E-01
3.950	69.	1.7250E-01
4.050	55.	1.3750E-01
4.150	57.	1.4250E-01
4.250	45.	1.1250E-01
4.350	27.	6.7500E-02
4.450	24.	6.0000E-02
4.550	20.	5.0000E-02
4.650	13.	3.2500E-02
4.750	8.	2.0000E-02
4.850	12.	3.0000E-02
4.950	4.	1.0000E-02
5.050	6.	1.5000E-02
5.150	5.	1.2500E-02
5.250	1.	2.5000E-03
5.350	3.	7.5000E-03
TOTAL	4000.	
M0		
M1		
M2		
M3		
M4		
10.0000E-01	2.9213E 00	9.0213E 00
		2.9299E 01
		9.9637E 01

DELTAX = 0.100

NEIGHBOR 3

X MIDPT	COUNT	P-BAR
1.750	1.	5.0000E-03
1.850	1.	5.0000E-03
1.950	0.	0.
2.050	1.	5.0000E-03
2.150	5.	2.5000E-02
2.250	7.	3.5000E-02
2.350	11.	5.5000E-02
2.450	11.	5.5000E-02
2.550	9.	4.5000E-02
2.650	15.	7.5000E-02
2.750	42.	2.1000E-01
2.850	32.	1.6000E-01
2.950	48.	2.4000E-01
3.050	51.	2.5500E-01
3.150	58.	2.9000E-01
3.250	84.	4.2000E-01
3.350	78.	3.9000E-01
3.450	81.	4.0500E-01
3.550	91.	4.5500E-01
3.650	89.	4.4500E-01
3.750	114.	5.7000E-01
3.850	112.	5.6000E-01
3.950	101.	5.0500E-01
4.050	106.	5.3000E-01
4.150	102.	5.1000E-01
4.250	97.	4.8500E-01
4.350	92.	4.6000E-01
4.450	83.	4.1500E-01
4.550	54.	2.7000E-01
4.650	66.	3.3000E-01
4.750	54.	2.7000E-01
4.850	41.	2.0500E-01
4.950	51.	2.5500E-01
5.050	37.	1.8500E-01
5.150	45.	2.2500E-01
5.250	20.	1.0000E-01
5.350	26.	1.3000E-01
5.450	15.	7.5000E-02
5.550	16.	8.0000E-02
5.650	9.	4.5000E-02
5.750	9.	4.5000E-02
5.850	4.	2.0000E-02
5.950	7.	3.5000E-02
6.050	7.	3.5000E-02
6.150	6.	3.0000E-02
6.250	3.	1.5000E-02
6.350	2.	1.0000E-02
6.450	0.	0.
6.550	4.	2.0000E-02
6.650	2.	1.0000E-02

TOTAL 2000.

M0 M1 M2 M3 M4

10.0000E-01 3.9997E 00 1.6597E 01 7.1329E 01 3.1699E 02

X MIDPT	P-SUM
0.050	7.6250E-02
0.150	2.4375E-01
0.250	3.9167E-01
0.350	5.4042E-01
0.450	6.0250E-01
0.550	7.3750E-01
0.650	7.5000E-01
0.750	8.7792E-01
0.850	9.1958E-01
0.950	9.4042E-01
1.050	9.1000E-01
1.150	9.3458E-01
1.250	9.7250E-01
1.350	9.7833E-01
1.450	1.0400E 00
1.550	9.6125E-01
1.650	1.0271E 00
1.750	1.0154E 00
1.850	1.0654E 00
1.950	1.0179E 00
2.050	9.2375E-01
2.150	1.0771E 00
2.250	1.0321E 00
2.350	1.0042E 00
2.450	9.9042E-01
2.550	9.1667E-01
2.650	1.0129E 00
2.750	1.1083E 00
2.850	9.2708E-01
2.950	9.6458E-01
3.050	9.1292E-01
3.150	9.8167E-01
3.250	9.7625E-01
3.350	8.6708E-01
3.450	8.7667E-01
3.550	7.9708E-01
3.650	8.0458E-01
3.750	8.4083E-01
3.850	7.8750E-01
3.950	6.8750E-01
4.050	6.7583E-01
4.150	6.5583E-01
4.250	5.9917E-01
4.350	5.2750E-01
4.450	4.7500E-01
4.550	3.2167E-01
4.650	3.6417E-01
4.750	2.9167E-01
4.850	2.3500E-01
4.950	2.6500E-01
5.050	2.0000E-01
5.150	2.3750E-01
5.250	1.0250E-01
5.350	1.3750E-01
5.450	7.5000E-02
5.550	8.0000E-02
5.650	4.5000E-02
5.750	4.5000E-02
5.850	2.0000E-02
5.950	3.5000E-02
6.050	3.5000E-02
6.150	3.0000E-02
6.250	1.5000E-02
6.350	1.0000E-02
6.450	0.
6.550	2.0000E-02
6.650	1.0000E-02

