Math 347 Hw 7

1. $\exists q: X \to Y$ that is injective $\Leftrightarrow \exists w: Y \to X$ that is surjective

   [$\Rightarrow$] $|X| \leq |Y|$ for injective functions and $|Y| \leq |X|$ for surjective functions

   [$\Leftarrow$] vice versa

2. $f: A \to A'$ and $g: B \to B'$ are bijections

   a. Since $A \to A'$ is a bijection, $|A| = |A'|$. Also, $|B| = |B'|$

   $A \cap B = \emptyset$ and $A' \cap B' = \emptyset$ so $|A \cup B| = |A' \cup B'|$. Therefore, there exists a bijection.

   b. $A = \{1\}$, $B = \{1, 2\}$, $U = \{1, 2, 3\}$

   The proof uses the assumptions when $|A \cup B| = |A' \cup B'|$.

3. a. Since $A \subset \mathbb{N}$ and A is infinite, A is countably infinite. A and $\mathbb{N}$ both have cardinality aleph-nought, so $|A| = |\mathbb{N}|$

   b. Let $|\mathbb{N}| - |B| = c$. Let $A = \{c \ldots n\}$ for any $n \in \mathbb{N}$. $f: B \to A$ is a bijection and $A \subseteq \mathbb{N}$ so $|A| = |\mathbb{N}|$ and $|B| = |\mathbb{N}|$.

4. X and Y are denumerable sets

a. $X \cap Y = \emptyset$

Let there be a bijection from X to the set of even integers a bijection from Y to the set of odd integers. $X \cup Y$ is denumerable because even integers union odd integers is the set of integers and $|X \cup Y| = |\mathbb{Z}| = |\mathbb{N}|$

b. Let A and B be denumerable sets where $A \cap B = \emptyset$. Then, $|X \cup Y| < |A \cup B|$ so $|X \cup Y| < |\mathbb{N}|$ which means $|X \cup Y| = |\mathbb{N}|$ according to 3(b).

7.4.4. I created a Mobius strip, cut it in half, and got a longer Mobius strip. However, when I did it again, I got two intertwined Mobius strips.

7.5.4. $(a,b) \sim (c,d) \Leftrightarrow ad = bc$

a. Reflexive: $(a,b) \sim (a,b)$ because $ab = ab$

Symmetric: $(c,d) \sim (a,b)$ because $cb = da = ab = bc$

Transitive: Assume $(a,b) \sim (c,d)$ and $(c,d) \sim (e,f)$

so $ad = bc$ and $cf = de$

$c = \frac{de}{f} \Rightarrow ad = \frac{bde}{f} \Rightarrow af = de$

so $(a,b) \sim (e,f)$

b. $(2,3), (4,6), (6,9), (8,12)$

$(-3,7), (-6,14), (-9,21), (-12,28)$

$[\frac{a}{b}]$ is the equivalence class for (a,b)

Each rational number has its own equivalence class

c. $[(a,b)] \oplus [(c,d)] = [(ad+bc, bd)]$

$[(a,b)] \otimes [(c,d)] = [(ac, bd)]$

$[(a,b)] = [(c,d)] \Rightarrow (a,b) \sim (c,d) \Rightarrow F(a,b) = F(c,d)$

$\Rightarrow f([a,b]) = f([c,d])$

7.6.6. a. $f: \mathbb{Z} \times \mathbb{N}/\sim \to \mathbb{Q}$ $\quad f([(x,y)]) = \frac{x}{y}$

$[(x,y)] = \{\frac{a}{b} : \frac{a}{b} = \frac{x}{y}\}$ so $\frac{x}{y}$ is a unique number which comes from $[(x,y)]$

b. $f([(a,b)] \oplus [(c,d)]) = f([(ad+bc, bd)])$

$= \frac{ad+bc}{bd} = \frac{a}{b} + \frac{c}{d} = f([(a,b)]) + f([(c,d)])$

$f([(a,b)] \otimes [(c,d)]) = f([(ac,bd)])$

$= \frac{ac}{bd} = f([(a,b)]) \cdot f([(c,d)])$