1\. a. Prove $f^{-1}(f(x)) = x$ for all $x \in X$:

$f(g(y)) = f(f^{-1}(y)) = y$

$f^{-1}(f(x)) = f^{-1}(y) = x$

b. $f: X \to Y$ because f is bijective

$f^{-1}: Y \to X$

2\. $\exists g: B \to A$ so $f \circ g = id_B$ and $g \circ f = id_A \Rightarrow$ f is bijective (Use prop 3)

(Use prop 1, 2)

3\. $3 \Rightarrow 2$: There can be the same x_j for different y_i.

$2 \Rightarrow 1$: elements not distinct

4\. $\exists f: X \to Y$ bijection $\Rightarrow |X| = |Y|$

f is a bijection, so f is injective and surjective. $\exists f: X \to Y$ injective, $\exists f: Y \to X$ surjective, $\exists g: X \to Y$ injective, $\exists g: Y \to X$ surjective so $|X| \leq |Y|$ and $|Y| \leq |X|$ so $|X| = |Y|$