DELTA E = 0.100

EIGENVALUES

EIGENVALUES NORMA LIZED BY 2\*SQRTF(N\*B\*\*2) = 4.4721E 00

E	MIDPT	COUNT	P-BAR*(PI/2)
-1.550	2.	3.1416E-03	
-1.450	4.	6.2832E-03	
-1.350	10.	1.5708E-02	
-1.250	26.	4.0841E-02	
-1.150	53.	8.3252E-02	
-1.050	114.	1.7907E-01	
-0.950	231.	3.6285E-01	
-0.850	315.	4.9480E-01	
-0.750	410.	6.4403E-01	
-0.650	455.	7.1471E-01	
-0.550	487.	7.6498E-01	
-0.450	523.	8.2153E-01	
-0.350	568.	8.9221E-01	
-0.250	595.	9.3462E-01	
-0.150	587.	9.2206E-01	
-0.050	591.	9.2834E-01	
0.050	650.	1.0210E 00	
0.150	619.	9.7232E-01	
0.250	567.	8.9064E-01	
0.350	578.	9.0792E-01	
0.450	532.	8.3566E-01	
0.550	489.	7.6812E-01	
0.650	481.	7.5555E-01	
0.750	384.	6.0319E-01	
0.850	304.	4.7752E-01	
0.950	205.	3.2201E-01	
1.050	117.	1.8378E-01	
1.150	61.	9.5819E-02	
1.250	22.	3.4558E-02	
1.350	16.	2.5133E-02	
1.450	3.	4.7124E-03	
1.550	1.	1.5708E-03	

TOTAL 10000.

M0

M1

M2

M3

M4

1.5708E 00 -2.3246E-03 4.6883E-01 -2.4816E-03 3.1156E-01

DELTAE = 0.100

EIGENVALUES( 1 ), ( 5 )

EIGENVALUES NORMIALIZED BY 2\*WFB\*SQRTF(N) = 4.4721E 00

E	MIDPT	COUNT	P-BAR	
0.050	2.	5.0000E-03		
0.150	14.	3.5000E-02		
0.250	69.	1.7250E-01		
0.350	180.	4.5000E-01		
0.450	342.	8.5500E-01		
0.550	550.	1.3750E 00		
0.650	682.	1.7050E 00		
0.750	707.	1.7675E 00		
0.850	594.	1.4850E 00		
0.950	433.	1.0825E 00		
1.050	229.	5.7250E-01		
1.150	114.	2.8500E-01		
1.250	48.	1.2000E-01		
1.350	26.	6.5000E-02		
1.450	7.	1.7500E-02		
1.550	3.	7.5000E-03		
TOTAL	4000.			
M0		M1	M2	
10.0000E-01	7.2752E-01	5.7891E-01	4.9545E-01	4.5127E-01
		M3	M4	

DELTAE = 0.100

EIGENVALUES( 2), ( 4)

EIGENVALUES NORMALED BY 2\*WFB\*SQRTF(N) = 4.4721E 00

E	MIDPT	COUNT	P-BAR
0.050	427.	1.0675E 00	
0.150	555.	1.3875E 00	
0.250	747.	1.8675E 00	
0.350	828.	2.0700E 00	
0.450	664.	1.6600E 00	
0.550	413.	1.0325E 00	
0.650	250.	6.2500E-01	
0.750	87.	2.1750E-01	
0.850	24.	6.0000E-02	
0.950	3.	7.5000E-03	
1.050	2.	5.0000E-03	

TOTAL 4000.

