HW 9

2.1.3. $\left\{ \frac{(-1)^n}{2n} \right\}$ Converges to 0 since the limit at infinity is 0

2.1.6. $\left\{ \frac{n}{n^2+1} \right\}$ $\lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{1}{n+\frac{1}{n}} = 0$

It is convergent

2.1.7. $\{X_n\}$

a. $\lim X_n = 0 \iff \lim |X_n| = 0$

[$\Rightarrow$] X_n converges to 0, $|X_n|$ must also converge to 0 since all values get closer to 0

[$\Leftarrow$] All values $|X_n|$ converge to 0, so the distance gets smaller so even if there were no abs. value X_n would still converge

b. $\{|(-1)^n|\}$ converges, $\{(-1)^n\}$ diverges

2.1.10. $\lim_{n\to\infty} X_n = \inf\{x_n : n \in \mathbb{N}\}$

Monotone decreasing since X_n is bounded by 1 as $\frac{n+1}{n} > 1$ for all $n \in \mathbb{N}$

Using Proposition 2.1.10, the limit converges to 1

2.1.15. $x_n := \begin{cases} n & \text{if } n \text{ is odd} \\ 1/n & \text{if } n \text{ is even} \end{cases}$

a. Yes, its lower bound is 0 since $x_n > 0$ for all $n \in \mathbb{N}$

b. No

2.1.16. $\lim_{i\to\infty} x_{n_i} = a \quad \lim_{i\to\infty} x_{m_i} = b$

A sequence converges to only one number. All subsequences of that sequence must also converge to that number or else that sequence doesn't converge