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Homework 1: Math 347

Problem 1. $L := A \Rightarrow (B \vee C)$ $\quad R := \neg C \Rightarrow (A \Rightarrow C)$

Prove $L \Rightarrow R$:

Assume L and $\neg R$ to be True. Then, R is False, meaning $\neg C$ must be True and $A \Rightarrow C$ False, so A is True and C is False. L becomes True $\Rightarrow$ (B v False). L can be True if B is True so this isn't a contradiction.

Prove $R \Rightarrow L$:

Assume R to be True. Also assume A to be True since $A \Rightarrow (B \vee C)$ and the only way for $R \Rightarrow L$ to be False is if R is True and L is False. If A is True, $\neg C \Rightarrow$ (True $\Rightarrow C$) is only True when C is True. L becomes True $\Rightarrow$ (B v Tr which makes L True no matter what B is. Therefore, $R \Rightarrow L$ is True.

Problem 2: $L := (A \Rightarrow B) \Rightarrow C$ $\quad R := A \Rightarrow (B \Rightarrow C)$

Prove $L \Rightarrow R$ Assume L and $\neg R$ to be True. Then, A is True and $B \Rightarrow C$ is False so B is True and C is False. L becomes (True $\Rightarrow$ True) $\Rightarrow$ False which is False. We assumed L is True so we have a contradiction.

Prove $R \Rightarrow L$:

Assume R and $\neg L$. $\neg L \iff (A \Rightarrow B) \wedge \neg C$ $\iff (\neg A \vee B) \wedge \neg C$, so C must be False. $R \iff \neg A \vee (B \Rightarrow C) \iff \neg A \vee (\neg B \vee C)$. For $\neg L$ to be True, either $\neg A$ or B must be True, when $\neg A$ is True, R is also True. Therefore, this isn't a contradiction so $R \Rightarrow L$ is not a tautology.

Problem 3: Let $L := (A \Rightarrow B) \wedge ((A \Rightarrow C) \Rightarrow D)$
and $R := (A \Rightarrow (B \Rightarrow C)) \Rightarrow D$
We are trying to prove $L \Rightarrow R$. We can see right away that if we set A to be False, L will be True. $A \Rightarrow (B \Rightarrow C)$ will also be True, so if we set D to be False, then True $\Rightarrow$ False so $L \Rightarrow R$ is not a tautology. ▨

2.1:12. a. If someone was playing pool, then Colin was not early. This is true.
b. If no-one was playing pool, then Colin was early. This is false.
c. i. Colin was not early.
ii. Can't conclude anything.

13. a. Zaphod does not have two heads.
b. Either both statements are false or both are True. Since the entire thing is false, both must be false. Ford is not tired and Zaphod does not have two heads.

18. Any expression can be expressed using only $\wedge$, $\vee$, and $\neg$. Implications such as $P \Rightarrow Q$ can be converted to $\neg P \vee Q$. Any variations can be expressed using $\neg$. $\vee$ can even be converted to $\wedge$ and vice versa. For example, $P \wedge Q \Leftrightarrow \neg(\neg P \vee \neg Q)$, so in reality, we only need ($\vee$ or $\wedge$) and $\neg$ to express ?.

2.2:14. César is a liar. Alain is forced to say he is a truth teller regardless of whether he is a truth teller or liar. Therefore, Ben's must be telling the truth.

16. a. If Ben doesn't play tennis and Ben doesn't play ping-pong and Ben hasn't appeared in at least one picture of the May 1992 Time Magazine, then Ben's father is not an artist or Ben's father does indeed have friends in Asia.

b. If Ben plays tennis or ping-pong, or has appeared in at least one picture of the May 1992 Time Magazine, then Ben's father is an artist and does not have any friends in Asia.

c. Ben's father is an artist and ~~Ben's father~~ doesn't have any friends in Asia and Ben doesn't play tennis nor ping-pong and hasn't appeared in at least one picture of the May 1992 Time Magazine.

d. Look for a picture of Ben in the May 1992 Time Magazine on the time.com Vault. If I see Ben, I can conclude B is True. Otherwise, start trailing Ben and ask one of his friends if he plays tennis or ping-pong. If he does, B is True. Else, go to Peru and observe Ben's father's whereabouts. If his occupation is an artist, B is True. Else, observe whether Ben's father has any friends in Asia. Bug his phone calls and if that doesn't work, walk up to him speaking various Asian languages. If he cannot comprehend you, B is False. If he can, the B is True.