M0 M1 M2 M3 M4

10.0000E-01 3.4005E-01 1.4947E-01 7.5826E-02 4.2551E-02

DELTAE = 0.100                    EIGENVALUES( 3 ), ( 3 )  
 EIGENVALUES NORMIALIZED BY 2\*WFB\*SQRTF(N) = 4.4721E 00  
 E MIDPT COUNT P-BAR  
 0.050 836. 4.1800E 00  
 0.150 619. 3.0950E 00  
 0.250 342. 1.7100E 00  
 0.350 137. 6.8500E-01  
 0.450 48. 2.4000E-01  
 0.550 13. 6.5000E-02  
 0.650 4. 2.0000E-02  
 0.750 0. 0.  
 0.850 1. 5.0000E-03  
 TOTAL 2000.  
 M0 M1 M2 M3 M4  
 10.0000E-01 1.5015E-01 3.5120E-02 1.0830E-02 4.0521E-03

DELTAE = 0.050

EIGENVALUES

EIGENVALUES NORMALIZED BY 2\*SQRTF(N\*B\*\*2) = 4.4721E 00

E MIDPT COUNT P-BAR\*(PI/2)

-1.575	1.	3.1416E-03
-1.525	1.	3.1416E-03
-1.475	2.	6.2832E-03
-1.425	2.	6.2832E-03
-1.375	2.	6.2832E-03
-1.325	8.	2.5133E-02
-1.275	10.	3.1416E-02
-1.225	16.	5.0265E-02
-1.175	24.	7.5398E-02
-1.125	29.	9.1106E-02
-1.075	48.	1.5080E-01
-1.025	66.	2.0735E-01
-0.975	92.	2.8903E-01
-0.925	139.	4.3668E-01
-0.875	135.	4.2412E-01
-0.825	180.	5.6549E-01
-0.775	197.	6.1889E-01
-0.725	213.	6.6916E-01
-0.675	222.	6.9743E-01
-0.625	233.	7.3199E-01
-0.575	246.	7.7283E-01
-0.525	241.	7.5712E-01
-0.475	282.	8.8593E-01
-0.425	241.	7.5712E-01
-0.375	285.	8.9535E-01
-0.325	283.	8.8907E-01
-0.275	300.	9.4248E-01
-0.225	295.	9.2677E-01
-0.175	292.	9.1735E-01
-0.125	295.	9.2677E-01
-0.075	318.	9.9903E-01
-0.025	273.	8.5765E-01
0.025	331.	1.0399E 00
0.075	319.	1.0022E 00
0.125	307.	9.6447E-01
0.175	312.	9.8018E-01
0.225	292.	9.1735E-01
0.275	275.	8.6394E-01
0.325	284.	8.9221E-01
0.375	294.	9.2363E-01
0.425	287.	9.0164E-01
0.475	245.	7.6969E-01
0.525	239.	7.5084E-01
0.575	250.	7.8540E-01
0.625	241.	7.5712E-01
0.675	240.	7.5398E-01
0.725	219.	6.8801E-01
0.775	165.	5.1836E-01
0.825	150.	4.7124E-01
0.875	154.	4.8381E-01
0.925	129.	4.0527E-01
0.975	76.	2.3876E-01
1.025	70.	2.1991E-01
1.075	47.	1.4765E-01
1.125	36.	1.1310E-01
1.175	25.	7.8540E-02
1.225	17.	5.3407E-02
1.275	5.	1.5708E-02
1.325	7.	2.1991E-02
1.375	9.	2.8274E-02
1.425	2.	6.2832E-03
1.475	1.	3.1416E-03
1.525	1.	3.1416E-03

TOTAL 10000.

