HW 11

2.4.2. $|x_{n+1} - x_n| \leq C|x_n - x_{n-1}|$

Since $0 < C < 1$, the terms must be strictly decreasing while also getting closer to each other because there is a factor of C everytime.

2.4.5. The elements must be getting closer to each other. 0 is the midpoint for $x_k < 0$ and $x_n > 0$ so the sequence must converge to 0.

2.4.8. True, because the elements must eventually get arbitrarily closer.

2.5.3. a. $\sum_{n=1}^{\infty} \frac{3}{9n+1}$ diverges using p-series

b. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges using p-series

c. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ $\quad \lim_{n\to\infty} \frac{1/(n+1)^2}{1/n^2} = 0$

converges

d. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ diverges using p-series

e. $\sum_{n=1}^{\infty} ne^{-n^2} = \sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

$$\lim_{n\to\infty} \frac{\frac{n+1}{e^{(n+1)^2}}}{\frac{n}{e^{n^2}}} = \lim_{n\to\infty} \frac{(n+1)e^{n^2}}{ne^{(n+1)^2}} = \lim_{n\to\infty} \frac{e^{n^2}}{e^{n^2+2n+1}}$$

$$= \lim_{n\to\infty} e^{-2n-1} = 0 \text{ ; converges}$$

2.5.6. a. upper bond exists

b. there is no upper bond

2.5.10. $\left| \sum_{n=1}^{\infty} x_n \right| \leq \sum_{n=1}^{\infty} |x_n|$

Negative numbers are turned into positive numbers before summing them

2.5.11. $0 < \lim_{n \to \infty} \frac{a_n}{b_n} < \infty$

$\sum a_n / \sum b_n$ is a constant, so the n terms must cancel. Then, they have the same n terms

2.5.14. $\sum x_n$ converges, $x_n \geq 0$ ~~for~~ for all n

$$\lim_{n \to \infty} \frac{|x_{n+1}|}{|x_n|} < 1$$

$$\lim_{n \to \infty} \left(\frac{x_{n+1}}{x_n}\right)^2 < 1$$ so $\sum x_n^2$ converges