

## Homework 2: Math 347

**Instructions:** You may use any results from the book or lecture in your proofs. You are welcome and encouraged to work together with other students on these problems, but the final write up should be done alone. If people submit solutions that look highly similar, then they will get automatic zeros on the problem.

For problems 1 and 2:  $P(x)$ ,  $Q(x)$ ,  $C(x, y)$  are logical statements depending on the variables  $x$  and  $y$  from the nonempty set  $S$ .

**Problem 1.** For the following pair of logical statements  $L$  and  $R$ , prove (or give a counterexample) for the implications  $L \Rightarrow R$  and  $R \Rightarrow L$ :

- (a)  $\forall(x \in S) : [P(x) \vee Q(x)]$     and     $[\forall(x \in S) : P(x)] \vee [\forall(y \in S) : Q(y)]$
- (b)  $\forall(x \in S) : [P(x) \wedge Q(x)]$     and     $[\forall(x \in S) : P(x)] \wedge [\forall(y \in S) : Q(y)]$

**Problem 2.** For the following pair of logical statements  $L$  and  $R$ , prove (or give a counterexample) for the implications  $L \Rightarrow R$  and  $R \Rightarrow L$ :

- (a)  $\forall(x \in S)\exists(y \in S) : C(x, y)$     and     $\exists(y \in S)\forall(x \in S) : C(x, y)$
- (b)  $\forall(x \in S)\exists(y \in S) : [P(x) \Rightarrow Q(y)]$     and     $\exists(y \in S)\forall(x \in S) : [P(x) \Rightarrow Q(y)]$

**Problem 3.** For each of the following properties, name two functions  $f : \mathbb{R} \rightarrow \mathbb{R}$ . One should and one should not satisfy the given property and you should prove your answer.

- (a)  $\forall(\epsilon > 0)\exists(\delta > 0)\forall(x \in \mathbb{R}) : |x| < \delta \Rightarrow |f(x)| < \epsilon$
- (b)  $\exists(\epsilon > 0)\forall(\delta > 0)\forall(x \in \mathbb{R}) : |x| < \delta \Rightarrow |f(x)| < \epsilon$

**From Donaldson and Pantanto:**

2.3: 7, 13, 17, 18