M0

M1

M2

M3

M4

1.5708E 00 -2.8584E-03 4.6769E-01 -2.6658E-03 3.0916E-01

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DELTAE = 0.050                    EIGENVALUES( 1 ), ( 5 )  
EIGENVALUES NORMIALIZED BY 2\*WFB\*SQRTF(M) = 4.4721E 00

E	MIDPT	COUNT	P-BAR	
0.025	1.	5.0000E-03		
0.075	1.	5.0000E-03		
0.125	3.	1.5000E-02		
0.175	11.	5.5000E-02		
0.225	19.	9.5000E-02		
0.275	50.	2.5000E-01		
0.325	66.	3.3000E-01		
0.375	114.	5.7000E-01		
0.425	143.	7.1500E-01		
0.475	199.	9.9500E-01		
0.525	233.	1.1650E 00		
0.575	317.	1.5850E 00		
0.625	323.	1.6150E 00		
0.675	359.	1.7950E 00		
0.725	377.	1.8850E 00		
0.775	330.	1.6500E 00		
0.825	316.	1.5800E 00		
0.875	278.	1.3900E 00		
0.925	266.	1.3300E 00		
0.975	167.	8.3500E-01		
1.025	134.	6.7000E-01		
1.075	95.	4.7500E-01		
1.125	65.	3.2500E-01		
1.175	49.	2.4500E-01		
1.225	33.	1.6500E-01		
1.275	15.	7.5000E-02		
1.325	15.	7.5000E-02		
1.375	11.	5.5000E-02		
1.425	4.	2.0000E-02		
1.475	3.	1.5000E-02		
1.525	2.	1.0000E-02		
1.575	1.	5.0000E-03		
	TOTAL	4000.		
	M0	M1	M2	
	10.0000E-01	7.2752E-01	5.7789E-01	4.9330E-01
			M3	M4
			4.4799E-01	

DELTAE = 0.050

EIGENVALUES( 2), ( 4)

EIGENVALUES NORMIALIZED BY 2\*WFB\*SQRTF(N) = 4.4721E 00

E	MIDPT	COUNT	P-BAR
0.025	227.	1.1350E 00	
0.075	200.	1.0000E 00	
0.125	244.	1.2200E 00	
0.175	311.	1.5550E 00	
0.225	360.	1.8000E 00	
0.275	387.	1.9350E 00	
0.325	413.	2.0650E 00	
0.375	415.	2.0750E 00	
0.425	355.	1.7750E 00	
0.475	309.	1.5450E 00	
0.525	239.	1.1950E 00	
0.575	174.	8.7000E-01	
0.625	147.	7.3500E-01	
0.675	103.	5.1500E-01	
0.725	55.	2.7500E-01	
0.775	32.	1.6000E-01	
0.825	13.	6.5000E-02	
0.875	11.	5.5000E-02	
0.925	2.	1.0000E-02	
0.975	1.	5.0000E-03	
1.025	2.	1.0000E-02	

TOTAL 4000.

M0	M1	M2	M3	M4
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10.0000E-01	3.3934E-01	1.4896E-01	7.5307E-02	4.2099E-02
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DELTAE = 0.050

EIGENVALUES( 3), ( 3)

EIGENVALUES NORMIALIZED BY 2\*WFB\*SQRTF(N) = 4.4721E 00

E	MIDPT	COUNT	P-BAR
0.025	422.	4.2200E 00	
0.075	414.	4.1400E 00	
0.125	343.	3.4300E 00	
0.175	276.	2.7600E 00	
0.225	204.	2.0400E 00	
0.275	138.	1.3800E 00	
0.325	88.	8.8000E-01	
0.375	49.	4.9000E-01	
0.425	30.	3.0000E-01	
0.475	18.	1.8000E-01	
0.525	8.	8.0000E-02	
0.575	5.	5.0000E-02	
0.625	4.	4.0000E-02	
0.675	0.	0.	
0.725	0.	0.	
0.775	0.	0.	
0.825	1.	1.0000E-02	
TOTAL		2000.	
M0	M1	M2	M3
10.0000E-01	1.4765E-01	3.4467E-02	1.0504E-02
			3.8709E-03

DELTA A = 0.050

EIGENVECTOR COMPONENTS

A MIDPT	COUNT	P-BAR
0.025	3733.	1.4932E 00
0.075	3653.	1.4612E 00
0.125	3591.	1.4364E 00
0.175	3740.	1.4960E 00
0.225	3551.	1.4204E 00
0.275	3535.	1.4140E 00
0.325	3290.	1.3160E 00
0.375	3249.	1.2996E 00
0.425	3050.	1.2200E 00
0.475	2942.	1.1768E 00
0.525	2759.	1.1036E 00
0.575	2562.	1.0248E 00
0.625	2315.	9.2600E-01
0.675	1991.	7.9640E-01
0.725	1774.	7.0960E-01
0.775	1471.	5.8840E-01
0.825	1256.	5.0240E-01
0.875	825.	3.3000E-01
0.925	533.	2.1320E-01
0.975	180.	7.2000E-02

TOTAL 50000.

M0 M1 M2 M3 M4

10.0000E-01 3.7577E-01 2.0017E-01 1.2495E-01 8.5587E-02

DELTAA = 0.050

EIGENVECTORS( 1 ), ( 5 )

A	MIDPT	COUNT	P-BAR	
0.025	1543.	1.5430E 00		
0.075	1422.	1.4220E 00		
0.125	1432.	1.4320E 00		
0.175	1480.	1.4800E 00		
0.225	1391.	1.3910E 00		
0.275	1397.	1.3970E 00		
0.325	1318.	1.3180E 00		
0.375	1357.	1.3570E 00		
0.425	1235.	1.2350E 00		
0.475	1144.	1.1440E 00		
0.525	1093.	1.0930E 00		
0.575	1064.	1.0640E 00		
0.625	927.	9.2700E-01		
0.675	791.	7.9100E-01		
0.725	718.	7.1800E-01		
0.775	583.	5.8300E-01		
0.825	486.	4.8600E-01		
0.875	339.	3.3900E-01		
0.925	211.	2.1100E-01		
0.975	69.	6.9000E-02		
	TOTAL	20000.		
	M0	M1	M2	
	10.0000E-01	3.7586E-01	2.0018E-01	1.2482E-01
			M3	M4
			8.5389E-02	

DELTAA = 0.050

EIGENVECTORS( 2), ( 4)

A	MIDPT	COUNT	P-BAR
0.025	1443.	1.4430E 00	
0.075	1485.	1.4850E 00	
0.125	1460.	1.4600E 00	
0.175	1488.	1.4880E 00	
0.225	1463.	1.4630E 00	
0.275	1423.	1.4230E 00	
0.325	1324.	1.3240E 00	
0.375	1255.	1.2550E 00	
0.425	1201.	1.2010E 00	
0.475	1199.	1.1990E 00	
0.525	1113.	1.1130E 00	
0.575	1005.	1.0050E 00	
0.625	934.	9.3400E-01	
0.675	790.	7.9000E-01	
0.725	711.	7.1100E-01	
0.775	572.	5.7200E-01	
0.825	512.	5.1200E-01	
0.875	328.	3.2800E-01	
0.925	219.	2.1900E-01	
0.975	75.	7.5000E-02	

TOTAL 20000.

M0

M1

M2

M3

M4

10.0000E-01 3.7574E-01 2.0012E-01 1.2502E-01 8.5738E-02

DELTAA = 0.050 EIGENVECTORS( 3 ), ( 3 )

A MIDPT	COUNT	P-BAR
0.025	747.	1.4940E 00
0.075	746.	1.4920E 00
0.125	699.	1.3980E 00
0.175	772.	1.5440E 00
0.225	697.	1.3940E 00
0.275	715.	1.4300E 00
0.325	648.	1.2960E 00
0.375	637.	1.2740E 00
0.425	614.	1.2280E 00
0.475	599.	1.1980E 00
0.525	553.	1.1060E 00
0.575	493.	9.8600E-01
0.625	454.	9.0800E-01
0.675	410.	8.2000E-01
0.725	345.	6.9000E-01
0.775	316.	6.3200E-01
0.825	258.	5.1600E-01
0.875	158.	3.1600E-01
0.925	103.	2.0600E-01
0.975	36.	7.2000E-02
TOTAL	10000.	
M0	M1	M2
10.0000E-01	3.7563E-01	2.0023E-01
		1.2507E-01
		8.5682E-02



